4D TRACK RECONSTRUCTION USING QUANTUM COMPUTING

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4th Annual Meeting of the International Muon Collider Collaboration and the 2nd MuCol Annual Meeting,
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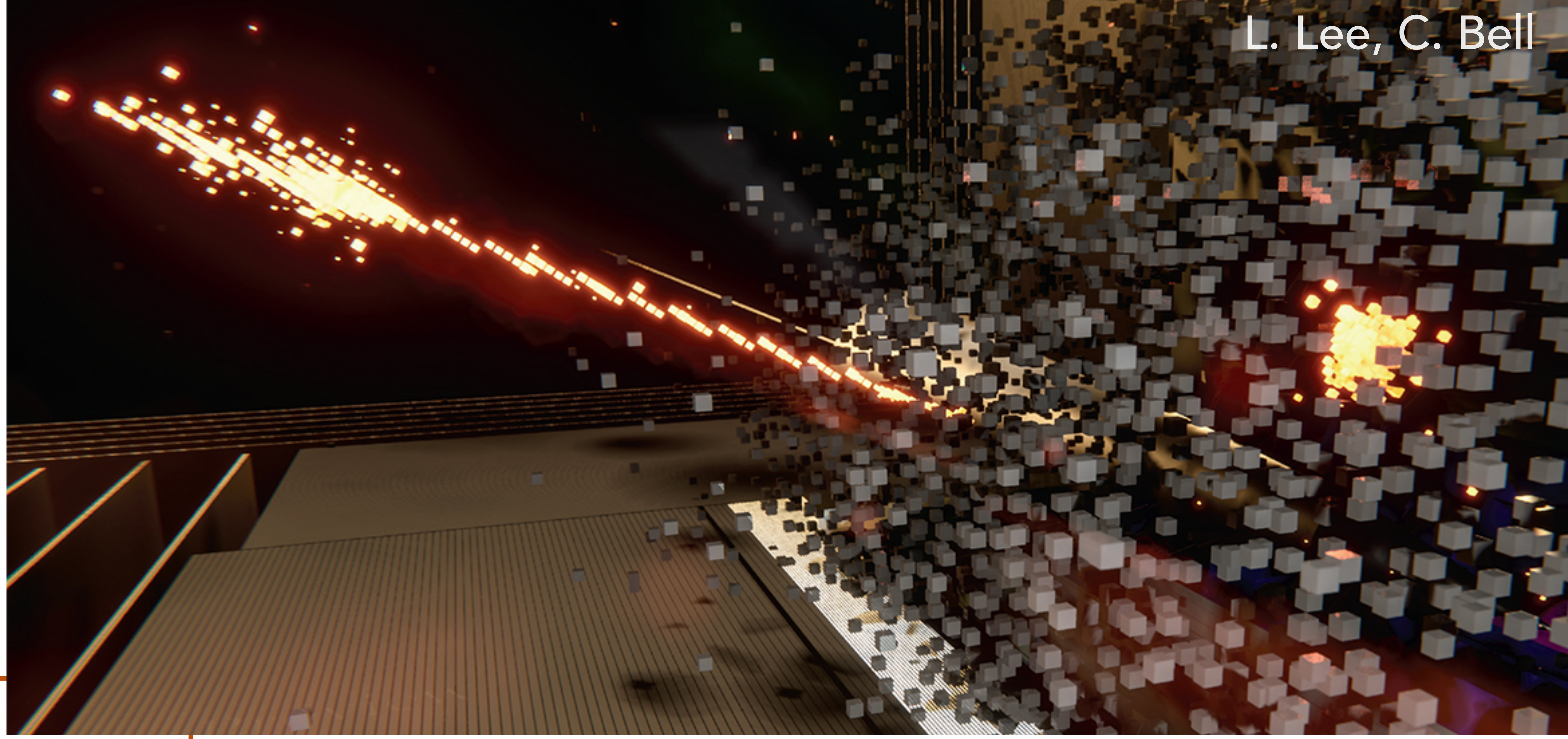
INTRODUCTION



- Reminder: beam-induced background at muon colliders.
  - Tracker with timing capability important.

Detector Reference	Hit Density [mm^{-2}] MCD	ATLAS ITk	ALICE ITS3
Pixel Layer 0	3.68	0.643	0.85
Pixel Layer 1	0.51	0.022	0.51

arXiv:2303.08533

- Computing landscape at the timescale of muon collider.
- Building on our previous work using quantum algorithm for tracking in LUXE experiment: arXiv:2304.01690
  - Extend to 4D tracking including timing information in muon collider.

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Quantum Algorithms for Charged Particle Track Reconstruction in the LUXE Experiment

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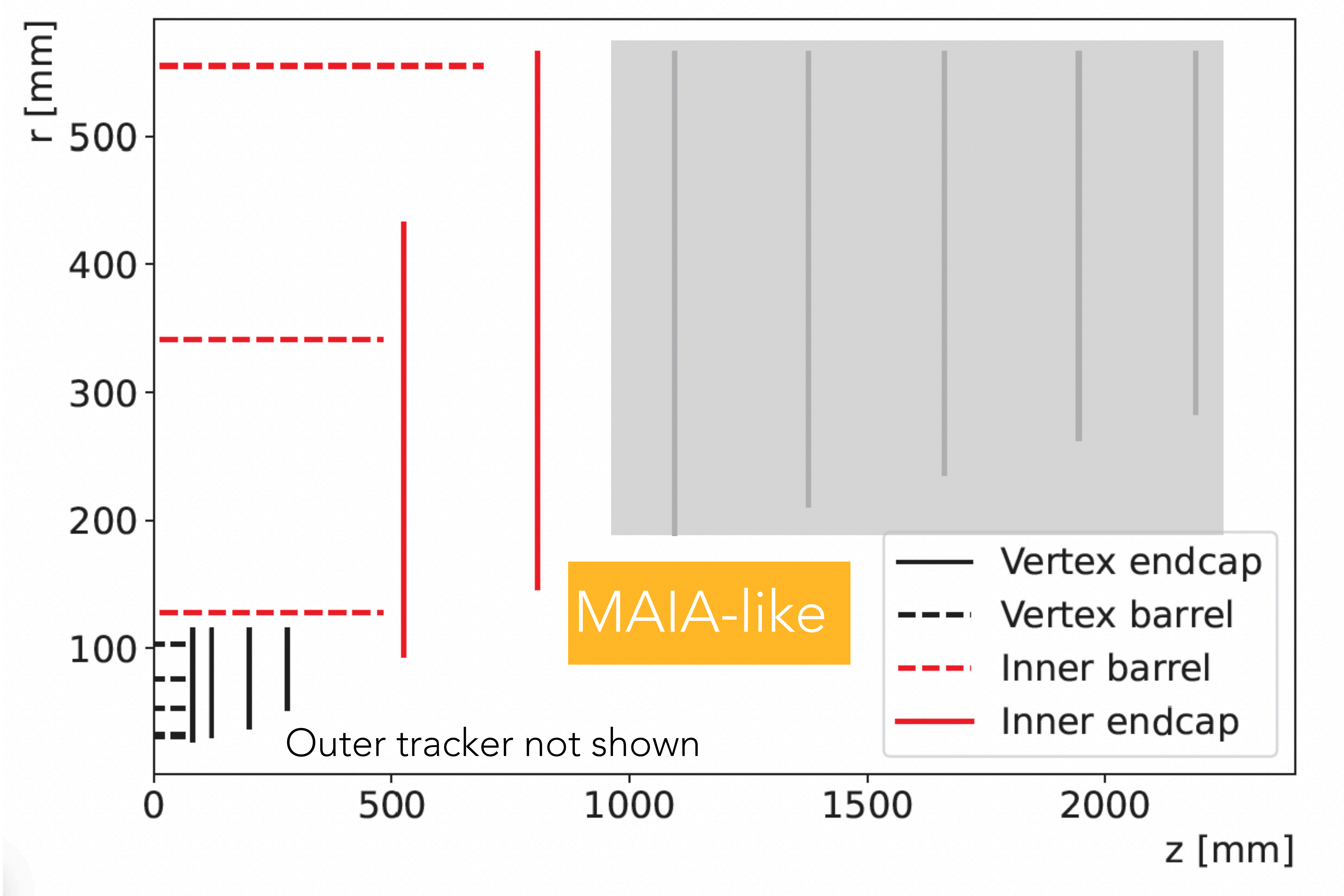
SAMPLE

- Signal generated with particle gun, origin at $(0,\ 0,\ \mathcal{N}(\mu = 0,\ \sigma = 1.5\text{mm}))$
  - **Single muon** with $p_{\mathrm{T}} \in [2.5,10]\ \text{GeV}$, or $\beta > 0.99$.
  - 1.1 TeV **slow-moving** long-lived (stable in the tracker) charged particle with kinetic energies $\in [250,3900]\ \text{GeV}$ or $0.22 < \beta < 0.97$.
    - An example of slow particles that could be missed by a tight time window cut.
- Overlaid with beam-induced background for 10 TeV collider from FLUKA.

TRACKING REGION

- Consider only vertex detector, inner barrel and first two layers of inner endcap.



	Vertex Detector	Inner Tracker	Outer Tracker
Sensor type	pixels	macro-pixels	micro-strips
Barrel Layers	4	3	3
Endcap Layers (per side)	4	~~7~~ 2	4
Cell Size	25 µm × 25 µm	50 µm × 1 mm	50 µm × 10 mm
Sensor Thickness	50 µm	100 µm	100 µm
Time Resolution	30 ps	60 ps	60 ps
Spatial Resolution	5 µm × 5 µm	7 µm × 90 µm	7 µm × 90 µm

arXiv:2502.00181

METHOD: QUBO

Quadratic
Unconstrained
Binary
Optimisation

- Triplet classification.
  - Find the optimal set of triplets that form valid tracks by minimising a QUBO, given by the states of T_i, T_j.

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j, \quad T_i, T_j \in \{0,1\}$$

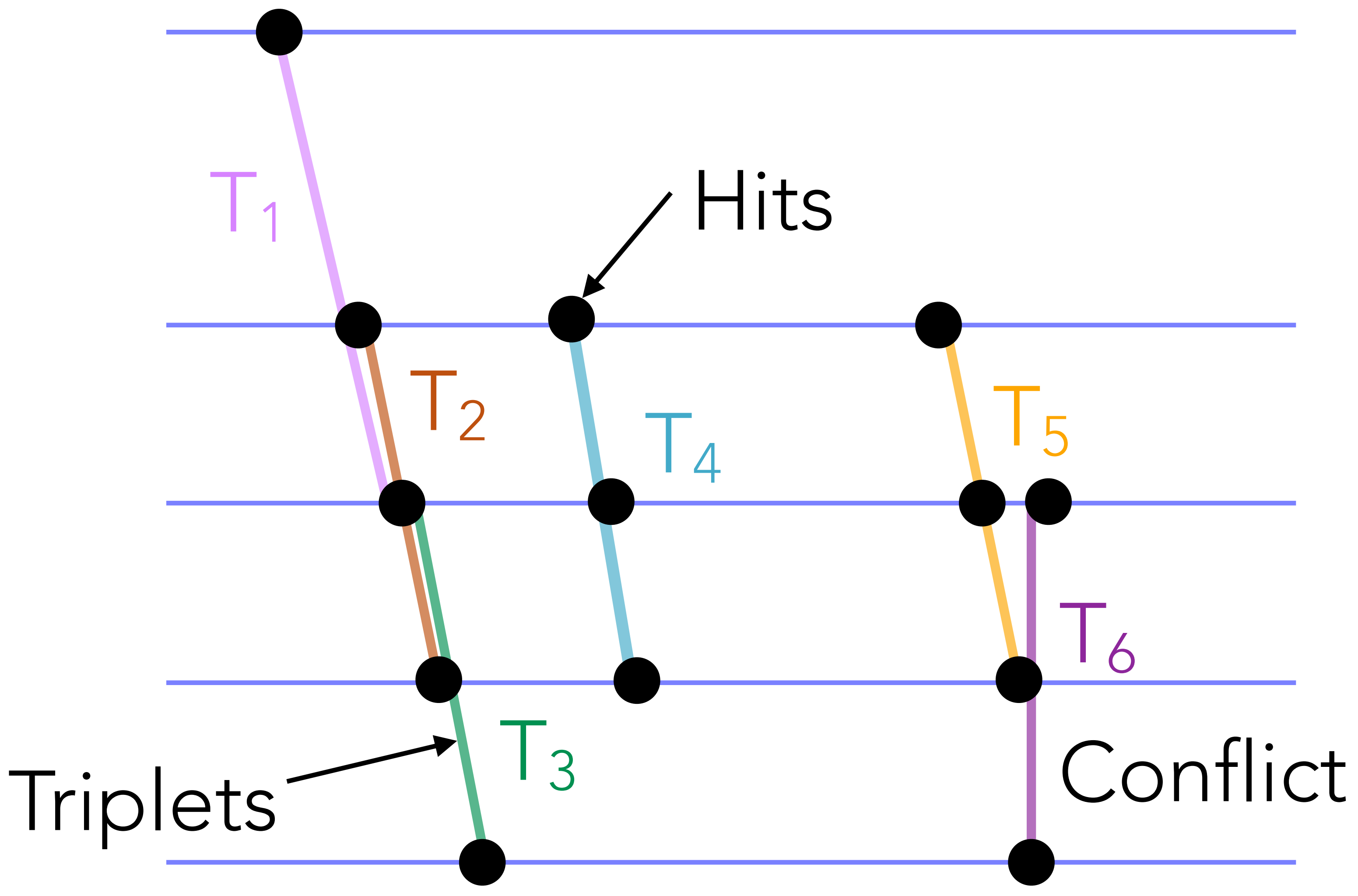
$$O(a,b,T) = \begin{pmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{pmatrix}^T \begin{pmatrix} a_0 & 0 & \cdots & 0 \\ b_{10} & a_1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ b_{n0} & b_{n1} & \cdots & a_n \end{pmatrix} \begin{pmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{pmatrix}$$

Weighting triplet T_i with quality a_i

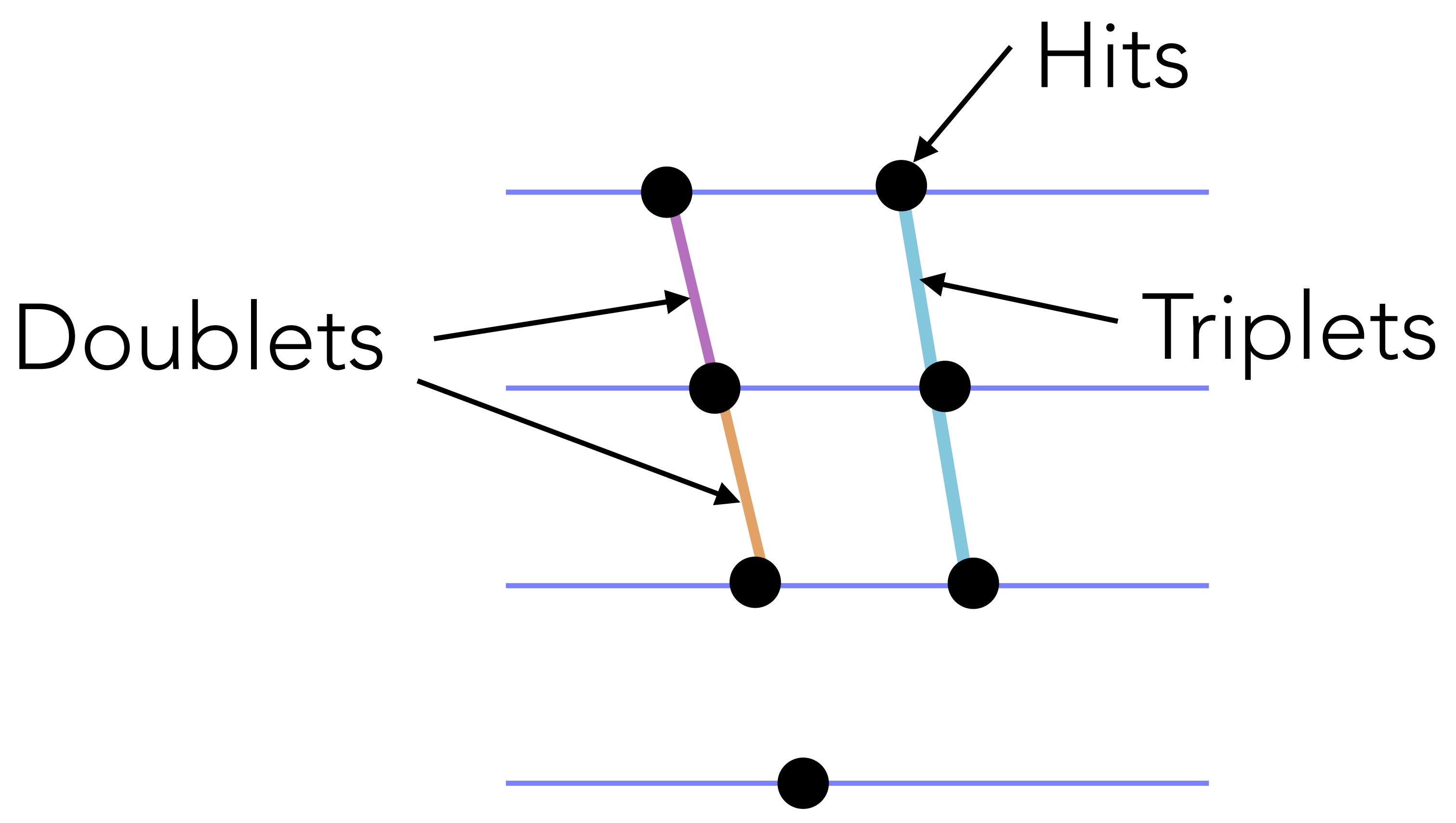
Compatibility b_{ij} between two triplets

$$b_{ij} = \begin{cases} -S(Ti, Tj), & \text{if } (T_i, T_j) \text{ form a quadruplet}, \\ \zeta & \text{if } (T_i, T_j) \text{ are in conflict}, \\ 0 & \text{otherwise.} \end{cases}$$

Find T_i, T_j that minimises QUBO!



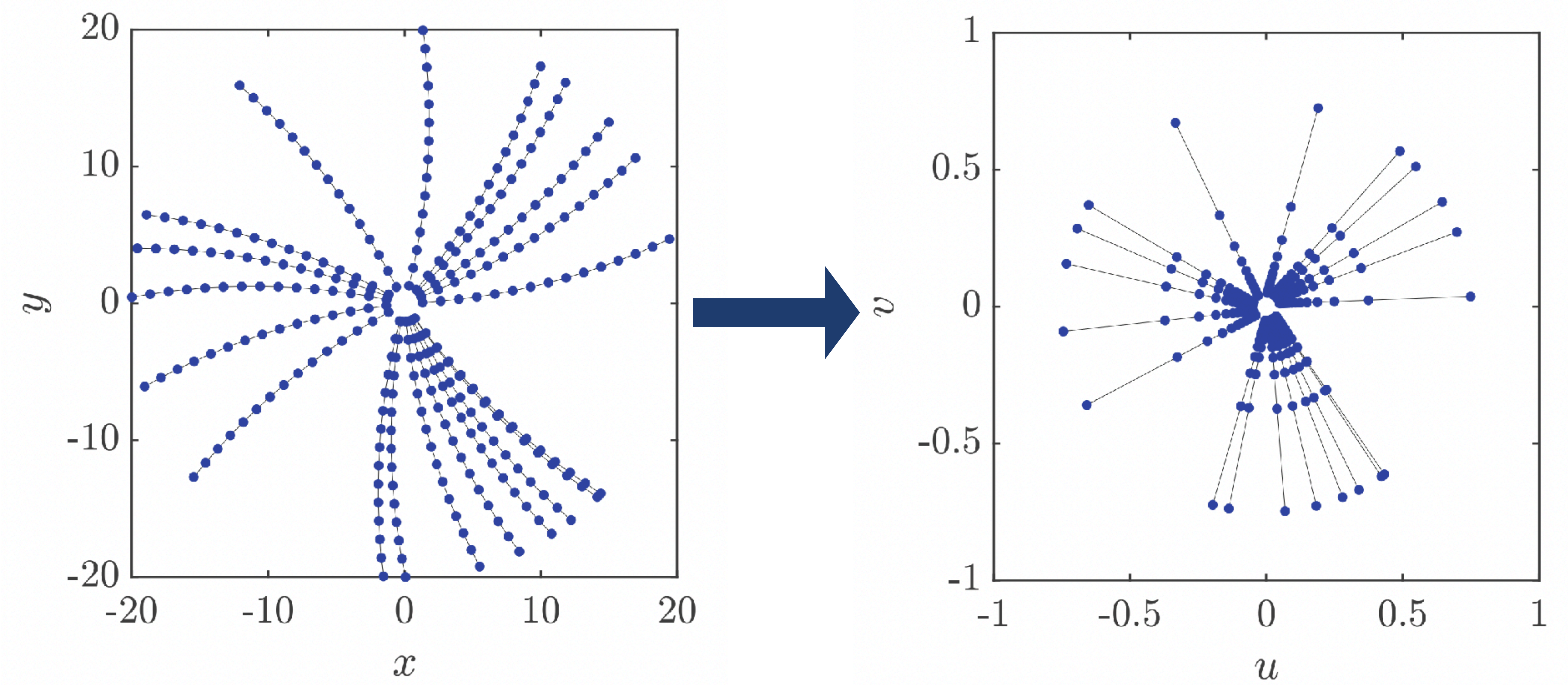
TRIPLET PRE-SELECTION



- Form doublets then triplets. Consider only doublets/triplets that are signal-like.

- Conformal mapping

$$u = \frac{x}{x^2 + y^2}, v = \frac{y}{x^2 + y^2}$$

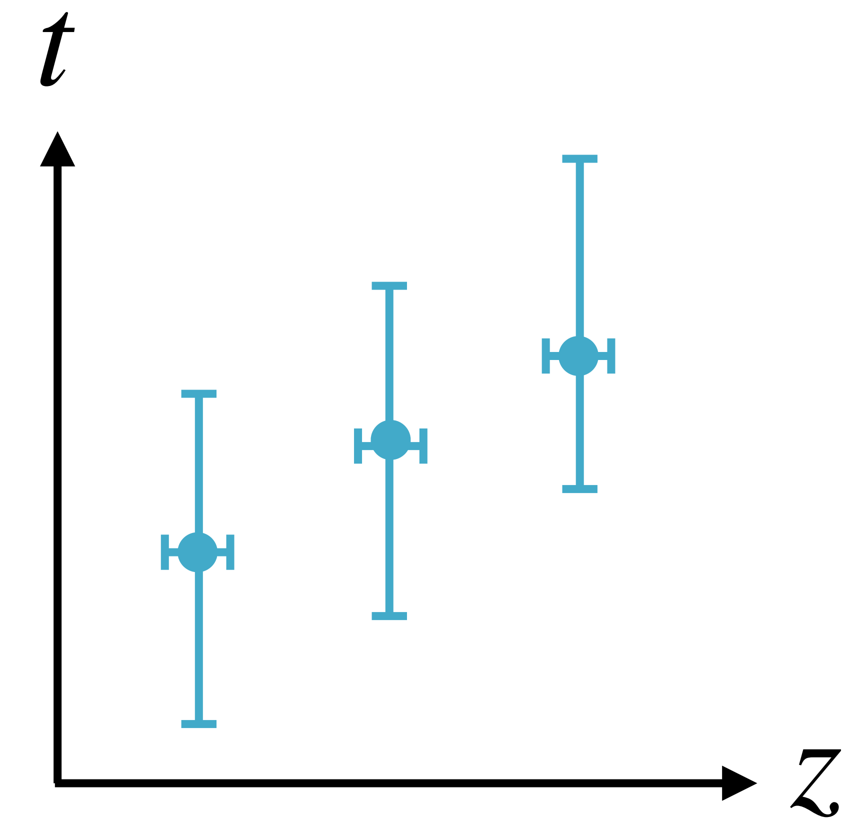


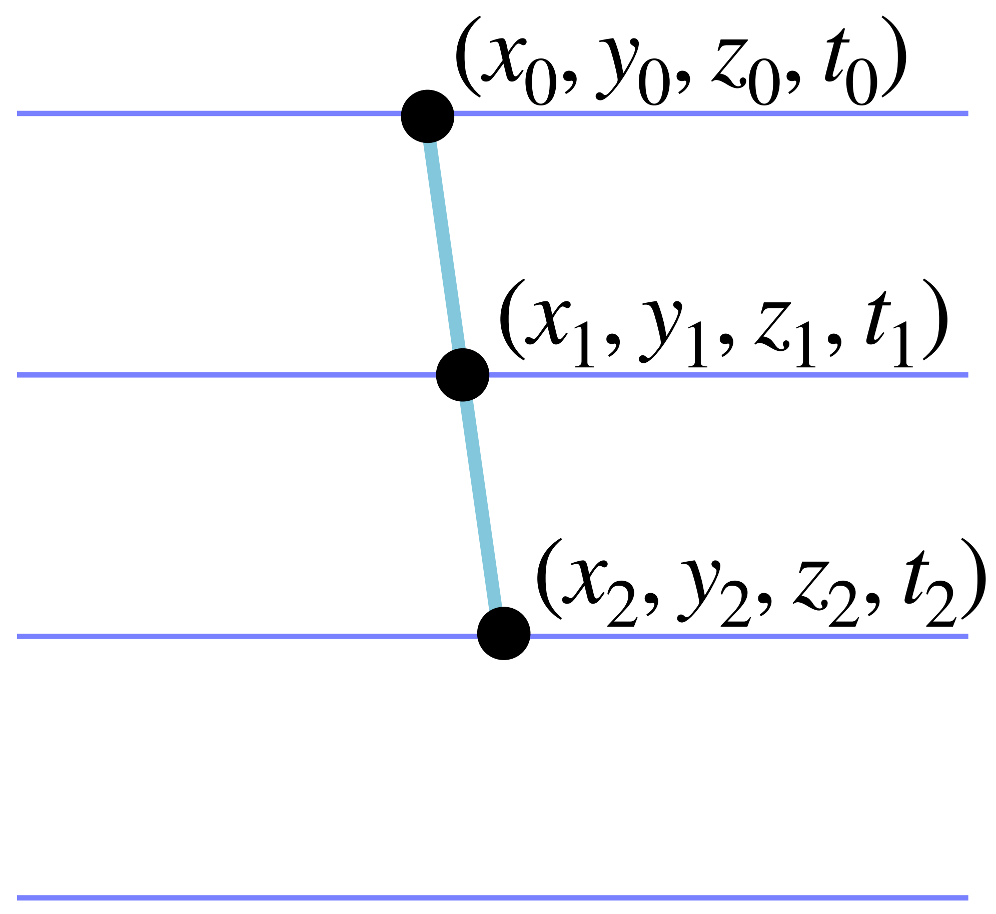
https://doi.org/10.1007/978-3-030-65771-0

- Doublets/triplets pre-selection: straight segments in conformal space and in $r - z$ compatible with originating from PV.

- **Only spatial information!**

QUBO COEFFICIENTS



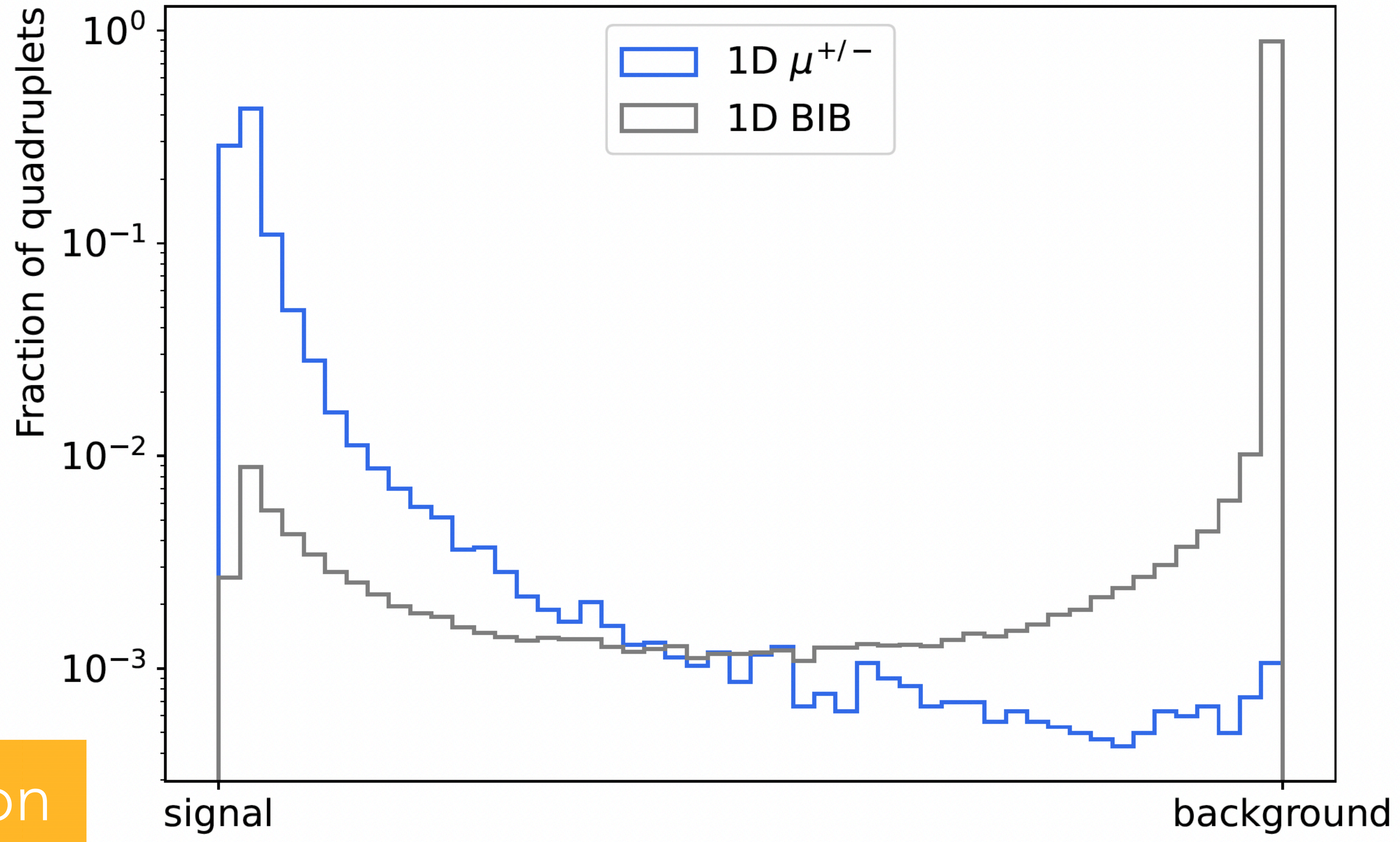


- Pre-selected triplets are further evaluated to form longer track segments.

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j, \quad T_i, T_j \in \{0,1\}$$

- QUBO coefficients: 1D (timing only), 3D (spatial) and 4D (spatial+timing).
  - Built from timing and/or spatial compatibility and normalised to the same range.

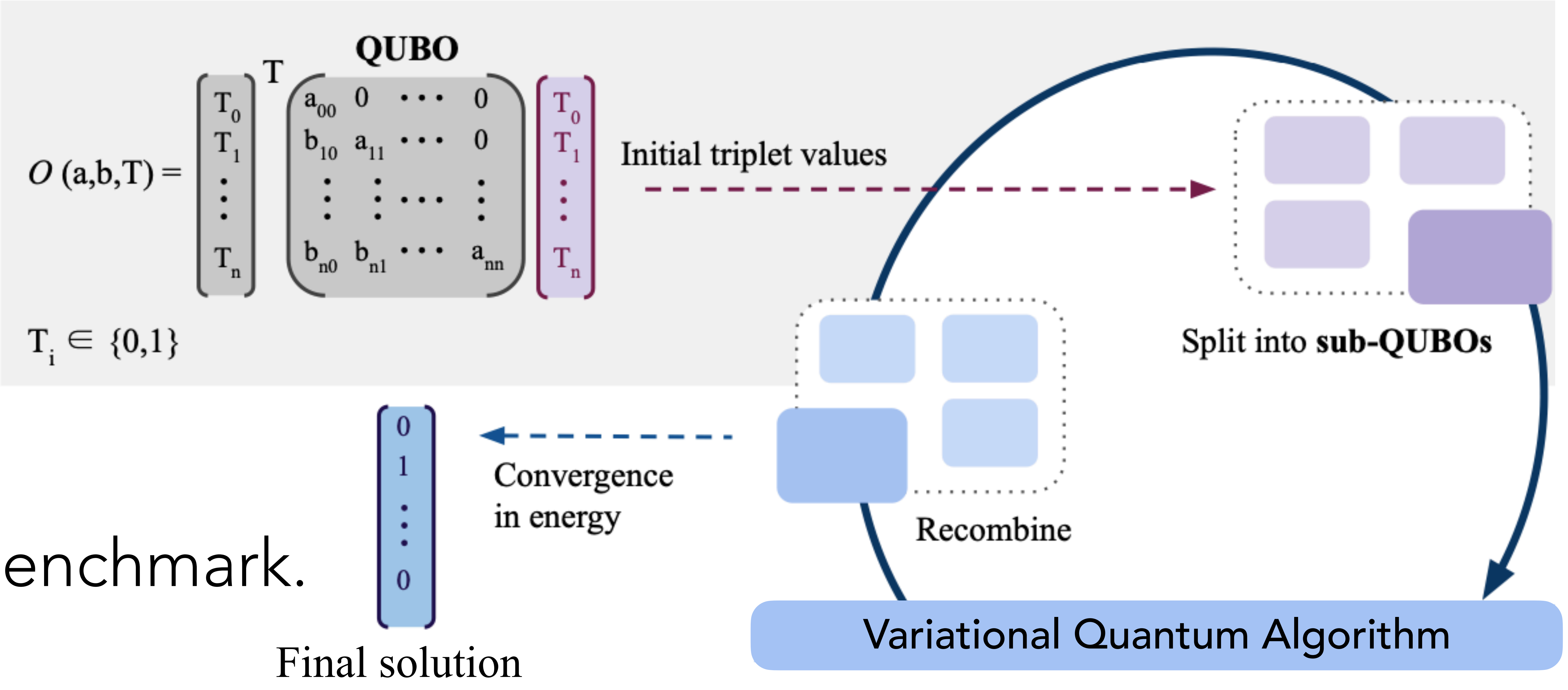
This is on top of a tight spatial pre-selection



QUBO SOLUTION

- The QUBO is mapped to an Ising Hamiltonian, where minimising it corresponds to finding the ground state. #qubits = #triplets.

- The ground state is found using **Variational Quantum Eigensolver (VQE)** .
  - QUBO is **partitioned** into smaller sub-QUBOs of size 7 and solved iteratively.

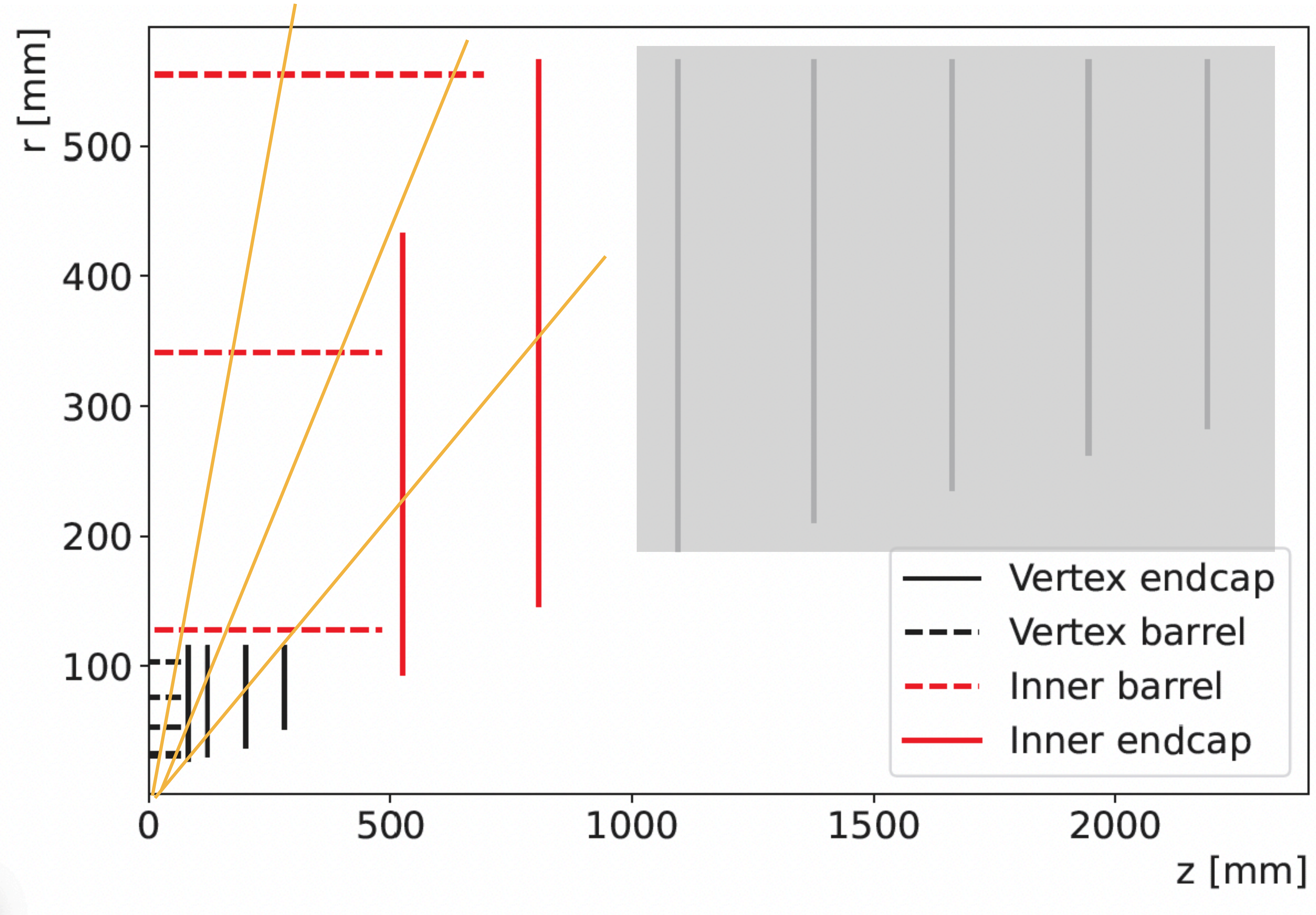


- Exact solutions via **matrix diagonalisation** serve as a benchmark.

TRACK SELECTION

- Track candidates formed by connecting selected triplets, ≥ 6 **hits.**

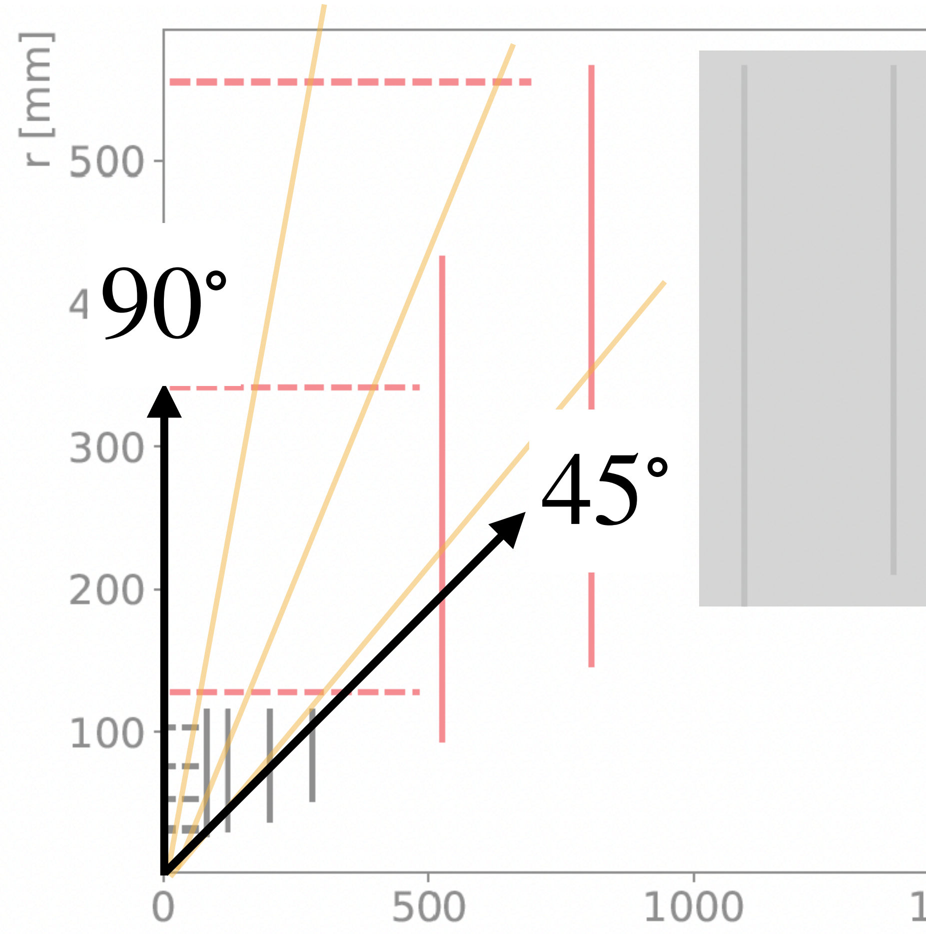
- Tracks are fitted with Kalman Filter within the ACTSTracking software. Standard ambiguity solving applied.



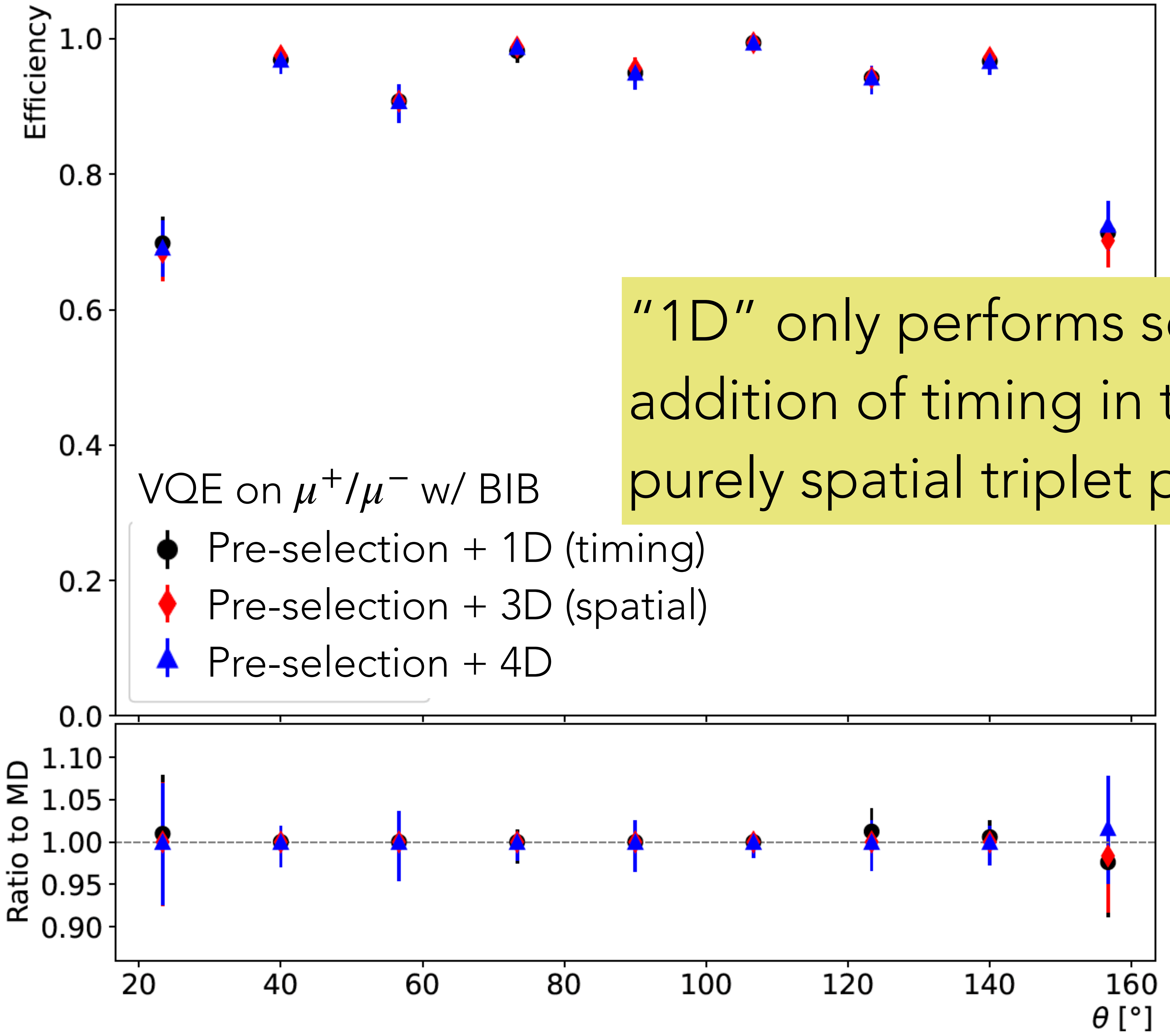
- Performance metrics:
  - $\text{Efficiency} = \frac{N^{\text{matched*}}_{\text{tracks}}}{N^{\text{generated}}_{\text{tracks}}}$
  - $\text{Fake rate} = \frac{N^{\text{fake}}_{\text{tracks}}}{N^{\text{reconstructed}}_{\text{tracks}}}$ or average number of fake tracks per event.

VQE: variational quantum eigensolver

RESULTS



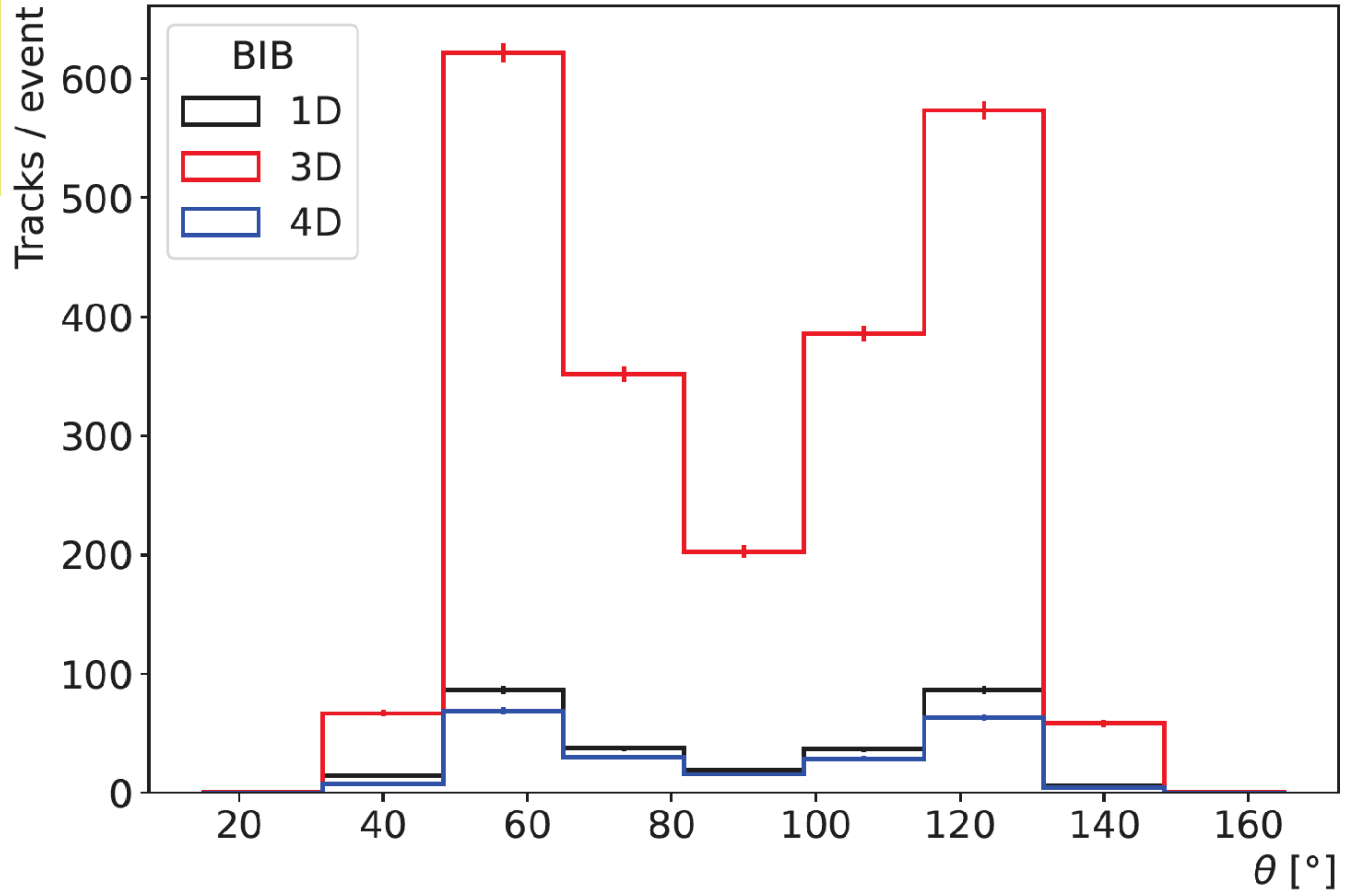
- Similar results from VQE as the benchmark (matrix diagonalisation).



"1D" only performs so well due to addition of timing in the QUBO to purely spatial triplet pre-selection.

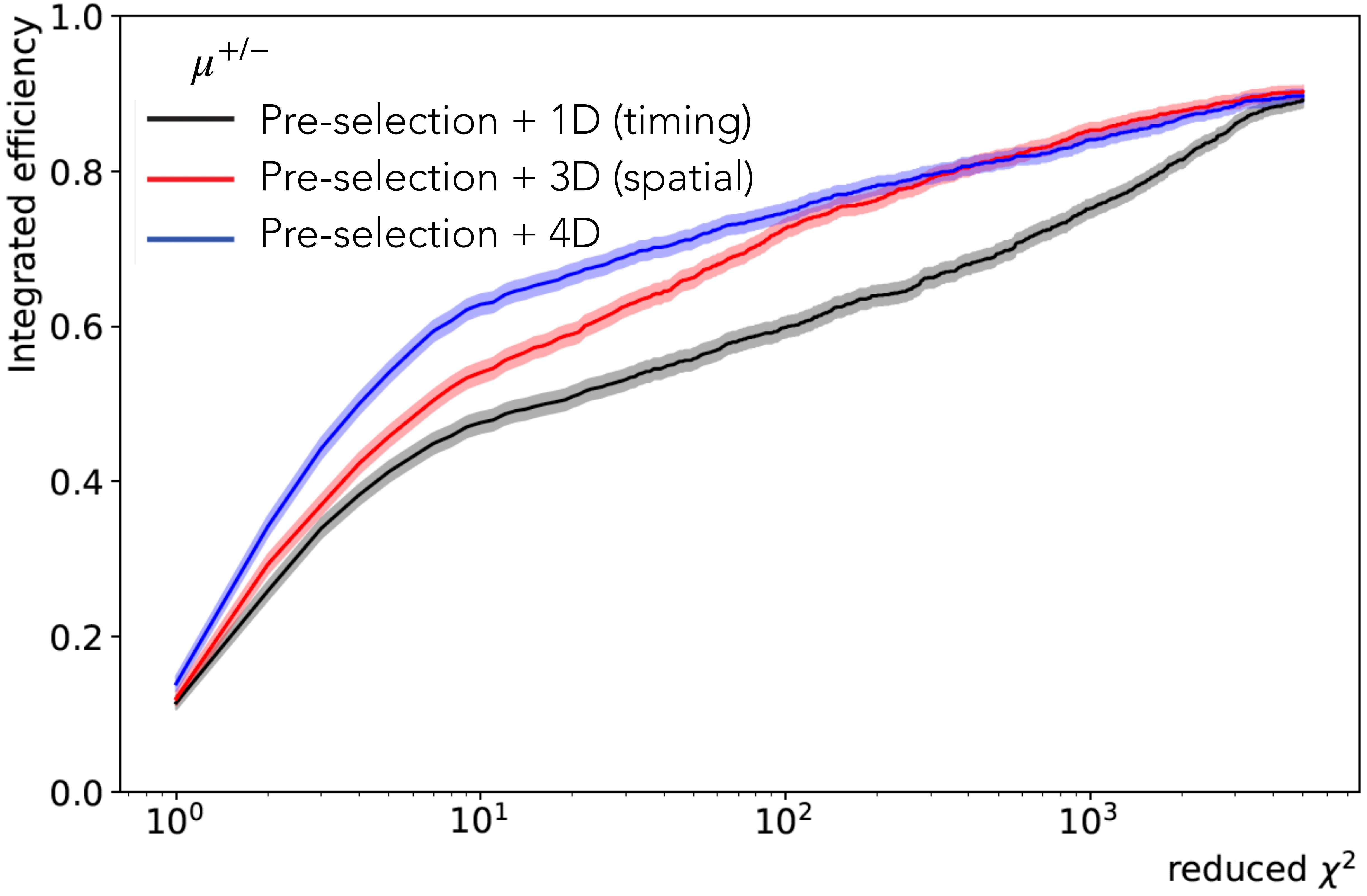
- Without timing information, "3D" shows a large number of fake tracks.

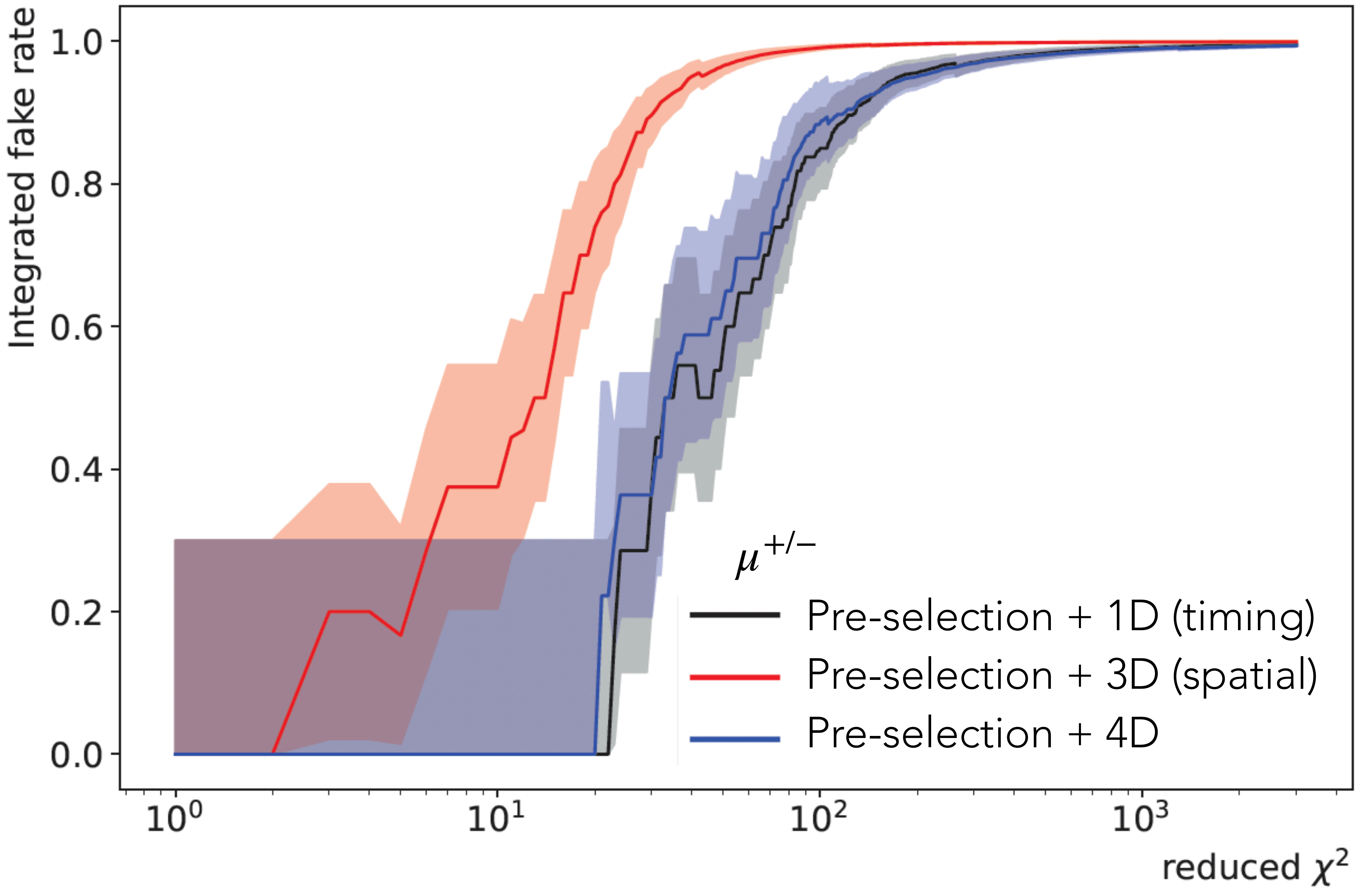
<# fake tracks >



EFFECT OF χ^2/NDF CUT

- With a χ^2 cut, the impact of adding timing information becomes clear.





RESULTS

- Fake track rejected completely with χ^2/ndf < 10 requirement.
- Using better track cleaning, e.g. based on track parameter, could improve the efficiencies shown here.

	$\mu^{+/-}$ Efficiency [%]		
χ^2	1D	3D	4D
10	$47.7^{+1.3}_{-1.3}$	$54.1^{+1.3}_{-1.3}$	$62.8^{+1.3}_{-1.3}$

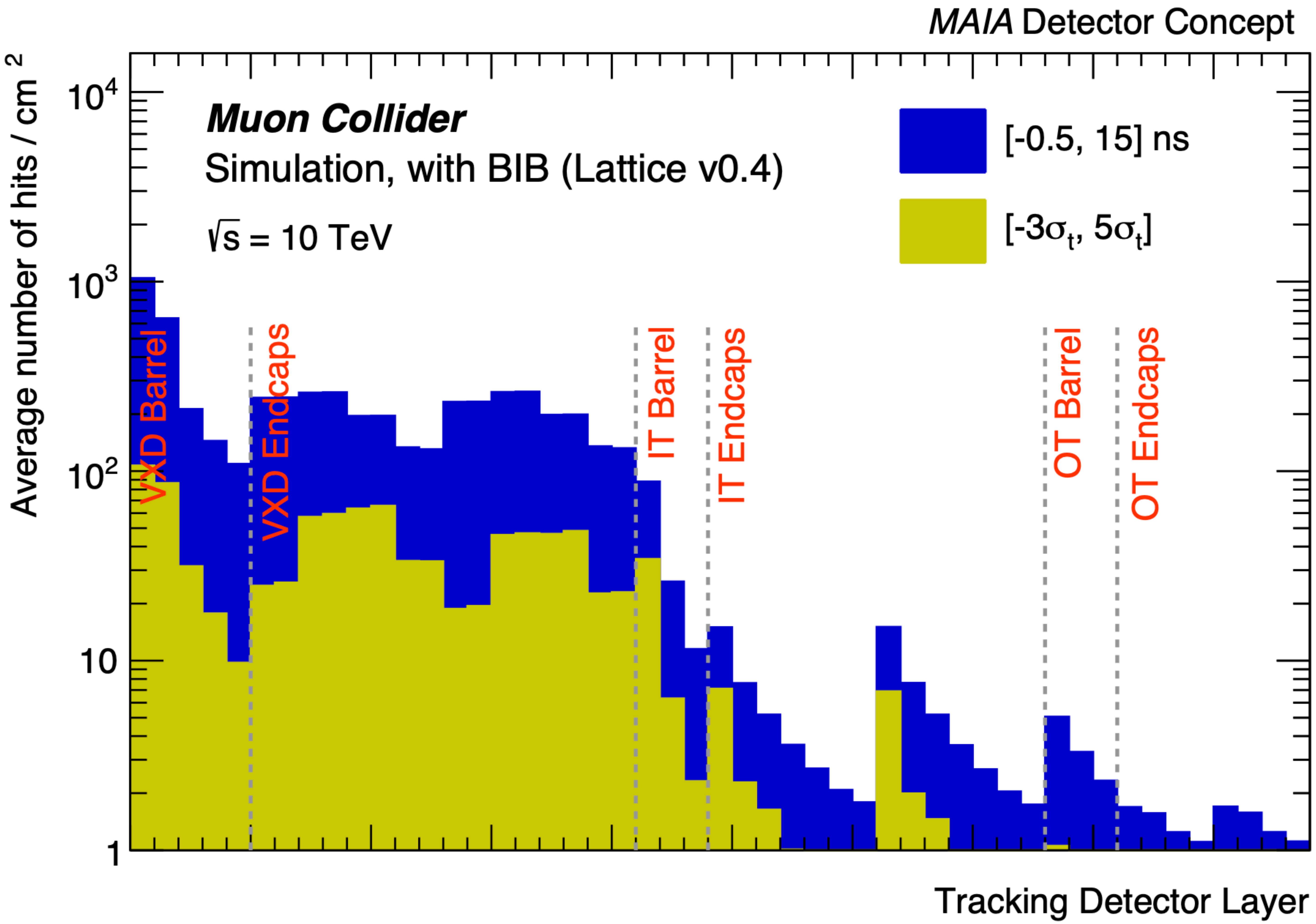
	1.1 TeV LLCP Efficiency [%]		
χ^2	1D	3D	4D
10	$59.9^{+1.3}_{-1.3}$	$70.2^{+1.2}_{-1.3}$	$81.8^{+1.0}_{-1.1}$

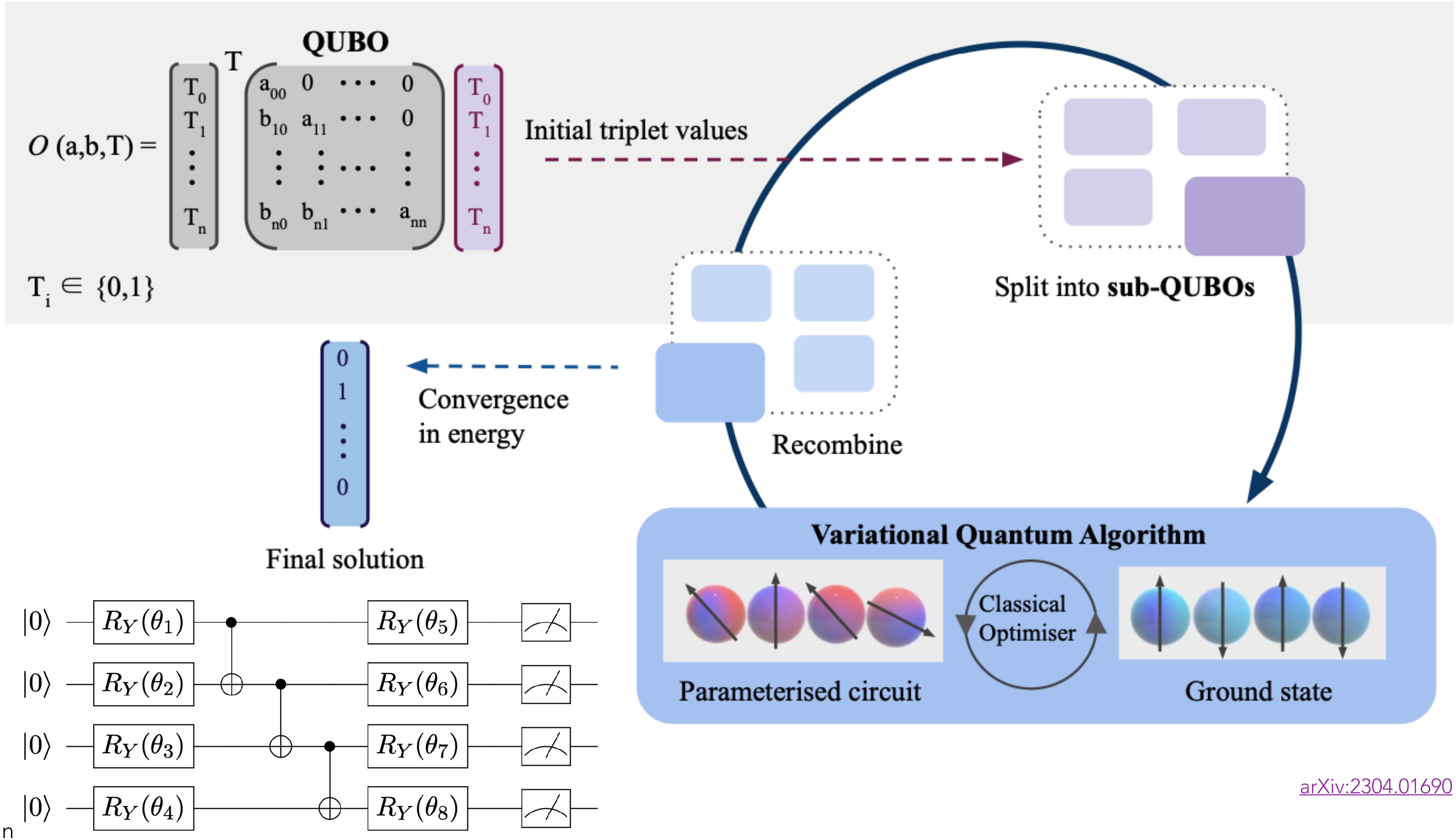
	Mean Fake Tracks per Event		
χ^2	1D	3D	4D
10	$0.0^{+0.3}_{-0.0}$	$0.3^{+0.2}_{-0.2}$	$0.0^{+0.3}_{-0.0}$

SUMMARY

- Demonstrated 4D tracking at muon colliders with a quantum computing approach.
  - Triplets passing spatial 3D pre-selection are further classified with 1D, 3D or 4D information in the QUBO.
- Currently limited to section of tracker closer to the IP most susceptible to BIB.
  - Outlook: a promising possibility would be to use this for seeding, in combination with combinatorial Kalman Filter.

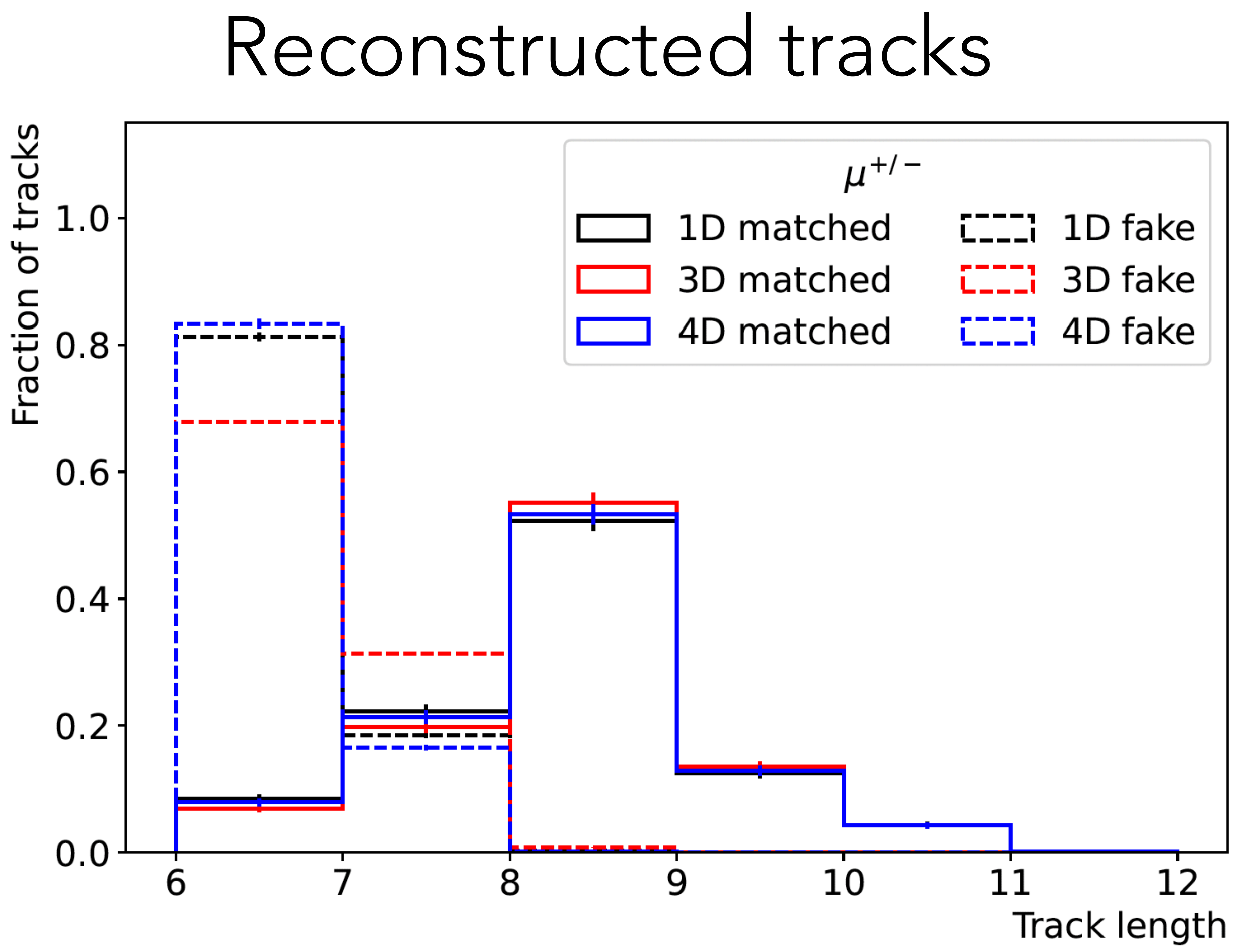
BACK-UP

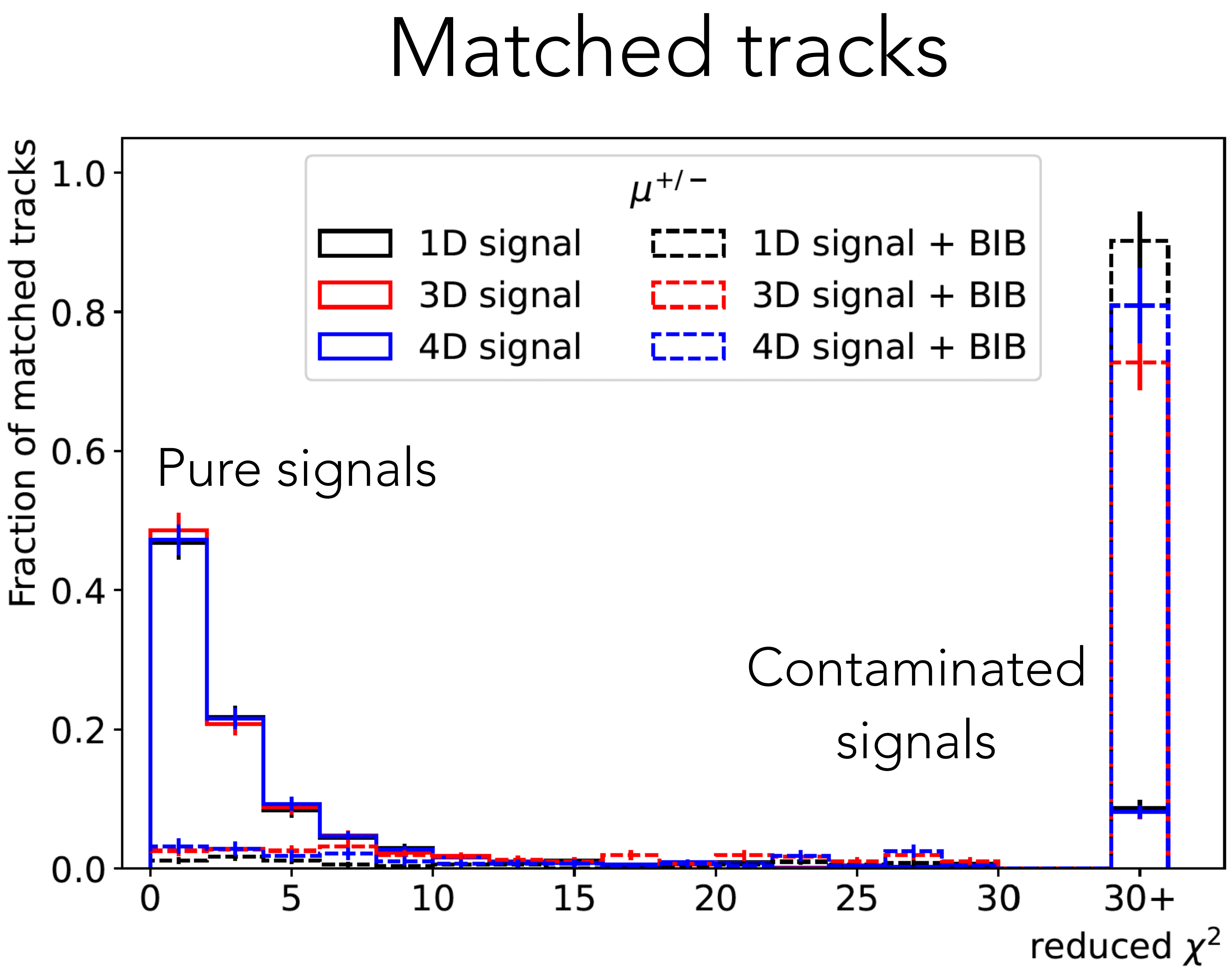

MAIA Detector Concept
Muon Collider
Simulation, with BIB (Lattice v0.4)
$\sqrt{s}$ = 10 TeV
[-0.5, 15] ns
[$-3\sigma_t$, $5\sigma_t$]
Average number of hits / cm 2
10^4
10^3
10^2
10
1
VXD Barrel
VXD Endcaps
IT Barrel
IT Endcaps
OT Barrel
OT Endcaps
Tracking Detector Layer



arXiv:2304.01690

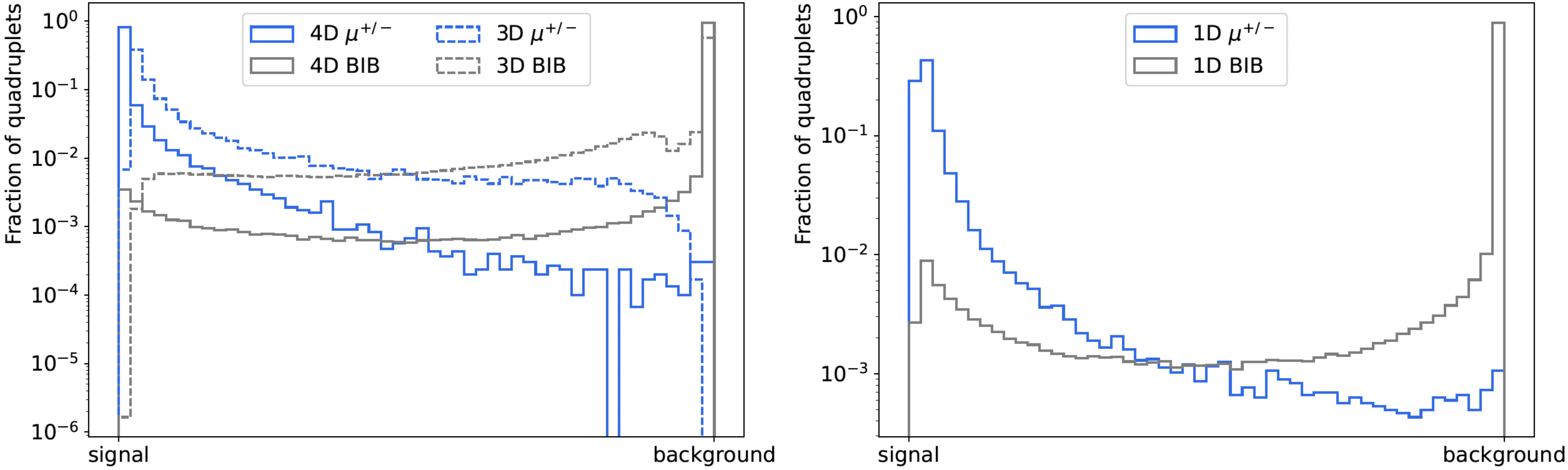
TRACK SELECTION

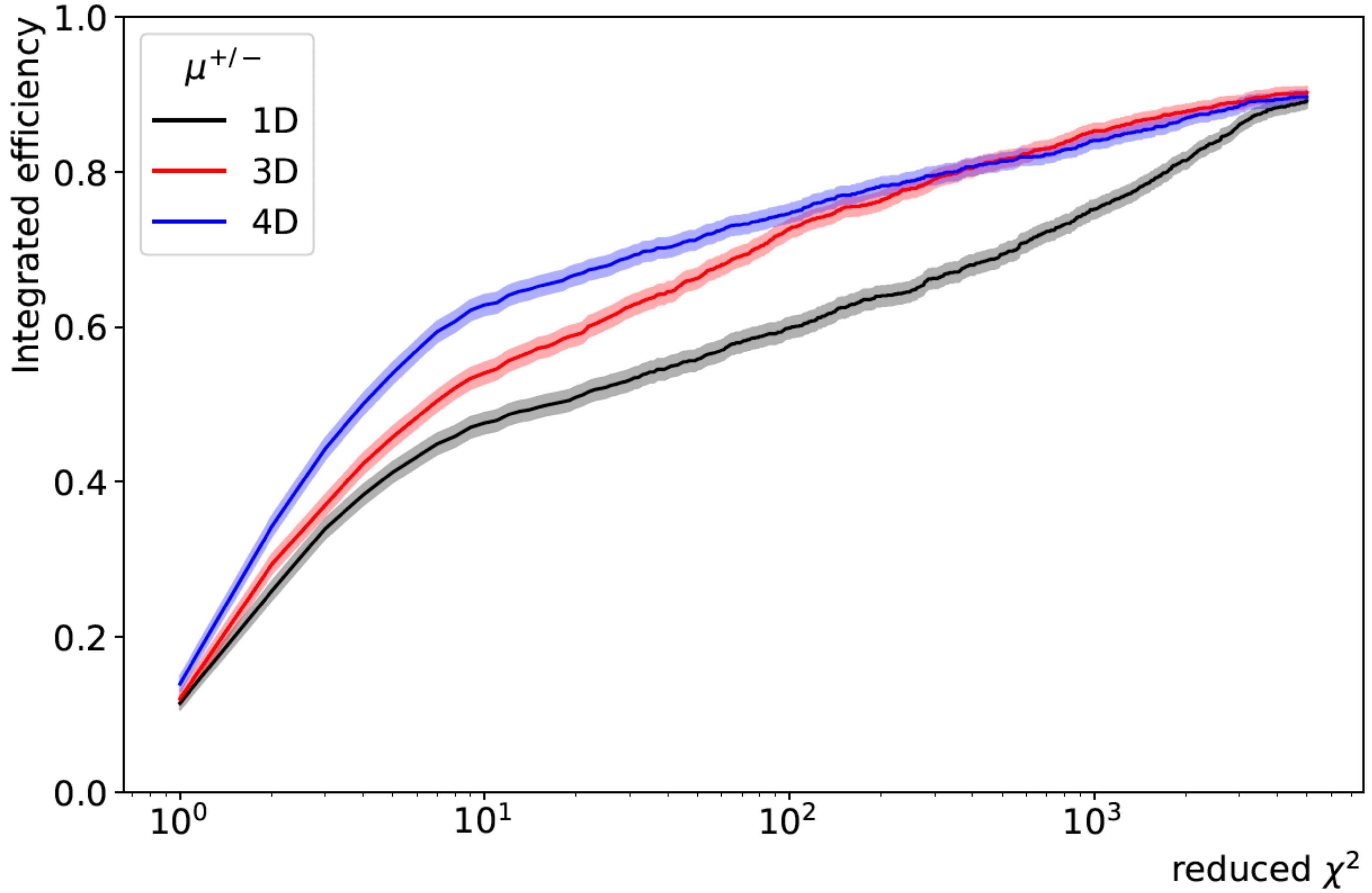


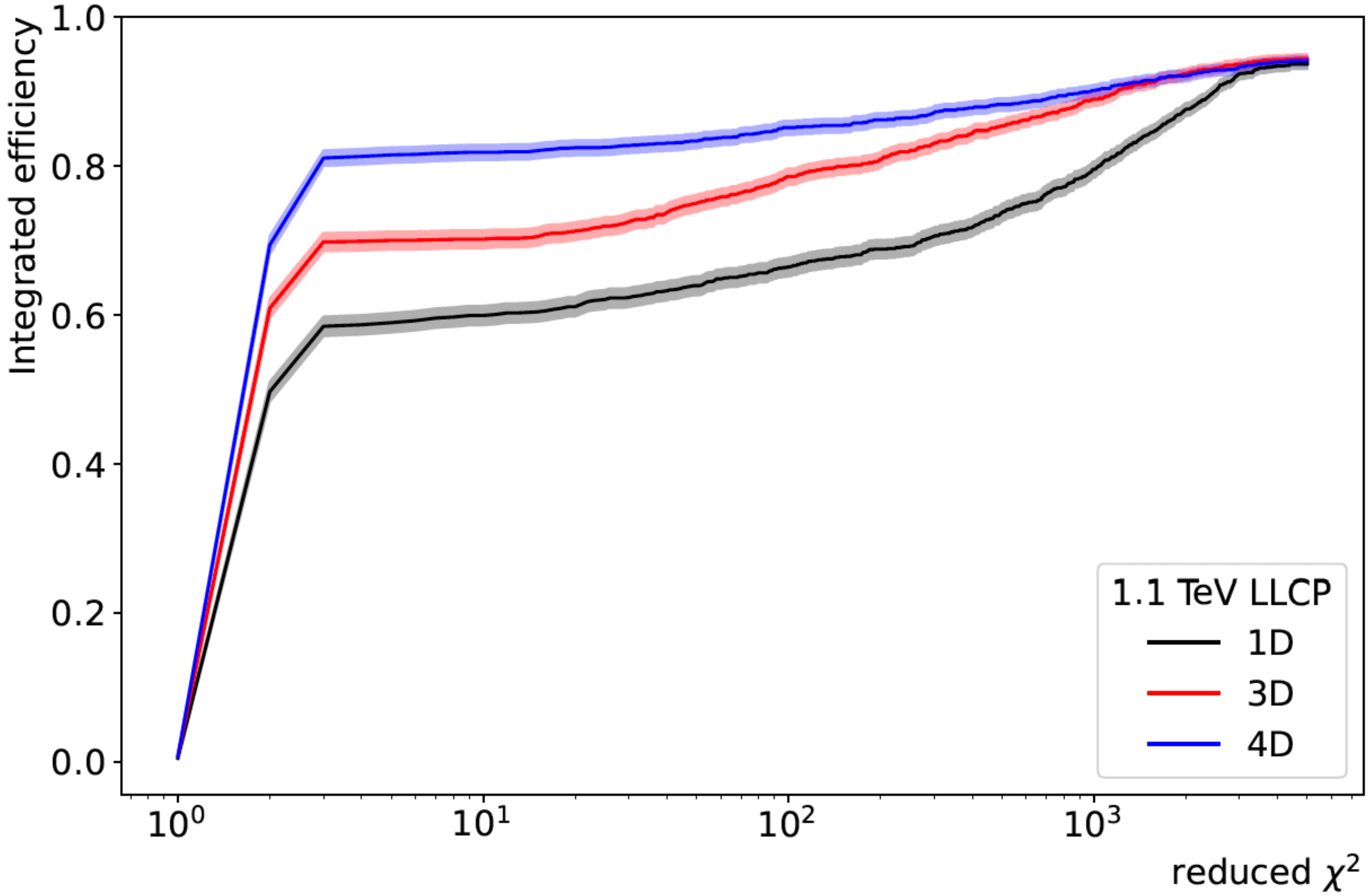


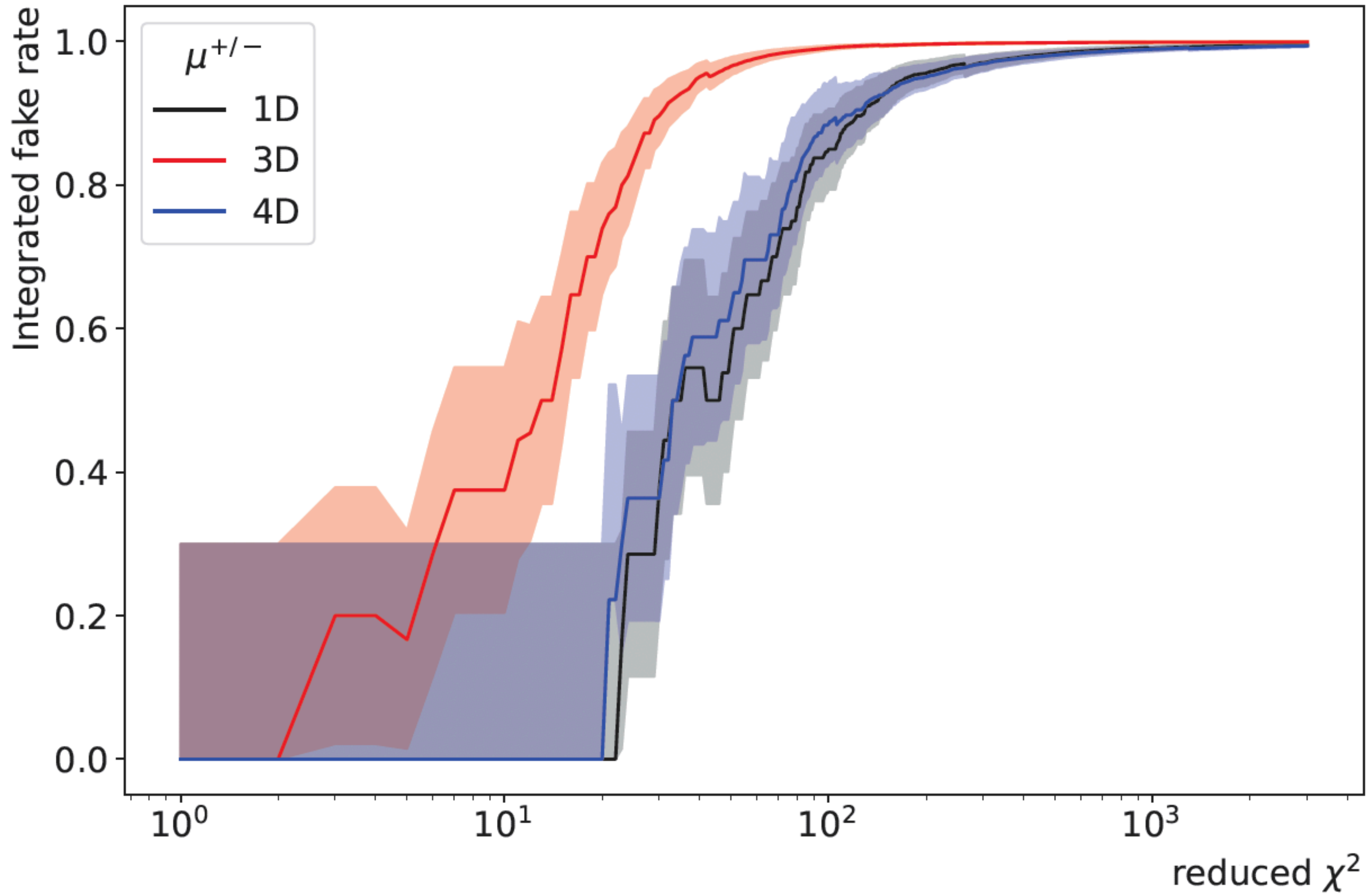
QUBO COEFFICIENTS

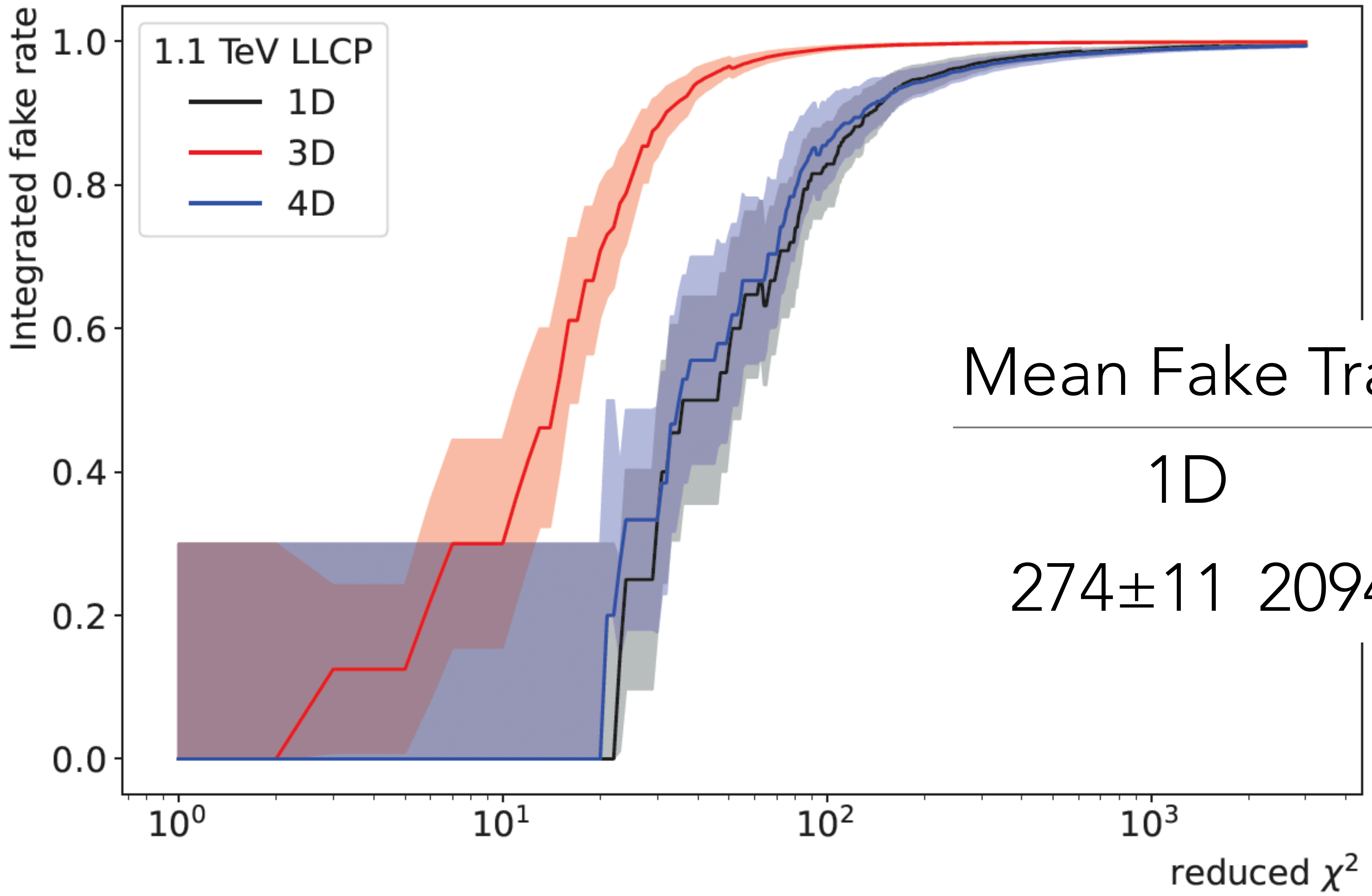
- $a_i \in [-0.5, 0.5]$
- $b_{ij} \in [-1, -0.9]$ or 2 (conflict) or 0 (otherwise).











Mean Fake Tracks per Event

1D	3D	4D
274±11	2094±41	209±20

PRE-SELECTION