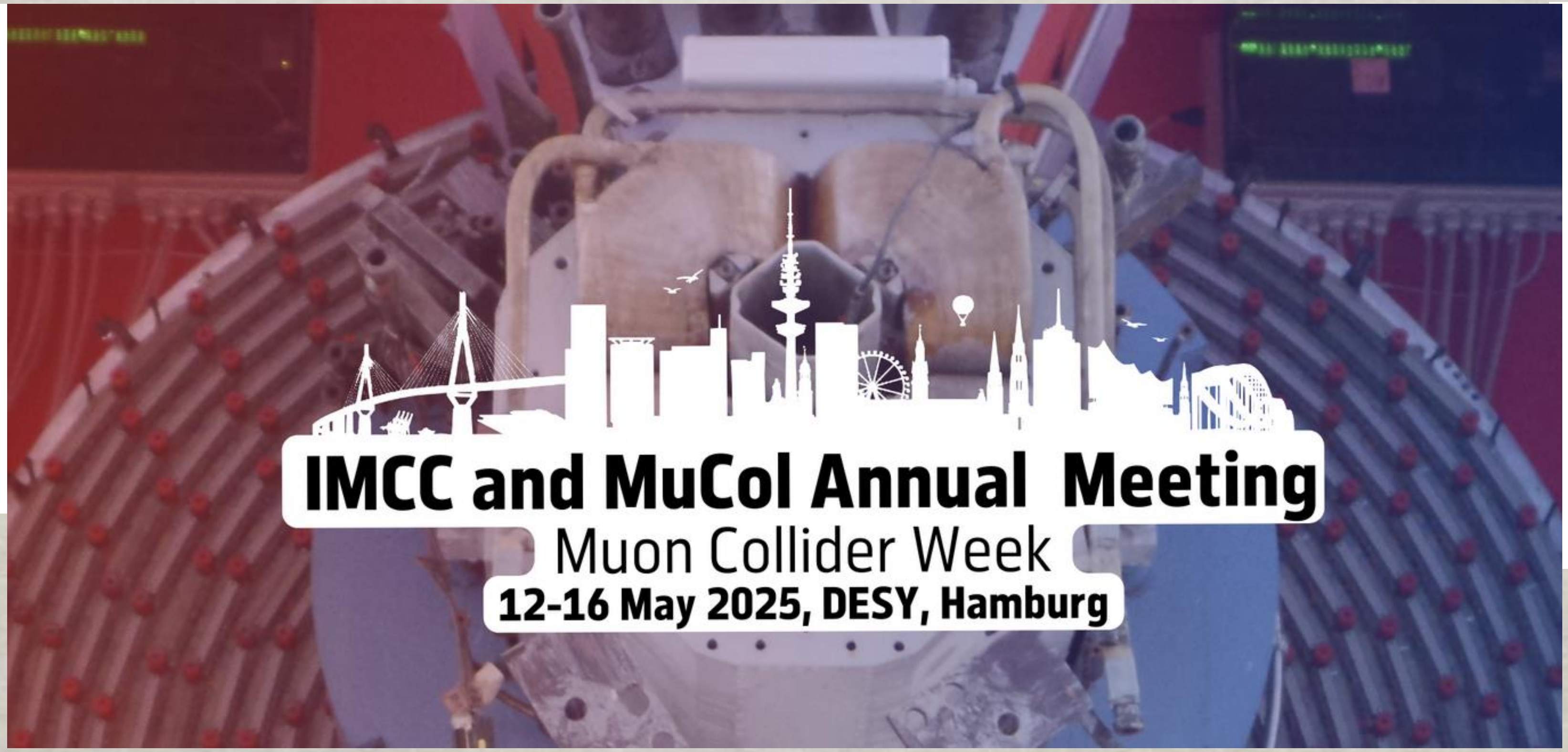
Electroweak Symmetry Restoration @ High Energies

-- Muon Collider as a Case Study

Tao Han
Pitt PACC, University of Pittsburgh

IMCC @ DESY, May 13, 2025



Electroweak Symmetry Breaking:

The Longitudinal & Goldstone Bosons

For an on-shell vector: $\epsilon(k)^{\mu}_{L} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^{\mu}}{m_W}$ + O(M_W/E)

trivial "scalarization" (for any vector state)

symmetry breaking residual

Goldstone-boson Equivalence Theorem

At high energies E>>M_W, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!) $\mathcal{M}(W^i_L W^j_L \to W^i_L W^j_L) \approx \mathcal{M}(\omega^i \omega^j \to \omega^i \omega^j)$

At high energies:

W^i_L → Correspond to the broken generators

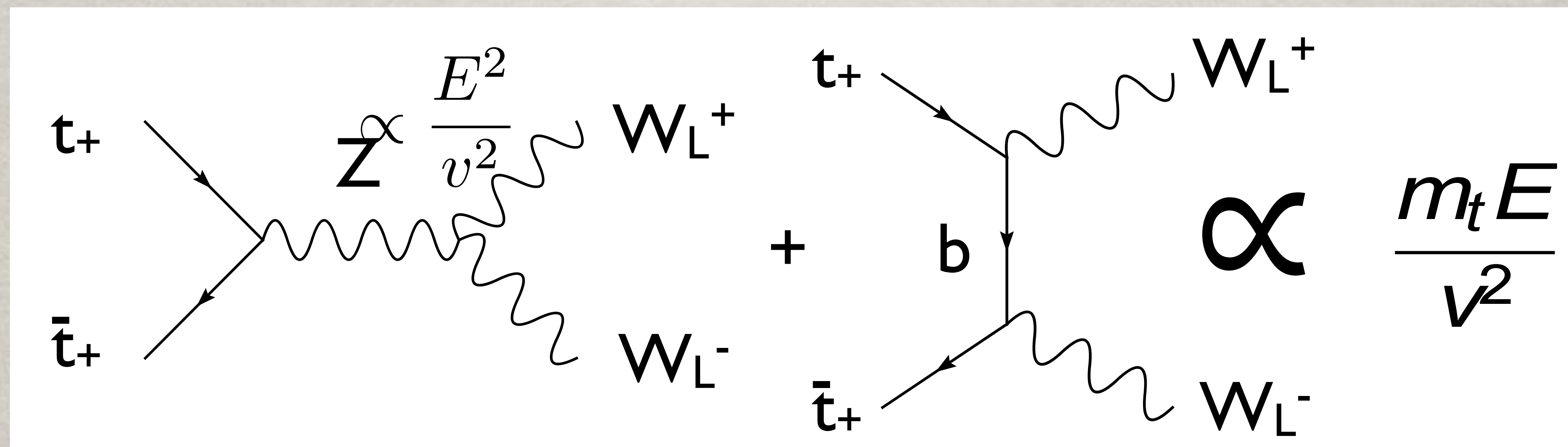
- "incomplete representation": $U = \exp\{i\omega^i \tau^i / v\}$
- nothing to say about the "Higgs boson"
  → The Higgs mechanism DOES NOT require a Higgs boson!

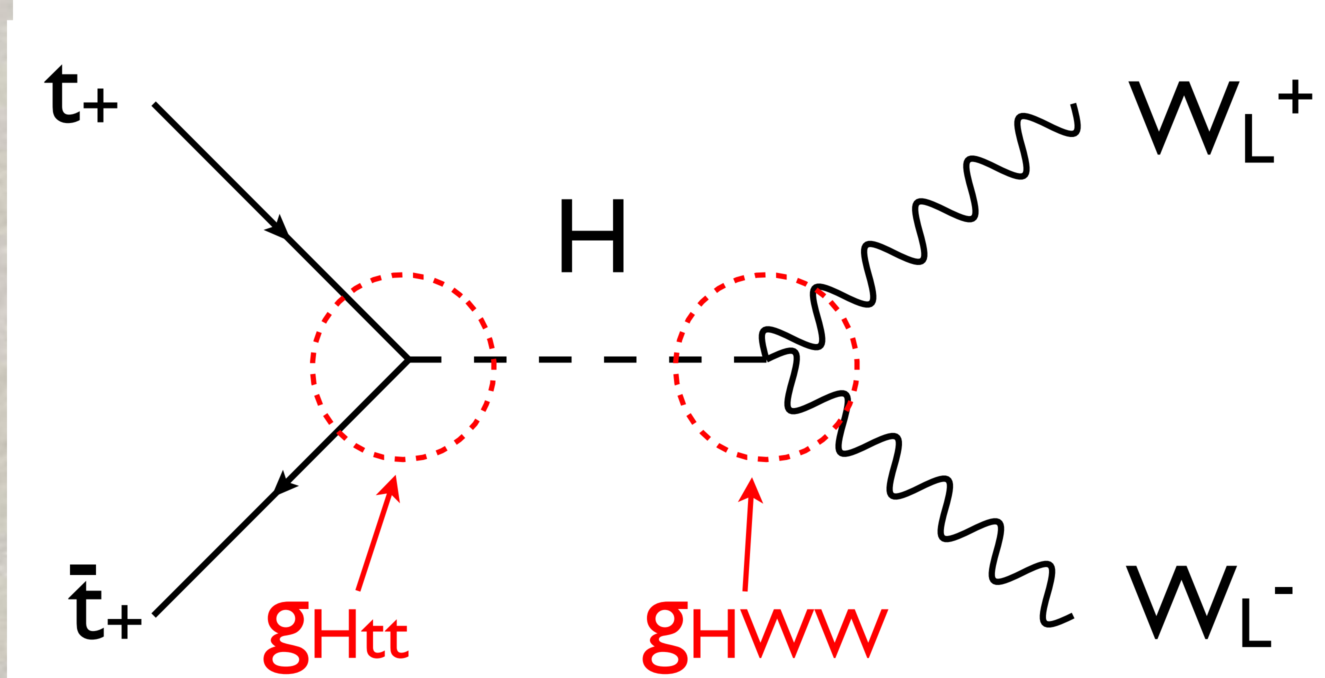
Lee, Quigg, Thacker (1977); Chanowitz, Gailard (1984); J. Chen, TH, B. Tweedie, arXiv:1611.00788; Coumo, L. Vecchi, A. Wulzer, arXiv:1911.12366

High-energy behavior: V_L & the role of H

$$\epsilon(k)^\mu_L = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W}$$

bad high-energy behavior!





Higgs boson save the day:

$$\propto \frac{m_t m_H}{v^2}$$

"UV complete"

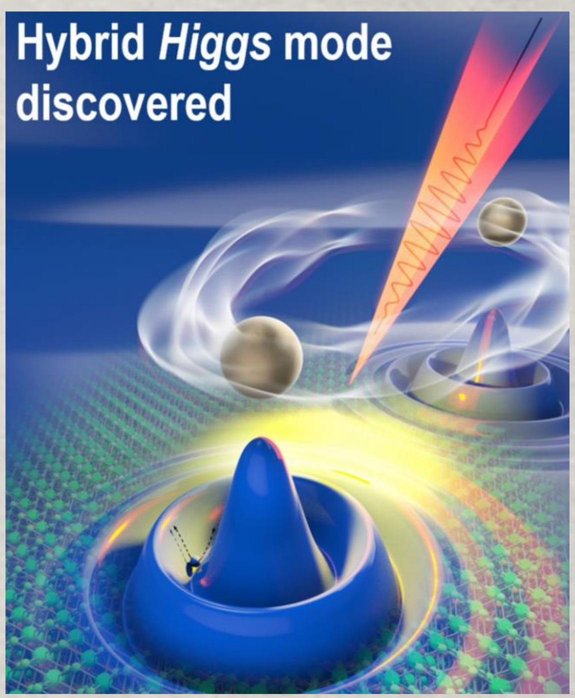
D. Dicus & V. Mathur (1973);
Lee, Quigg, Thacker (1977).

The Higgs Boson

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]



Laser @ 10^{12} Hz (2021, Ames Lab)

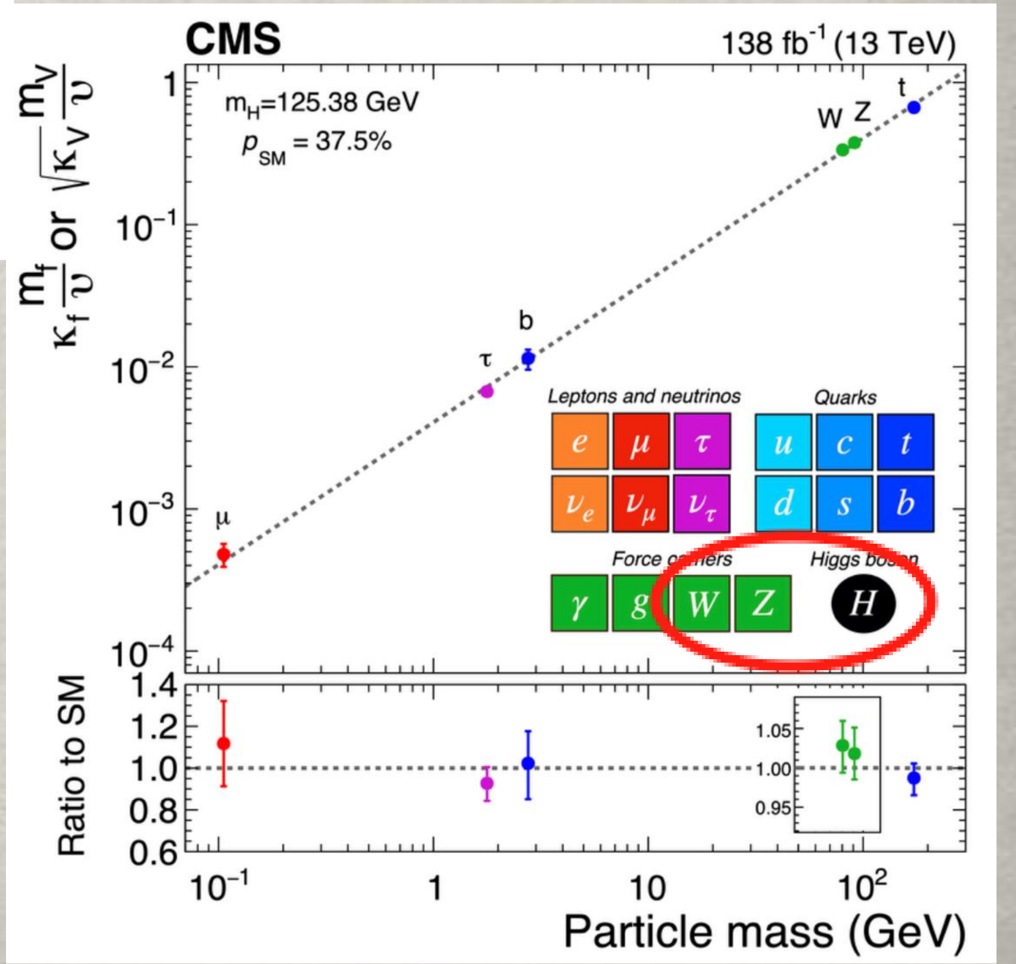
Weak interactions at very high energies: The role of the Higgs-boson mass

Benjamin W. Lee,* C. Quigg,† and H. B. Thacker
Fermi National Accelerator Laboratory,‡ Batavia, Illinois 60510
(Received 20 April 1977)

At energies very large compared with the Higgs-boson mass the trilinear term in the interaction Lagrangian (3.9) becomes ineffectual (contact terms dominate pole graphs at the tree level), so the theory displays an asymptotic O(4) symmetry. The fields w_1, w_2, z, and h form a four-vector in

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ z \\ h \end{pmatrix}$$

or $U = \exp\{i\omega^i\tau^i/v\}$



EW Symmetry Restoration (EWSR)

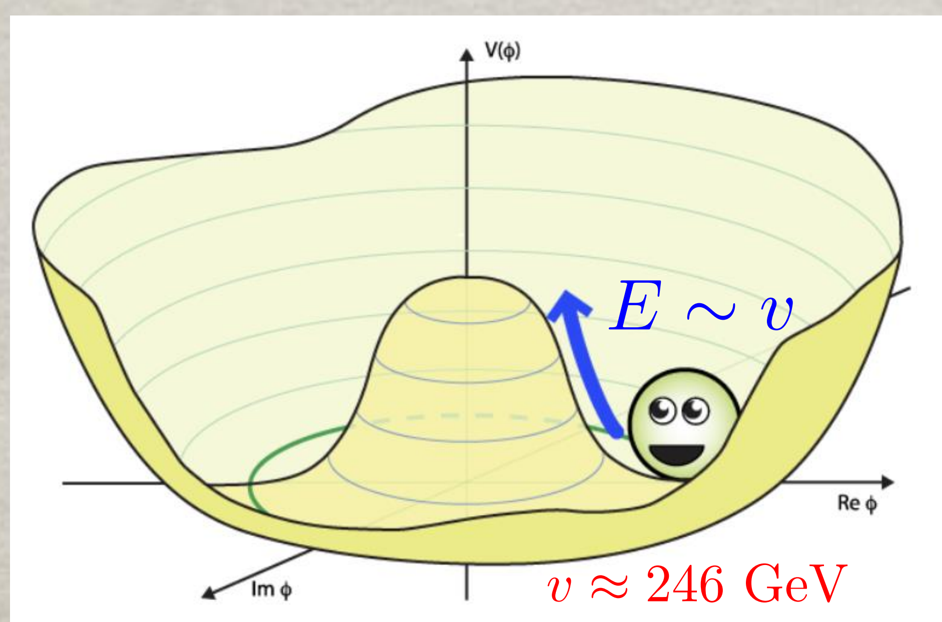
$$\frac{v}{E}: \quad \frac{v\ (250\ \text{GeV})}{10\ \text{TeV}} \approx \frac{\Lambda_{QCD}\ (300\ \text{MeV})}{10\ \text{GeV}} \qquad v/E,\ m_t/E,\ M_W/E \to 0!$$

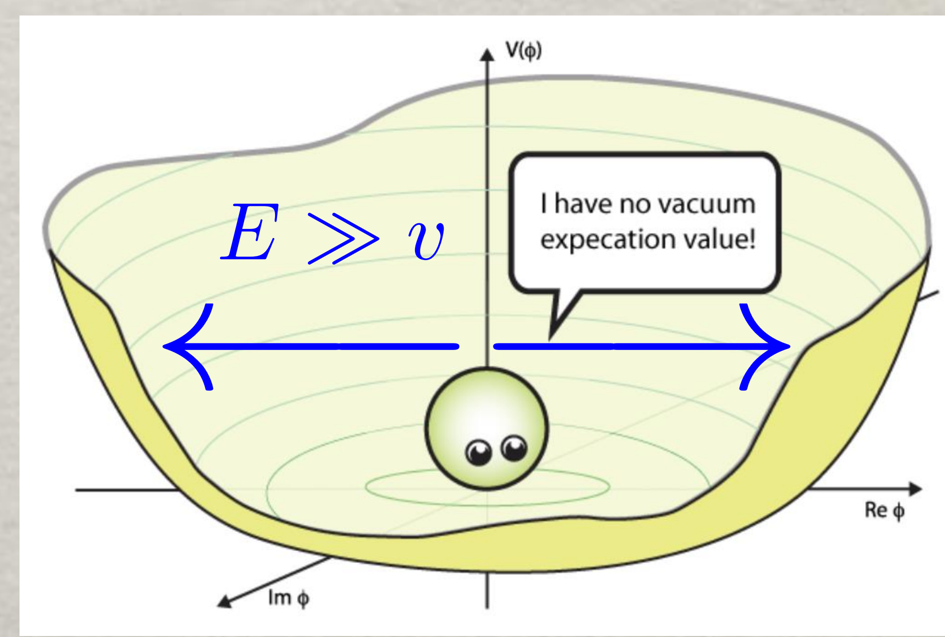
(i) the physics of the transverse gauge bosons $(W_T^\pm, Z_T, \gamma)$ and fermions is described by a massless theory in the unbroken phase;

(ii) the longitudinal gauge bosons $(W_L^\pm Z_L)$ are scalarized as Goldstone bosons $(\omega^\pm, \omega^0)$, and join the Higgs boson to restore the unbroken $O(4)$ symmetry $(\omega^\pm, \omega^0, H)$ in the Higgs sector.

parametrically measured by: $\delta = \dfrac{M_W}{2E_W}$

R. Capdevilla, TH, arXiv:2412.12336;





Radiation Amplitude Zeros (RAZs)

VOLUME 43, NUMBER 11 PHYSICAL REVIEW LETTERS 10 SEPTEMBER 1979

Magnetic Moment of Weak Bosons Produced in pp and $p\bar{p}$ Collisions

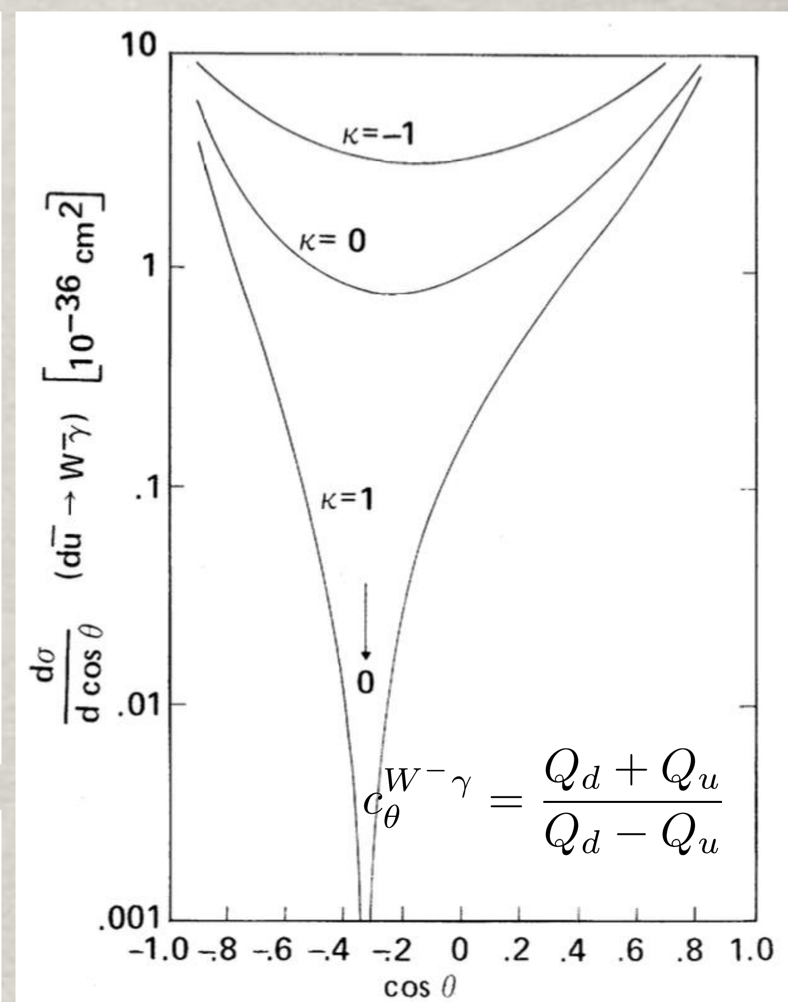
K. O. Mikaelian and M. A. Samuel
Physics Department, Oklahoma State University, Stillwater, Oklahoma 74074

and

D. Sahdev
Physics Department, Case Western Reserve University, Cleveland, Ohio 44106

(Received 5 June 1979)

We suggest that the reactions $pp \to W^{\pm}\gamma X$ and $p\bar{p} \to W^{\pm}\gamma X$ are good candidates for measuring the magnetic moment parameter κ in $\mu_W = (e/2M_W)(1+\kappa)$. The angular distribution of the W bosons in $p\bar{p} \to W^{\pm}\gamma X$ is particularly sensitive to this parameter. For the gauge-theory value of $\kappa = 1$, we have found a peculiar zero in $d\sigma(d\bar{u} \to W^-\gamma)/d\cos\theta$ at $\cos\theta = -\frac{1}{3}$, the location of this zero depending on the quark charge through $\cos\theta = -(1+2Q_d)$. A similar zero occurs in $d\sigma(u\bar{d} \to W^+\gamma)/d\cos\theta$. We can offer no explanation for this behavior.



VOLUME 72, NUMBER 25 PHYSICAL REVIEW LETTERS 20 JUNE 1994

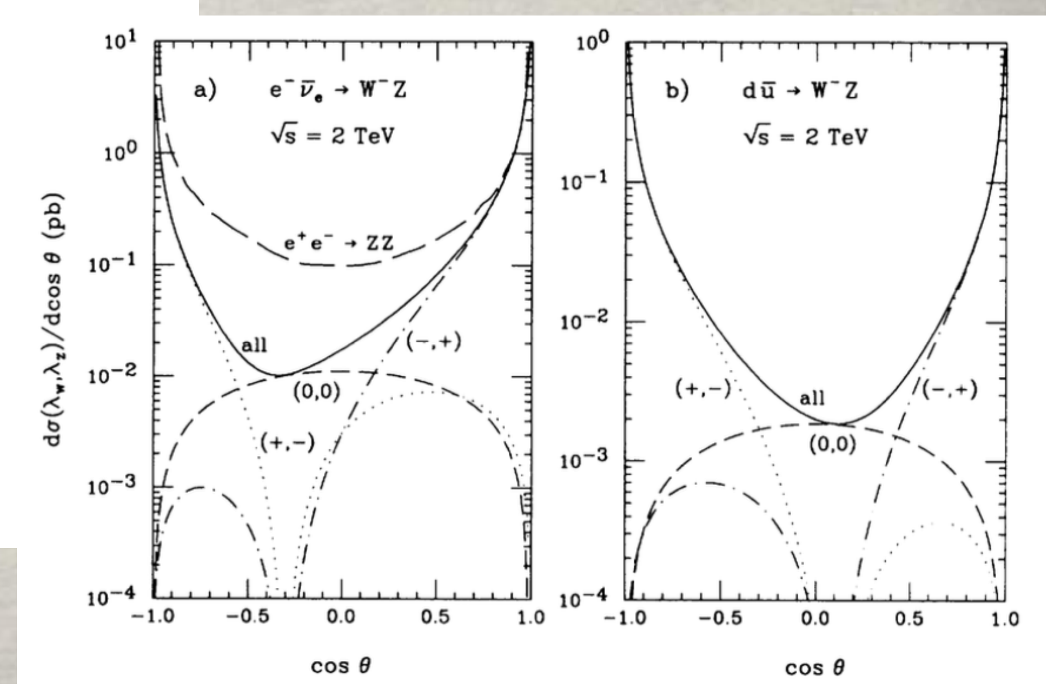
Amplitude Zeros in $W^{\pm}Z$ Production

U. Baur
Department of Physics, Florida State University, Tallahassee, Forida 32306

T. Han and J. Ohnemus
Department of Physics, University of California, Davis, California 95616

(Received 9 March 1994)

We demonstrate that the standard model amplitude for $f_1\bar{f}_2 \to W^{\pm}Z$ at the Born level exhibits an approximate zero located at $\cos\theta = (g_-^{f_1}+g_-^{f_2})/(g_-^{f_1}-g_-^{f_2})$ at high energies, where the $g_-^{f_i}$ $(i=1,2)$ are the left-handed couplings of the Z boson to fermions and θ is the center of mass scattering angle of the W boson. The approximate zero is the combined result of an exact zero in the dominant helicity amplitudes $\mathcal{M}(\pm,\mp)$ and strong gauge cancellations in the remaining amplitudes. For non-standard WWZ couplings these cancellations no longer occur and the approximate amplitude zero is eliminated.



$$c_\theta^{W^-Z_T} = \frac{g_-^d + g_-^u}{g_-^d - g_-^u}$$

Gauge / scalar separation:

R. Capdevilla, TH, arXiv:2412.12336

$$f_1\bar{f}_2 \to W^\pm\gamma,\quad f_1\bar{f}_2 \to W^\pm Z,\quad f_1\bar{f}_2 \to W^\pm H.$$

$$\mathcal{M}^{W\gamma}_{\pm\mp} \approx -\frac{geV_{12}}{\sqrt{2}}\frac{(\lambda_{\rm w}-c_\theta)}{s_\theta}\left[Q_{(1-2)}c_\theta - Q_{(1+2)}\right],$$
$$\mathcal{M}^{WZ}_{\pm\mp} \approx \frac{gg_zV_{12}}{\sqrt{2}}\frac{(\lambda_{\rm w}-c_\theta)}{s_\theta}\left[g_-^{(1-2)}c_\theta - g_-^{(1+2)}\right],$$
$$\mathcal{M}^{WZ}_{00} \approx -\frac{g_z^2V_{12}}{2\sqrt{2}}s_\theta g_-^{(1-2)} = \frac{g^2V_{12}}{2\sqrt{2}}s_\theta,$$
$$\mathcal{M}^{WH}_{0} \approx \frac{g^2V_{12}}{2\sqrt{2}}s_\theta,$$

- Gauge sector: Radiation Amplitude Zeros (RAZs)

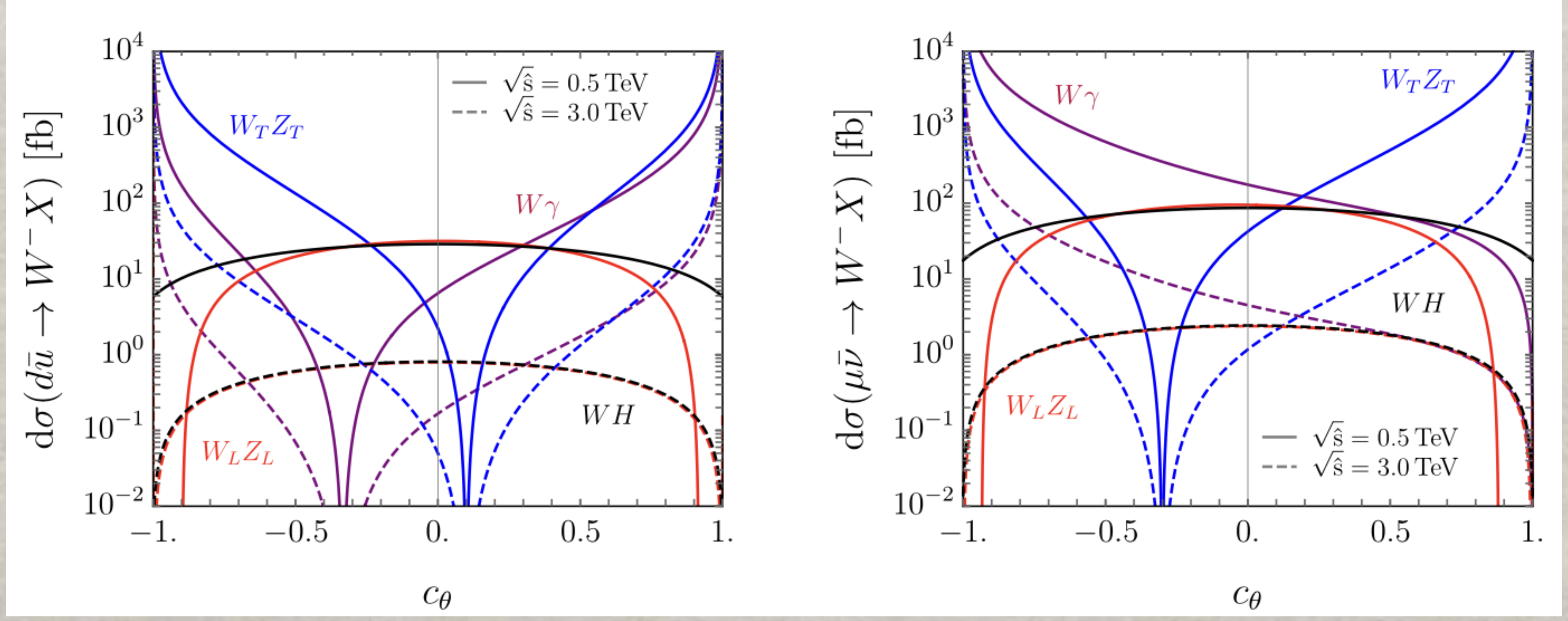
EM: $c_\theta^{W^-\gamma} = \dfrac{Q_d+Q_u}{Q_d-Q_u}$. EW (transverse): $c_\theta^{W^-Z_T} = \dfrac{g_-^d+g_-^u}{g_-^d-g_-^u}$

Mikaelian, Samual (1979)

$$c_{\theta_0} = \begin{cases} -1/3\ (\approx 0.1) & \text{for } d\bar{u}\to W_T^-\gamma\ (W_T^-Z_T), \\ 1\ (\approx -0.3) & \text{for } \ell^-\bar{\nu}\to W_T^-\gamma\ (W_T^-Z_T), \end{cases}$$

U. Baur, TH, JO, (1994)

- Higgs scalar sector: $\mathcal{M}^{W_LZ_L}(\delta \ll 1) \approx \mathcal{M}^{W_Lh}(\delta \ll 1)$



Test EWSR @ LHC / muon Collider

R. Capdevilla, TH, arXiv:2412.12336; Huang, Lewis, Lane, Liu, arXiv:2009.09429

Massless gauge sector & Higgs sector:

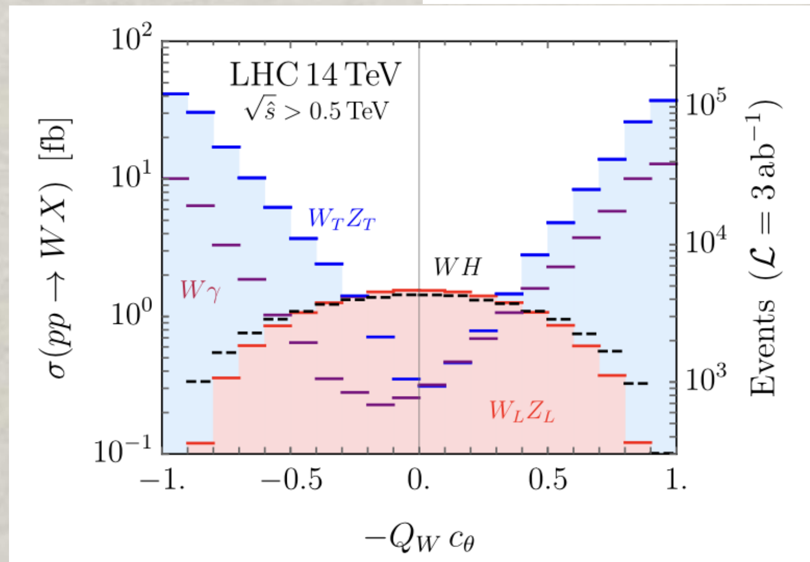
$$r_{Z\gamma} = \frac{\sigma(WZ)}{\sigma(W\gamma)}\,, \quad r_{ZH} = \frac{\sigma(WZ)}{\sigma(WH)}$$

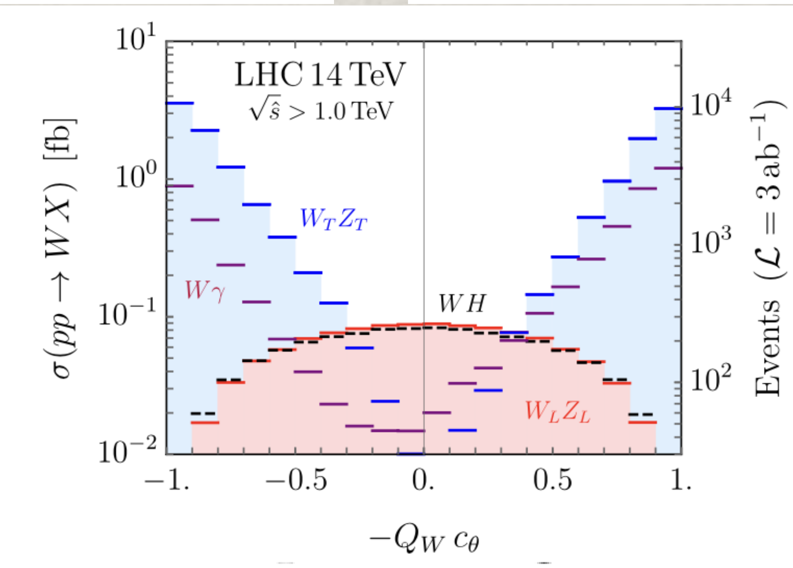
For $\boldsymbol{\delta} = M_W/2E \ll 1$:

$$\frac{\sigma(W_T Z_T)}{\sigma(W_T\gamma)} \approx \frac{g_z^2}{e^2}\,\frac{(g_-^{f_1})^2+(g_-^{f_2})^2}{Q_1^2+Q_2^2}$$

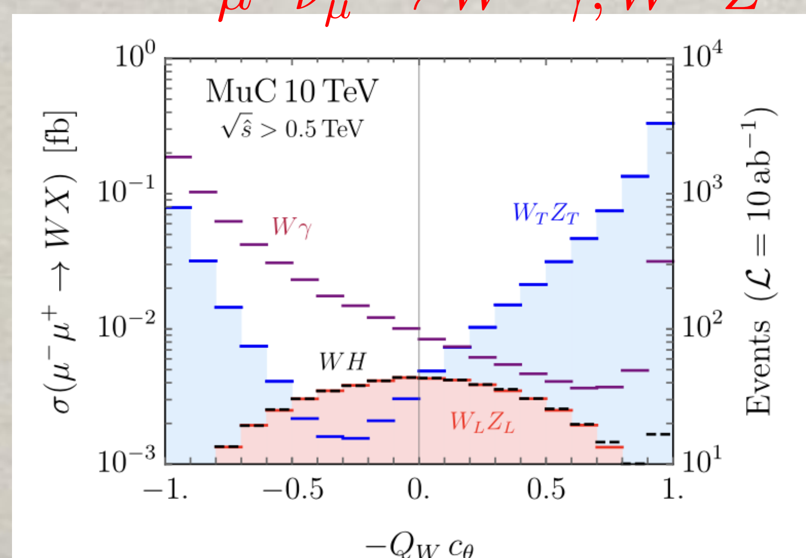
$$\sigma(W_L^\pm Z_L) \sim \sigma(W_L^\pm H),$$

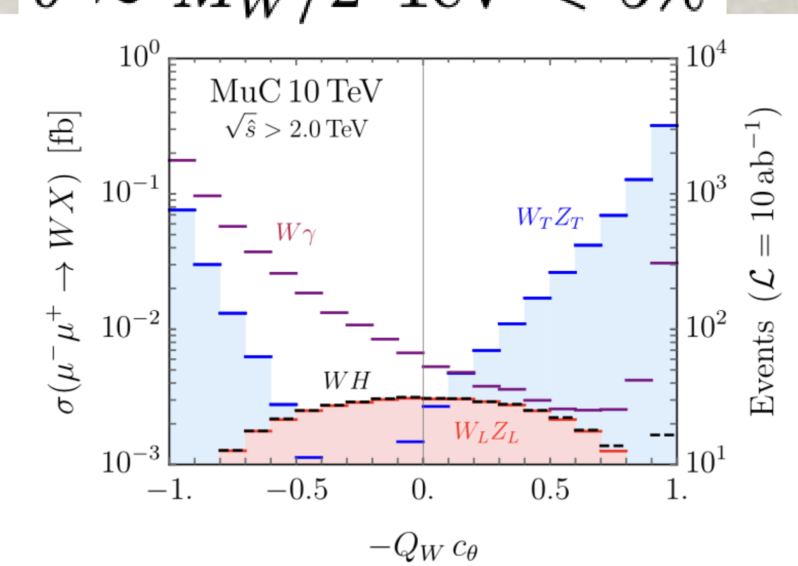
$$\text{or } \sigma(\omega^\pm\omega^0) \sim \sigma(\omega^\pm H)$$





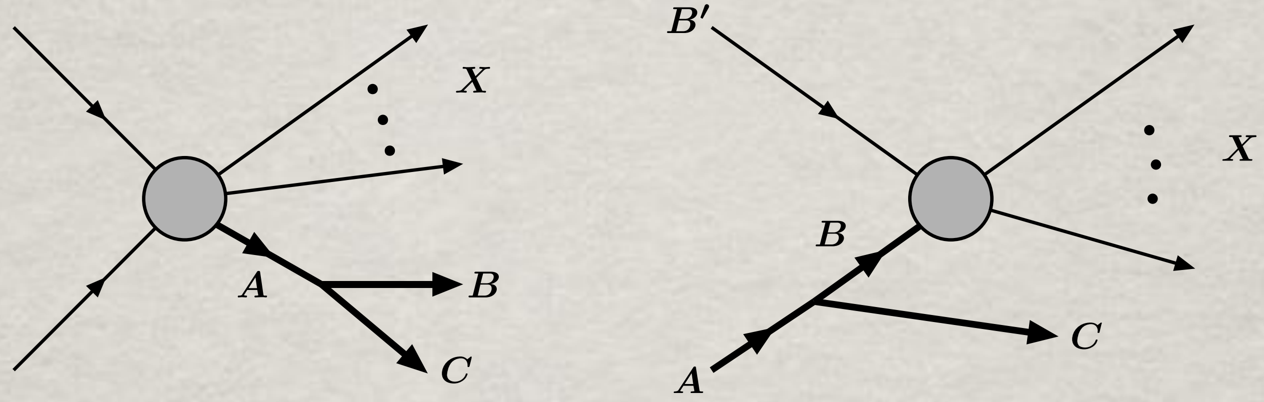
$\mu^\pm\nu_\mu \to W^\pm\gamma, W^\pm Z$ $\quad \delta \approx M_W/2\text{ TeV} < 5\%$





Other Aspects of EWSR

Splitting: the dominant phenomena

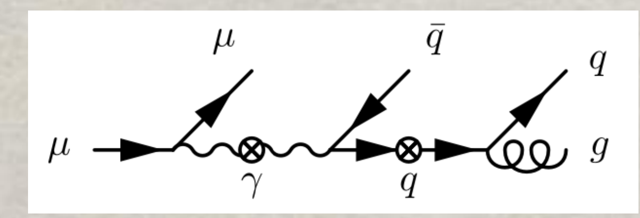


$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A\to B+C}$$

For the factorization to be valid:

- Power corrections suppressed: $M_W^2/Q^2 \ll 1$
- Log corrections (RGE) large: $\alpha_2 \ln^2(Q^2/M_W^2) \sim \mathcal{O}(1)$

EW "partons" dynamically generated



$$\frac{\mathrm{d} f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$

EW shower/jets:

$$\begin{aligned} W^* &\to q\bar{q}g \;..., \;\ell^\pm\nu\gamma \;... \\ t^* &\to b\bar{b}W^*,\; tZ^*,\; th^* \;... \\ \nu^* &\to \ell^\pm W^* \;... \qquad \to \text{EW jets} \end{aligned}$$

What do we learn in testing EWSR?

"endlessly confirm the correctness of SM" ?!

- Carlo Rubia

SMEFT BSM vs. HEFT BSM

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi^+ \\ v + H + i\phi^0 \end{pmatrix},$$

$$\mathcal{L}_{\mathrm{SMEFT},\mu\phi} = -\sum_{n=1}^{\infty} \frac{c_{\varphi}^{(2n+4)}}{\Lambda^{2n}} (\varphi^\dagger \varphi)^{n+2}$$

$$U = e^{2i\phi^a T_a/v} \quad \text{with} \quad \phi^a T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix},$$

$$\mathcal{L}_{Uh} = \frac{v^2}{4} \mathrm{tr}[D_\mu U^\dagger D^\mu U] F_U(H) + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H)$$

weakly coupled (SUSY) strongly coupled (composite)

new scale ~ ***Λ*** nearby scale ~ **4π υ**

At the LHC: Higgs coupling SM-like ~ 10%

(light) Fermion Yukawa's wide open:

$$-\sum_{n=1}^{\infty} \frac{c_{\ell\varphi}^{(2n+4)}}{\Lambda^{2n}} (\varphi^\dagger \varphi)^n (\bar{\ell}_L \varphi \mu_R + \mathrm{h.c.})$$

$$-\frac{v}{\sqrt{2}} \left[\bar{\ell}_L Y_\ell(H) U P_- \ell_R + \mathrm{h.c.}\right]$$

$$Y_\ell(H) = \frac{\sqrt{2} m_\mu}{v} + \sum_{k\geq 1} y_{\ell,k} \left(\frac{H}{v}\right)^k$$

E. Celada, TH et al., arXiv:2312.13082

A Few Remarks

- Goldstone boson Equivalence Theorem:
  → process-dependent & need to be quantified:
  parametrically measured by
  Goldstone couplings g_2 , y_t … and y_f … >> $\delta = M_W/2E$!

GET Violation in fermion splitting f -> f V_L:

$$\frac{v^2}{k_T^2}\ \frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$$

- Analogous to QCD:

Theoretical treatment in symmetric phase *q, g*
→ observation in broken phase massive hadrons.
Theoretical treatment in symmetric phase $W^{0,1,2}$, *B*
→ observation in broken phase $W^{\pm}$, *Z*, γ; *(t, b) (e,* ν*)* …

- Perhaps, only at the high-T environment (e.g., in the early universe) would we experience the (real) EW symmetry restoration.

Further Remarks

In approaching EWSR, how much the SM UV completion tells us?

- QED is UV complete, but doesn't go beyond O(GeV):

  e.g. $(g\text{-}2)_e$ versus $(g\text{-}2)_\mu$
- QCD is UV complete, could be dynamically extrapolated to an exponentially high scale Q:

$$\alpha_s(Q^2) \approx 1/\ln(Q^2/\Lambda^2_{QCD}) \Rightarrow \Lambda_{QCD} \approx Q\exp(-1/2\alpha_s)$$

  but new physics comes in at $v \sim 250\ \mathrm{GeV}$
- The SM with the Higgs IS UV complete,

  --- to be tested by EWSR

but what confidence do we have to extrapolate it to $O(M_{PL})$?

→UV completion needs NOT to be a completion!

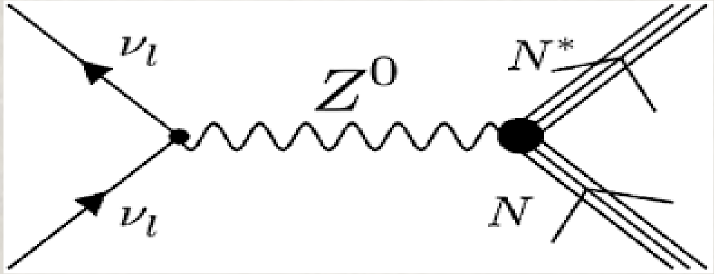
i.e. **Go for BSM & beyond EWSR!**

Back up slides

Electroweak Gauge Theory

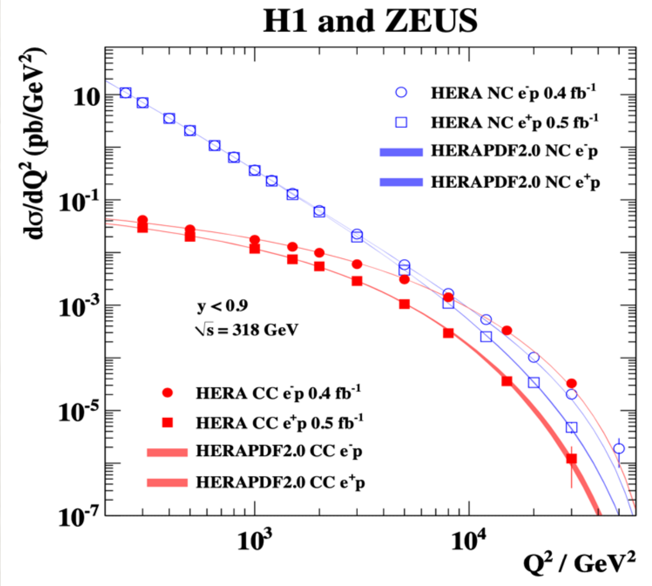
- In our daily life, we live in a deeply broken phase, no sign of EW gauge symmetry! The "weak" force:

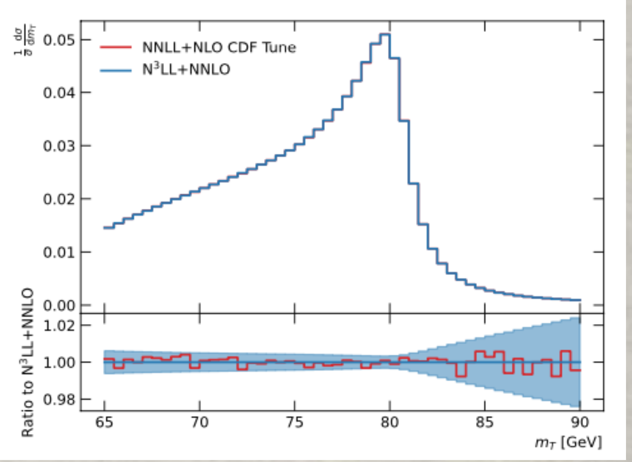
$$\mathcal{M}(n \to p^+ e^- \bar\nu) \sim G_F\ \bar p \mathcal{O}^\mu n\ \bar e \mathcal{O}_\mu \nu, \quad G_F \approx 1/(300\ \mathrm{GeV})^2$$



- More discoveries:
- Weak neutral current (1973):
- EW unification: HERA
- $W^\pm$,Z^0 discovery: SppS (1983)
  LEP, Tevatron, LHC …
- EW Gauge Theory at Work:
  - Non-Abelian gauge structure
  - Gauge coupling universality

  EW precision ~ 10^{-3}





$\Delta M_W/M_W \sim 10^{-4}$

- @100 GeV, still a broken/superconducting phase:

London penetration depth (the Meissner effect):

$$\kappa \sim M_W^{-1} \approx 10^{-9}\ \mathrm{nm}, \quad \text{in } 10^{-24}\mathrm{s}.$$

$$M^2W^+_\mu W^-_\mu - \tfrac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac12\partial_\mu A_\nu\partial_\mu A_\nu - \tfrac12\partial_\mu H\partial_\mu H -$$
$$\tfrac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \tfrac12\partial_\mu\phi^0\partial_\mu\phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} +$$
$$\tfrac{2M}{g}H + \tfrac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$\tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) +$$
$$m_u^\lambda(\bar u_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa] + \tfrac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1-$$

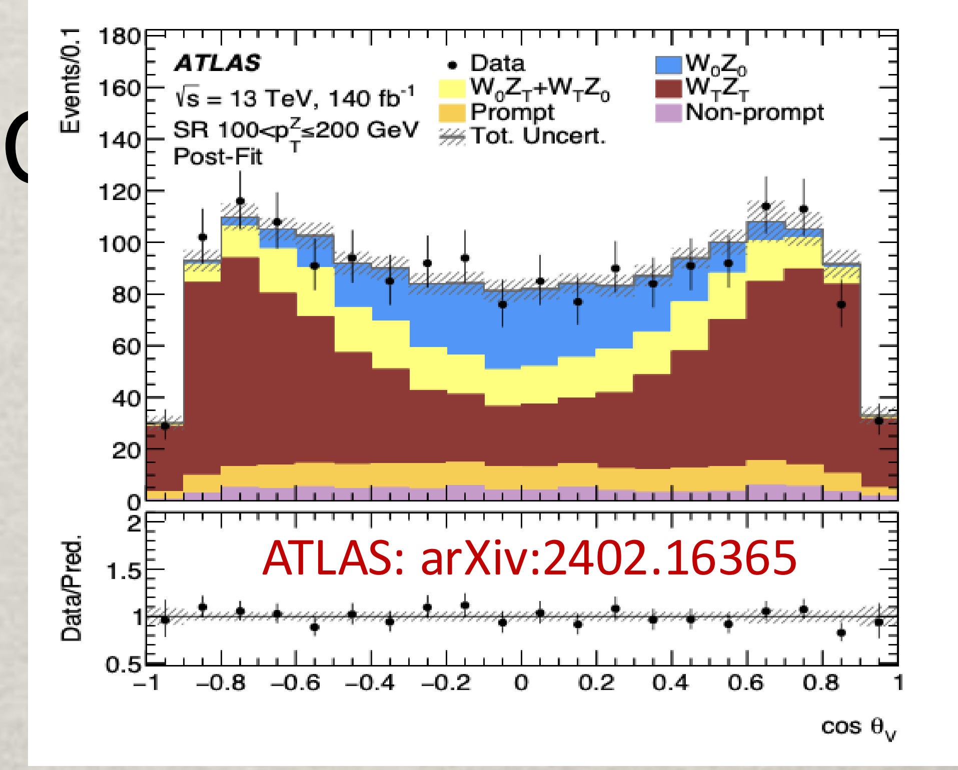
Longitudinal Gauge Boson & its Mass

- Observation of W_L , Z_L?
  first from the top decay t → W^+b:

$$\frac{d\Gamma(t \to bW_{T,L})}{d\cos\theta^*} \sim 1 \pm \cos^2\theta^*$$

$$\Gamma_{W_L}/\Gamma_{W_T} \approx m_t^2/2M_W^2 \approx 2.$$

- $W_L^{\pm}$ Z_L at the LHC in high energies:



$W^{\pm}W^{\pm}$ → $W^{\pm}W^{\pm}$ observed @ LHC
but only for W_TW_T

ATLAS: arXiv:1906.03203, CMS: arXiv:2009.09429

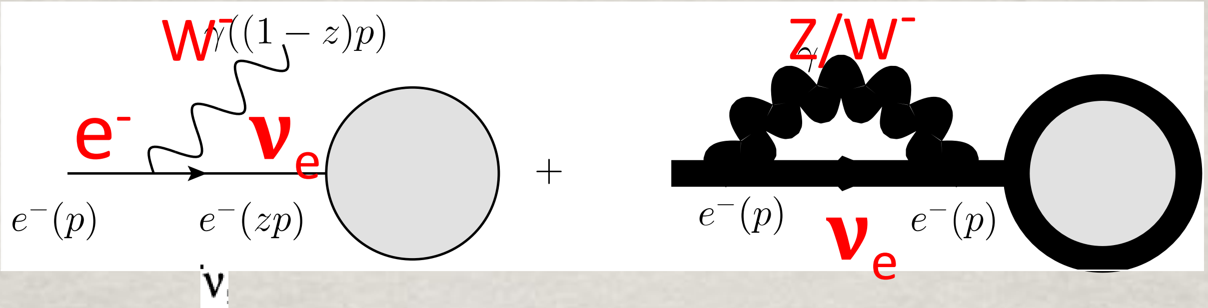
V_L wavefunction for a massive vector :

$$\epsilon_L^\mu(p) = \frac{E}{M}(\beta, \hat{p}) = \frac{p^\mu}{M} - \frac{1}{1+\beta}\frac{M}{E}n^\mu$$

trivial "scalarization"
(for any vector state)

symmetry breaking
residual

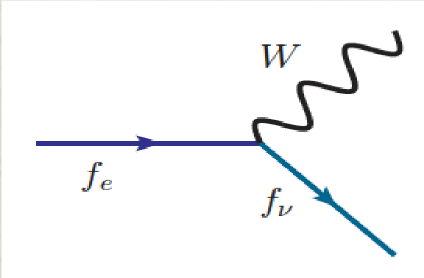
EW Evolution beyond Leading Log



Incomplete cancellation for non-inclusive process in SU(2):
SU(2) "color" (e,ν) distinguishable, unlike QCD!

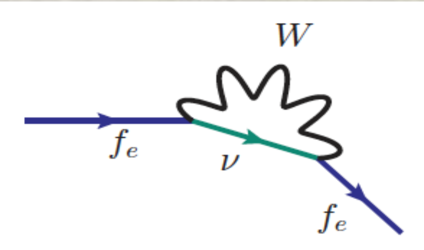
$$d\mathcal{P}_{\nu\leftarrow e} = d\mathcal{P}_{e\leftarrow\nu} \sim \frac{(T^{\pm})^2}{1-z} = \frac{(1/\sqrt{2})^2}{1-z}$$

$$d\mathcal{P}_{e\leftarrow e} = d\mathcal{P}_{\nu\leftarrow\nu} \sim \frac{(T^3)^2}{1-z} = \frac{(1/2)^2}{1-z}$$



$$dV_{\nu\leftarrow e} = dV_{e\leftarrow\nu} = 0$$

$$dV_{e\leftarrow e} = dV_{\nu\leftarrow\nu} \sim -\int dz \frac{C_2(\mathbf{2})}{1-z} = -\int dz \frac{3/4}{1-z}$$



→ Bloch-Nordsieck theorem violation!

→ "Factorization theorem":

- sufficiently inclusive **processes,**
- and infrared safe-**observables**

Not there yet! Much more to learn @ HE !