Energy driven results at the Muon Collider

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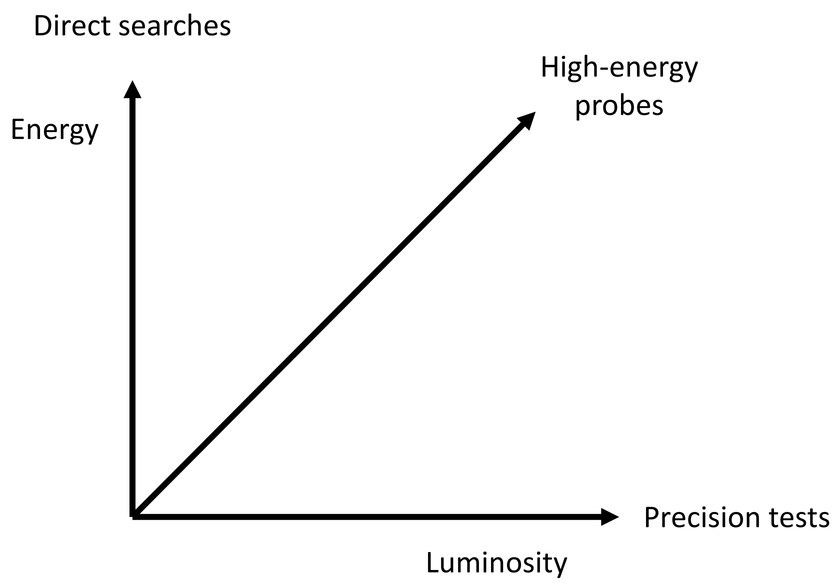
Chen, Glioti, Rattazzi, Ricci, Wulzer **2202.10509**
De Blas, Franceschini, Glioti, Marzocca, Wang, Wulzer, **2505.xxxxx & 2505.xxxxx**
(to appear soon)

High energy probes



Consider the SM as an **Effective Theory**

Standard Model

Possible deviations due to new **heavy** states

$$\mathcal{L} = \mathcal{L}^{d\leq 4} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\mathcal{L}^{d=6} + \dots$$

By dimensional analysis, d=6 operators can give contribution that grow with the energy

$$\mathcal{A} = \mathcal{A}_{\rm SM} + \frac{E^2}{\Lambda^2}\mathcal{A}_{d=6} + \dots$$

Higher collider **energy** → Higher **reach** on new physics

High energy probes

Muon collider can **indirectly** probe energies much higher than the collision energies

$$\frac{\Delta O_{\rm BSM}}{O_{\rm SM}} \sim \frac{E^2}{\Lambda^2}$$

$$\boxed{1\% \text{ at } E = 10 \text{ TeV} \implies \Lambda \sim 100 \text{ TeV}}$$

Caveat: achieving percent precision requires theoretical control of EW radiation

In particular **Sudakov Double Logs**

$$\boxed{\frac{\alpha_w}{4\pi} \log^2 \frac{E^2}{m_W^2} \sim 0.25 \text{ at } 10 \text{ TeV}}$$

Double logs require **resummation**

EW Sudakov at the MuCol

Electroweak Double Logs are **special**

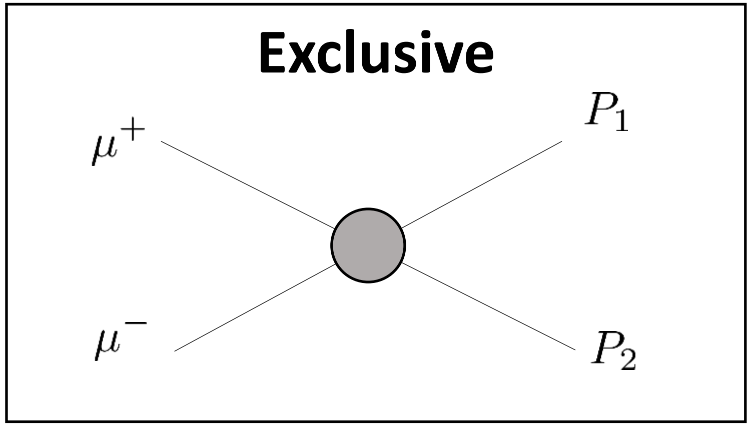
- The EW group is **non-abelian** and **spontaneously broken**:
  - Double Logs do **not cancel out** even for inclusive observables
  - Initial/final states are **not** EW **singlets**
- EW theory is **weakly coupled**
- IR cutoff (m_W) is **physical**
- Double Logs are **extremely large** at a Muon Collider

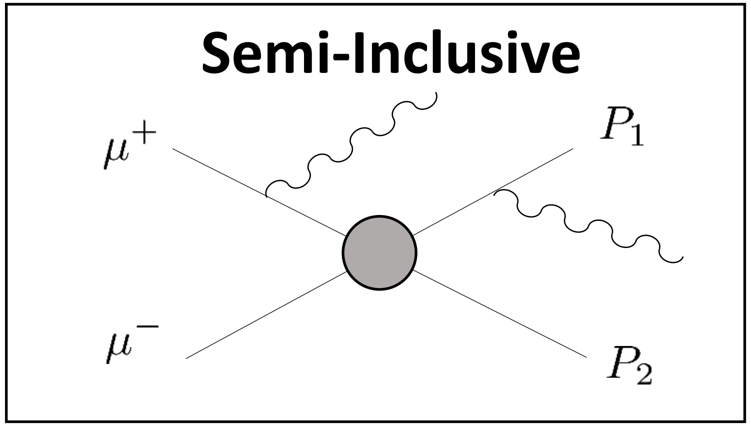
BN Theorem violation
Ciafaloni, Ciafaloni, Comelli (2000)

Several different independent and calculable observables can be measured varying the level of "inclusiveness"

Which observables?

We consider **two representative observables** for which we know how to perform the DL resummation





- **Two hard** final particles with **definite** EW **color**
- **Veto** on soft/collinear EW **radiation**
- Inclusive on soft photons/gluons

- **Two hard** final particles with **definite** EW **color**
- All **radiation** is **allowed** (up to some hardness threshold)
- "**Semi**" since we don't **sum** over external legs colors

IREE strategy

Our resummation strategy is based on an **InfraRed Evolution Equation** (IREE)

Fadin, Lipatov, Martin, Melles, 1999

$\lambda = E^2$ Born obs.

$\big\downarrow$ IREE

$\lambda = m_W^2$ Resummed obs.

- Introduce an unphysical **IR cutoff** λ

$$\lambda < \min \left| \frac{(k_i \cdot q)(k_j \cdot q)}{(k_i \cdot k_j)} \right|$$

- Compute derivative of the observable wrt λ through **diagrammatic** techniques

$$\frac{d}{d\lambda} \mathcal{O}^{\lambda} = \mathcal{K} \cdot \mathcal{O}^{\lambda}$$

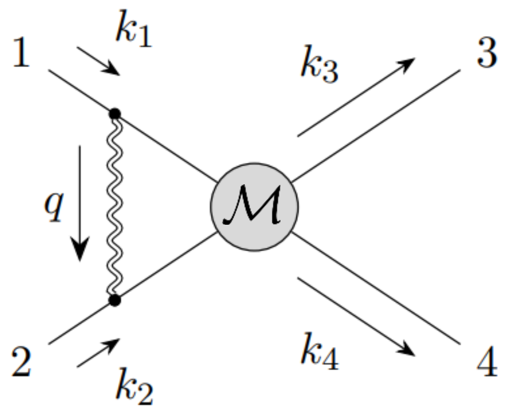
- Solve the IREE with the **boundary condition**

$$\mathcal{O}^{\lambda = E^2} \equiv \mathcal{O}^{\text{Born}}$$

Exclusive observables

Exclusive on EW radiation, but inclusive on soft photons

$$\boxed{\mu^+\mu^- \to P_1 P_2 + n\gamma}$$



IREE defined at the level of **Amplitudes**

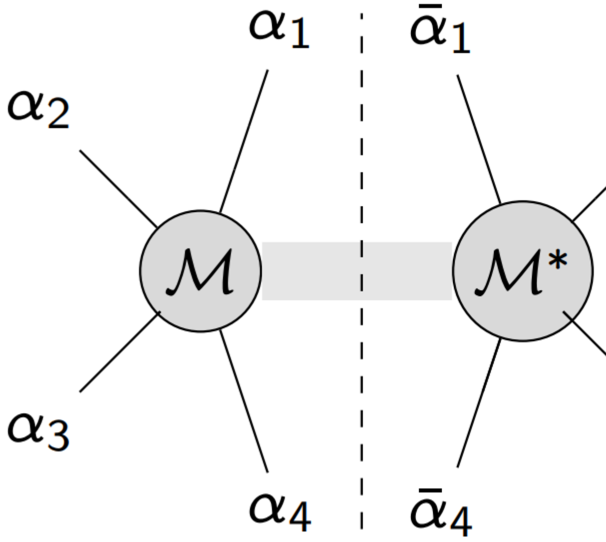
Kernel is **diagonal**

$$\frac{d\mathcal{M}^\alpha_\lambda}{d\log^2(E^2/\lambda)} = -\frac{1}{2}\mathcal{K}\,\mathcal{M}^\alpha_\lambda\,, \qquad \mathcal{K} = \frac{1}{16\pi^2}\sum_i \left[g^2 c_i + g'^2 y_i^2\right]$$

Semi-inclusive observables

$$\mu^+\mu^- \to P_1 P_2 + n(W,\, Z,\, \gamma)$$

P1 and P2 carry ~90% of COM energy



$$\equiv \mathcal{D}_\lambda^{\alpha\bar{\alpha}}$$

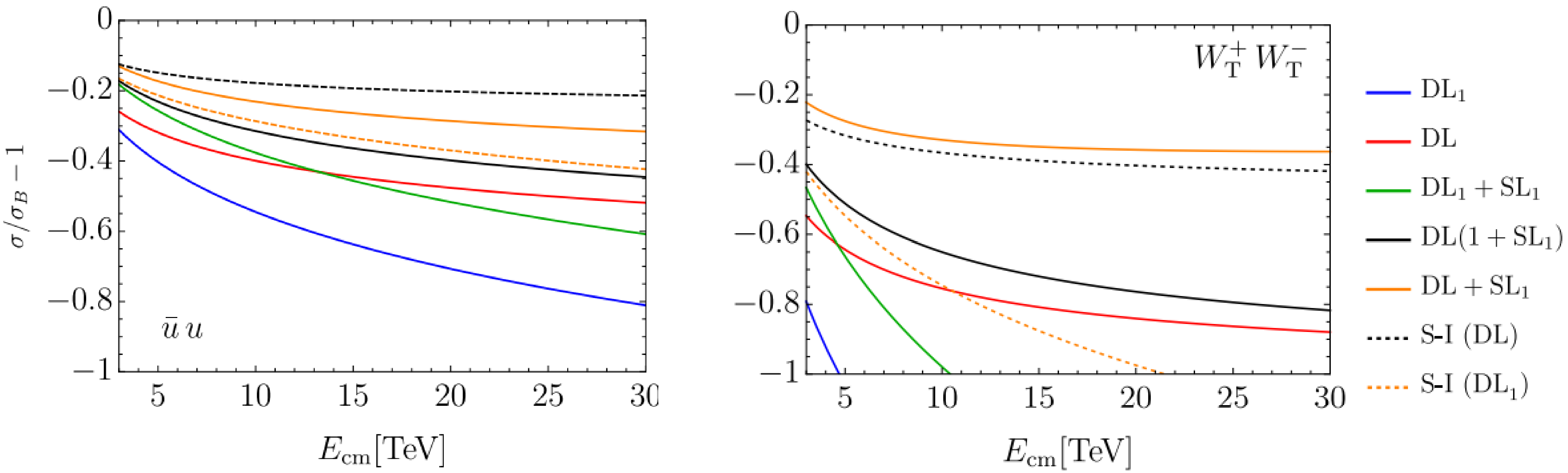
IREE defined at the level of **Density Matrix**

$$\frac{d\mathcal{D}_\lambda^{\alpha\bar{\alpha}}}{d\log^2(E^2/\lambda)} = -\mathcal{K}_{\beta\bar{\beta}}^{\alpha\bar{\alpha}}\,\mathcal{D}_\lambda^{\beta\bar{\beta}}$$

Kernel is **off-diagonal**

All **Born hard processes mix** with each other

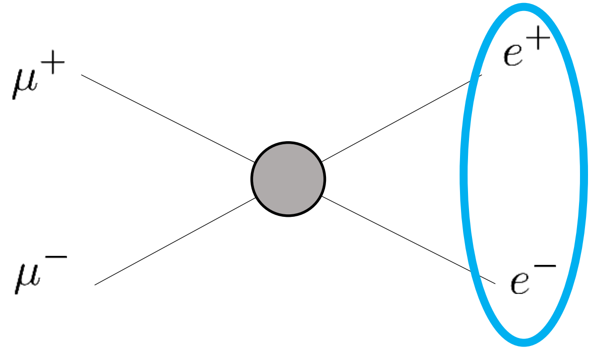
Effect of Double and Single Logs

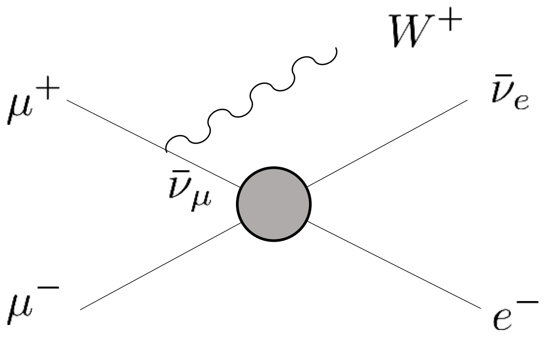


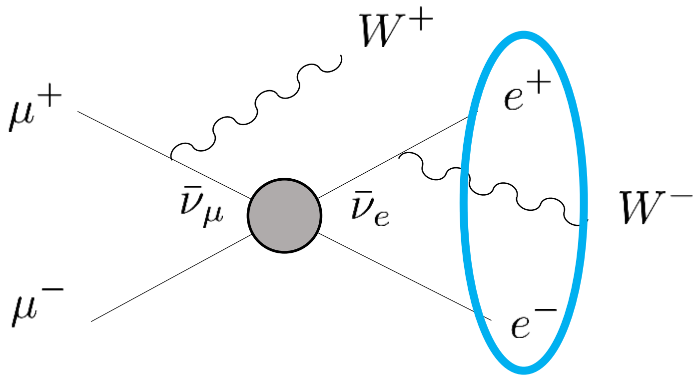
Single-Logs (virtual only) added at fixed-order from Denner, Pozzorini (2000)

Radiation for BSM

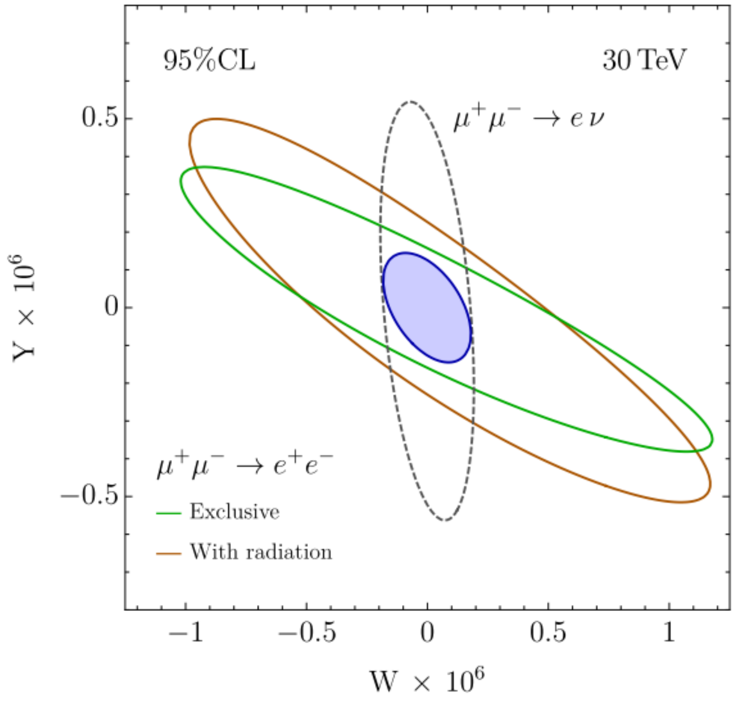
Thanks to the large double **logs processes with soft emission** become **as likely as** processes allowed at **tree-level**







Hard processes sensitive to **different combinations** of new physics parameters



Energy Growing effects

To recap, we considered the following 2 → 2 processes

	Exclusive	Semi-Inclusive	Semi-Inclusive (charged)
Difermion	$\mu\mu \to l\bar{l}$ $\quad \mu\mu \to q\bar{q}$	$\mu\mu \to l\bar{l} + X$ $\quad \mu\mu \to q\bar{q} + X$	$\mu\mu \to l\nu + X$ $\quad \mu\mu \to u\bar{d} + X$
Diboson	$\mu\mu \to Zh$ $\quad \mu\mu \to WW$	$\mu\mu \to Zh + X$ $\quad \mu\mu \to WW + X$	$\mu\mu \to Wh + X$ $\quad \mu\mu \to WZ + X$

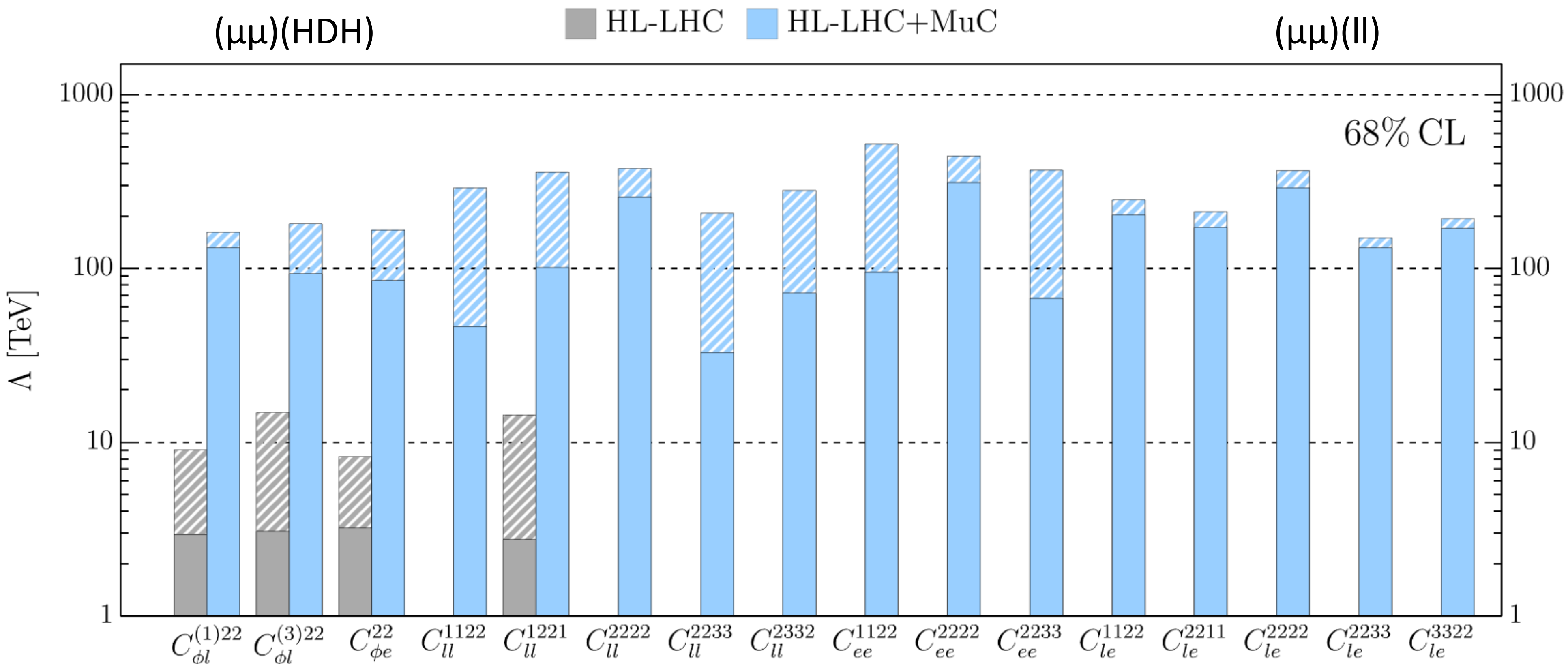
Analytic expressions for the cross sections and Mathematica notebook for
DL Resummation available

https://github.com/aglioti/muColSudakov

Some things are still WIP, will be finalized soon

Results on effective operators

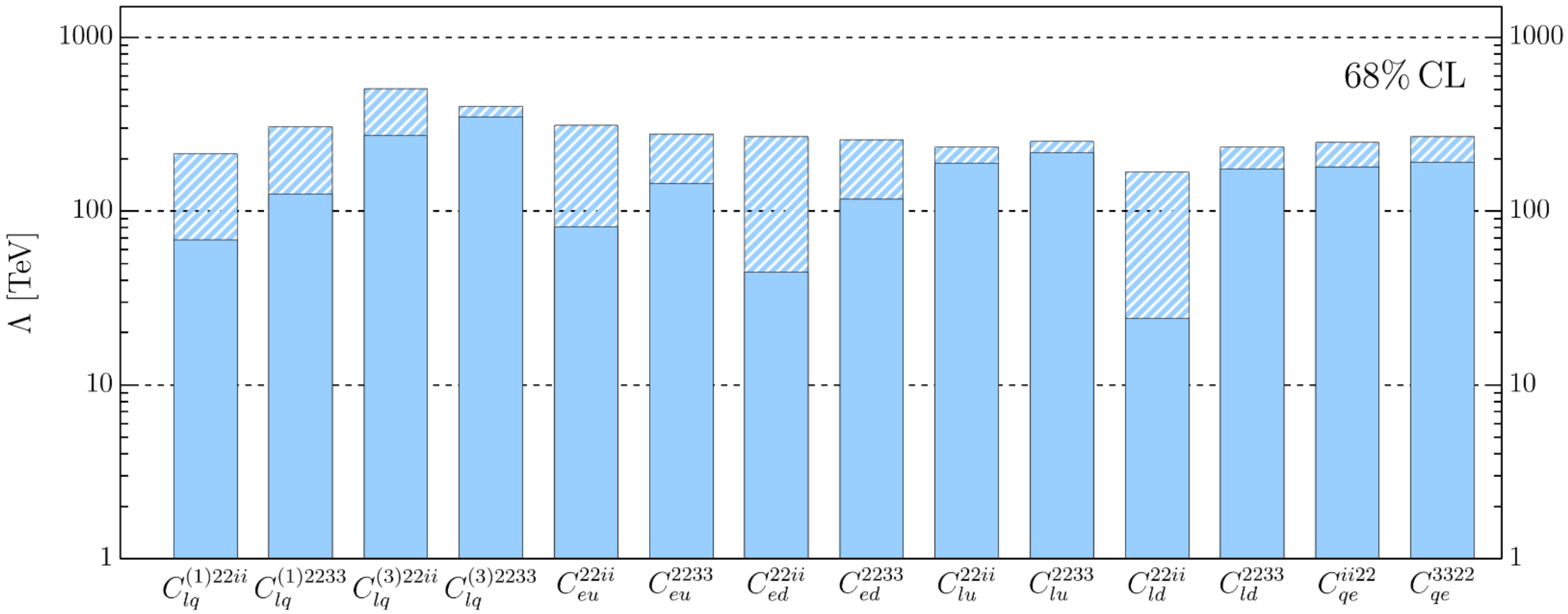
Diboson & Dilepton



Results on effective operators

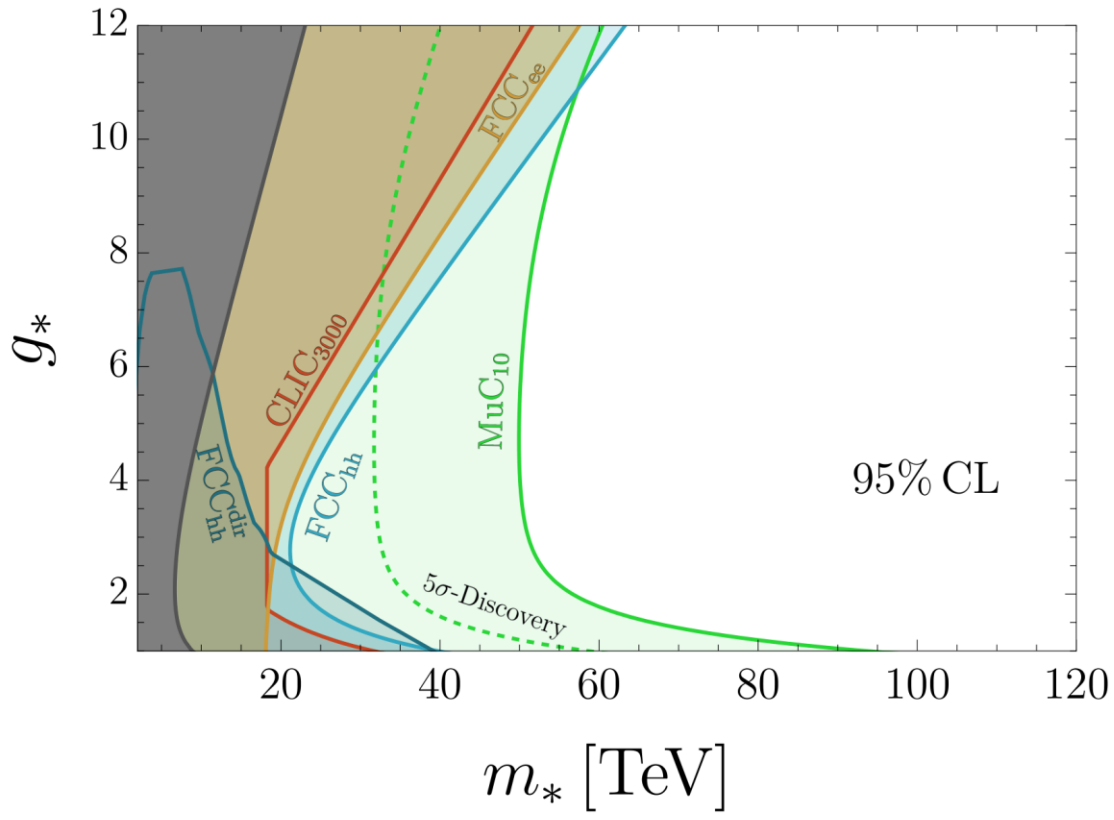
Diquark

(μμ)(qq)

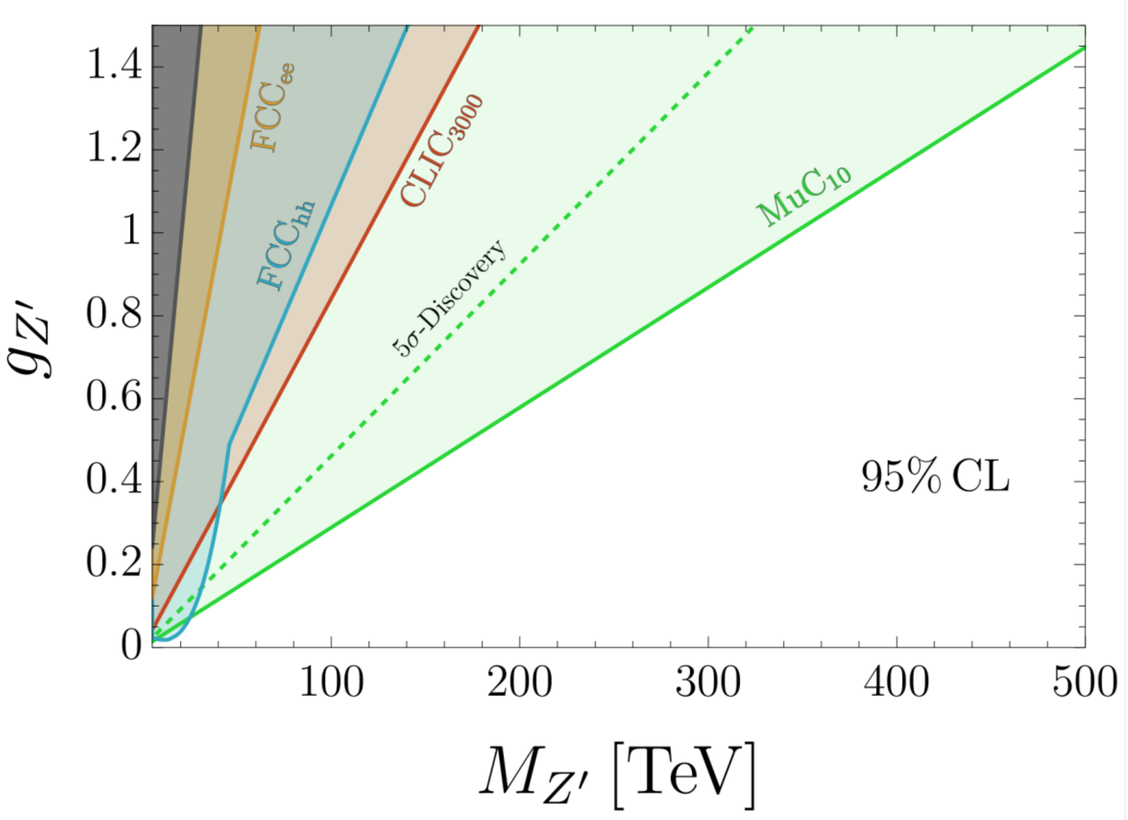


Reach on BSM models

Composite Higgs



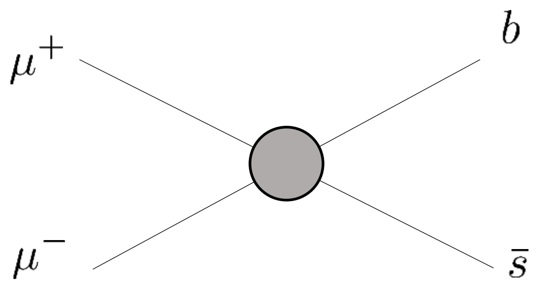
Y-universal Z'



Flavor at MuCol

We can apply the same strategy on **flavor-violating** 4-fermion operators

For example, the famous **bsμμ contact interaction** can be probed by



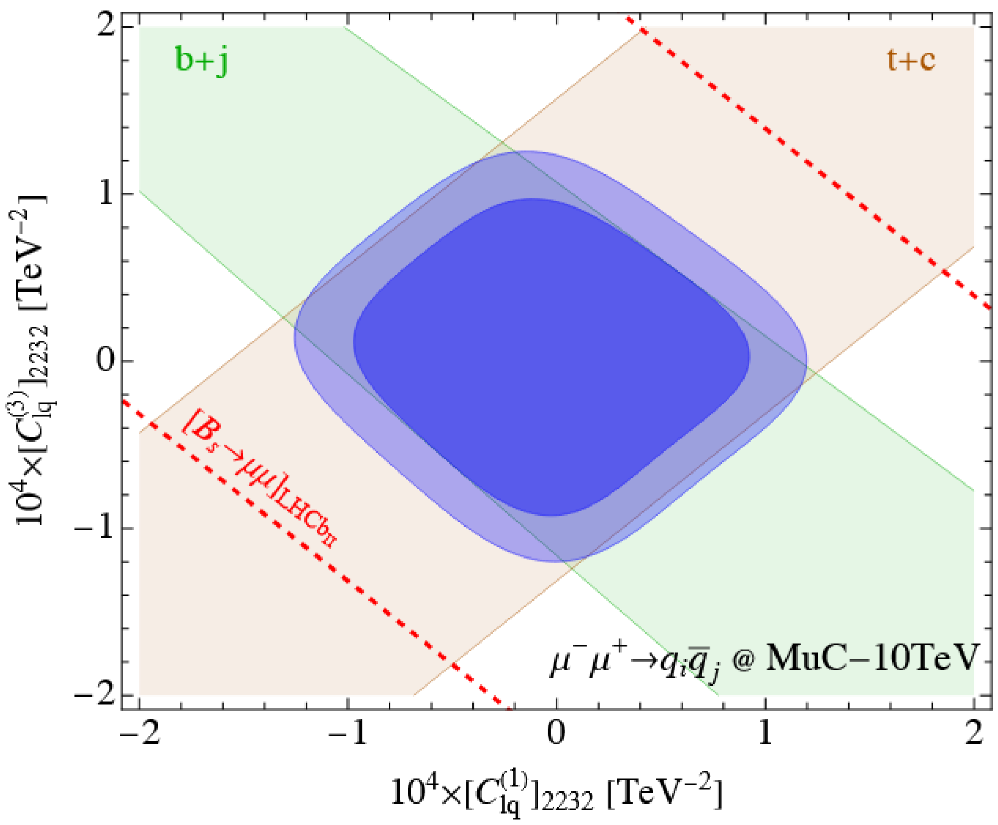
Even in this case we can exploit the **energy growth** to probe higher new physics scales

However, no (or small) interference with SM → $\delta\sigma/\sigma_{\rm SM} \sim E^4/\Lambda^4$

NP reach is comparable with low energy precision measurements

The MuCol is also a flavor machine!

Flavor at MuCol

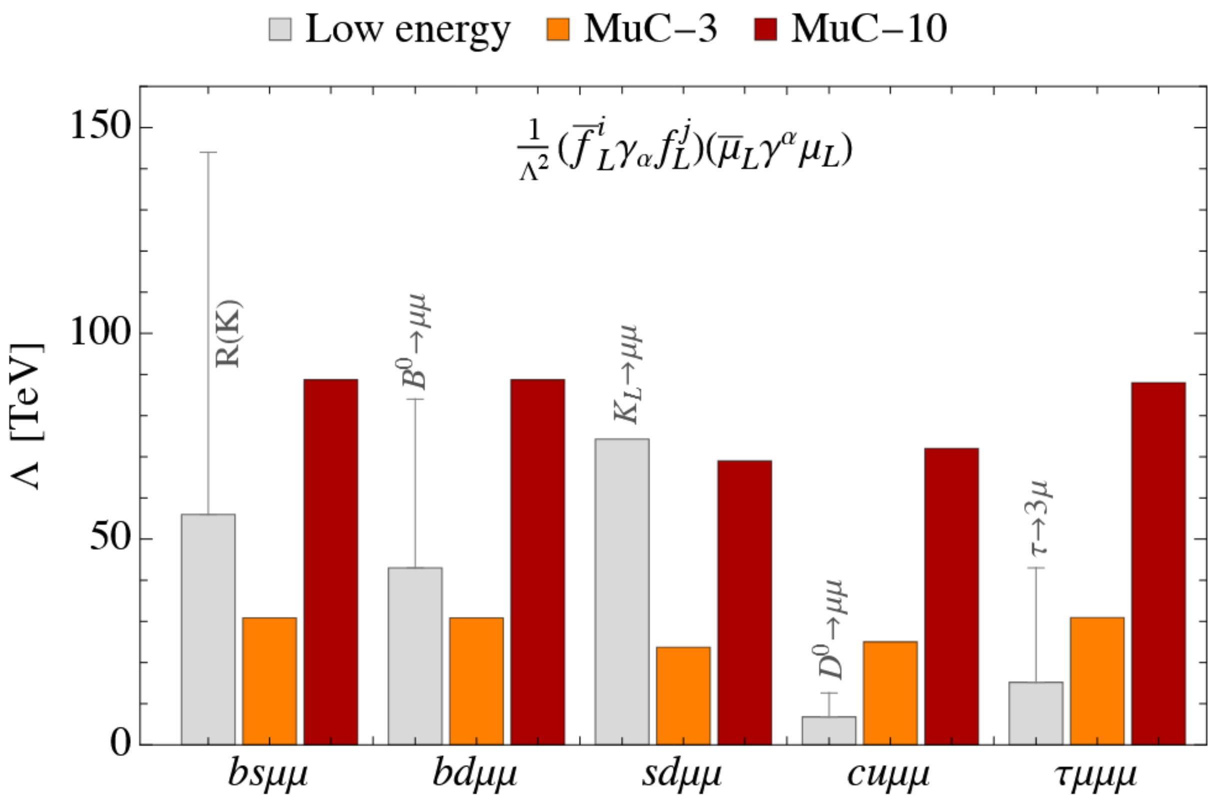


Different final states probe the parameter space in different directions

This analysis for now only contains the exclusive predictions

Adding also semi-inclusive and charged states would improve the bounds even more (WIP)

Flavor: high vs low energy



Conclusions

- A **10 TeV Muon Collider** can indirectly probe New Physics up to **100+ TeV**

- Large radiation effect allows to **probe New Physics** in a **richer way** compared to tree-level prediction

- This also extends to **flavor violating observables**, where the Muon Collider is competitive with other specific experiments

- Furthermore, the study of EW radiation is an amazing playground of fundamental QFT questions

A lot of physics to be done!