### Physics Simulations for a Muon Collider with Whizard

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QCD ranges

 $\ll 100 \text{ MeV}$ 

short-range interaction (nuclear)

 $f_{\pi}=100~{
m MeV}$ 

massive particles, chiral pT

 $4\pi f_{\pi} = 1 \text{ GeV}$ 

transition region (strong int.)

 $\gg 1 \; \text{GeV}$ 

massless jets, QCD gauge theory

### EW ranges

#### $\ll 100 \text{ GeV}$

short-range interaction (LEFT) v = 250 GeVmassive particles, massive-EW pT

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### EW ranges

### $\ll 100 \text{ GeV}$

short-range interaction (LEFT) v = 250 GeVmassive particles, massive-EW pT  $4\pi v = 3 \text{ GeV}$ transition region  $\gg 3 \text{ GeV}$ massless jets, SM gauge theory

EW Approximations may be derived from

- 1. unitary-gauge Feynman rules (EWSB, massive),
  - on-shell simple initial state  $(\mu^+\mu^-)$
  - tree-level + NLO Feynman graphs
  - exclusive final states + QCD/QED shower
- 2. 
   generic-gauge Feynman rules (symmetric, massless)
  - composite initial state
  - (semi-)inclusive final states, EW jets
  - EW shower

 $\mu$  collider: explores transition and EW jet region



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### The WHIZARD MC generator and muon collisions

High-energy lepton collider physics (ILC/CLIC)

- Matrix-element code generator, multi-particle integration / events
- ILC/CLIC samples for physics studies (Pythia shower/hadrons)
- Support for lepton-collider beam structure / polarization
- SM and BSM models (UFO support)

#### NLO QCD and NLO electroweak: Whizard 3

- NLO fixed-order support w/ OpenLoops, Recola, GoSam
- SM NLO for BSM physics (GoSam)

#### Muon collider physics coverage: progress

- NLO fixed-order (massive particles + radiation)
- Muon beam structure (massless approach)
- SM: EW symmetry restoration

## The WHIZARD 3 Team (2025)

U Siegen WK, Pia Bredt, Nils Kreher, Tobias Striegl

### DESY

Jürgen Reuter, Maximilian Löschner, Krzysztof Mękała

#### KIT Marius Höfer

U Würzburg Thorsten Ohl

U Warsaw A. Filip Żarnecki

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Matrix Elements for Hard Processes (LO)

### $\mathsf{EW}+\mathsf{QCD}$ in tree approximation: <code>O'Mega</code>

automated perturbative helicity-amplitude calculation for multi-leg off-shell processes with interfering resonances [hep-ph/0102195]

- avoids redundant common subexpressions altogether: no Feynman-graph expansion
   ⇒ factorial growth of # terms reduced to power law
- color-flow formalism (phantom 9th gluon) [JHEP 10 (2012) 022]
- helicity evaluation in parallel: OpenMP multi-core

#### EW + QCD 1-loop (fixed order): GoSam, Recola, OpenLoops

- Helicity amplitudes (virtual + real) including UV renormalization
- IR + collinear divergences: gluons and photons (and light fermions)
- FKS subtraction for ME, PowHeg matching for events)

## Phase-Space Mapping and Sampling

VAMP = Whizard's built-in ML algorithm for phase space

- Invertible, parameterized mappings with adaptive optimization (⇒ normalizing-flow ansatz)
- Multi-channel decomposition with adaptive optimization for weights (⇒ attention / transformer ansatz)
- 3. Initial setup not from random noise  $\Rightarrow$  use ME singularity structure and phase-space topology

Important for electroweak / multi-particle MC problems:

- CPU resources: learning must not exceed production effort
- Whizard: HPC Parallel evaluation: OpenMP, MPI
- Whizard: GPU support under study

Multi-TeV muon collider environment:

- ▶ fixed-order approach useful up into transition region, but NLO corrections increase
- reduced soft Bremsstrahlung as compared to e<sup>+</sup>e<sup>-</sup> collider

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[P. Bredt, W. Kilian, J. Reuter, P. Stienemeier, 2208.09438] <code>WHIZARD+RECOLA</code>,  $G_\mu$  scheme,  $m_\mu=0.1056...$  GeV

$\mu^+\mu^- \to X, \sqrt{s} = 3  {\rm TeV}$	$\sigma_{\sf LO}^{\sf incl}$ [fb]	$\delta_{\sf EW}$ [%]	$\delta_{\rm ISR}$ [%]
$W^{+}W^{-}$	$4.6591(2) \cdot 10^2$	+4.0(2)	+13.82(4)
ZZ	$2.5988(1) \cdot 10^{1}$	+2.19(6)	+15.71(4)
HZ	$1.3719(1) \cdot 10^{0}$	-1.51(4)	+30.24(3)
$W^+W^-Z$	$3.330(2) \cdot 10^{1}$	-22.9(2)	+2.90(9)
$W^+W^-H$	$1.1253(5) \cdot 10^{0}$	-20.5(2)	+7.10(8)
ZZZ	$3.598(2) \cdot 10^{-1}$	-25.5(3)	+5.24(8)
HZZ	$8.199(4) \cdot 10^{-2}$	-19.6(3)	+8.39(8)
HHZ	$3.277(1) \cdot 10^{-2}$	-25.2(1)	+7.58(7)
$W^{+}W^{-}W^{+}W^{-}$	$1.484(1) \cdot 10^0$	-33.1(4)	-1.3(1)
$W^+W^-ZZ$	$1.209(1) \cdot 10^{0}$	-42.2(6)	-1.8(1)
$W^+W^-HZ$	$8.754(8) \cdot 10^{-2}$	-30.9(5)	-0.1(1)
$W^+W^-HH$	$1.058(1) \cdot 10^{-2}$	-38.1(4)	+1.7(1)
ZZZZ	$3.114(2) \cdot 10^{-3}$	-42.2(2)	+0.8(1)
HZZZ	$2.693(2) \cdot 10^{-3}$	-34.4(2)	+1.4(1)
HHZZ	$9.828(7) \cdot 10^{-4}$	-36.5(2)	+2.2(1)
HHHZ	$1.568(1) \cdot 10^{-4}$	-25.7(2)	+5.7(1)

with 
$$\delta_{EW} = \sigma_{NLO}^{incl} / \sigma_{LO}^{incl} - 1$$
 and  $\delta_{ISR} = \sigma_{LO,LL-ISR}^{incl} / \sigma_{LO}^{incl} - 1$   
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	777	2 508(2) . 10-1	_25 5(3)	+5.24(8)
$\alpha \sum \sum_{l=1}^{\infty} (p_k + p_l)^2$				+8.39(8)
$\Lambda_{\rm EW,Sud} \sim -$	$-\frac{1}{8\pi}\sum_{k}\sum_{k}P^{k}(k)P^{k}(k)$	$M_{\rm ev}^2 \Rightarrow \text{virtual } V$		+7.58(7)
$k, l \neq k a = \gamma, Z, W$				-1.3(1)
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with $\delta_{\sf EW} = \delta$	$\sigma_{ m NLO}^{ m incl}/\sigma_{ m LO}^{ m incl}-1$ and $\delta_{ m ISR}=c$	$\sigma_{\rm LO  II - ISB}^{\rm incl} / \sigma_{\rm LO}^{\rm incl} - 1$	L	

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	$W^+W$ 38.1(4)			+1.7(1)	
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[P. Bredt, W. Kilian, J. Reuter, P. Stienemeier, 2208.09438]

<u>Fixed order differential distributions</u>:  $d\sigma(\mu^+\mu^- \rightarrow HZ)/d\cos\theta_H$ 



- bulk of Born cross section in the central part
- ▶ QED (hard) real-emission contribution dominant close to beam axis, phase-space cut on hard photons occuring at NLO:  $E_{\gamma} < 0.7\sqrt{s}$
- ▶ virtual corrections at  $\theta_H = 90^\circ$  agrees approx. with EW HZ Sudakov factor

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Collinear initial-state radiation:  $\sim (\alpha/\pi)^k \log^k \left(\frac{s}{m_{\mu}^2}\right)$ 

For NLO-EW: collinear factorization in NLLA

[Frixione, ... 1909.03886; 1911.12040; 2105.06688; 2207.03265]



Whizard: revised expressions for numerical stability (T. Striegl)

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Muon beam structure at high energy: PDF ansatz

- 1. EVA: Radiation of (massive) W and Z bosons with on-shell projection for VBS processes
- 2. EWPDF: splitting of massless SM partons, extension of DGLAP Han/Ma/Xie [2007.14300, 2103.09844]
- $\Rightarrow$  implemented in Whizard; to be published



#### To be developed further:

- Beyond the transition region, SM symmetry is effectively unbroken
- Make use of EW jet picture in calculations
- Enable matching and merging schemes to connect high-energy (massless) hard amplitudes with low-energy (massive) event structure and observables
- Watch for multi-scale structure of EW processes
- $\Rightarrow$  talk by Tao Han

### Final-state effects

- ace
- Jets: QCD integrated FastJet interface
- extend to: EW jets and proper matching to physical states
- ▶ Polarized decays (e.g., W, Z, H, t) vs full off-shell matrix elements
- Resonance selection for shower initialization
- Parton shower + hadronization: PYTHIA6 (integrated)
- Parton shower + hadronization: Pythia 8 (interface or via event file)
- Event file formats: ILC-like (legacy, LCIO/Key4HEP) and LHC-like (legacy, LHE, HepMC)

### Conclusions and Outlook

- Whizard is a viable tool for physics studies at a muon collider
- Any perturbative SM (NLO) and BSM processes can be handled
- High-energy phase (beyond 3 TeV):

new approaches required, different from  $e^+e^-$  and LHC algorithms (but straightforward in principle)

EWPDF framework already supported



#### References

- WHIZARD 2: WK/Ohl/Reuter, EPJ C71 (2011) 1742
- O'Mega: Moretti/Ohl/Reuter, hep-ph/0102195
- VAMP: Ohl, CPC 120 (1999) 13 and Brass/WK/Reuter, EPJ C79 (2019) 4344
- NLO@MuCol: Bredt/WK/Reuter JHEP 12 (2022) 138
- WHIZARD 3: Stienemeier et al., 2104.11141; Bredt 2212.04393; main paper to be completed

#### Links

- WHIZARD Portal: https://whizard.hepforge.org/
- Launchpad Page: https://launchpad.net/whizard
- gitlab repo: https://gitlab.tp.nt.uni-siegen.de/whizard/public

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