A Charge-Agnostic Design for Initial 6D Muon Cooling

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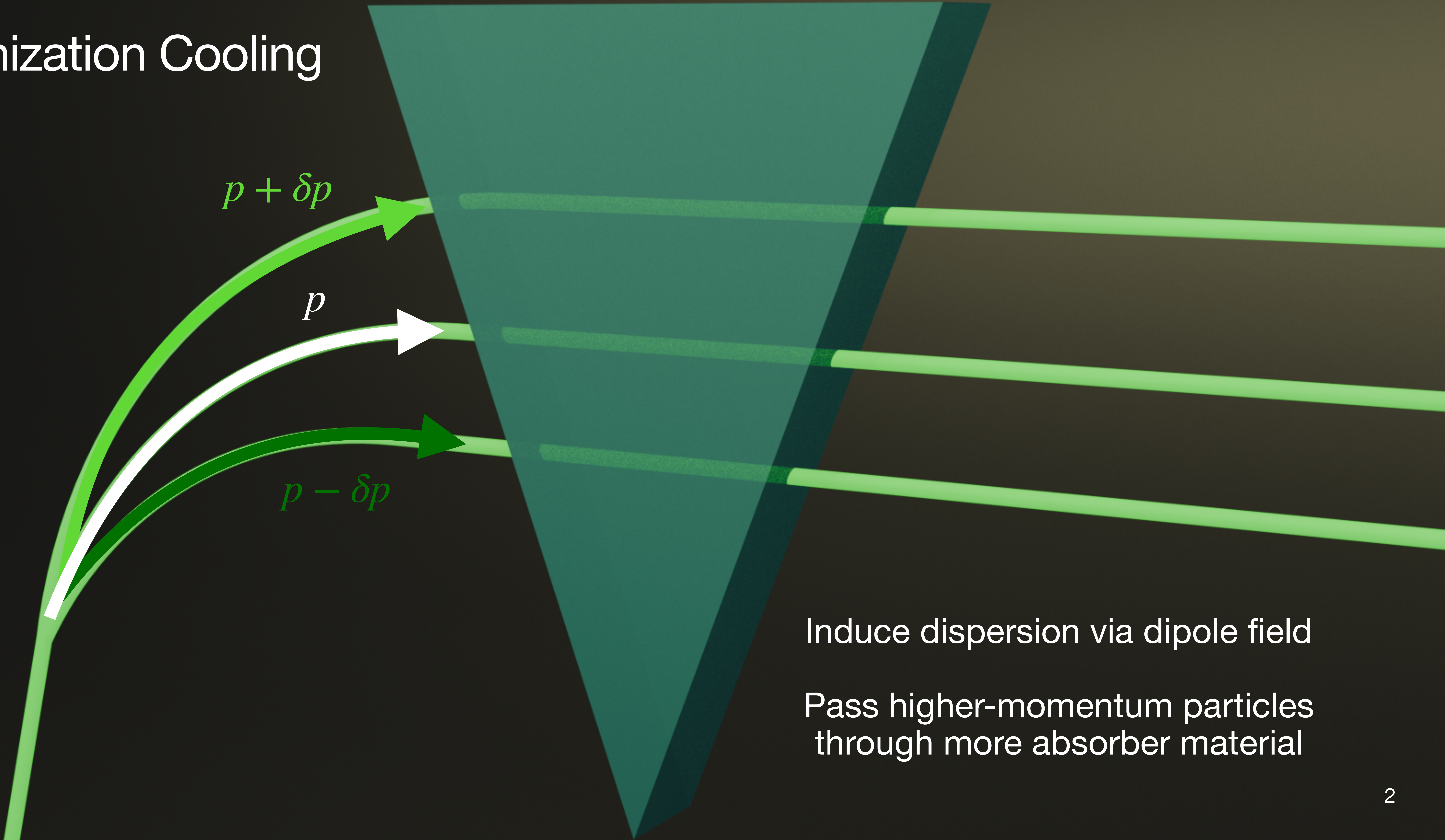
IMCC and MuCol Annual Meeting 2025

Ionization Cooling



Induce dispersion via dipole field

Pass higher-momentum particles through more absorber material

Ionization Cooling

This cooling scheme is **charge-dependent**

$\Longrightarrow$ Need for *two* cooling channels (one for μ^+, one for μ^-)

Introducing: HFOFO

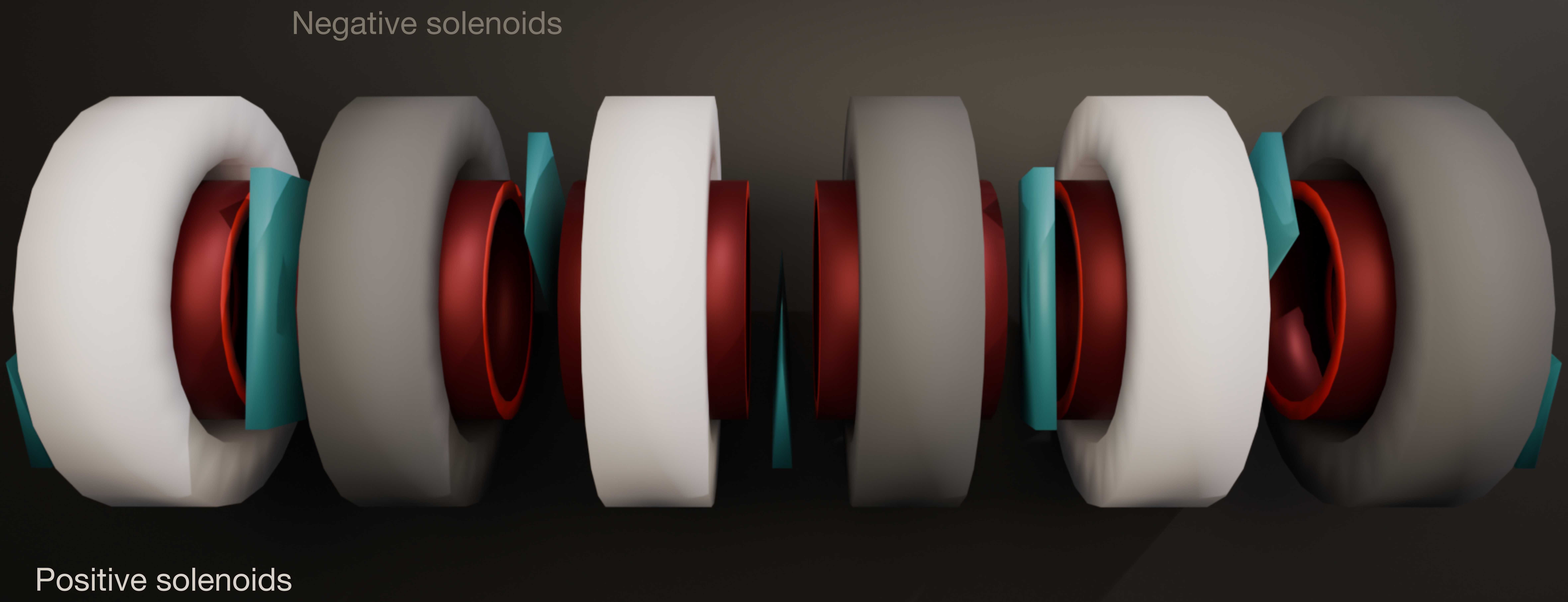
Presented here is the Helical FOFO Snake (**HFOFO**) design:

Yuri Alexahin's concept from the MAP era

Charge-agnostic dispersion generation $\Longrightarrow$ simultaneous cooling of *both signs*

HFOFO would enable a *single* initial cooling channel!

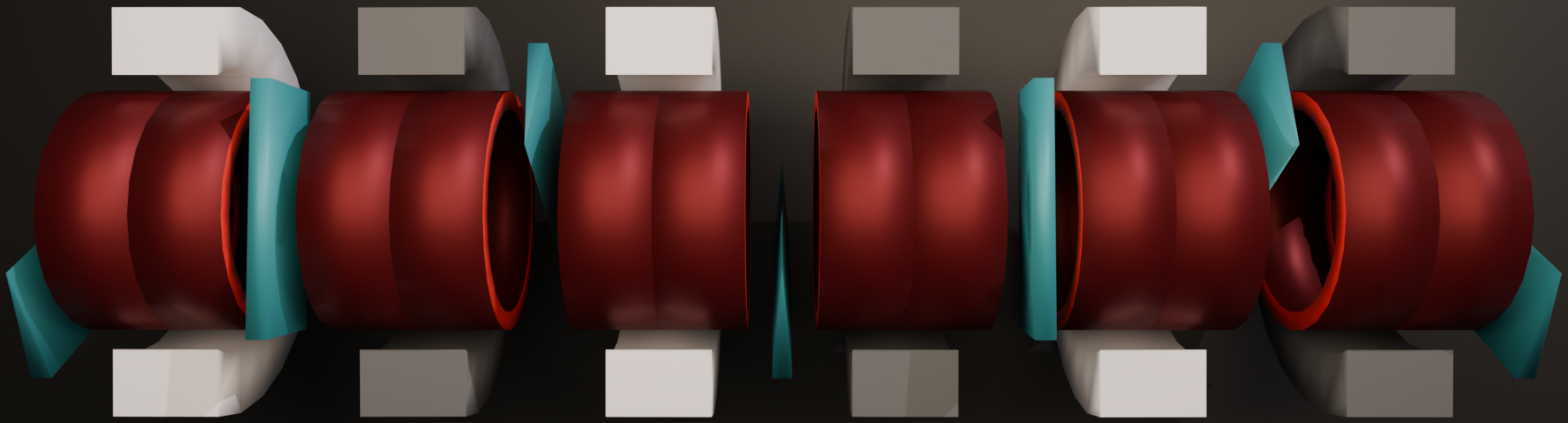
Introducing: HFOFO



Introducing: HFOFO

6 solenoids = 1 period (4.2 m)

Full channel = 30 periods



Basic Principles

HFOFO operates on 3 basic principles:

Alternating solenoid focusing

Simultaneous focusing of both signs of muon

Periodic rotating dipole fields

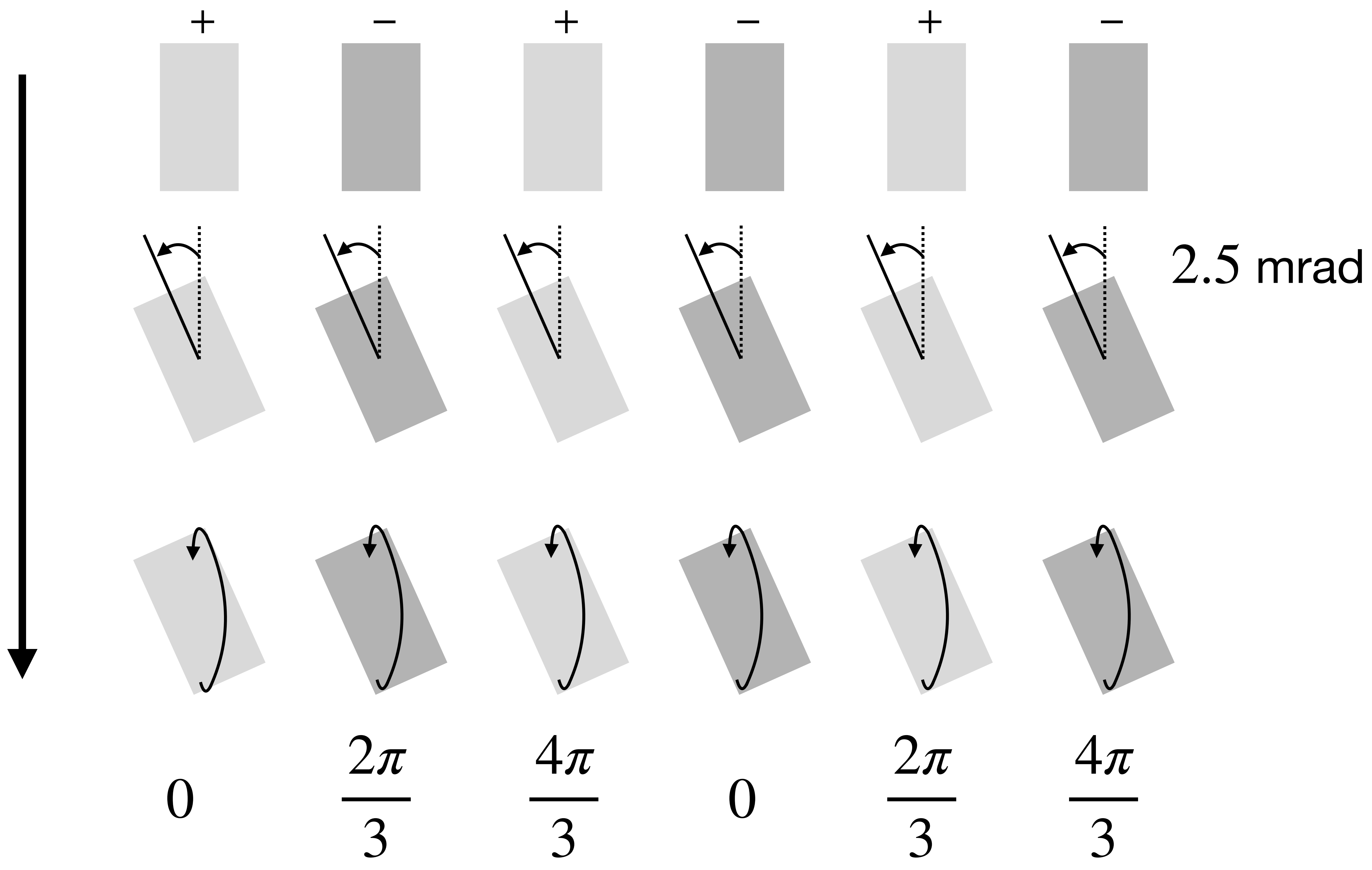
Charge-agnostic dispersion generation

Resonant dispersion generation

Large localized path lengthening

Solenoid Focusing

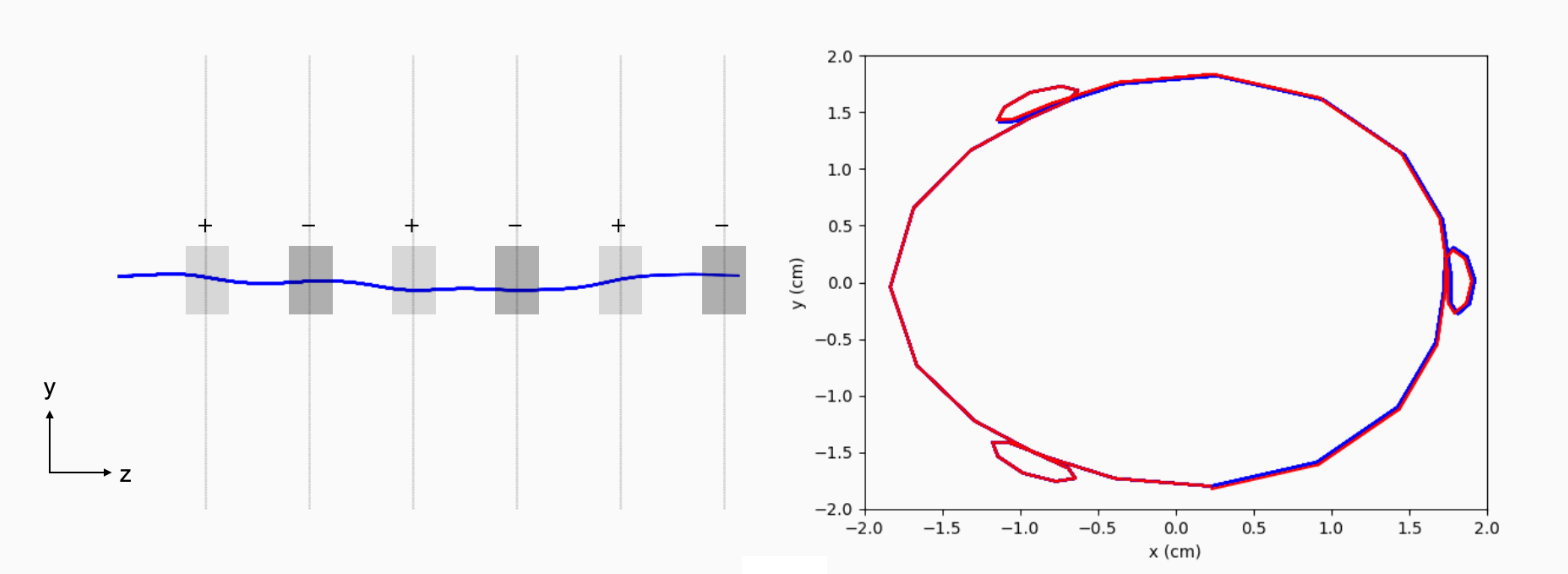
- Alternating-polarity solenoids for focusing
    - This is called a FOFO lattice
    - Does not depend on muon sign
- Solenoids are periodically inclined to create **rotating dipole field**



μ^+ and μ^- have *same orbit* but with a *half-period longitudinal offset*

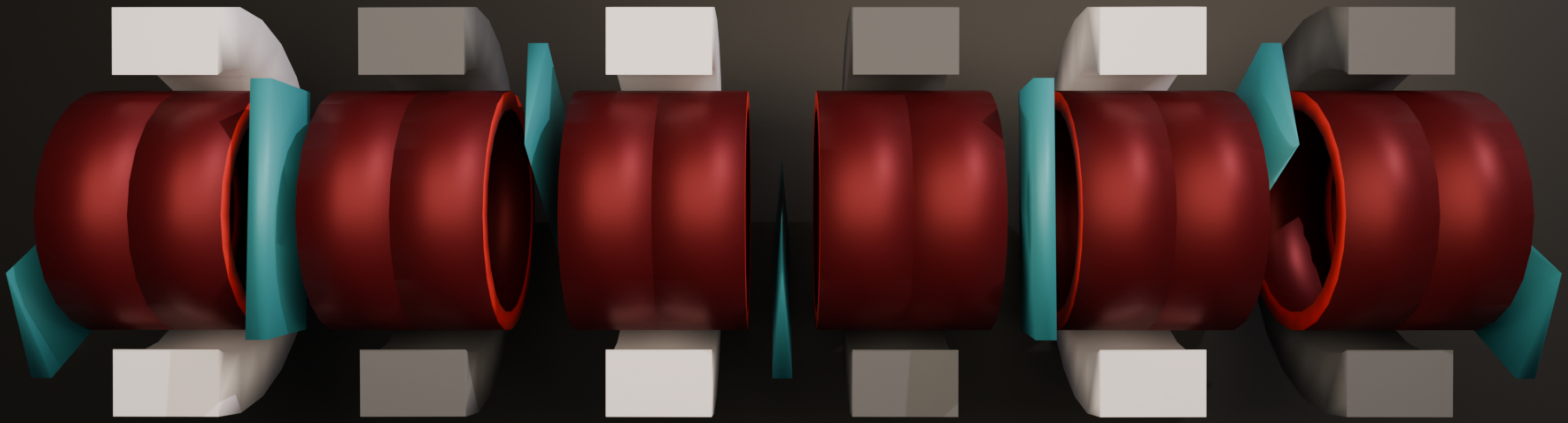
Solenoid Focusing

μ^+ and μ^- orbits along single period (animation)



Lattice Periodicity

Groups of 3 focusing units with same solenoid and absorber orientations repeated *twice per period* to treat both signs



Dispersion

Traditional schemes:

Solenoid tilting along one axis only

Generate *upward dipole field*

$\Longrightarrow$ Charge-specific dispersion

HFOFO:

Solenoid tilting along two axes, with twice per period repetition of rotations

Generate *rotating dipole field*

$\Longrightarrow$ Dispersion independent of sign

Dispersion

Beta tune near *resonance* $\implies$ momentum compaction is *positive:*

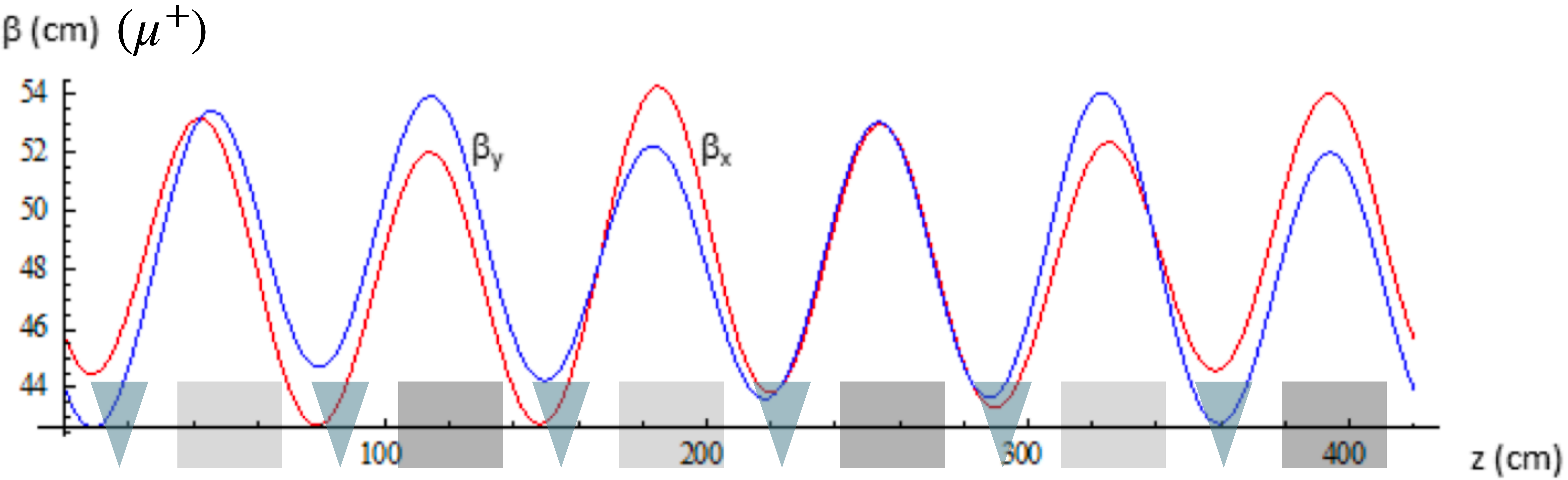
$$\alpha_c = \frac{\Delta L/L}{\Delta p/p} > 0$$

$\therefore$ Higher-momentum particles travel longer paths

Having **near-resonance** conditions allows for *large dispersion* — ideal for cooling!

Beta Function

Wedge absorbers placed at *minima of beta function* (between solenoids)



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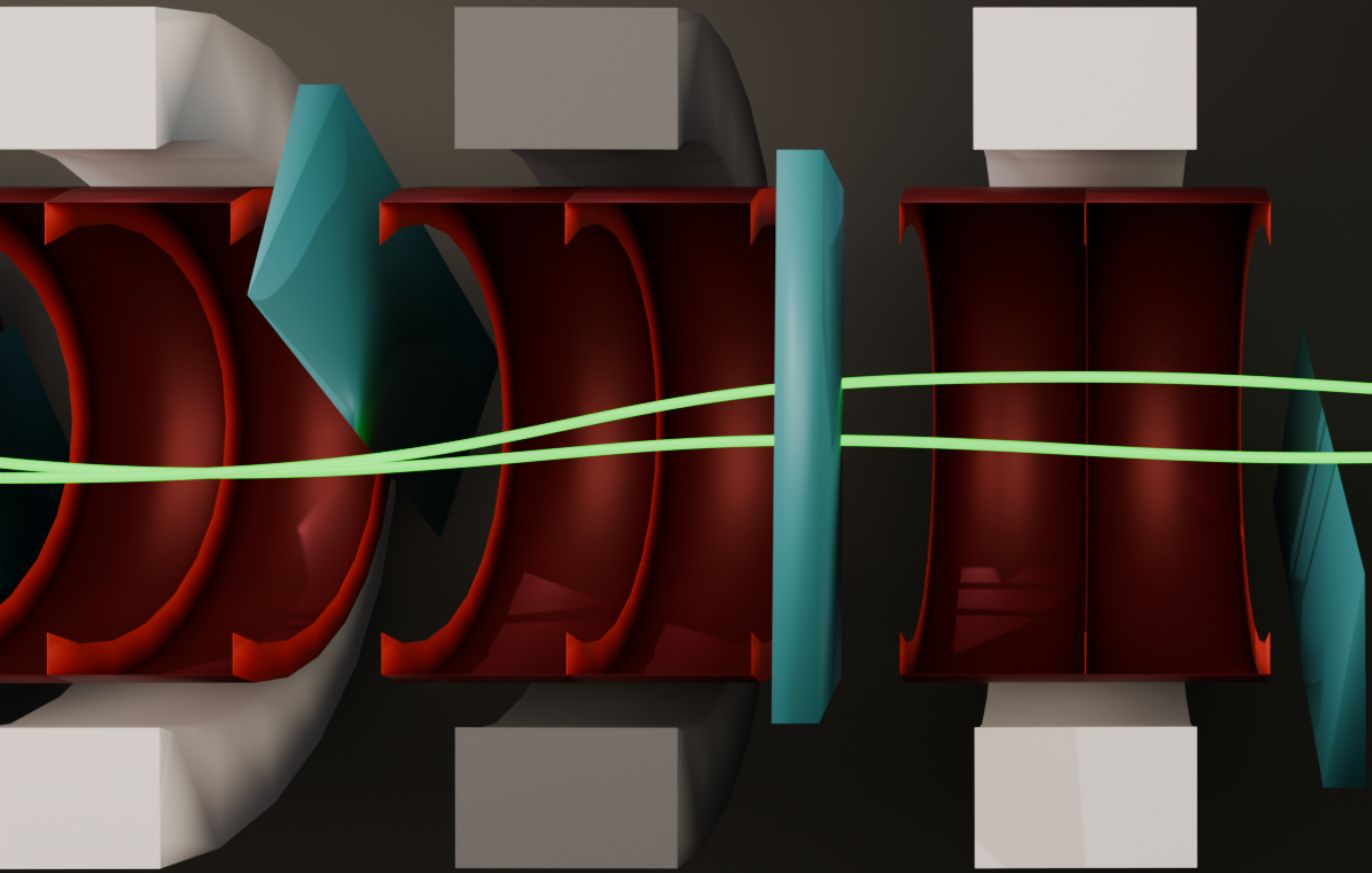
RF Cavities



Cooling degrades longitudinal momentum

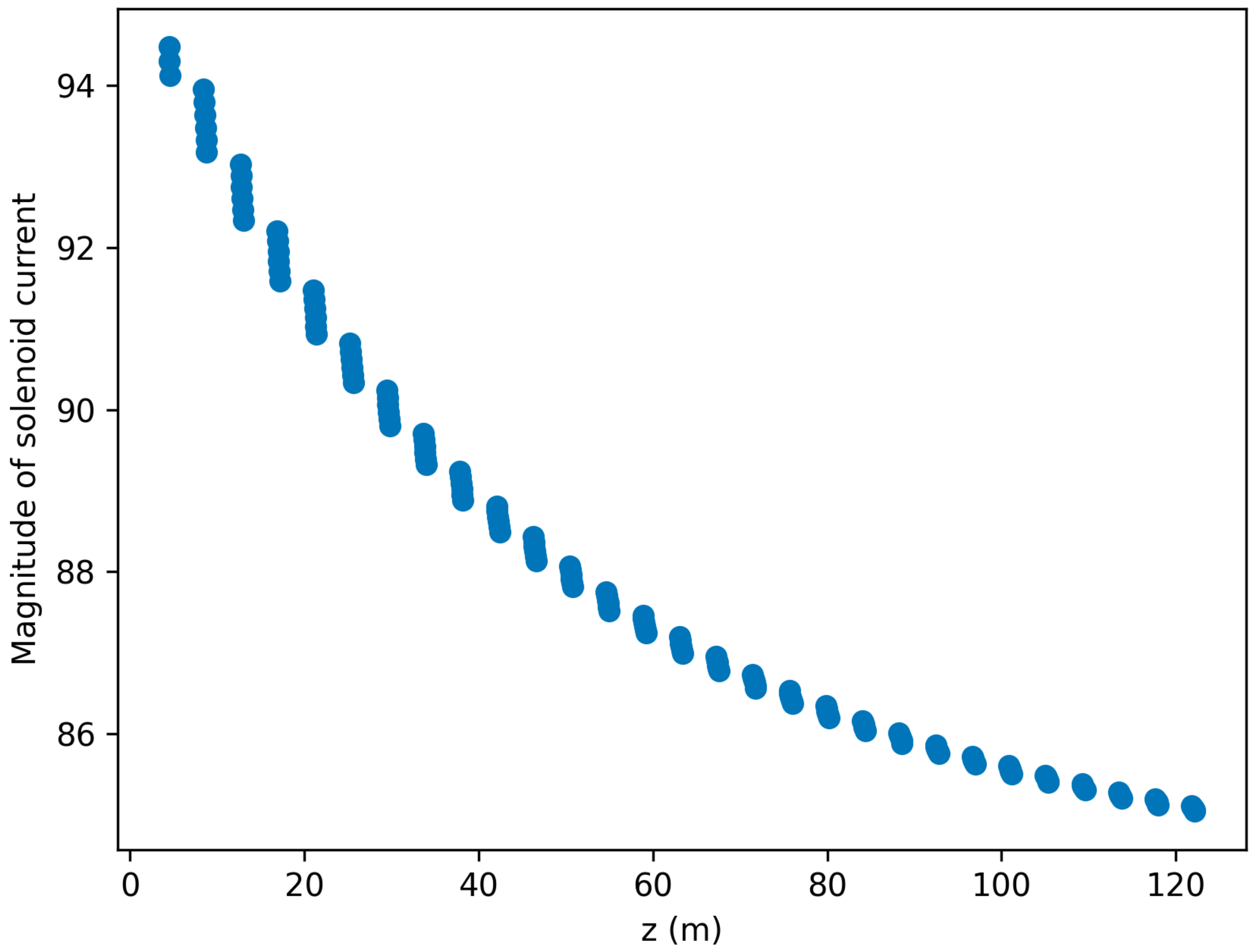
$\Longrightarrow$ Use RF cavities to *restore energy*

f_{RF} = **325 MHz**

GH_2 enables higher RF gradient and contributes to cooling

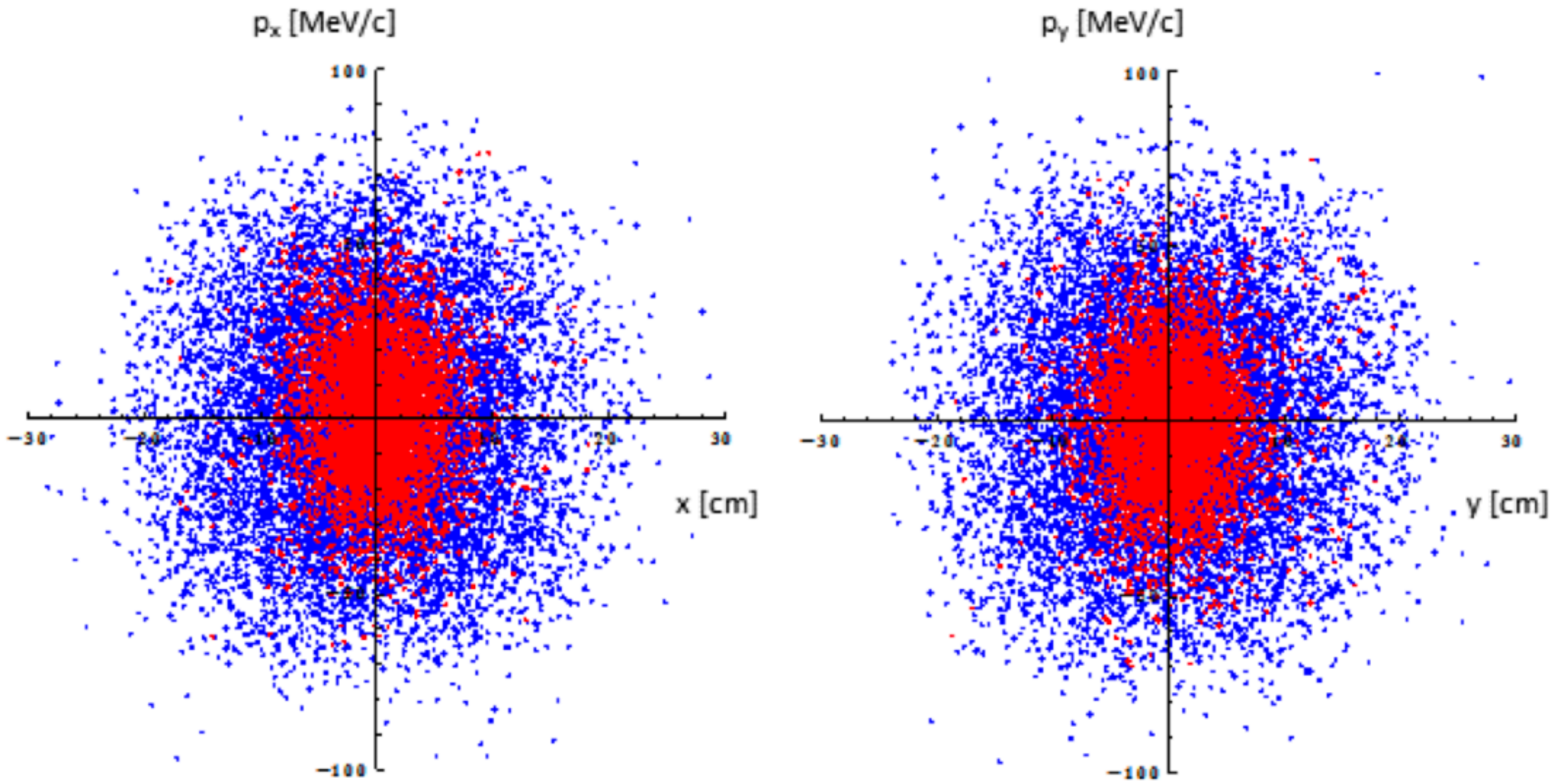
Momentum Evolution

- Average front-end momentum is ~250 MeV/c
- Momentum is degraded along channel to ~**200 MeV/c**
  - Ideal for ionization cooling
- Achieved by *reducing solenoid current* and adjusting RF phase with z



Yuri’s Results

Initial and final phase space distributions (μ^+)



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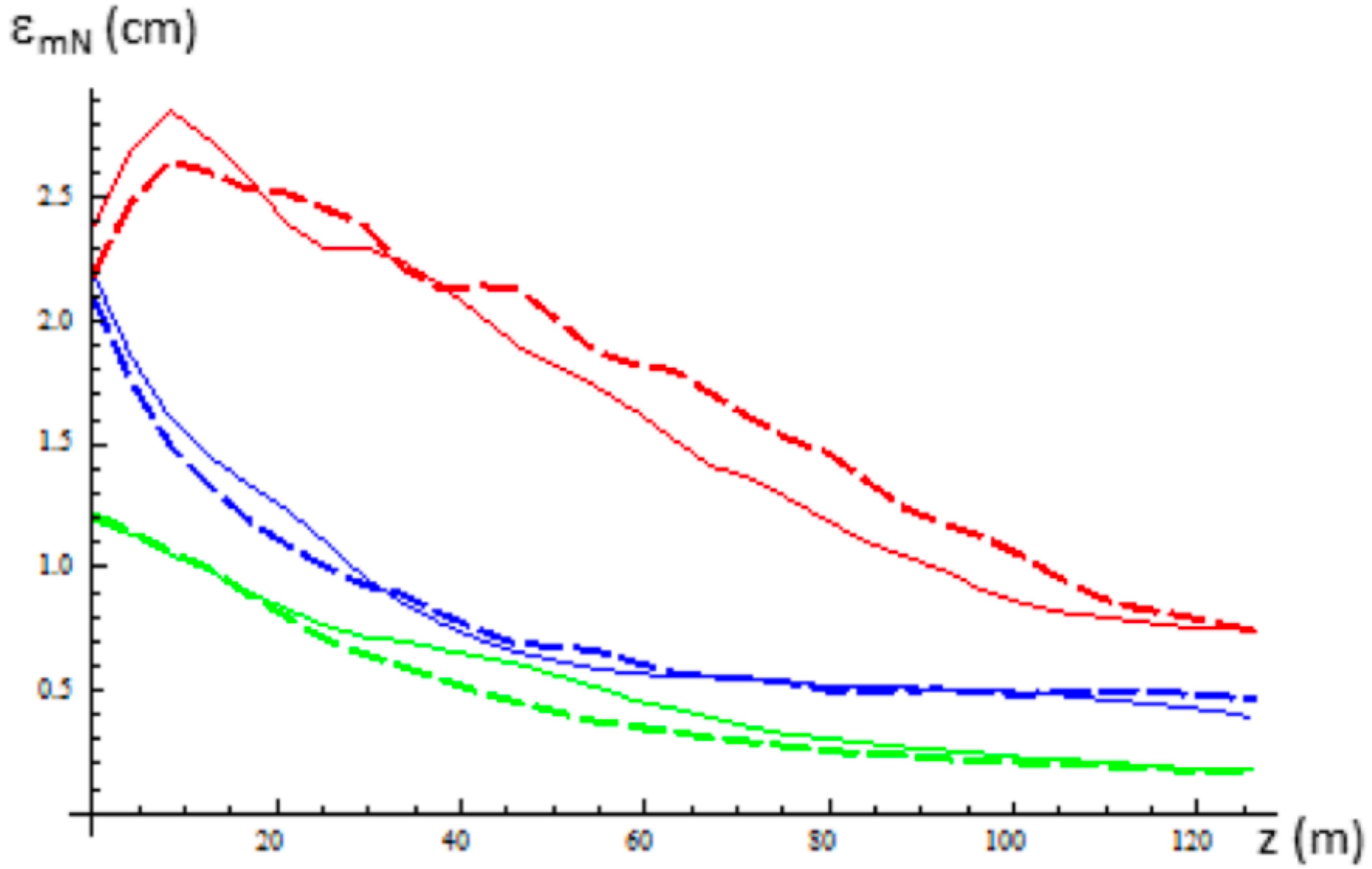
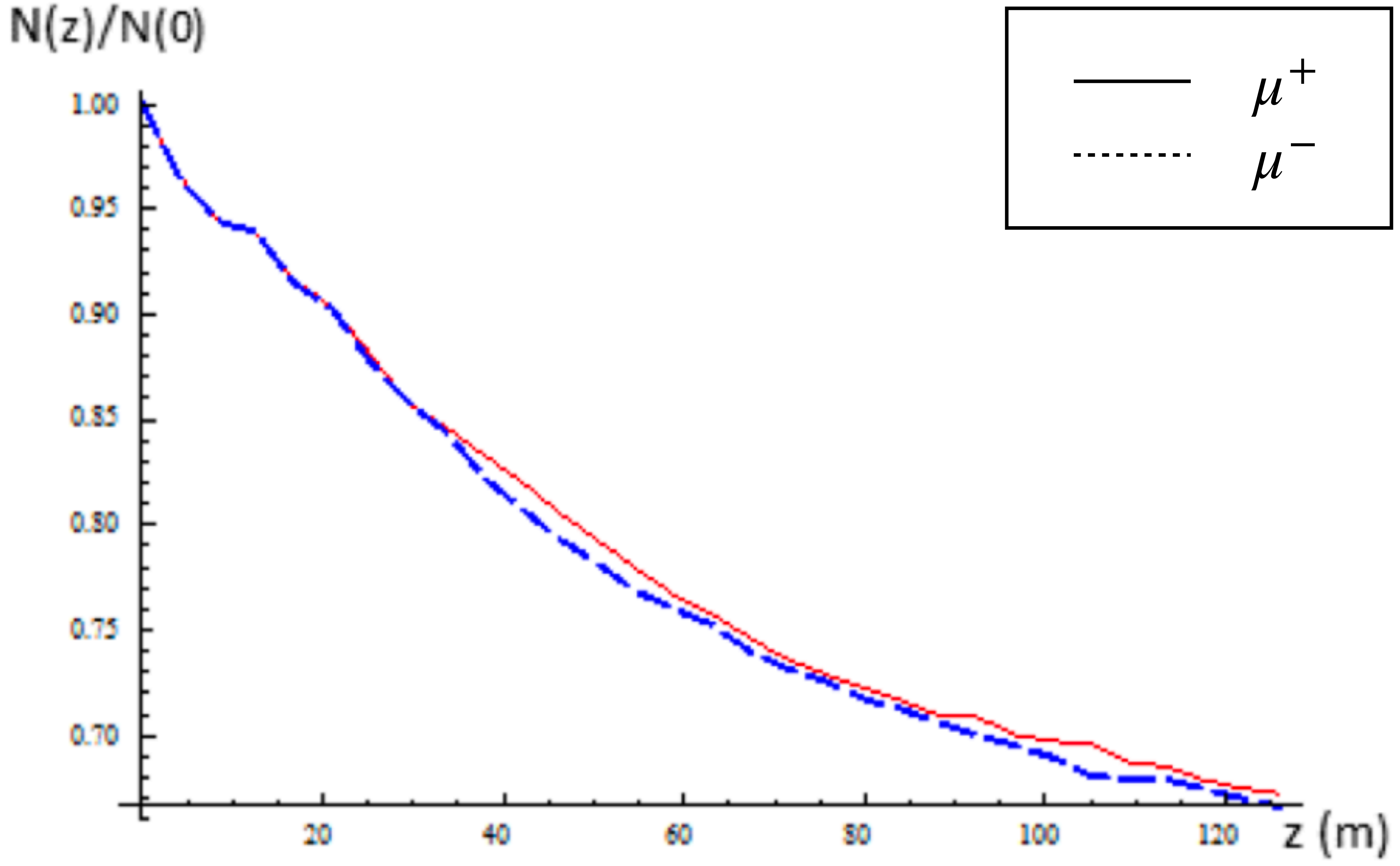
Yuri’s Results

Normal mode emittances and beam intensity along channel





Factor of **112.8** total emittance reduction!
36.4 transverse, 3.1 longitudinal

65% transmission rate

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What is Next?

Yuri demonstrated ~*two orders of magnitude* in emittance reduction for *both* μ^+ and μ^-, without significant losses

Yet since the MAP project, there has been minimal work on HFOFO

… until now!

Our Work

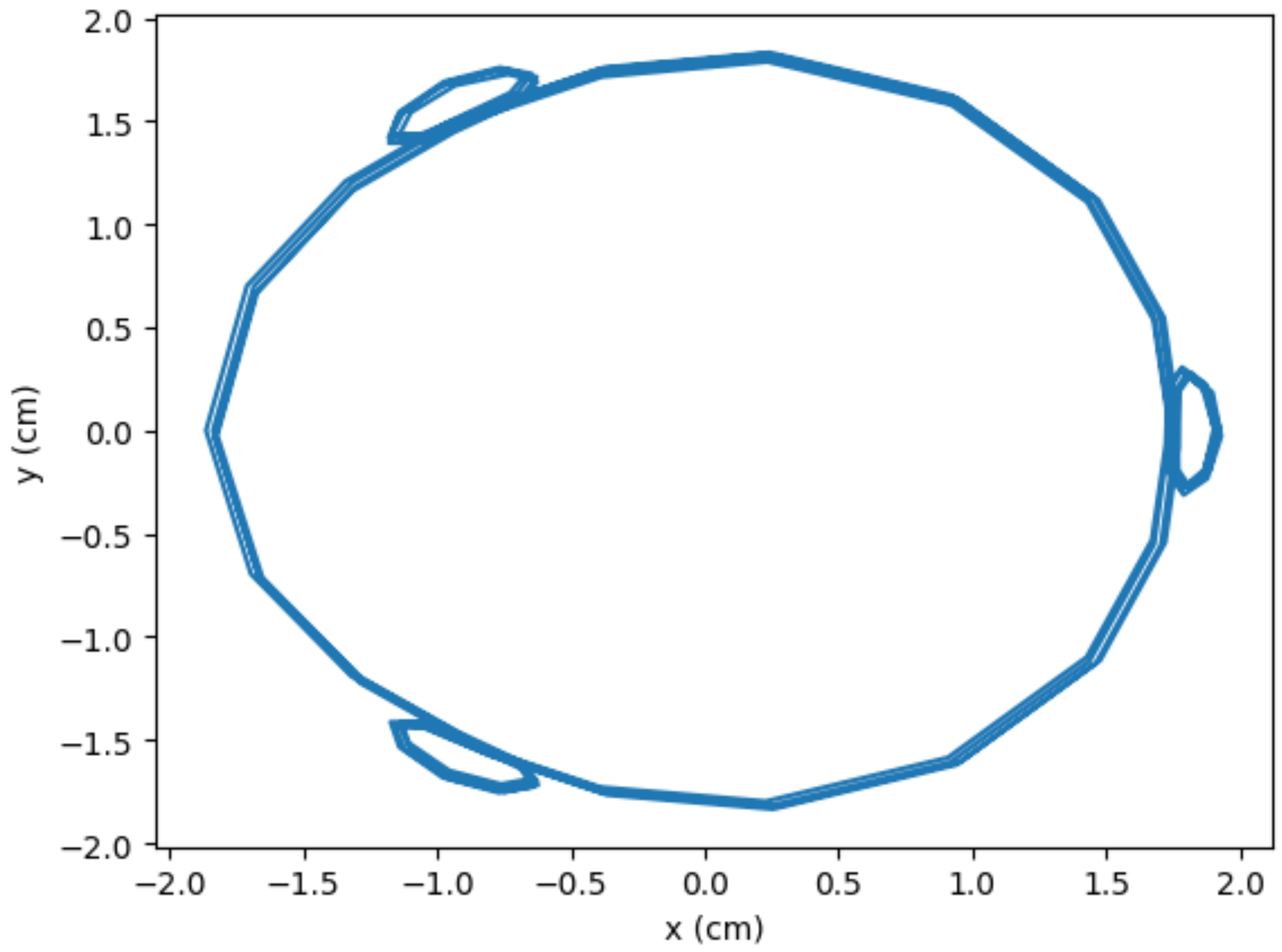
Yuri's design is highly nontrivial, so we are rebuilding HFOFO from the basics

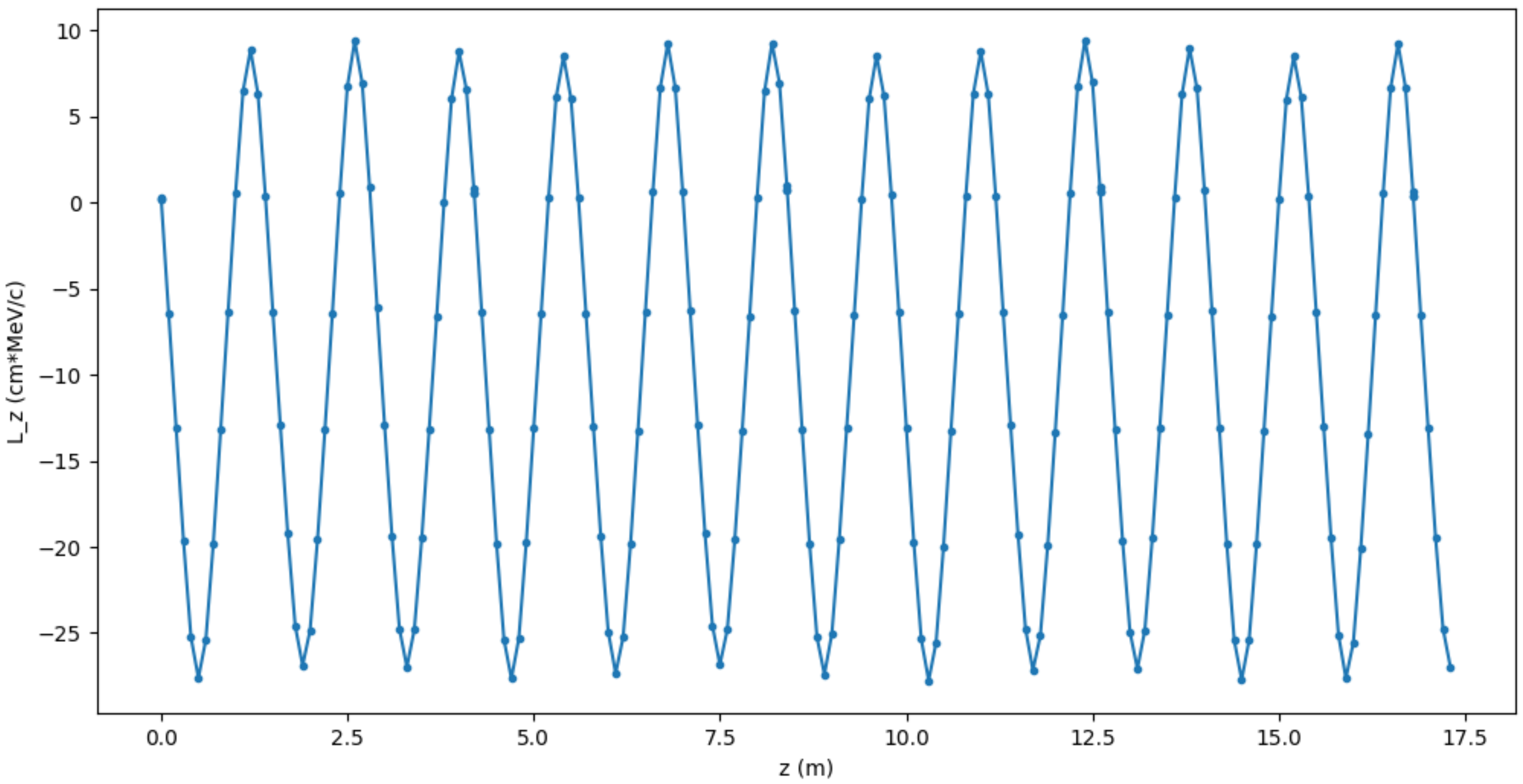
Goal: study simplified channel $\Longrightarrow$ *approximate Hamiltonian*

Study beam optics in constant-momentum channel → Replicate Yuri's results with RF and cooling → Optimize

Our Work

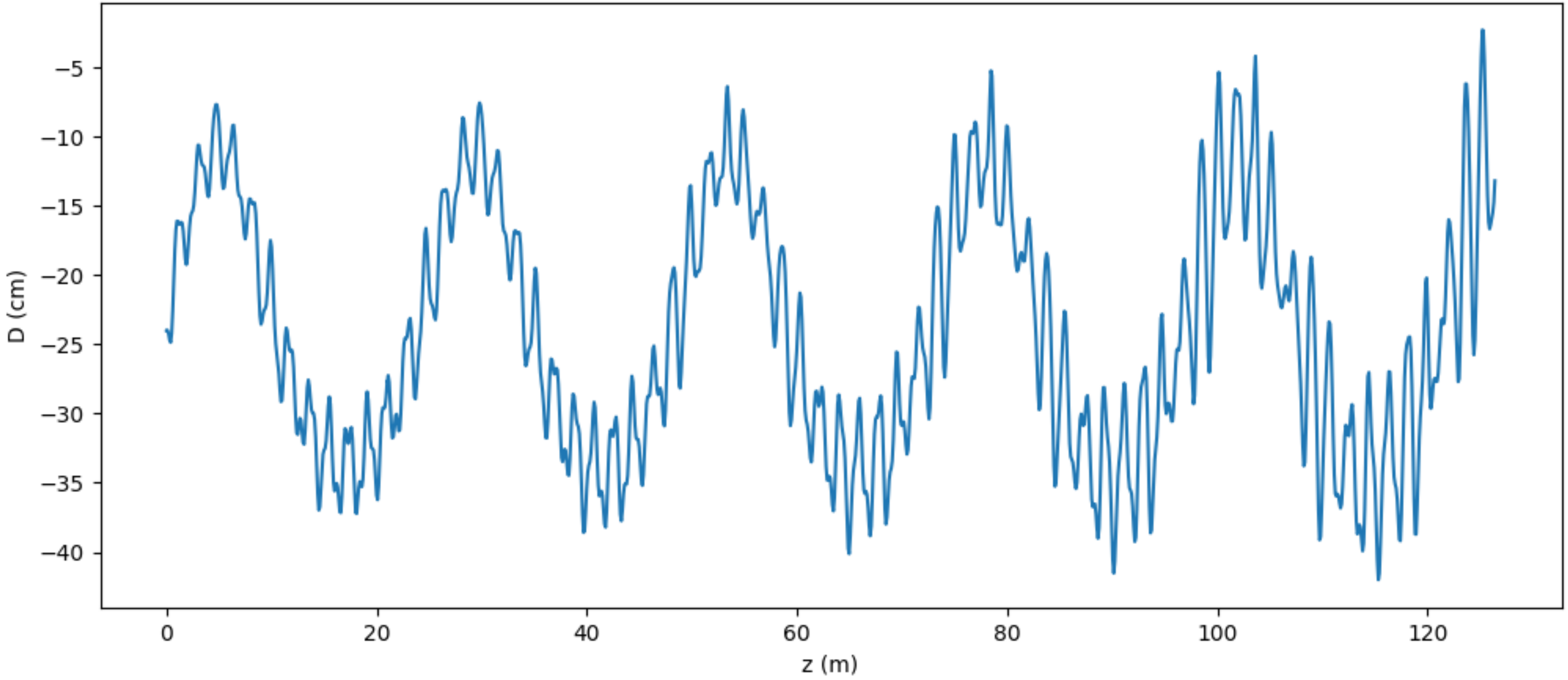
Matching 200 MeV/c reference orbit for simplified channel (μ^+)





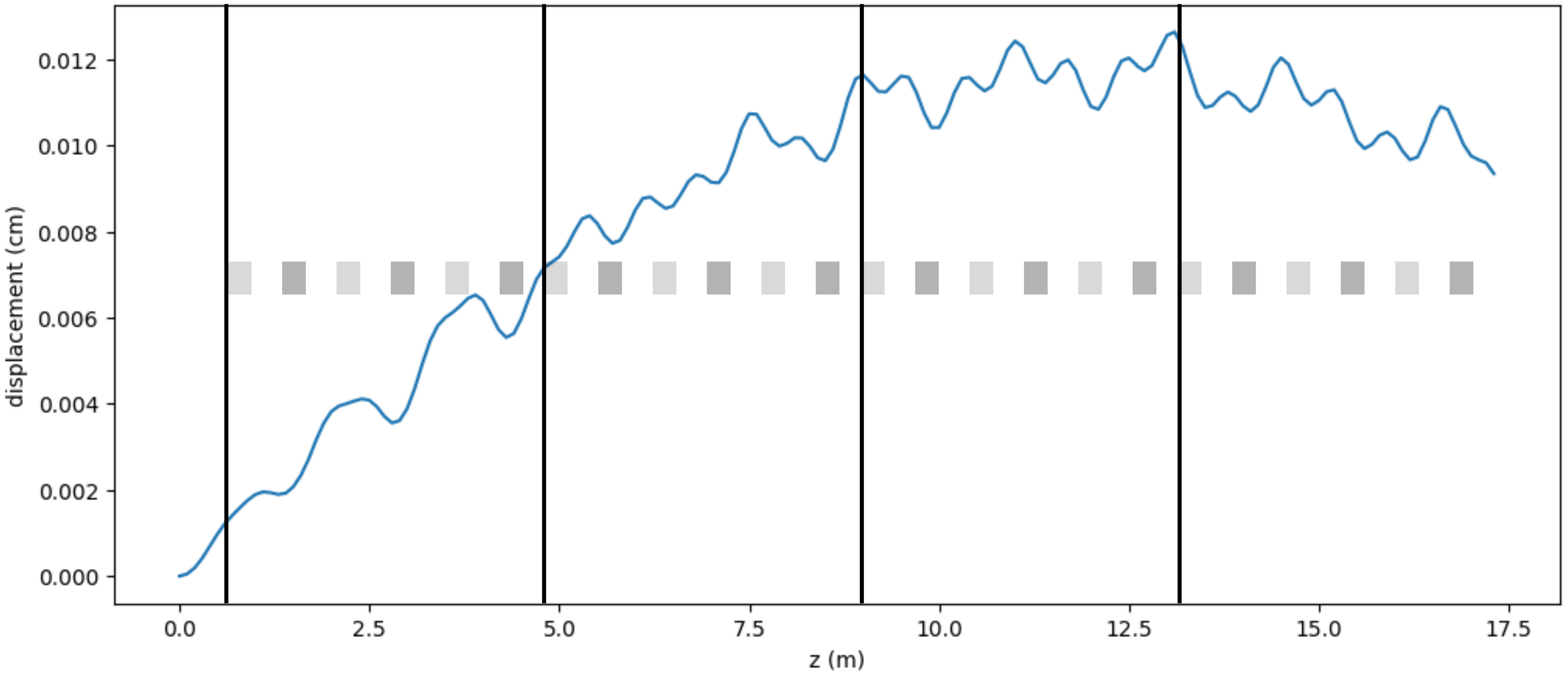
Our Work

Dispersion along full simplified channel (μ^+)



Our Work

Dispersion along isolated region of simplified channel (μ^+)



Our Work

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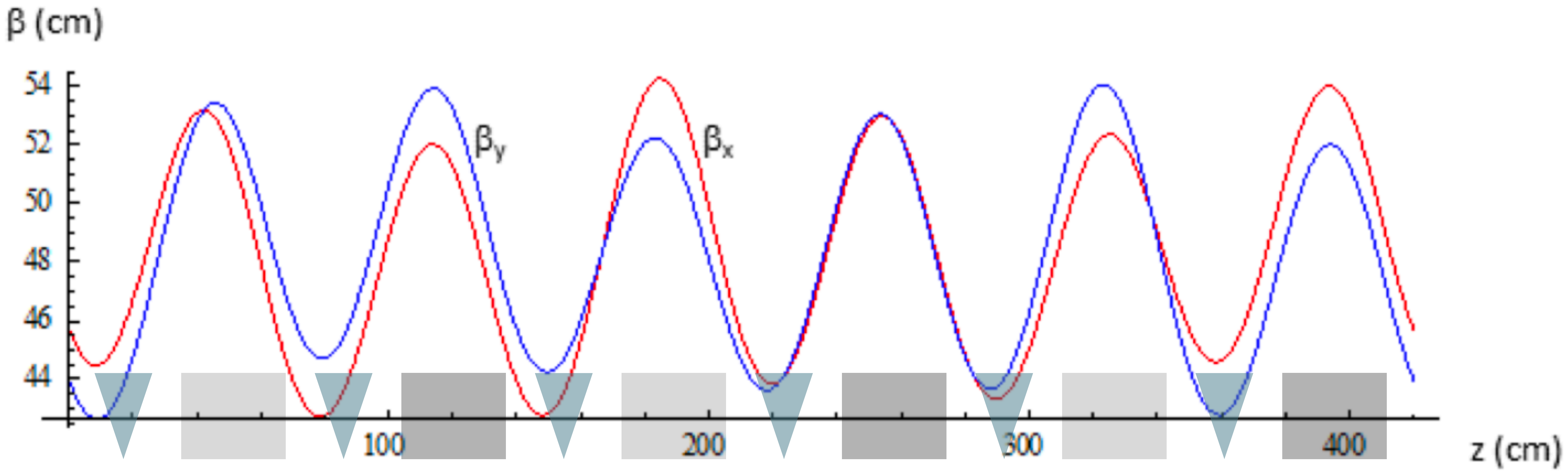
Goal: study simplified channel $\Longrightarrow$ *approximate Hamiltonian*

Optimization Ideas

- Reduce length of lattice elements along channel and increase solenoid current to preserve transverse beta tune
- Increase betatron phase advance for each focusing unit (nominally 74°) to reduce beta function at minima
- **Other ideas?**



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Thank You

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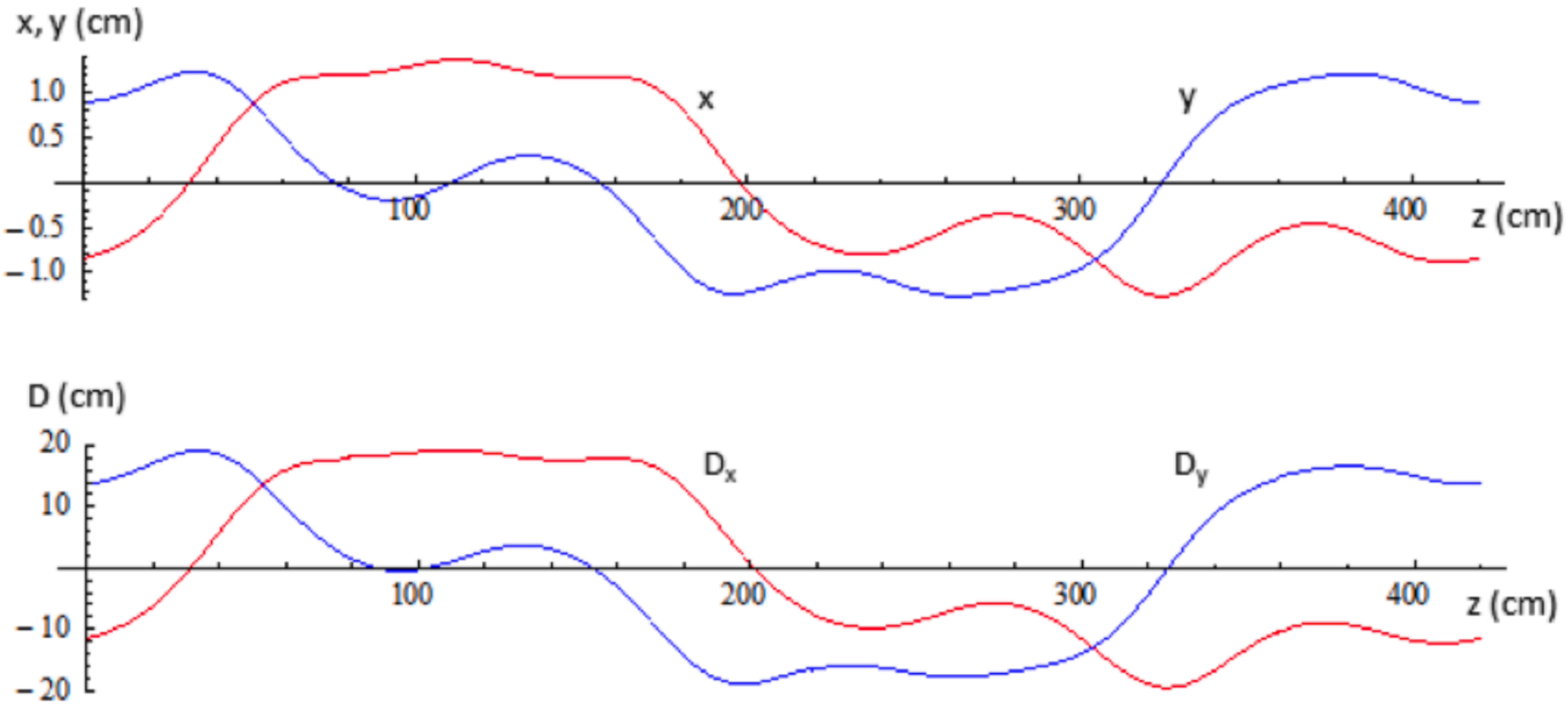
Supplementary

Yuri's Results

Trajectory and dispersion in transverse plane



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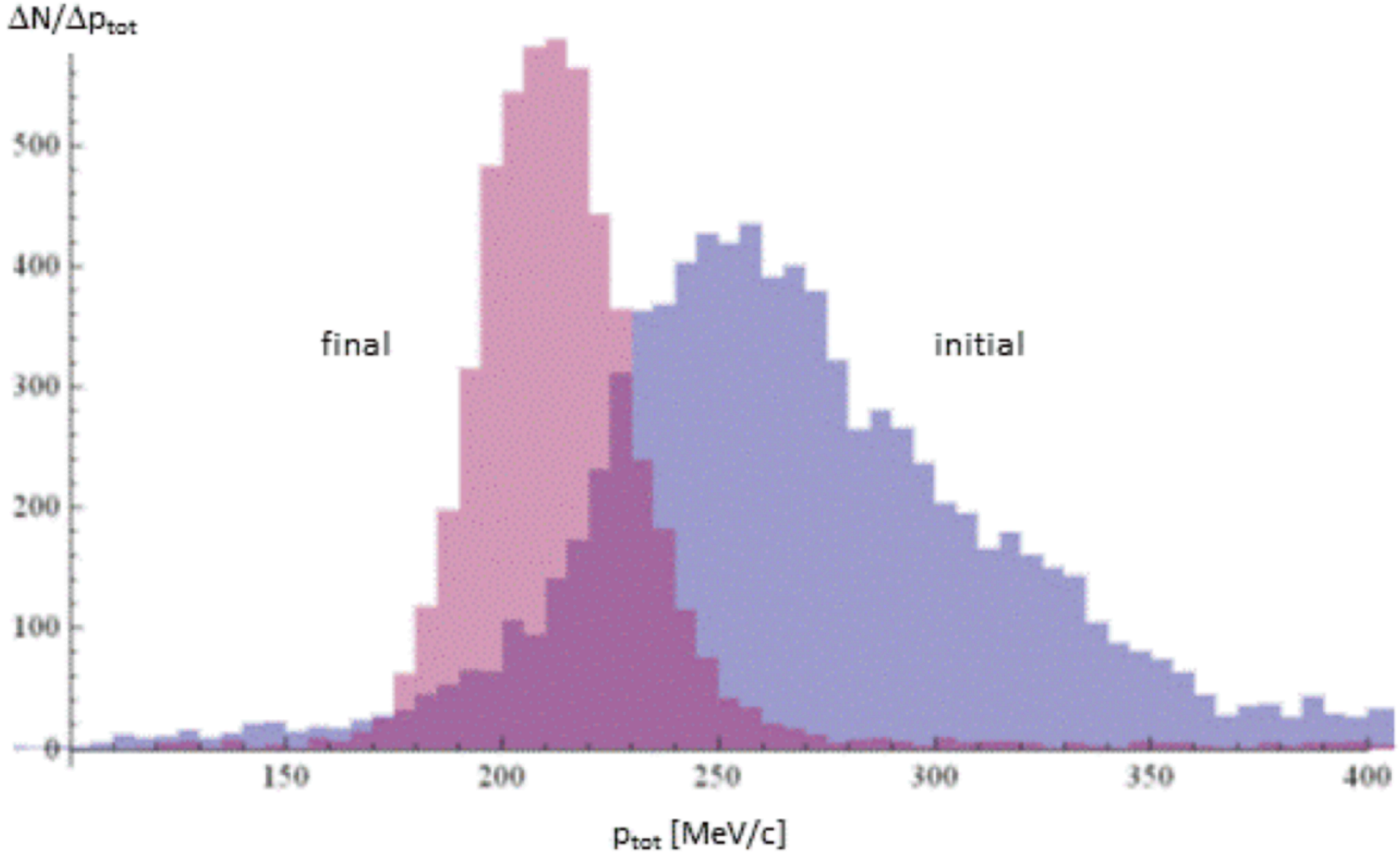
Yuri’s Results

Initial and final total momentum distributions (μ^+)



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Yuri's Results

μ^+

	N_{total}	ϵ_{mN} (cm)			ϵ_{6D} (cm³)
Initial	11755	1.19	2.19	2.38	6.22
Final	5378	0.19	0.36	0.76	0.051

μ^-

	N_{total}	ϵ_{mN} (cm)			ϵ_{6D} (cm³)
Initial	12396	1.22	2.10	2.19	5.59
Final	5896	0.16	0.46	0.72	0.051