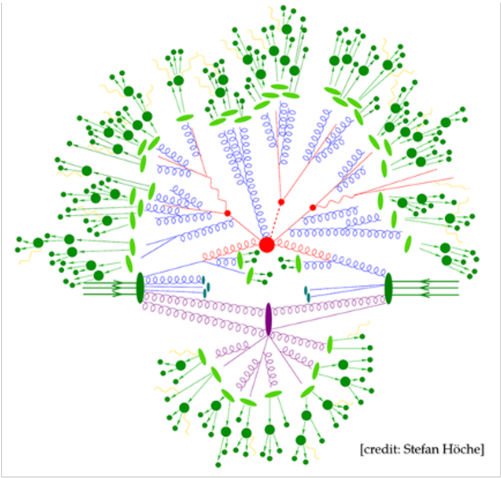
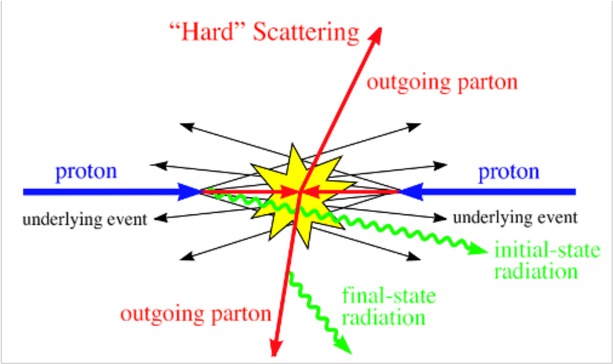
Probing Electroweak NLO Corrections and Sudakov Logarithms at Multi-TeV Muon Colliders

Yang Ma
Centre for Cosmology, Particle Physics and Phenomenology (CP3)
Université catholique de Louvain (UCLouvain)
May 13, 2025

Lesson from the LHC: tools needed

Copy needed for an EW version





Standard practice for phenomenology studies at Tevatraon/LHC

- Hard interaction: fixed-order calculation (in QCD $\equiv 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \ldots$)
- All-order radiative corrections: parton shower emissions
- Hadronisation from confinement property (quarks and gluons cannot live individually for long)
- Multiple parton interactions (MPI)
- Hadron decays

The "factorization formalism"

- Hadrons are composite and contains "partons" inside them
- The parton distributon functions (PDF) describes the probability to find a parton with a particular momentum from the beam particle.

$$\sigma(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(ab \to X)$$

Lessons from the LHC: NLO

What do we learn from the QCD machine ?

$$\delta_X = \frac{\sigma_X}{\sigma_{\text{LO QCD}}}$$

$t\bar{t}H$: $\delta(\%)$	8 TeV	13 TeV	100 TeV
NLO QCD	$25.9^{+5.4}_{-11.1} \pm 3.5$	$29.7^{+6.8}_{-11.1} \pm 2.8$ $(24.2^{+4.8}_{-10.6} \pm 4.5)$	$40.8^{+9.3}_{-9.1} \pm 1.0$
LO EW	1.8 ± 1.3	1.2 ± 0.9 (2.8 ± 2.0)	0.0 ± 0.2
LO EW no γ	-0.3 ± 0.0	-0.4 ± 0.0 (-0.2 ± 0.0)	-0.6 ± 0.0
NLO EW	-0.6 ± 0.1	-1.2 ± 0.1 (-8.2 ± 0.3)	-2.7 ± 0.0
NLO EW no γ	-0.7 ± 0.0	-1.4 ± 0.0 (-8.5 ± 0.2)	-2.7 ± 0.0
HBR	0.88	0.89 (1.87)	0.91

NLO QCD is inevitable

$t\bar{t}Z$: $\delta(\%)$	8 TeV	13 TeV	100 TeV
NLO QCD	$43.2^{+12.8}_{-15.9} \pm 3.6$	$45.9^{+13.2}_{-15.5} \pm 2.9$ $(40.2^{+11.1}_{-15.0} \pm 4.7)$	$50.4^{+11.4}_{-10.9} \pm 1.1$
LO EW	0.5 ± 0.9	0.0 ± 0.7 (2.1 ± 1.6)	-1.1 ± 0.2
LO EW no γ	-0.8 ± 0.1	-1.1 ± 0.0 (-0.3 ± 0.0)	-1.6 ± 0.0
NLO EW	-3.3 ± 0.3	-3.8 ± 0.2 (-11.1 ± 0.5)	-5.2 ± 0.1
NLO EW no γ	-3.7 ± 0.1	-4.1 ± 0.1 (-11.5 ± 0.3)	-5.4 ± 0.0
HBR	0.95	0.96 (2.13)	0.85

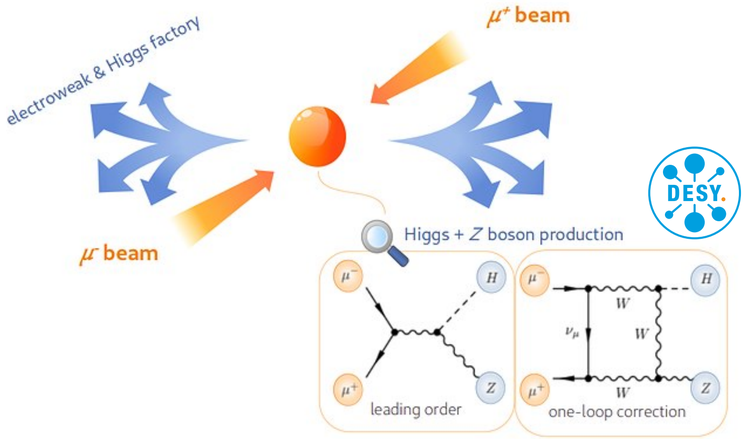
S. Frixione, V. Hirschi, D. Pagani, H.S. Shao, M. Zaro, 1504.03446

$t\bar{t}W^+$: $\delta(\%)$	8 TeV	13 TeV	100 TeV
NLO QCD	$40.8^{+11.2}_{-12.3} \pm 2.9$	$50.1^{+14.2}_{-13.5} \pm 2.4$ $(59.7^{+18.9}_{-17.7} \pm 3.1)$	$156.4^{+38.3}_{-35.0} \pm 2.4$
LO EW	0	0	0
LO EW no γ	0	0	0
NLO EW	-6.9 ± 0.2	-7.7 ± 0.2 (-19.2 ± 0.7)	-9.3 ± 0.2
NLO EW no γ	-7.1 ± 0.2	-8.0 ± 0.2 (-20.0 ± 0.5)	-9.6 ± 0.1
HBR	2.41	3.88 (7.41)	21.52

NLO EW is needed

Next-to-leading order at the muon collider

The muon collider could also be a precision machine



Lesson from the LHC: the NLO QCD is inevitable, so is the NLO EW at muC

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma_{\rm LO}^{\rm incl}$ [fb]	$\sigma_{\rm NLO}^{\rm incl}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2)\cdot 10^2$	$4.847(7)\cdot 10^2$	$+4.0(2)$
ZZ	$2.5988(1)\cdot 10^1$	$2.656(2)\cdot 10^1$	$+2.19(6)$
HZ	$1.3719(1)\cdot 10^0$	$1.3512(5)\cdot 10^0$	$-1.51(4)$
HH	$1.60216(7)\cdot 10^{-7}$	$5.66(1)\cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2)\cdot 10^1$	$2.568(8)\cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5)\cdot 10^0$	$0.895(2)\cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2)\cdot 10^{-1}$	$2.68(1)\cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4)\cdot 10^{-2}$	$6.60(3)\cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1)\cdot 10^{-2}$	$2.451(5)\cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6)\cdot 10^{-8}$	$0.86(7)\cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1)\cdot 10^0$	$0.993(6)\cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1)\cdot 10^0$	$0.699(7)\cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8)\cdot 10^{-2}$	$6.05(4)\cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1)\cdot 10^{-2}$	$0.655(5)\cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2)\cdot 10^{-3}$	$1.799(7)\cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2)\cdot 10^{-3}$	$1.766(6)\cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7)\cdot 10^{-4}$	$6.24(2)\cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1)\cdot 10^{-4}$	$1.165(4)\cdot 10^{-4}$	$-25.7(2)$

$\mu^+\mu^- \to X, \sqrt{s} = 10$ TeV	$\sigma_{\rm LO}^{\rm incl}$ [fb]	$\sigma_{\rm NLO}^{\rm incl}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$5.8820(2)\cdot 10^1$	$6.11(1)\cdot 10^1$	$+3.9(2)$
ZZ	$3.2730(4)\cdot 10^0$	$3.401(4)\cdot 10^0$	$+3.9(1)$
HZ	$1.22929(8)\cdot 10^{-1}$	$1.0557(8)\cdot 10^{-1}$	$-14.12(7)$
HH	$1.31569(5)\cdot 10^{-9}$	$42.9(4)\cdot 10^{-9}$ *	
W^+W^-Z	$9.609(5)\cdot 10^0$	$5.86(4)\cdot 10^0$	$-39.0(2)$
W^+W^-H	$2.1263(9)\cdot 10^{-1}$	$1.31(1)\cdot 10^{-1}$	$-38.4(5)$
ZZZ	$8.565(4)\cdot 10^{-2}$	$5.27(8)\cdot 10^{-2}$	$-38.5(9)$
HZZ	$1.4631(6)\cdot 10^{-2}$	$0.952(6)\cdot 10^{-2}$	$-34.9(4)$
HHZ	$6.083(2)\cdot 10^{-3}$	$2.95(3)\cdot 10^{-3}$	$-51.6(5)$
HHH	$2.3202(4)\cdot 10^{-9}$	$-1.0(2)\cdot 10^{-9}$ *	

Best Paper Awards 2023

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

P. Bredt, W. Kilian, J. Reuter and P. Stienemeier, 2208.09438

$\mathcal{O}(10\% - 100\%)$ corrections in the differential cross sections

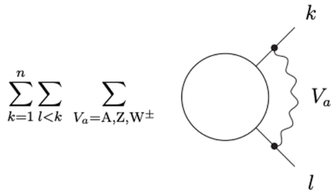
What is inside the NLO EW correction?

One loop virtual correction = Sudakov log terms $\mathcal{O}(-\alpha \log^{1,2}(s/m_W^2))$

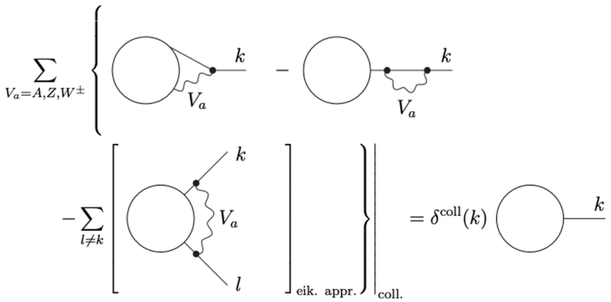
$\mathcal{O}(\alpha)$

+ const. term $\mathcal{O}(1)$

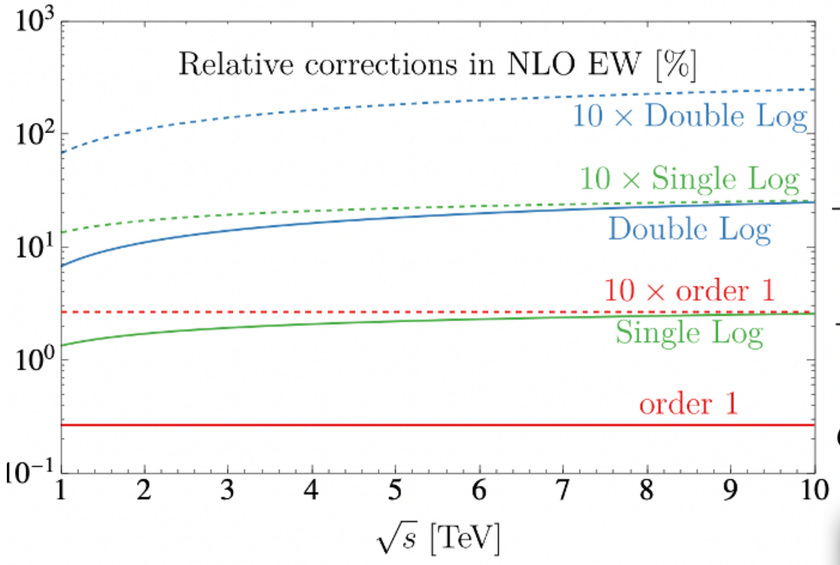
+ mass-suppressed terms $\mathcal{O}(m_W^2/s)$



Double Logs



Single Logs



$\rightarrow \frac{\alpha}{4\pi s_W^2}\log^2\left(s/m_W^2\right)$

$\rightarrow \frac{\alpha}{4\pi s_W^2}\log\left(s/m_W^2\right)$

$\mathcal{O}(1) \rightarrow \frac{\alpha}{4\pi s_W^2} \sim 0.3\%$

- Double Logs do not cancel out even for inclusive observables
- IR cutoff (m_V) is physical
- Double Logs are extremely large at a Muon Collider

Example: $e^+e^- \rightarrow t\bar{t}$

A. Denner, S. Pozzorini, hep-ph/0010201

$$\delta^{\rm RL,ew}_{e^+e^-\to t\bar{t}} = -4.68\, L(s) + 0.86 \left(\log\frac{t}{u}\right) l(s) + 0.50\, l_Z + 14.0\, l_C - 5.30\, l_{\rm Yuk} + 8.80\, l_{\rm PR},$$

$$\delta^{\rm LR,ew}_{e^+e^-\to t\bar{t}} = -4.25\, L(s) + 1.72 \left(\log\frac{t}{u}\right) l(s) + 0.29\, l_Z + 12.7\, l_C - 10.6\, l_{\rm Yuk} + 8.80\, l_{\rm PR},$$

EW Sudakov Logs

Master formula (Denner&Pozzorini)

Born amplitude: $\mathcal{M}_0^{i_1\dots i_n}(p_1,\dots,p_n)$

Denner Pozzorini '01

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\dots i_n}(p_1,\dots,p_n) = \mathcal{M}_0^{i'_1\dots i'_n}(p_1,\dots,p_n)\delta_{i'_1 i_1\dots i'_n i_n}$

other tree-level amplitudes — the logs

$$\delta = \boxed{\delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

eikonal approximation of soft EW boson exchange

- Leading Soft-Collinear: It depends only on s and it is the only term involving double logarithms.
- Subleading Soft-Collinear: The only one involving ratios of s with other invariants and also angular dependences.
- Collinear: In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.
- Parameter renormalis.

The logs inside the δ^i have always the form.

$$L(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log^2\frac{|r_{kl}|}{M^2}$$

$$l(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log\frac{|r_{kl}|}{M^2}$$

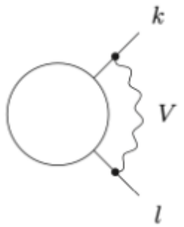
$$M = M_W, M_Z, m_f, \lambda, \dots$$
$$r_{kl} \equiv (p_k + p_l)^2$$

ASSUMPTIONS:

$$r_{kl} \equiv (p_k + p_l)^2 \simeq 2p_k p_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$$ the high-energy limit

$$r_{kl}/r_{k'l'} \simeq 1$$

All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.



Denner&Pozzorini

$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\text{SSC}} + \cdots$$

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

SDK_0

Remove some QED logs that invlove MW and IR scale to remove the IR singularities

$\mathrm{SDK}_{\mathrm{weak}}$

Cluster the photons with the charged particles. A pure version of EWSL.

$$\Delta^{s\to r_{kl}}$$

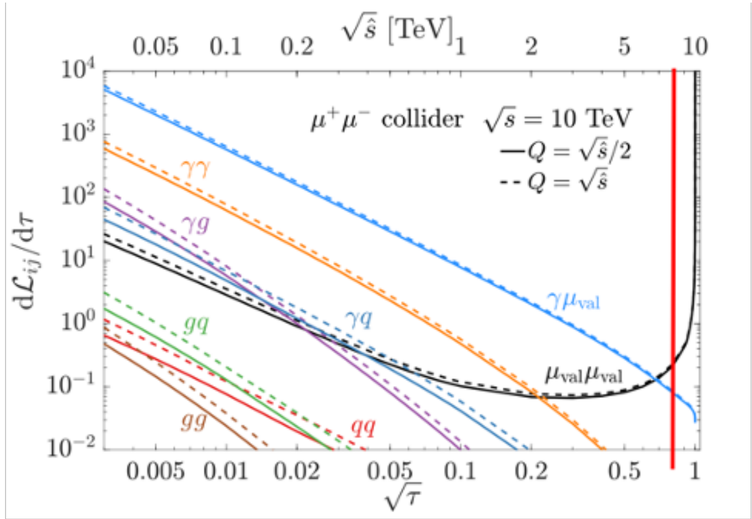
To include the $\log(|r_{kl}|/|r_{k'l'}|)$

D. Pagani, M. Zaro, 2110.03714

Calculation set-up

Kinematic cuts:

$\eta(X) < 2.44,\ p_T(X) > 100$ GeV, $\Delta R(X,Y) > 0.4$



T. Han, YM and K. Xie, 2007.14300, 2103.09844

Focus on annihilation

Cluster photon with charged particles for $\Delta R(X,\gamma) < 0.4$

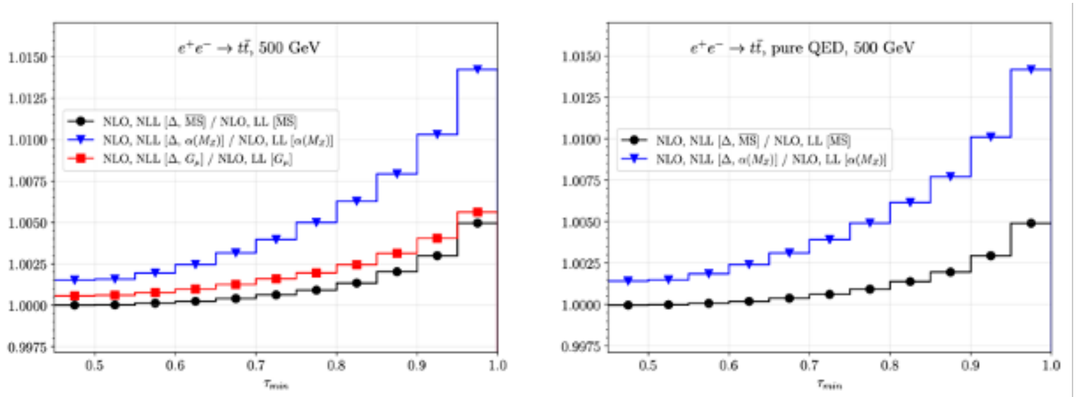
Initial State Radiation (ISR):

We use the leading-log (LL) muon PDF

$$\Gamma_{\rm LO}(z) = \frac{\exp\left(3\beta_S/4 - \gamma_E\beta_E\right)}{\Gamma\left(1+\beta_E\right)}\beta_E(1-z)^{\beta_E-1} - \frac{1}{2}\beta_H(1+z) + \mathcal{O}(\alpha^2)$$

Beta Scheme $\beta_E = \beta_S = \beta_H = e_e^2\beta$

Eta Scheme $\beta_E = \beta_S = e_e^2\beta\,,\qquad \beta_H = e_e^2\eta$ $\quad \eta = \frac{\alpha}{\pi}\log\frac{\mu^2}{m^2}\,,\qquad \beta = \frac{\alpha}{\pi}\left(\log\frac{\mu^2}{m^2} - 1\right)$

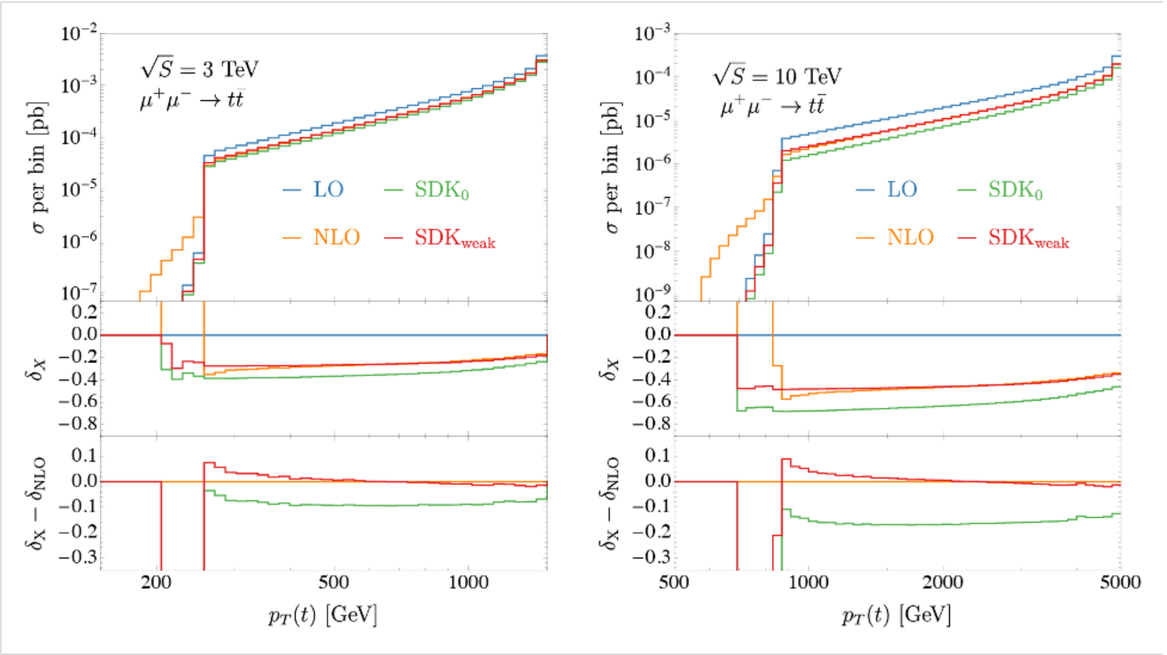


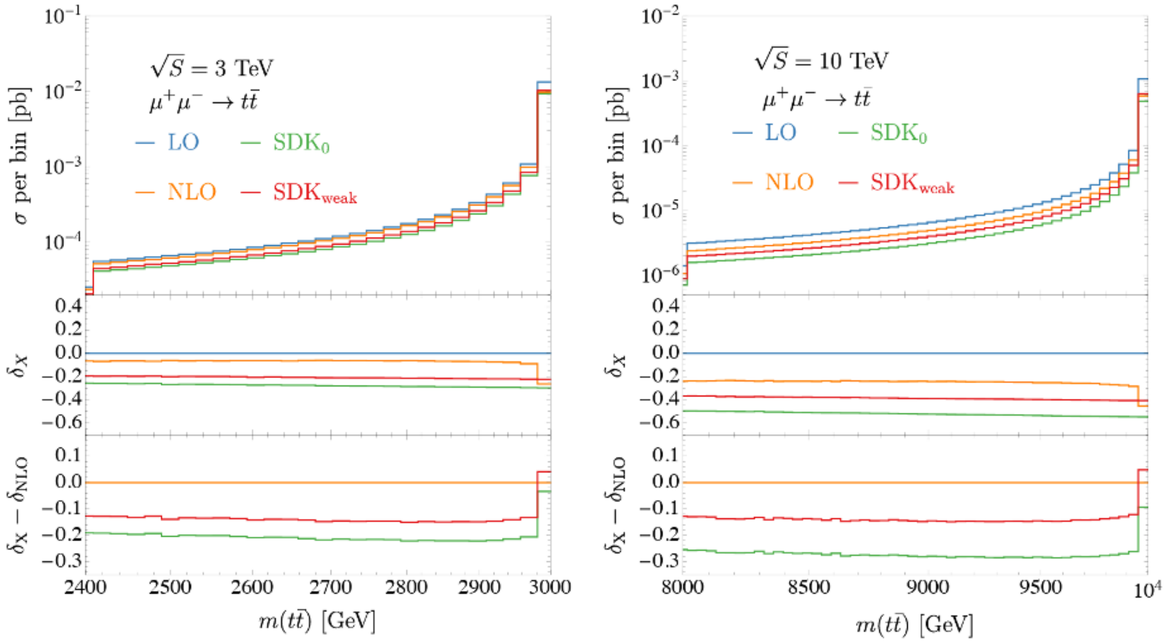
V. Bertone, M. Cacciari, S. Frixione, G. Stagnitto, M. Zaro and X. Zhao, 2207.03265

Better PDF is needed

Top pair production

EWSL works well in reproducing the full NLO EW





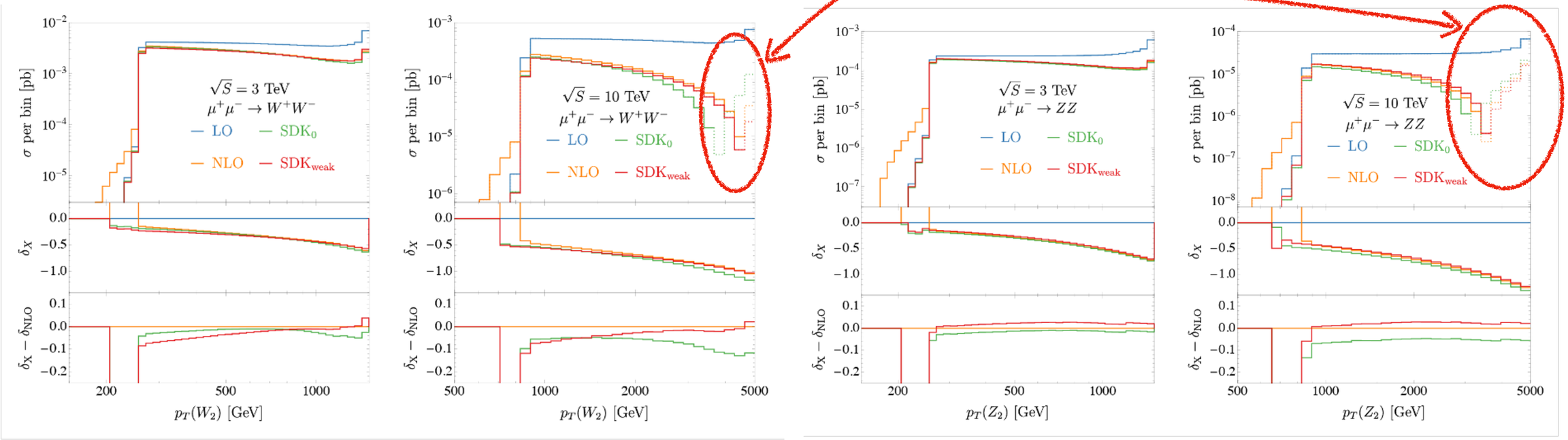
$$\delta_X \equiv \frac{\sigma_X - \sigma_{\rm LO}}{\sigma_{\rm LO}}$$

YM, D. Pagani, and M. Zaro, 2409.09129

Di-V production

$\mathrm{SDK}_{\mathrm{weak}}$ has better behaviors

Negative!

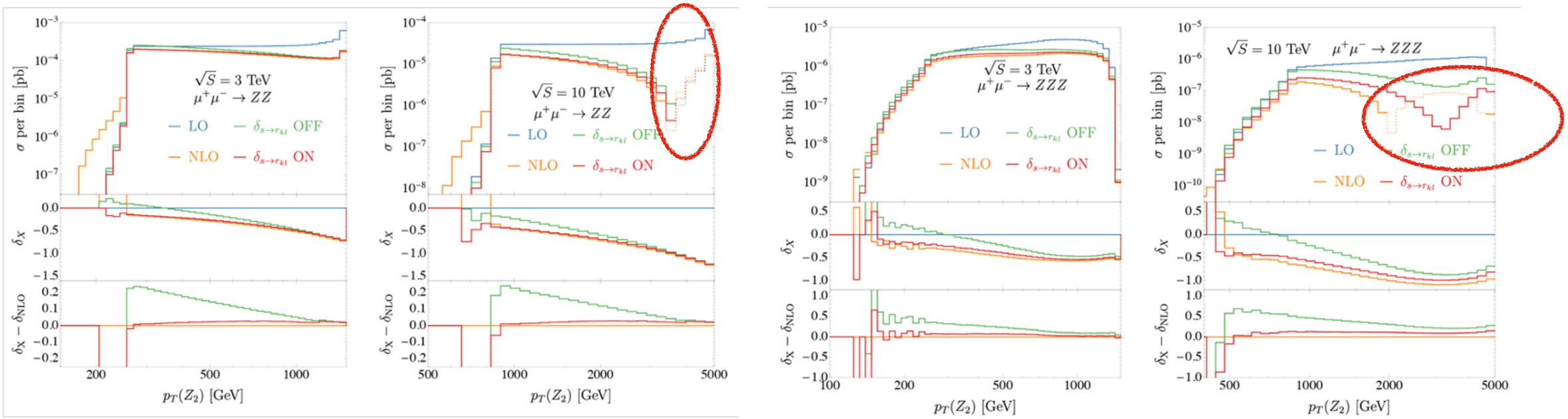


$$\delta_X \equiv \frac{\sigma_X - \sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}}$$

YM, D. Pagani, and M. Zaro, 2409.09129

Go beyond the DP EWSL

$\Delta^{s\to r_{kl}}$: To include the $\log\left(|r_{kl}|/|r_{k'l'}|\right)$



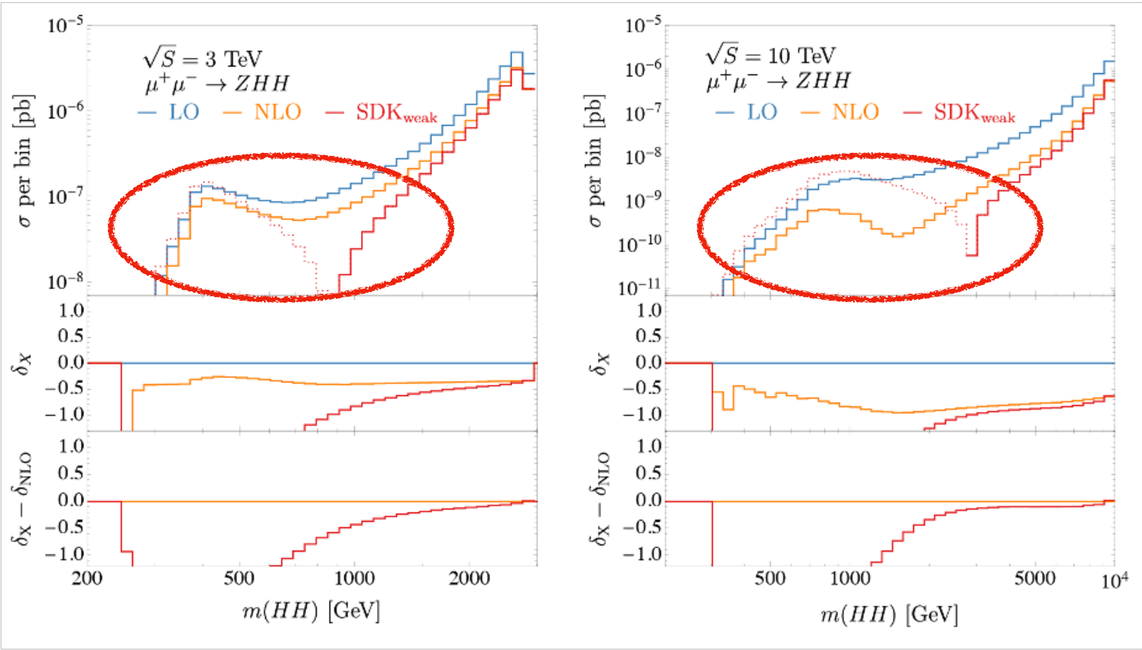
YM, D. Pagani, and M. Zaro, 2409.09129

The red lines better mimic the full NLO EW calculation

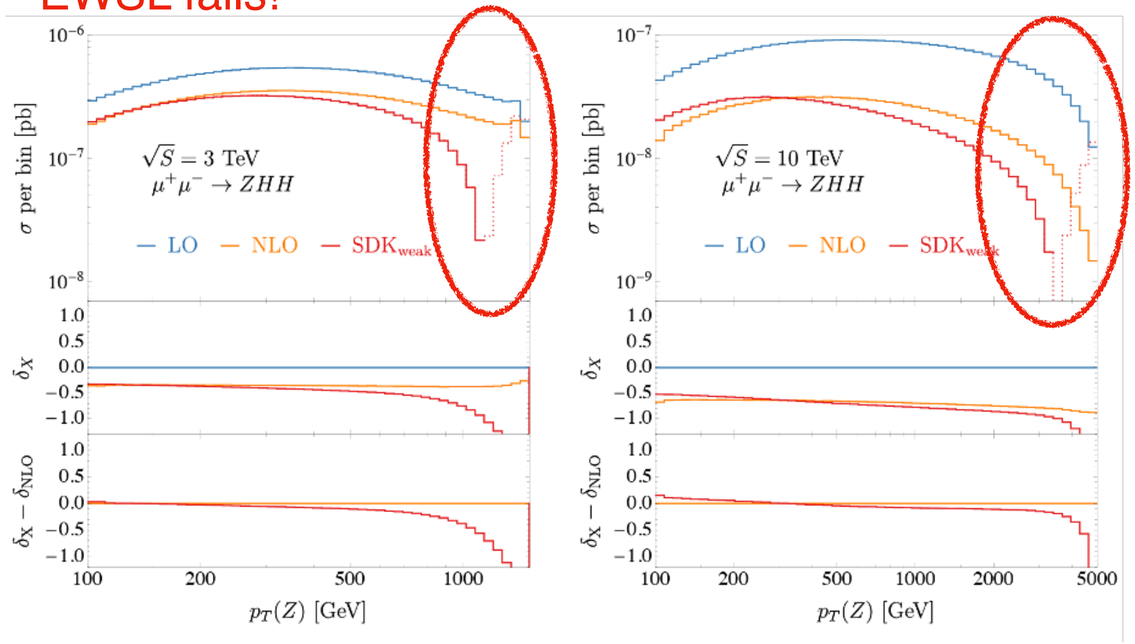
Does EWSL fail?

Yes, sometimes… e.g. ZHH production

The mass suppressed $\frac{v}{m_{HH}}$ dominates in the large $p_T(Z)$ and small m_{HH} region



EWSL fails!



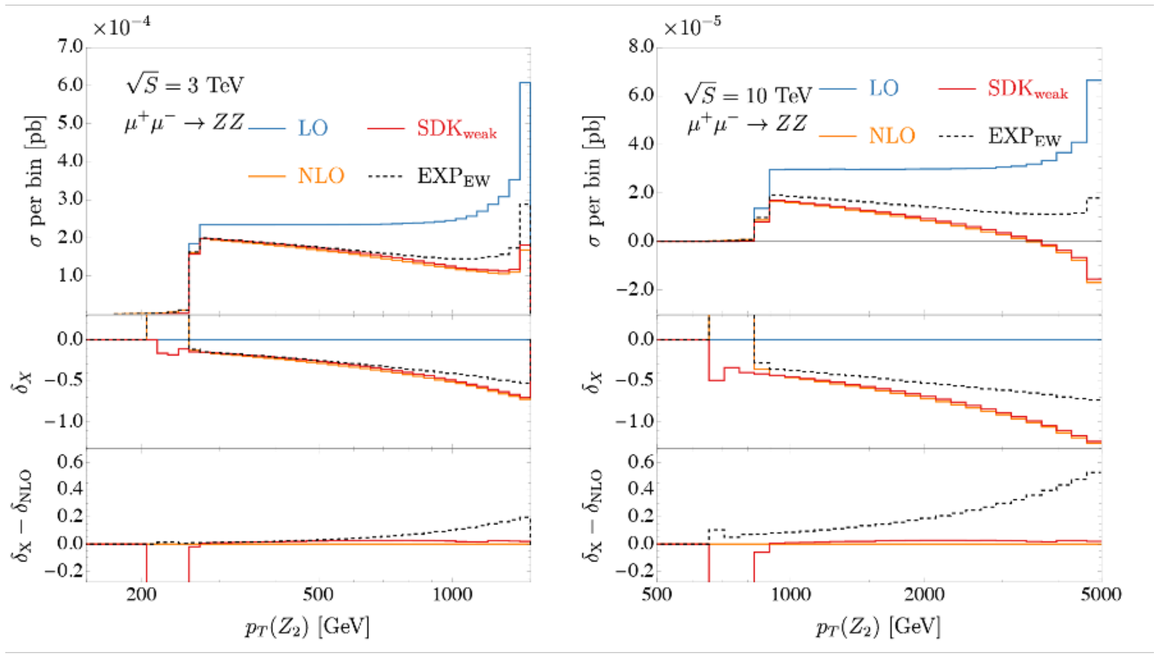
YM, D. Pagani, and M. Zaro, 2409.09129

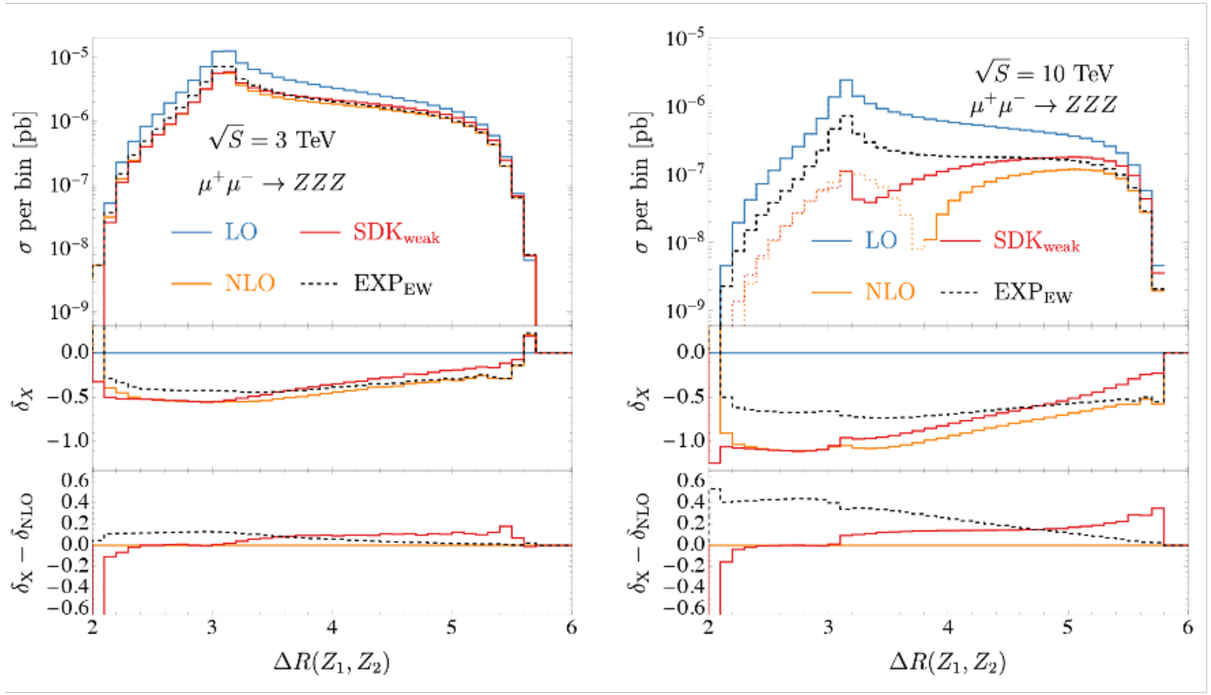
It also happens to VBF single Higgs production

Resuming the EW Sudakov Logs

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$

$$\delta_X \equiv \frac{\sigma_X - \sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}}$$



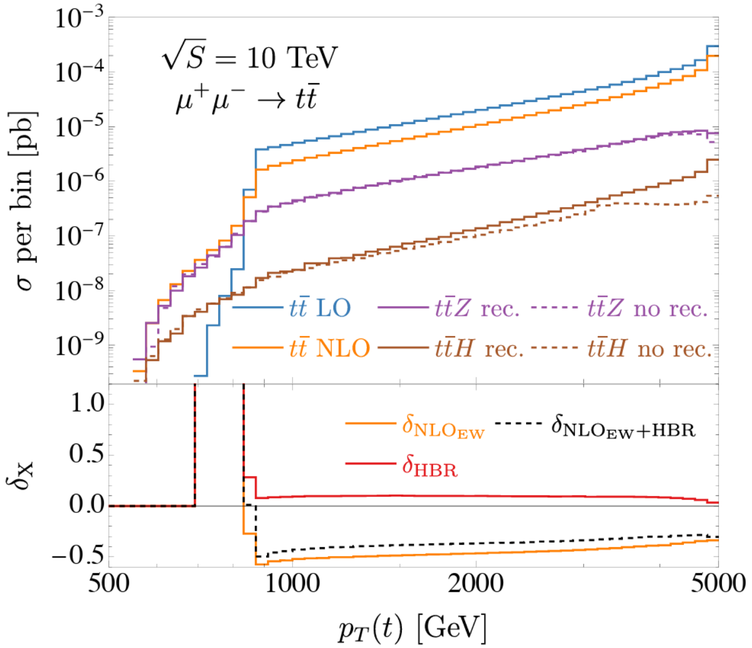


The negative cross sections are fixed

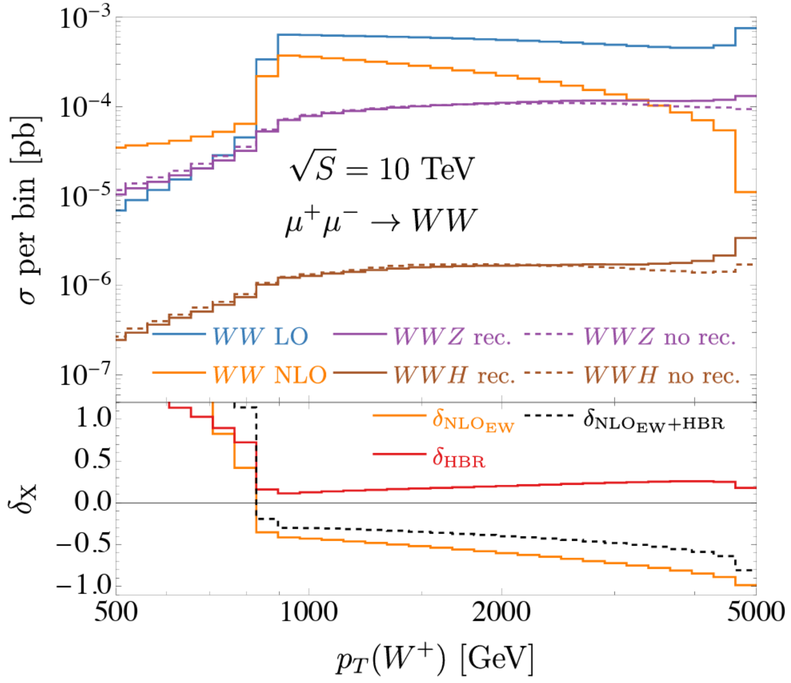
HBR at the muC

The heavy boson radiation is not so large

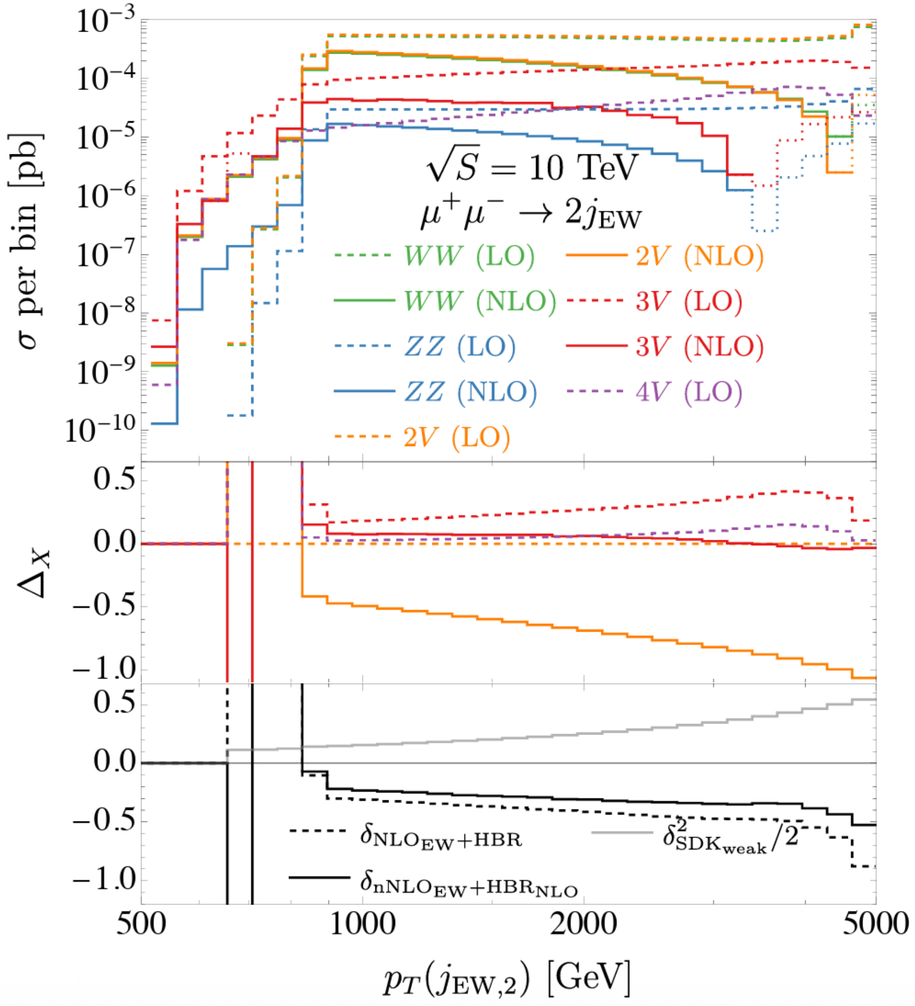
$\mu^+\mu^- \to t\bar{t}Z(H)$



$\mu^+\mu^- \to W^+W^-Z(H)$



$\mu^+\mu^- \to VV$



Summary

- Multi-TeV muon colliders bring us more physics opportunities at the new frontiers, as a discovery machine/precision machine
- The NLO EW is mandatory for the pheno studies at the muon colliders
- The EW Sudakov Logs can give good approximate for the NLO EW correction, **but not always**
- To cure the negative differential cross sections, resuming the EWSL is essential
- For the $\mu^+\mu^- \to X$ process, the HBR tend to be small