Update of vFFA Study

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Contents:

- Introduction to the vFFA
 - What is a vertical-excursion Fixed-Field Accelerator?
 - Why are we interested?
- Parameter studies of the vFFA for muon acceleration
- Remaining design objectives
- Further concepts

RCS challenges



A brief (non-exhaustive) list of problems:

- Ramping magnets
 - \rightarrow Rate of ramping can be limiting factor on muon survival
 - \rightarrow Ramping requires normal-conducting magnets
 - \rightarrow Construction and distribution of power converters, storing energy, etc..
- Changing path length (hybrid RCS)
 - Changing time of flight \rightarrow requirement for tunable RF ?

What if we didn't have to deal with any of these?



Fixed Field Accelerators

Time-independent magnetic fields means...

- No ramp times
 - Rate of acceleration limited only by RF
 - Mitigates engineering challenges of designing and powering fast-ramping dipoles
- All magnets can be superconducting
- Orbit position moves when energy changes

Spatial dependence of magnetic fields allows tune control

"Conventional" horizontal-excursion FFA (hFFA):

Orbits move outwards with increasing

energy Fields increase radially



For hFFA options for muon acceleration see presentation by J. Scott Berg in 2023 IMCC meeting

Vertical-Excursion Fixed Field Accelerators

Vertical-excursion FFA (vFFA):

- Higher energy orbits are vertically translated copies of lower energy orbits
- Zero chromaticity if fields increase with vertical coordinate (Z) following scaling law

 $B = B_0 e^{mZ}$

Zero path length difference means...

- \rightarrow Zero momentum compaction factor α_c
- \rightarrow Transitionless
- \rightarrow Quasi-isochronous for relativistic particles
 - \rightarrow on-crest acceleration can be used

Vertical-excursion FFA (vFFA):

Orbits move upwards with increasing energy Fields increase vertically



Properties of the vFFA

Non-planar closed orbit

Intrinsically coupled optics

Scaling laws \rightarrow optics determined by closed orbit and field index m



F-magnet is shaded in green; D-magnet is shaded in blue.

Analytic model of vFFA has been developed

"Scaling Fixed Field Accelerators: theory and modelling of horizontal- and vertical-excursion accelerators" – PhD thesis 2024

Properties of the vFFA

Closed orbit



Geometric constraints can be used to determine closed orbit from input parameters

Some important variables for each magnet: ρ , γ , r(radius of curvature, inclination of plane of curvature, radius wrt machine centre)

$$\begin{aligned} \theta_D &= \arctan\left(\frac{\sqrt{\left(\tan\frac{\pi}{N} - \cos\gamma_F \tan\theta_F\right)^2 + \sin^2\gamma_F \sin^2\theta_F \left(\tan\frac{\pi}{N}\cos\gamma_F \tan\theta_F + 1\right)^2}}{\sqrt{1 - \sin^2\gamma_F \sin^2\theta_F} \left(\tan\frac{\pi}{N}\cos\gamma_F \tan\theta_F + 1\right)}\right) \\ \sin\gamma_D &= \frac{\sin\theta_F}{\sin\theta_D} \sin\gamma_F, \\ \rho_F &= r_0 \frac{\tan\beta_F}{\sin\theta_F + (1 - \cos\theta_F)\cos\gamma_F \tan\beta_F}, \\ r_1 &= \rho_F \frac{\sin\theta_F}{\sin\beta_F}, \\ r_2 &= r_1 \frac{\cos\beta_F + \tan\theta_F\cos\gamma_F \sin\beta_F}{\cos\left(\frac{\pi}{N} - \beta_D\right) + \tan\theta_F\cos\gamma_F \sin\left(\frac{\pi}{N} - \beta_D\right)}, \\ \rho_D &= r_2 \frac{\sin\beta_D}{\sin\theta_D}, \\ r_3 &= r_2\cos\beta_D - \rho_D \left(1 - \cos\theta_D\cos\gamma_D\right). \end{aligned}$$

Properties of the vFFA

Magnet body Hamiltonian



x, z: horizontal and vertical transverse coordinates

m : normalised field gradient

 ρ : radius of curvature

 γ : inclination (angle of the plane of curvature in magnet)

m is a constant around the ring

 \rightarrow Sign of focusing terms can only be changed if sign of ρ changes

 \rightarrow reverse bends are required

Hamiltonian has normal quad + skew quad + geometric terms

vFFA parameter study (stage 1)

Analytic model of vFFA used to develop proposal for equivalent vFFA FODO ring to RCS1

Constraints:

MuCol Milestone 17 Report - WP5 - Tentative design of the FFA (2025)

- Footprint no larger than RCS1
- Peak field on orbit <8T
- Drift lengths > 1m
- Excursion < 5 cm

	RCS1	Stage 1 vFFA
Circumference [m]	5990	5990
Injection Energy [TeV]	0.06	0.06
Extraction Energy [TeV]	0.3	0.3
NC Ramped Magnets	Yes	No
SC Fixed magnets	No	Yes
Ramp Rate [T/s]	4200	0
Vertical Excursion [m]	0	0.048
Relative path length difference	0.0	0.0
Peak Dipole Field On Orbit [T]	1.8 (NC)	6.93
Peak Dipole Field (Good Field Region) [T]		12.52
Drift length [m]		1.18
Tune		(0.382, 0.079)



vFFA parameter study (stage 4)

Equivalent vFFA FODO ring to RCS4

Constraints:

- Footprint no larger than RCS4
- Peak field on orbit <16T
- Drift lengths > 1m
- Excursion < 10 cm

	RCS4	Stage 4 vFFA
Circumference [m]	35000	35000
Injection Energy [TeV]	1.5	1.5
Extraction Energy [TeV]	5	5
NC Ramped Magnets	Yes	No
SC Fixed magnets	Yes	Yes
Ramp Rate [T/s]	565	0
Vertical Excursion [m]	0	0.099
Relative path length difference	1.71×10^{-6}	0.0
Peak Dipole Field On Orbit [T]	16	13.59
Peak Dipole Field (Good Field Region) [T]		29.04
Drift length [m]		1.03
Tune		(0.460, 0.057)





vFFA parameter study conclusions

Simple vFFA FODO lattices have been proposed based on analytic formalism

Equivalent vFFA lattices to RCS1, RCS4 exist with

- No ramped magnets
 - Fully SC \cdot No issues for power converter, energy storage
- No requirement for RF frequency ramping
- Zero momentum compaction
- Fixed tunes
- Possibility of **on-crest acceleration** (potential $\sqrt{2}$ improvement in accelerating voltage for given cavity power)
- Excursion less than: 5 cm (vFFA1), 10 cm (vFFA4)
- On-orbit dipole fields less than: **7 T** (vFFA1), **13.6 T** (vFFA4)

Remaining Design Objectives

Longitudinal behaviour of the vFFA has not been studied in this regime

- How will bunch structure change after passing through isochronous machine?
 - Is there a need to perturb isochronicity to preserve longitudinal properties of bunch?
- 6D simulation is needed

Large change in orbit position can be problematic for acceleration

- High-frequency RF cavities have limited aperture
- Off-axis beam in RF cavities will excite high order modes (HOMs)
- Coupling between transverse and longitudinal planes
 - Increases need for distributed RF

High normalized field gradient $m \rightarrow$ rapid growth of fields away from axis Coupled optics \rightarrow how do we design an extraction system?

Two takeaways:

- Full 6D numerical simulation of vFFA lattices must be completed
- Are there ways to mitigate some of the other problems?



vFFA1 Simulation

vFFA1 has been simulated using the FFA code FIXFIELD

- Analytic model was able to predict closed orbit, position magnets, and identify valid lattice in previously untested region of parameter space
- Disagreement between analytic model and simulation on range of stable m-values $_{\rightarrow}$ excursion of simulation is larger than analytic model (m ~21/m numerical, m ~30/m analytic)
- However, demonstrated success of analytic model for finding new lattices and use of analytic model insights to optimize lattice

6D tracking studies now underway!

vFFA1 simulation to be used as testbed for of **further concepts**

Coupling in the vFFA

Magnet body Hamiltonian

$$\mathcal{H} \simeq \frac{p_x^2}{2} + \frac{p_z^2}{2} - \frac{1}{\rho + \frac{\sin\gamma}{m}} \left[\cos\gamma \left(m + \frac{2\sin\gamma}{\rho} \right) xz - \frac{1}{2}m \left(x^2 - z^2 \right) \sin\gamma \right] + \frac{1}{\rho \left(\rho + \frac{\sin\gamma}{m} \right)} \left(x^2 \cos^2\gamma + z^2 \sin^2\gamma \right) \right]$$

geometric terms – always present skew quad term – proportional to $\cos \gamma$ normal quad term – proportional to $\sin \gamma$

If $\gamma = 90^{\circ}$, we can remove the coupling in this Hamiltonian!

However... cells need at least 2 magnets.

Closed orbit equations



Decoupled insertions

 Inclination of both magnets can be 90° if the net bending angle of the cell is zero



- A straight insertion with a 90° inclination enables the complete negation of coupling over the length of the straight
- Analytic theory for vFFA straight cell design has been completed
- N.B. a vFFA FODO straight with a 90° inclination is equivalent to an hFFA FODO straight – hFFA straight cells have been tested experimentally (J.-B. Lagrange, *Study of zerochromaticity in FFAG accelerators*. PhD thesis, Kyoto U., 2012.)

Decoupled insertions

When optics are decoupled:

• Orbit can be kicked/bumped in one plane without affecting the other

 \rightarrow Injection and extraction design becomes much less restrictive

• Betatron oscillations can be induced in the dispersive plane

 \rightarrow Possibility of mitigating transverse/longitudinal issues from RF by tuning cell phase advance between cavities

 \rightarrow Enables design of dispersion suppressors

Dispersion suppression

In a decoupled straight:



3 regions with different dispersions $D_Z = \frac{1}{m}$ Initial region: D_1 is at nominal value for arcs Target region : D_3 is at desired value (can be zero) Dispersion suppressor : $D_2 = (D_1 + D_3)/2$

Allows separation of arc excursion from RF constraints:

More freedom in choice of arc parameters · lower demands on aperture of RF · lessen severity of HOM issues

Conclusions

- vFFA equivalents to RCS1, RCS4 have been proposed using analytic model
 - vFFA RCS1 equivalent has been simulated numerically

• vFFA FODO rings can be built with

 Equivalent footprint to RCS designs · Achievable dipole fields · Fully superconducting magnets · Small orbit excursion
...for an energy-efficient, zero-chromatic, isochronous accelerator

• Coupling effects can be negated in straights

- Could help solve injection/extraction issues + dispersion suppression
- Use of dispersion suppression allows
 - Dispersion-suppressed RF insertions · Increased freedom in choice of normalised field index in arcs (possibility to lower peak fields)

The future



vFFA offers promising alternative to RCS

- Able to circumvent a number of key issues
- Analytic design tool enables **development of new lattices**
- Further development of the concept is needed
 - Tradeoffs and potential mitigation strategies have been identified
 - End-to-end 6D simulation must be completed
- Fully SC energy-efficient rapid acceleration is possible !!

The future





(Saint-Genis Pouilly in 2050 when we build the muon vFFA)

13/05/2025