Probing Z/W Pole Physics at High-energy Muon Colliders

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based on arXiv: 2503.19073 (updating)

- Motivation
- Theoretical framework SMEFT
- Analysis method
- Numerical results
- Summary

Motivation

High-energy Muon collider

Enormous physical potential, despite many technical challenges

High energy & high Luminosity

Energy frontier & Precision frontier

3 TeV run (up to 10 TeV & 30 TeV), 1 ab^{-1} (up to 10 ab^{-1} & 90 ab^{-1})

 \rightarrow Directly searching for BSM (Beyond Standard Model)

- However, MuCol may fall short on some aspect of EW precision program e.g. lack of Z-pole run (compared with future e^+e^- colliders)
- All is not lost

It can be made up by exploiting other EW measurements

• Manifested with SMEFT (Standard Model Effective Field Theory)

New physics described by a serious of high-order operators

Impose different couplings on SM gauge couplings

Motivation

Two aspects for EW analysis under SMEFT

- Contributions from many higher dimensional operators have **energy enhancements**
 - + e.g. WW/WZ/ZH processes D. Buttazzo, R. Franceschini, A. Wulzer, arXiv: 2012.11555
 - + However, the subsequent decays of W or Z to fermions may not
- VBF (vector boson fusion) processes increase with the collider energy
 - + High energy $\xrightarrow{\text{effectively}}$ high precision



T. Han, W. Kilian et.al. arXiv: 2108.05362



A. Costantini, F. D. Lillo et.al. arXiv: 2005.10289

Motivation

Two aspects for EW analysis under SMEFT

- Contributions from many higher dimensional operators have **energy enhancements**
 - + e.g. WW/WZ/ZH processes D. Buttazzo, R. Franceschini, A. Wulzer, arXiv: 2012.11555
 - + However, the subsequent decays of W or Z to fermions may not
- **VBF** (vector boson fusion) processes increase with the collider energy
 - + High energy $\xrightarrow{\text{effectively}}$ high precision

Thus, in this study

- To further investigate the potential of EW measurements at MuCol
 - + Phenomenological analyses to the fusion processes of two vector bosons
- VBF→ 2f processes*
 - + At least one W boson, i.e. $WW/WZ/W\gamma$, into a pair of fermions
 - + Along with a $\nu_{\mu}\bar{\nu}_{\mu}$ or $\nu_{\mu}\mu^{+}/\bar{\nu}_{\mu}\mu^{-}$ pair coming from the incoming $\mu^{+}\mu^{-}$

^{*} Although can be controversial, for convenience, "VBF→ 2f " is simply used to denote the two-vector-boson-to-two-fermion processes.

• SMEFT lagrangian:

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{i} \frac{c_{i}^{(6)}}{\Lambda^{2}} \mathcal{O}_{i}^{(6)} + \cdots,$$

where Λ is potential new physics scale.

• dim-6 operators involved:

$$\begin{split} & \mathcal{O}_{Hq}^{(1)} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{q}\gamma^{\mu}q) \,, \\ & \mathcal{O}_{Hq}^{(3)} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}^{i}H)(\bar{q}\sigma^{i}\gamma^{\mu}q) \,, \\ & \mathcal{O}_{Hu} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{u}\gamma^{\mu}u) \,, \\ & \mathcal{O}_{Hd} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{d}\gamma^{\mu}d) \,, \end{split}$$

$$\begin{split} & \mathcal{O}_{Hl}^{(1)} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{\ell}\gamma^{\mu}\ell) \,, \\ & \mathcal{O}_{Hl}^{(3)} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}^{i}H)(\bar{\ell}\sigma^{i}\gamma^{\mu}\ell) \,, \\ & \mathcal{O}_{He} = (H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{e}\gamma^{\mu}e) \,, \end{split}$$

where, $H^{\dagger}i\overleftrightarrow{D}_{\mu}H = H^{\dagger}(iD_{\mu}H) - (iD_{\mu}H^{\dagger})H$, $H^{\dagger}i\overleftrightarrow{D}_{\mu}^{i}H = H^{\dagger}\sigma^{i}(iD_{\mu}H) - (iD_{\mu}H^{\dagger})\sigma^{i}H$, 6/19

• Effective lagrangian:

$$\begin{split} \mathcal{L} &\supset -\frac{g}{c_W} Z_\mu \left[\sum_{f=u,d,e,v} \bar{f}_L \gamma^\mu (T_f^3 - s_W^2 Q_f + \delta g_L^{Zf}) f_L + \sum_{f=u,d,e} \bar{f}_R \gamma^\mu (-s_W^2 Q_f + \delta g_R^{Zf}) f_R \right] \\ &- \frac{g}{\sqrt{2}} \left[W_\mu^+ \bar{u}_L \gamma^\mu (V_{\rm CKM} + \delta g_L^{Wq}) d_L + W_\mu^+ \bar{\nu}_L \gamma^\mu (1 + \delta g_L^{W\ell}) e_L + \text{h.c.} \right] \,, \end{split}$$

• *Vff* coupling modification by Wilson Coefficients:

$$\begin{split} \delta g_L^{Ze} &= -\left(c_{H\ell}^{(1)} + c_{H\ell}^{(3)}\right) \frac{v^2}{2\Lambda^2}, \quad \delta g_R^{Ze} = -c_{He} \frac{v^2}{2\Lambda^2}, \quad \delta g_L^{Wl} = c_{Hl}^{(3)} \frac{v^2}{\Lambda^2}, \\ \delta g_L^{Zu} &= -\left(c_{Hq}^{(1)} - c_{Hq}^{(3)}\right) \frac{v^2}{2\Lambda^2}, \quad \delta g_R^{Zu} = -c_{Hu} \frac{v^2}{2\Lambda^2}, \\ \delta g_L^{Zd} &= -\left(c_{Hq}^{(1)} + c_{Hq}^{(3)}\right) \frac{v^2}{2\Lambda^2}, \quad \delta g_R^{Zd} = -c_{Hd} \frac{v^2}{2\Lambda^2}, \quad \delta g_L^{Wq} = c_{Hq}^{(3)} \frac{v^2}{\Lambda^2}, \end{split}$$

* $hV\!f\!f$ contact interactions are not considered in this study

*Top quark not included, left to the future

Processes:



(d):

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*Top quark not included, left to the future

• Processes:

$$\mu^{-}\mu^{+} \rightarrow b\bar{b}\nu_{\mu}\bar{\nu}_{\mu}, \quad \mu^{-}\mu^{+} \rightarrow c\bar{c}\nu_{\mu}\bar{\nu}_{\mu}, \quad \mu^{-}\mu^{+} \rightarrow \tau^{-}\tau^{+}\nu_{\mu}\bar{\nu}_{\mu},$$
$$\mu^{-}\mu^{+} \rightarrow cs\nu_{\mu}\mu, \quad \mu^{-}\mu^{+} \rightarrow \tau\nu_{\tau}\nu_{\mu}\mu$$

• Typical diagrams for $\mu^-\mu^+ \rightarrow \tau^+ \nu_\tau \bar{\nu}_\mu \mu^-/h$

 $\bar{
u}_{\mu}$ μ^+ asymmetric processes ν_{μ} ν_{τ} μ^+ $\bar{\nu}_{\mu}$ W^+ W u_{μ} ν_{τ} $\dot{\nu}_{\mu}$ ν_{τ} Z(a) to (c): $2f \rightarrow 2f$ with additional $\bar{\nu}_{\mu}$ μ^{-} μ^{-} μ^{-} μ^{-} μ initial or final Z/γ (a) (b) (c) $\bar{\nu}_{\mu}$ μ^+ μ^+ $\bar{\nu}_{\mu}$ μ^+ W^+ W^+ τ^+ μ^+ Z/γ^{W^+} ν_{τ} (d) to (e): u_{μ} Z/γ ν_{τ} diboson production W(f) to (g): **VBF** W^{-} $\bar{\nu}_{\mu}$ μ^{-} μ^{-} μ (d) (e) (f) (g) Xiaoze Tan (DESY & FDU) 13/05/2025 9/19

Analysis method

• Run scenarios @10TeV & 30TeV

$$\mathscr{L}_{int} = 10 \mathrm{ab}^{-1} \left(\frac{E_{cm}}{10 \mathrm{TeV}}\right)^2,$$

• Kinematic cuts:

 $p_T^j>20\,{\rm GeV}, |\eta_j|<2.44,$ for jets and $p_T^l>10\,{\rm GeV}, |\eta_\tau|<2.44, |\eta_\mu|<6$ for charged leptons

• **Tagging** (optimistic):

Process	Requirement	Efficiency
$\mu^-\mu^+ o b ar{b} u_\mu ar{ u}_\mu$	2 b-tags	0.64
$\mu^-\mu^+ o c \bar{c} u_\mu \bar{ u}_\mu$	2 c-tags	0.49
$\mu^-\mu^+ \to \tau^-\tau^+\nu_\mu\bar\nu_\mu$	2 au-tags	0.7
$\mu^-\mu^+ ightarrow cs u_\mu \mu$	1 c-tag and 1 s-tag	0.35
$\mu^-\mu^+ \to \tau \nu_\tau \nu_\mu \mu$	1τ -tag	0.84

*optimistic *c* & s tagging borrowed from H. Liang et.al. arXiv: 2310.03440 Few words about **backgrounds**:

- VBF $\rightarrow h \rightarrow b\bar{b}$ not considered:
 - + can be efficiently removed with invariant mass window cut
- $\mu^+\mu^- \rightarrow b\bar{b}$ with initial-state radiation (ISR) photons:
 - + suppressed when $M_{b\bar{b}} \ll \sqrt{s}\,$, impose upper bound for $M_{b\bar{b}} < 1 {\rm TeV}$

•
$$\mu^+\mu^- \rightarrow b\bar{b}\mu^+\mu^-$$
via $ZZ/Z\gamma/\gamma\gamma$ fusion:
• Only the very forward muons
undetectable
• assuming $\eta_{\mu} > 6$ untagged, missing
 $p_T > 20$ GeV cut can help
•



Analysis method

Binning analysis: $M_{f\bar{f}}$ distribution

• take $\mu^+\mu^- \rightarrow b\bar{b}\nu_\mu\bar{\nu}_\mu$ @ 10TeV run for example



 ${}^{*}c_{i} = 1 @ \Lambda = 1 \text{TeV}$ on this page

Analysis method



*bin7-9 not included when fit bounds due to background

Binning analysis (by invariant mass $M_{f\bar{f}}$):

Bin number	bin 1	bin 2	bin 3	bin 4	bin 5	bin 6	bin 7	bin 8	bin 9
Invariant mass (GeV)	[0, 100)	[100, 200)	[200, 400)	[400,600)	[600, 800)	[800, 1000)	[1000, 1500)	[1500, 2000)	$[2000, +\infty)$

Binning by angle $|\cos \bar{\theta}|$:

 $|\cos\bar{\theta}| \equiv \frac{1}{2} \left(\left| \cos\theta_{\mu^{-b}} \right| + \left| \cos\theta_{\mu^{+b}} \right| \right)$

Invariant Mass	b	$ar{b}$	0	cē	$ au^- au^+$			
$({ m GeV})$	$10{ m TeV}$	$30{ m TeV}$	$10{ m TeV}$	$30{ m TeV}$	$10{ m TeV}$	$30{ m TeV}$		
[600, 800)	0.5	0.45	0.45	0.45	0.4	0.45		
[800, 1000)	0.55	0.5	0.45	0.45	0.5	0.45		
[1000, 1500)	0.6	0.5	0.5	0.45	0.6	0.45		
[1500, 2000)	0.65	0.55	0.5	0.5	0.6	0.45		
$[2000, +\infty)$	0.7	0.55	0.55	0.55	0.6	0.55		

* impose $M_{b\bar{b}} < 1 {\rm TeV}$ when fitting

Numerical results

•
$$\Delta \chi^2 = 1$$
 contours from the 3-parameter fit to $\mu^- \mu^+ \rightarrow b \bar{b} \nu_\mu \bar{\nu}_\mu$



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Numerical resuluts

• Comparison of the 3-parameter fit for $\mu^-\mu^+ \rightarrow b\bar{b}\nu_\mu\bar{\nu}_\mu$ and



 \uparrow solid contours only use invariant mass bins, dashed contours add $|\cos \theta|$ bins



†solid contours only include symmetric processes, dashed contours add asymmetric process

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Numerical results

• 1 σ bounds and correlations

$10{ m TeV}$								10 Te	V		$10{ m TeV}$						
	68% CL 1σ bound Correlation			68% CL 1σ bound Corr					orrelation			, bound	Correlation				
	$(imes 10^{-2})$ matrix				$(\times 10^{-1})$	matrix				(×10	$^{-2})$	matrix					
	Individual	Global	$\left(c_{Hq}^{(1)}\right)_{33}$	$\left(c_{Hq}^{(3)}\right)_{33}$	$(c_{Hd})_{33}$		Individual	Global	$\left(c_{Hq}^{(1)}\right)_{22}$	$\left(c_{Hq}^{(3)}\right)_{22}$	$(c_{Hu})_{22}$		Individual	Global	$\left(c_{Hl}^{(1)}\right)_{33}$	$\left(c_{Hl}^{(3)}\right)_{33}$	$(c_{He})_{33}$
$\left(c_{Hq}^{(1)} ight)_{33}$	± 0.500	± 1.90	1			$\left(c_{Hq}^{(1)} ight)_{22}$	± 0.679	± 5.86	1			$\left(c_{Hl}^{(1)}\right)_{33}$	± 1.10	± 8.12	1		
$\left(c_{Hq}^{(3)}\right)_{33}$	± 0.526	± 3.47	0.871	1		$\left(c_{Hq}^{(3)}\right)_{22}$	± 0.310	± 0.349	-0.142	1		$\left(c_{Hl}^{(3)}\right)_{33}$	± 0.323	± 0.338	-0.00695	1	
$(c_{Hd})_{33}$	± 3.14	± 28.1	0.954	0.963	1	$(c_{Hu})_{22}$	± 1.61	± 13.8	0.907	-0.323	1	$(c_{He})_{33}$	± 1.35	± 10.0	0.986	0.0431	1

$30{ m TeV}$								$30\mathrm{Te}$	V		$30{ m TeV}$						
	68% CL 1σ bound Correlation		68% CL 1σ bound			Correlation		68%CL 1 σ bound			Correlation						
	$(\times 10^{-2})$ matrix				(×10	matrix			$(\times 10^{-2})$			matrix					
	Individual	Global	$\left(c_{Hq}^{(1)}\right)_{33}$	$\left(c_{Hq}^{(3)}\right)_{33}$	$(c_{Hd})_{33}$		Individual	Global	$\left(c_{Hq}^{(1)}\right)_{22}$	$\left(c_{Hq}^{(3)}\right)_{22}$	$(c_{Hu})_{22}$		Individual	Global	$\left(c_{Hl}^{(1)}\right)_{33}$	$\left(c_{Hl}^{(3)}\right)_{33}$	$(c_{He})_{33}$
$\left(c_{Hq}^{(1)}\right)_{33}$	± 0.158	± 0.629	1			$\left(c_{Hq}^{(1)}\right)_{22}$	± 0.224	± 1.97	1			$\left(c_{Hl}^{(1)} ight)_{33}$	± 0.350	± 2.72	1		
$\left(c_{Hq}^{(3)} ight)_{33}$	± 0.165	± 1.06	0.895	1		$\left(c_{Hq}^{(3)}\right)_{22}$	± 0.117	± 0.138	-0.765	1		$\left(c_{Hl}^{(3)} ight)_{33}$	± 0.125	± 0.134	0.0388	1	
$(c_{Hd})_{33}$	± 0.992	± 9.01	0.966	0.961	1	$(c_{Hu})_{22}$	± 0.526	± 4.61	0.701	-0.747	1	$(c_{He})_{33}$	± 0.453	± 3.53	0.946	0.156	1

Numerical results

• 1 σ constraints compared with e^+e^- collider



Summary

• High energy muon collider

enormous physical potential in both energy & luminosity frontier

• Lack of Z-pole run (compared with e^+e^- collider)

can be compensated by exploiting other EW measurements

• Within a subset of SMEFT for Vff coupling

precision for Wilson coefficients reach up to ~ 10^{-2} @ 10TeV, even ~ 10^{-3} @ 30TeV roughly comparable with Z-pole run e^+e^- collider ($\Lambda = 1$ TeV)

• Binned analysis can be helpful

illustrate the contribution of SMEFT operators in different region of distributions

• More efforts are needed

muon forward tagging, background analysis.....

Thank you!

$$\sigma(\ell^+\ell^- \to Zh) = 1220 \left[1 + \left(\frac{E_{\rm cm}}{0.78}\right)^2 C_W + \left(\frac{E_{\rm cm}}{0.96}\right)^4 C_W^2 \right] \times \left(\frac{10\,{\rm TeV}}{E_{\rm cm}}\right)^2 0.1\,{\rm ab}\,,$$
arXiv: 2012.11555

Mis-tagging: e.g. for *b* to be identified as *c*, which is around 20%. Given that we require both final state quarks to be tagged, we only expect a small mixing ($\leq 4\%$) between different signal events, which we simply ignore in our study.



The $\Delta \chi^2 = 1$ contours from the 3-parameter fit to $\mu^- \mu^+ \rightarrow b \bar{b} \nu_\mu \bar{\nu}_\mu$ at 10 TeV (top row) and 30 TeV (bottom row), obtained from three separate groups of invariant mass bins, which are bin 1-3 (red contours), bin 4-6 (green contours) and bin 7-9 (blue contours).



the $\Delta \chi^2 = 1$ contours from the 3-parameter but fit to $\mu^- \mu^+ \rightarrow c \bar{c} \nu_\mu \bar{\nu}_\mu$ at 10 TeV (top row) and 30 TeV (bottom row)



the $\Delta \chi^2 = 1$ contours from the 3-parameter but fit to $\mu^- \mu^+ \rightarrow \tau^- \tau^+ \nu_\mu \bar{\nu}_\mu$ at 10 TeV (top row) and 30 TeV (bottom row)



25/19