SFQED CONCEPTS

ANTON ILDERTON
UNIVERSITY OF EDINBURGH

SFQED WORKSHOP

DESY, HAMBURG 2–3 DECEMBER 2024

OUTLINE

1. QED ⟷ SFQED — BASIC SETUPS

PAUSE!

2. FURRY PICTURE — HOW WE CALCULATE / NON-LINEARITY / DRESSED STATES

PAUSE!

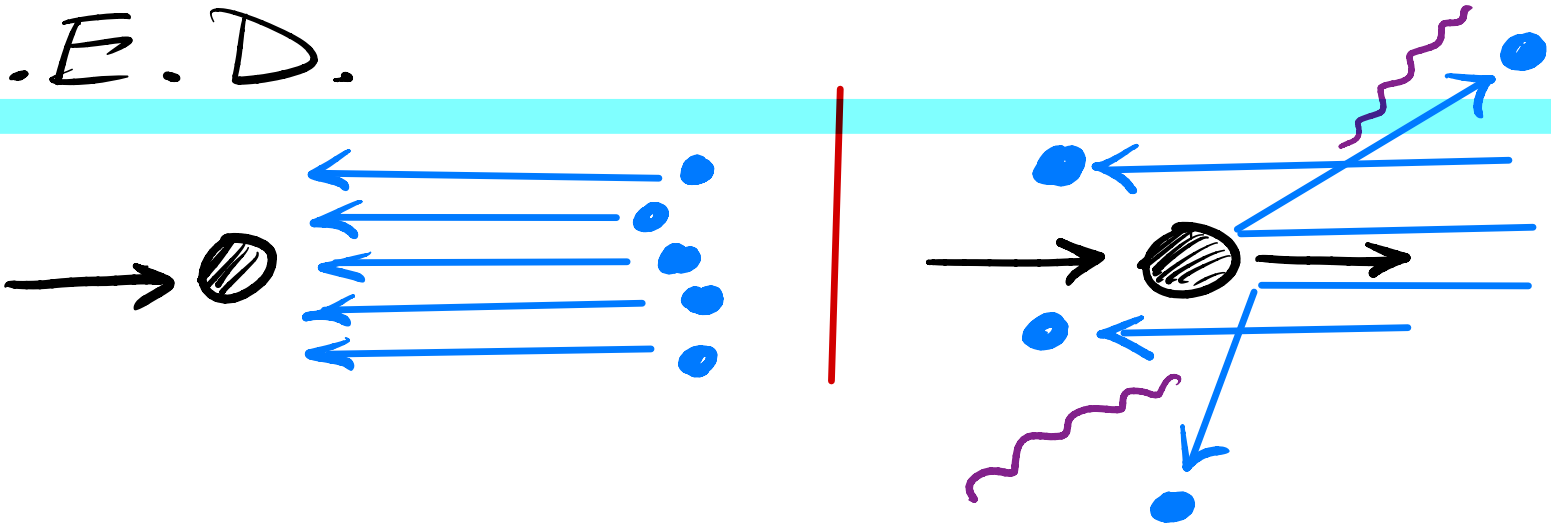
3. SCHWINGER EFFECT — INTEREST & RELEVANCE

PAUSE!

4. RITUS — NAROZHNYI — EXTREME INTENSITY

1. QED ↔ SFQED

Q.E.D.

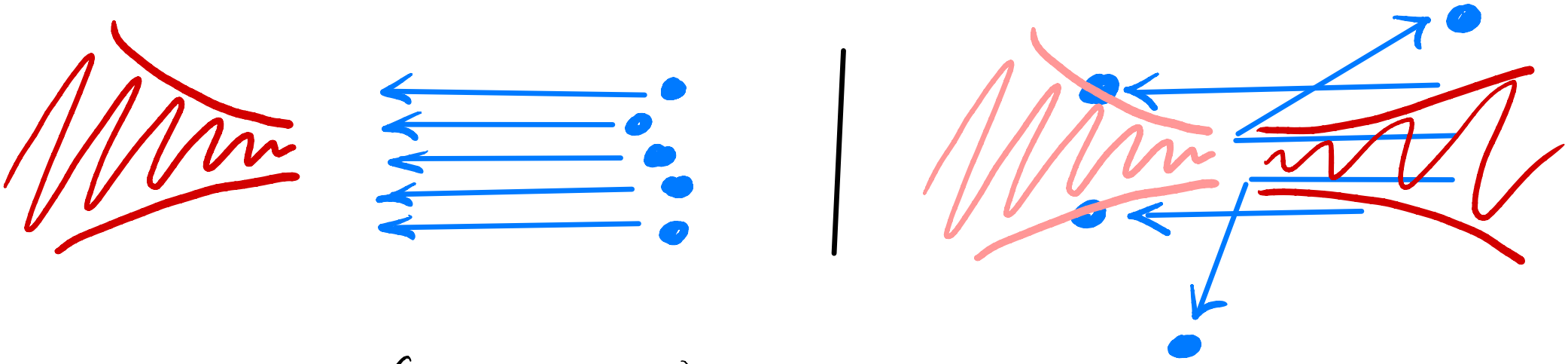


- COLLIDE (BEAMS OF) PARTICLES
- DETECT PARTICLES, MEASURE ENERGIES & ANGLES
- CROSS SECTION ETC FROM PROBABILITIES $\mathbb{P}$
- $\mathbb{P} = |A|^2$

A = AMPLITUDE FOR $n + m \longrightarrow n' + m'$

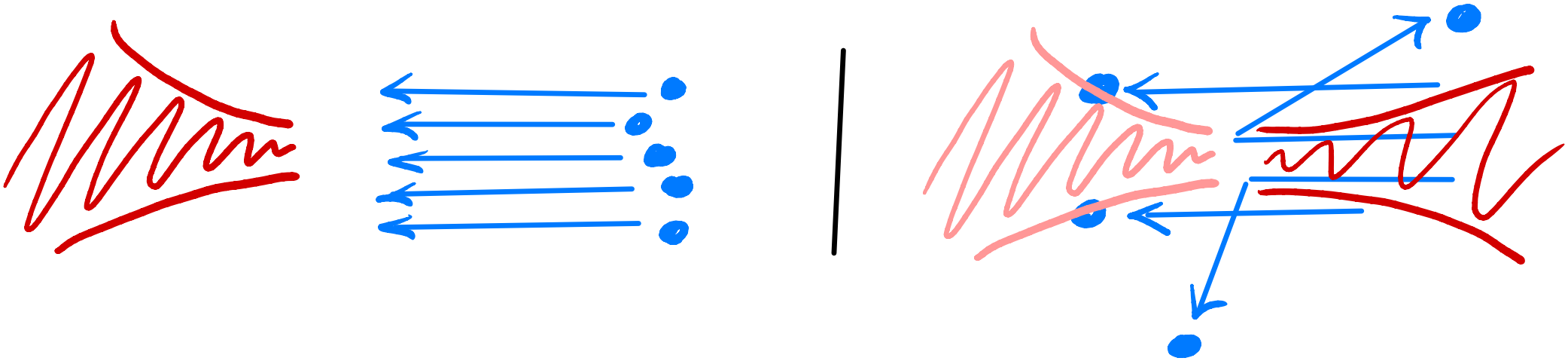
NOTE: DEFINITE NUMBERS OF PARTICLES

SFQED



- COLLIDE (BEAMS OF) PARTICLES WITH LASER PULSE
- DETECT PARTICLES, MEASURE ENERGIES & ANGLES
- CROSS SECTION ETC FROM PROBABILITIES $\mathbb{P}$ ...
- LASER $\neq$ NUMBER OF PHOTONS

SFQED



- STRONG CLASSICAL $\underline{E}, \underline{B}$:

LASER = COHERENT STATE OF PHOTONS.

- COLLIDING SOME # OF PARTICLES WITH COHERENT STATE

$\Rightarrow$ CALCULATIONS DIFFERENT!

Coherent States:

- Photonic state representing given classical $\underline{E}, \underline{B}$

$$|E\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \left(\frac{E}{\omega}\right)^n |n\rangle$$

$\left(\frac{E}{\omega}\right)^n$: characterises laser

$|n\rangle$: n-photon state

Roughly!!

$\Rightarrow$ every calculation must sum over all n.

- Every interaction generates factor of $\frac{e}{m} * \frac{E}{\omega} \equiv \xi$

$\Rightarrow$ every calculation / physical process involves sum

$$\sum_{n=0}^{\infty} \#(n)\, \xi^n$$

HOW DO WE DEAL WITH $\sum_{n=0}^{\infty} \#(n)\,\xi^n$?

- SUPPOSE ξ SMALL – PERTURBATION THEORY!

  $\sum_{n=0}^{\infty} \rightarrow \#\xi$. DONE!

- HOWEVER....

  ... FOR $I \gtrsim 10^{18}$ W/cm^2 @ OPTICAL, $\xi > 1$

☹ SUM CAN'T BE TRUNCATED/TREATED IN PERT. THEORY

> MUST BE TREATED EXACTLY
>
> TO PROPERLY CAPTURE INTERACTION WITH LASER!

- THIS IS WHAT WE MEAN BY "NON-LINEARITY" AT $\xi > 1$.

2. Furry Picture

Q : What is the Furry picture?

A : HOW WE CALCULATE. (DETAIL BELOW!)

Q : What [problem] does the Furry picture Solve?

A : COUPLING TO LASER (ξ) $\gg 1$ CANNOT BE TREATED IN STANDARD PERTURBATION THEORY.

Q : How does Furry solve this?

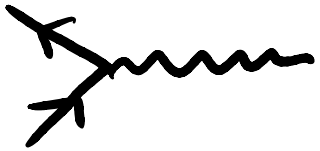
A : SLIGHTLY MODIFIES COMPUTATION SCHEME OF QED.

CALCULATIONS IN THE FURRY PICTURE

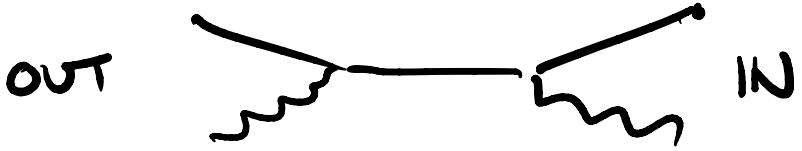
QED

1) TWO TYPES OF LINE

2) RULE FOR JOINING



3) DRAW ALL POSSIBLE PICS CONNECTING IN/OUT STATES



4) LINES & JOINS → FUNCTIONS & INTEGRALS

SFQED

OBVIOUS PROBLEM!

∞ MANY TERMS

ALL NUMBERS OF PHOTONS

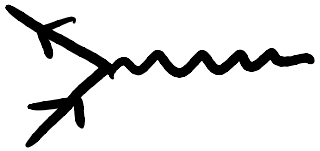
CAN'T TRUNCATE SERIES!

CALCULATIONS IN THE FURRY PICTURE

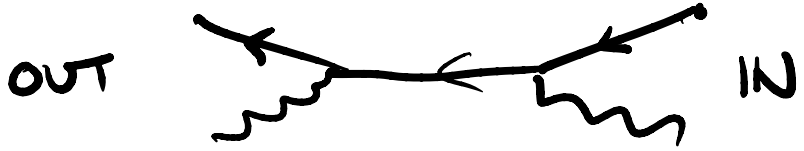
QED

1) TWO TYPES OF LINE

2) RULE FOR JOINING



3) DRAW ALL POSSIBLE PICS CONNECTING IN/OUT STATES



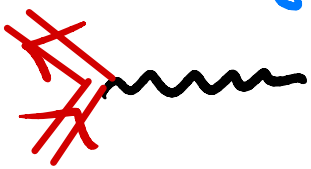
4) LINES & JOINS → FUNCTIONS & INTEGRALS

SFQED

1) TWO TYPES OF LINE

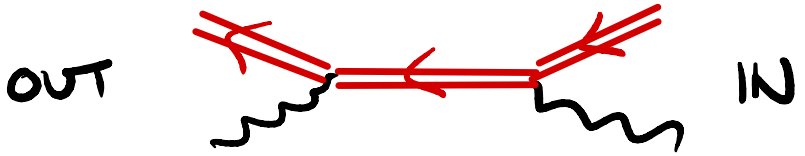
!

2) RULE FOR JOINING



SAME!

3) DRAW ALL POSSIBLE PICS CONNECTING IN/OUT STATES



SAME!

4) LINES & JOINS → FUNCTIONS & INTEGRALS

SAME!

3/6

CALCULATIONS IN THE FURRY PICTURE

- ONLY ONE CHANGES:  BECOMES 

- IF  IS AN ELECTRON PROPAGATING IN VACUUM

  THEN  IS AN ELECTRON PROPAGATING IN THE LASER

  (PUSHED AROUND BY LORENTZ FORCE!)

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right) \underset{x}{\longleftarrow}\!\!\!\!\!\!\underset{y}{} = \delta^4(x-y)$$

QED → SFQED

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - e\gamma^\mu A_\mu^{\text{LASER}} - m\right) \underset{x}{\Longleftarrow}\!\!\!\!\!\!\underset{y}{} = \delta^4(x-y)$$

☺ MODIFIED RULES SOLVE PROBLEM OF LARGE COUPLING TO LASER.

☺ " DO X, Y, Z, THEN COUPLING (ξ) ACCOUNTED FOR EXACTLY.

CALCULATIONS IN THE FURRY PICTURE

- SOME CAVEATS & DETAILS!
- STICKING POINT: ACTUALLY CALCULATING ⇐ ☹
- POSSIBLE IN SIMPLE (MEANS "HIGHLY SYMMETRIC") FIELDS

CONSTANT FIELDS
COULOMB FIELDS
PLANE WAVES
OTHER SPECIAL CASES

✓

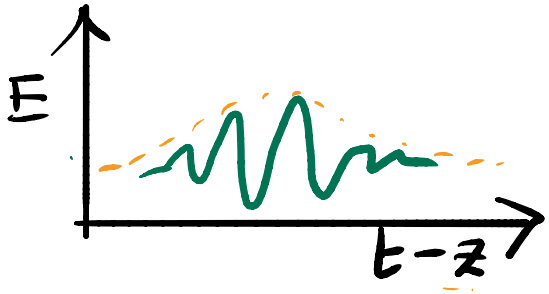
FOCUSED GAUSSIAN BEAMS
FLYING FOCUS
PARAXIAL BEAMS
BESSEL BEAMS

✗

- EIKONAL / HIGH-ENERGY APPROXIMATION
- → PLANE WAVE APPROXIMATION

CALCULATIONS IN THE FURRY PICTURE

- → PLANE WAVE APPROXIMATION
  - ↳ ⫦ KNOWN EXACTLY
  - ↳ ARBITRARY TEMPORAL PROFILE
  - ↳ EXTEND TO FOCUSSED FIELDS VIA APPROXIMATION SCHEMES + SIMULATIONS



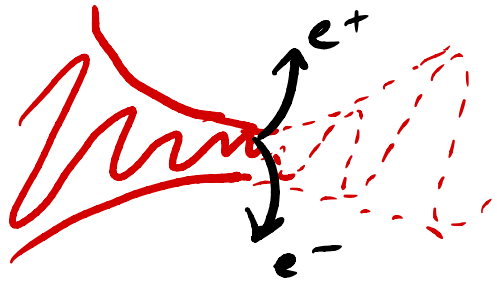
→ MORE FROM TOM BLACKBURN

- ⫦ ~ "VOLKOV SOLUTION"
- "DRESSED STATES" (NOT A GREAT NAME)

- MORE DETAILS ON FURRY SOON, AND FROM DANIEL SEIPT

3. Schwinger effect

MATTER FROM LIGHT



- PRACTICALLY:
  1) FOCUS (LASER) LIGHT
  2) LIGHT TURNS INTO e^+e^- PAIRS!

- IF MODEL LASER AS CONSTANT, HOMOGENEOUS ELECTRIC FIELD

THEN OBSERVABLES GO LIKE $e^{-\pi E_S/E} = e^{\frac{-\pi m^2}{eE}}$

NON-ANALYTIC
NON-PERTURBATIVE IN COUPLING TO FIELD eE!

Coo!!

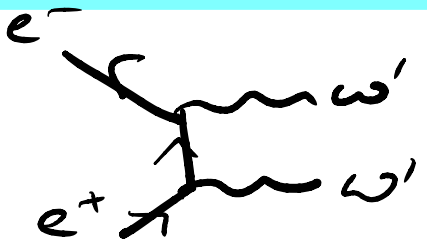
- MORE REALISTIC FIELD MODELS?

⇒ BOTH PERTURBATIVE & NON-PERTURBATIVE PARTS

MATTER FROM LIGHT

- (TEXTBOOK, LINEAR) BREIT-WHEELER

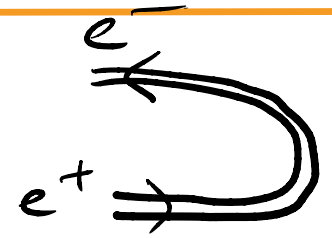
$$\gamma\gamma \to e^+e^-$$



$$\sigma \sim e^4\left(\frac{m}{\omega'}\right)^2\left[\log\frac{2\omega'}{m} - 1\right] \qquad \omega' \gg m$$

- LASER $\to e^+e^-$ (SCHWINGER)

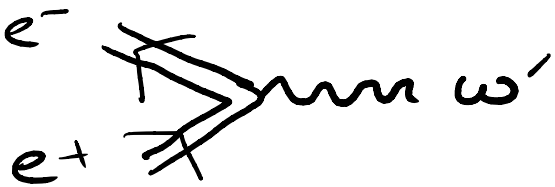
LASER MODELLED AS CONSTANT E $\quad P \sim e^{-\frac{\pi m^2}{eE}}$



- "NONLINEAR BREIT WHEELER"

$$\gamma + \text{LASER} \to e^+e^-$$

LASER MODELLED AS PLANE WAVE



$$P \sim \exp\left(\frac{-8}{3}\frac{m}{\xi\omega'}\right) = \exp\left(\frac{-8}{3}\frac{m^2}{eE}\frac{\omega}{\omega'}\right)$$

NON-ANALYTIC
NON-PERTURBATIVE IN COUPLING TO FIELD eE!

- SCHWINGER: INVARIANT $\longrightarrow$ "$\frac{E}{E_S}$" $\equiv \frac{eE_{LAB}}{m^2}$

  PROPERTY OF FIELDS ONLY

- $\chi = \xi \frac{\omega'}{m}$ INVARIANT $\longrightarrow$ $\chi = \frac{eE_{R.F.}}{m^2}$

  PROPERTY OF FIELDS & PARTICLE IN FIELD

NOT THE SAME!

NOT THE SAME!

- $\frac{E_{LAB}}{E_S} \ll 1$ AT LUXE.

"SCHWINGER-LIKE" BEHAVIOUR NOT RELEVANT.

- $\chi = \xi \frac{\omega'}{m} \simeq 1$

NON-PERT/NON-ANALYTIC BEHAVIOUR IN $\chi = \xi \frac{\omega'}{m}$ IS RELEVANT!

Q:

Why do theorists like Ben not like it when we use the expression
"*creating real electron positron pairs from the vacuum above the Schwinger limit?*"

A: 1) PAIRS ARE CREATED FROM LIGHT, NOT VACUUM

2) LUX2 IS NOT ABOVE THE SCHWINGER LIMIT

Q:

Can we say "*real electron-positron pairs above the Schwinger limit are created*", or is this misleading?

A: MISLEADING: NOT ABOVE THE SCHWINGER LIMIT

$$\frac{E_{LAB}}{E_S} \ll 1$$

Q:

What does the Dirac sea have to do with Schwinger pair creation?

A: OLD-FASHIONED "PICTURE" OF PAIR CREATION, PRE-DATES UNDERSTANDING OF ANTIMATTER.

NSUM NEED TO INVOKE IT.

Summary

☹ No Schwinger at Luxe.

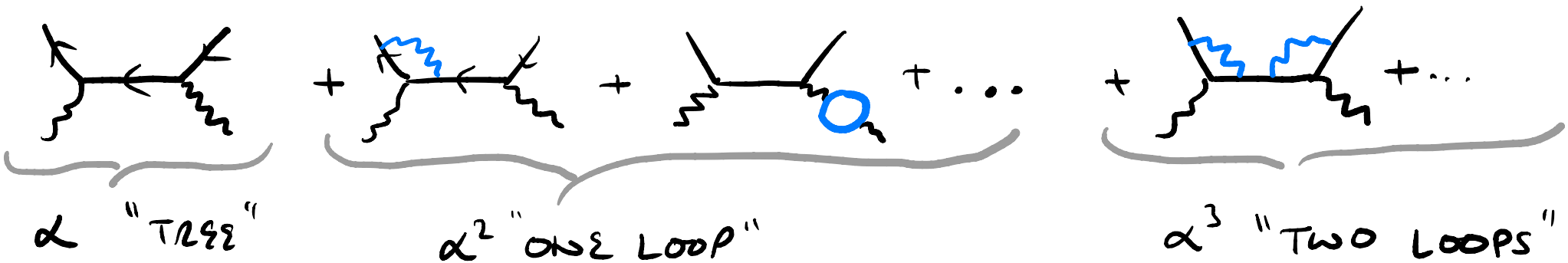
☺ Non-pert. / non-analytic aspects of pair production still accessible!

☺ Non-pert. / non-analytic dependence on coupling to laser still there!

4. Ritus - Narození

QED LOOP CORRECTIONS

- IN CALCULATIONS  GIVES FACTOR $e \leftrightarrow \sqrt{\alpha}$
- EXAMPLE : $e^- \gamma \to e^- \gamma$  α



α "TREE" α^2 "ONE LOOP" α^3 "TWO LOOPS"

- ANY OBSERVABLE O HAS AN EXPANSION LIKE

$$O \sim \alpha^n \left(c_0 + c_1 \alpha + c_2 \alpha^2 + \dots \right)$$

TREE 1-LOOP 2-LOOP

- ASYMPTOTIC BUT α SMALL SO CALCULATE TO FINITE ORDER AND STOP. PRECISION FRONTIER : 4 LOOPS?

SFQED LOOP CORRECTIONS

- IN CALCULATIONS  STILL GIVES $e \leftrightarrow \sqrt{\alpha}$
- α STILL SMALL. CALCULATIONS ORGANISED IN SAME WAY
  - EVERYTHING ELSE GIVES NONTRIVIAL FUNCTIONS OF ξ!
- TWO PARAMETERS IN THE SAME — α AND ξ.

- ANY OBSERVABLE O HAS AN EXPANSION LIKE

$$O \sim \alpha^n \left(C_0(\xi) + \alpha C_1(\xi) + \alpha^2 C_2(\xi) + \dots \right)$$

TREE 1-LOOP 2-LOOP

- PRECISION FRONTIER : 1 LOOP
- .... USUALLY ENOUGH..?

HANG ON....

SFQED LOOP CORRECTIONS

$$0 \sim \alpha^n \left(c_0(\xi) + \alpha c_1(\xi) + \alpha^2 c_2(\xi) + \dots \right)$$

- EXPLICIT CALCULATIONS SHOW :

$\xi \ll 1$: $c_1, c_2 \dots$ INDEPENDENT OF ξ

$\xi \gg 1$: $c_1(\xi) \sim \xi^{2/3}$

- RITUS-NAROZHNYI CONJECTURE : $\dfrac{c_{n+1}(\xi)}{c_n(\xi)} \sim \xi^{2/3}$

- TERMS IN SERIES GROW & GROW FOR $\alpha \xi^{2/3} > 1$

  - MOST CALCULATIONS FOR CCF $\Rightarrow$ $\boxed{\alpha \chi^{2/3} > 1}$

  - "SPIRIT OF THE LAW V. LETTER OF THE LAW"

Conjecture & Consequences

For $\alpha\chi^{2/3} > 1$ ....

1) Furry expansion not a good expansion!
2) Furry expansion must be resummed!

Q: where does the upper limit for theoretical predictions $\alpha\chi^{2/3}$ come from?

A: explicit calculations & a conjecture

Q: . Why do non-perturbative methods fail at some point?

A: clarify: Furry expansion fails (?) $\alpha\chi^{2/3} > 1$.

Conjecture & Consequences

For $\alpha \chi^{2/3} > 1$ .... [way beyond current laser tech!]

1) Furry expansion not a good expansion!
2) Furry expansion "must be resummed"!

- Maths : incredibly challensing!
- Physics : we know almost nothing about QED in this regime!
- Physics : strongly coupled QFT!?
- Physics : D.O.F.? Plasma? Bound states? .....

Q : How can we obtain.... experimental signatures in the regime of very high χ ?

A : New ideas & methods needed!!

HOPE THAT WAS USEFUL / INTERESTING!

THANK YOU!