Non-Gaussianities in ultra slow-roll inflation

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Based on [**arXiv:2406.02417 & 2412.14106**]
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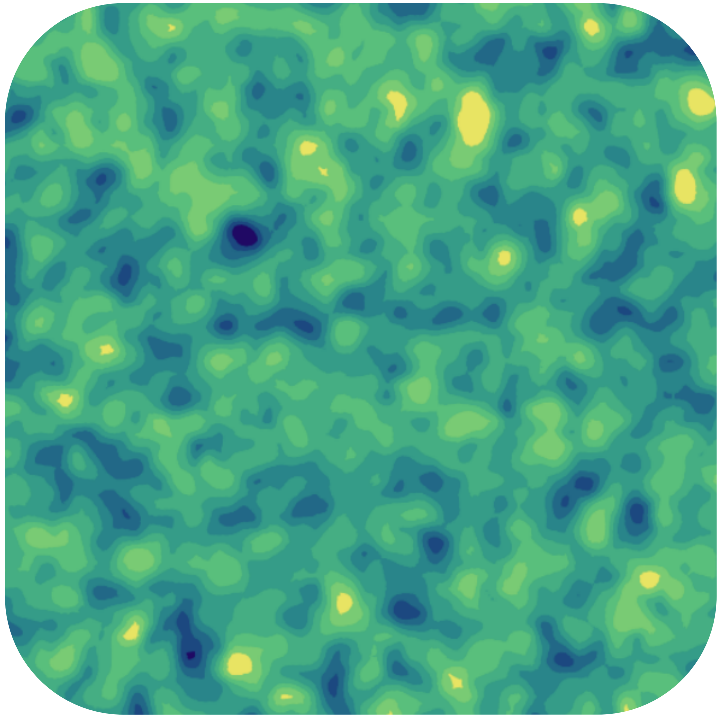
& T. Konstandin, J. Rey (DESY - QU)

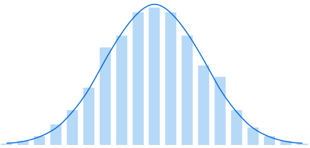
Fluctuations at CMB scales are Gaussian

Slow-roll inflation predicts inhomogeneities characterized by curvature perturbations $\mathcal{R} \leftrightarrow \zeta$

$$\mathcal{R}_k'' + (3+\epsilon-2\eta)\mathcal{R}_k' + \frac{k^2}{a^2H^2}\mathcal{R}_k = 0 \qquad \mathcal{P}_\mathcal{R} \equiv \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \sim 10^{-9} \qquad k_{\rm CMB} \simeq 0.05\ {\rm Mpc}^{-1}$$



→ **Gaussian variable**



$$\sigma^2 = \int {\rm d}\log k\, \mathcal{P}_\mathcal{R}(k)$$

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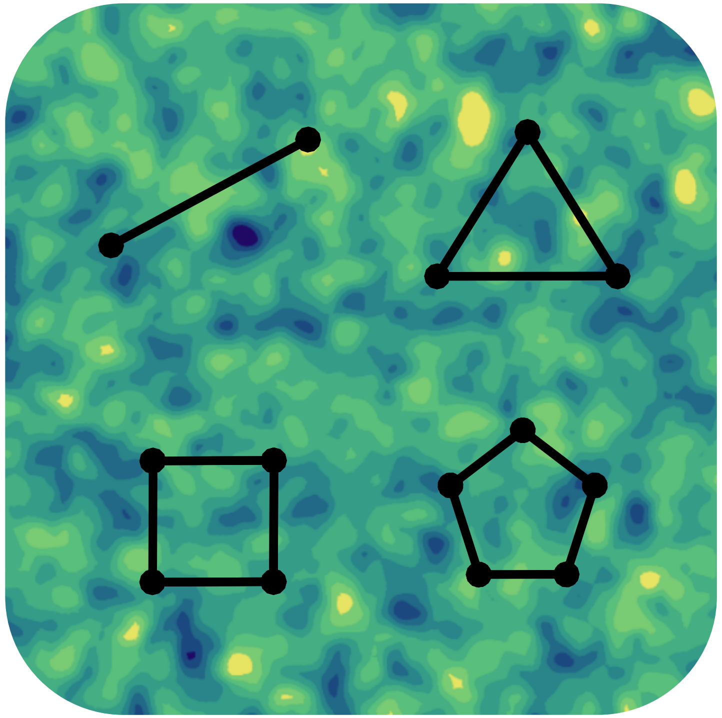

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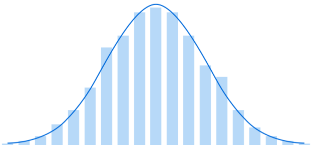

$$\sigma^2 \; = \; \int \mathrm{d}\log k \, \mathcal{P}_\mathcal{R}(k)$$

→ **Statistics fully determined by two-point function**

$$\langle \mathcal{R}_k \rangle = 0$$
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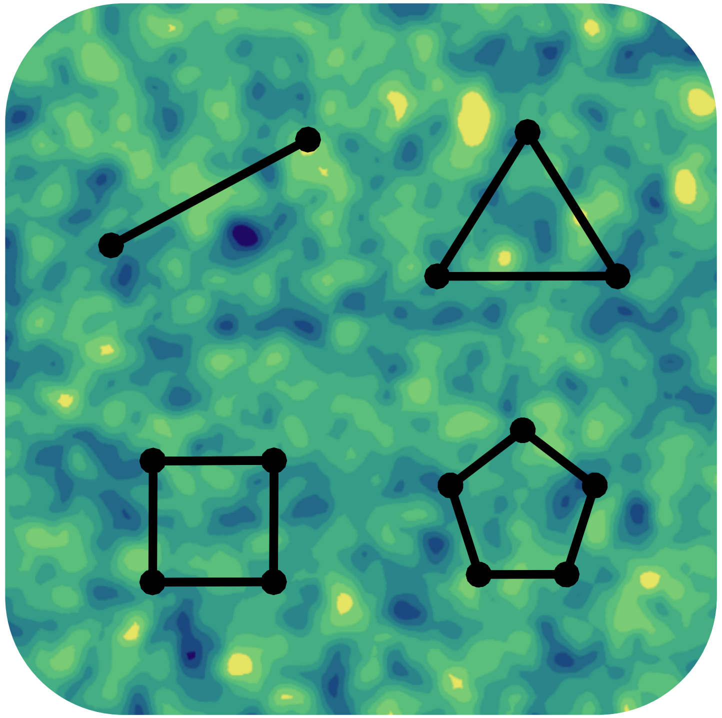

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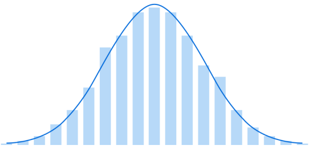

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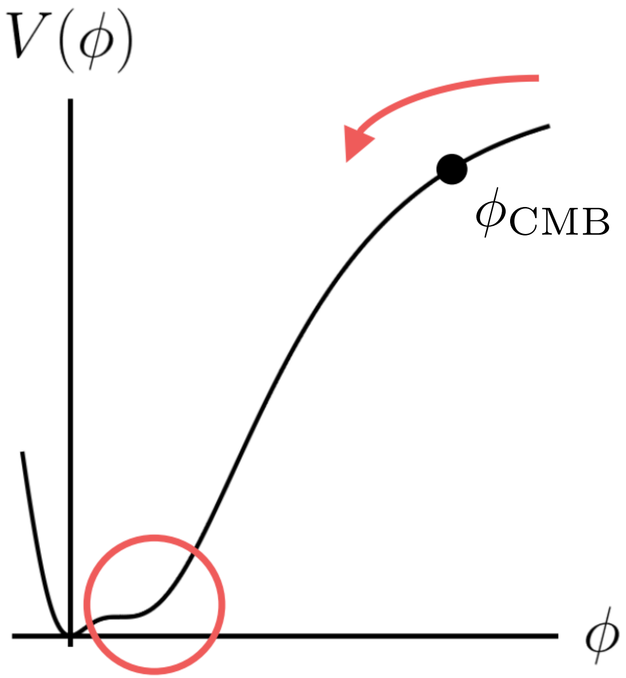
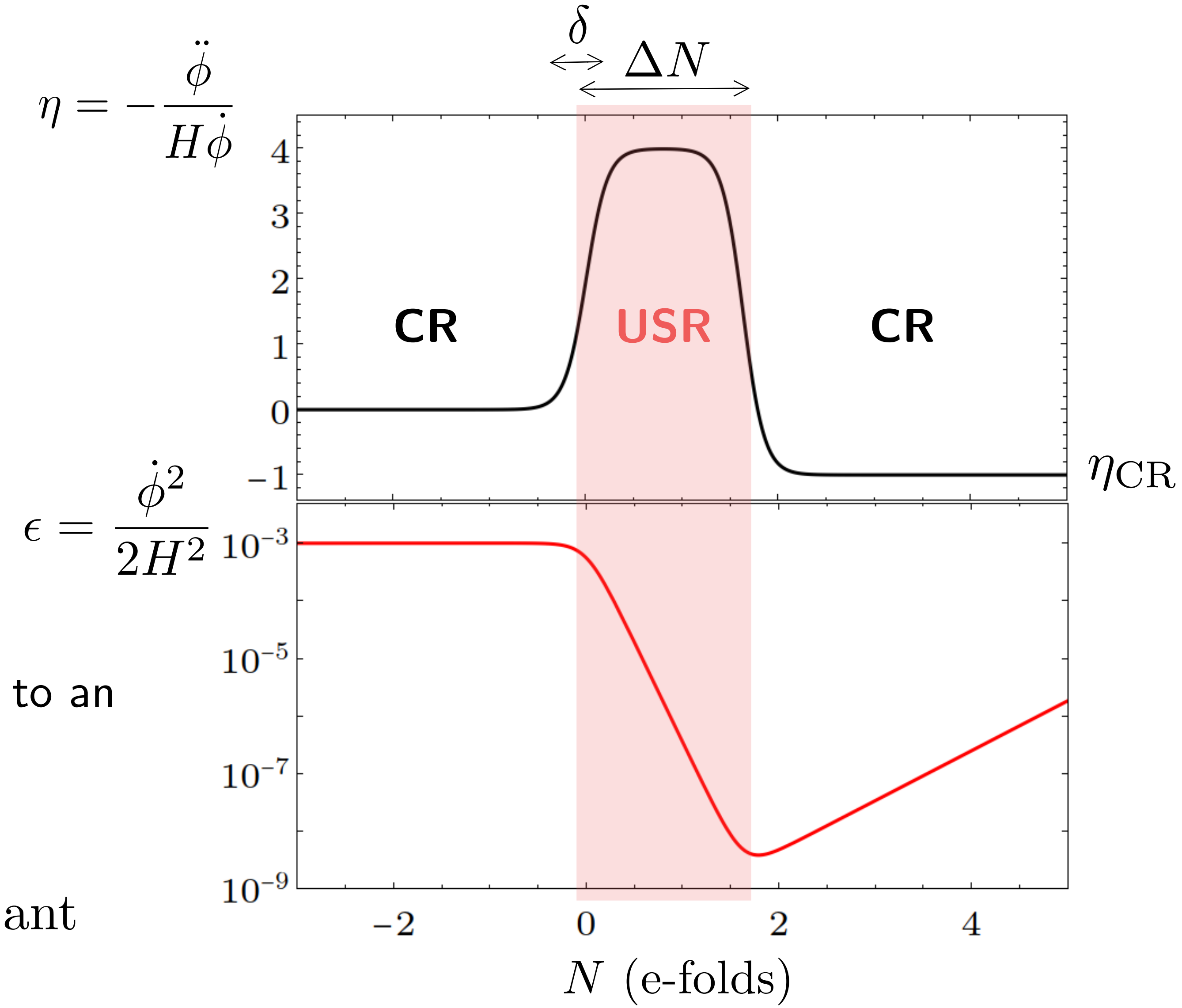
→ **What about much smaller scales** $k \gg k_\mathrm{CMB}$ **?**

A phase of Ultra-Slow Roll during inflation





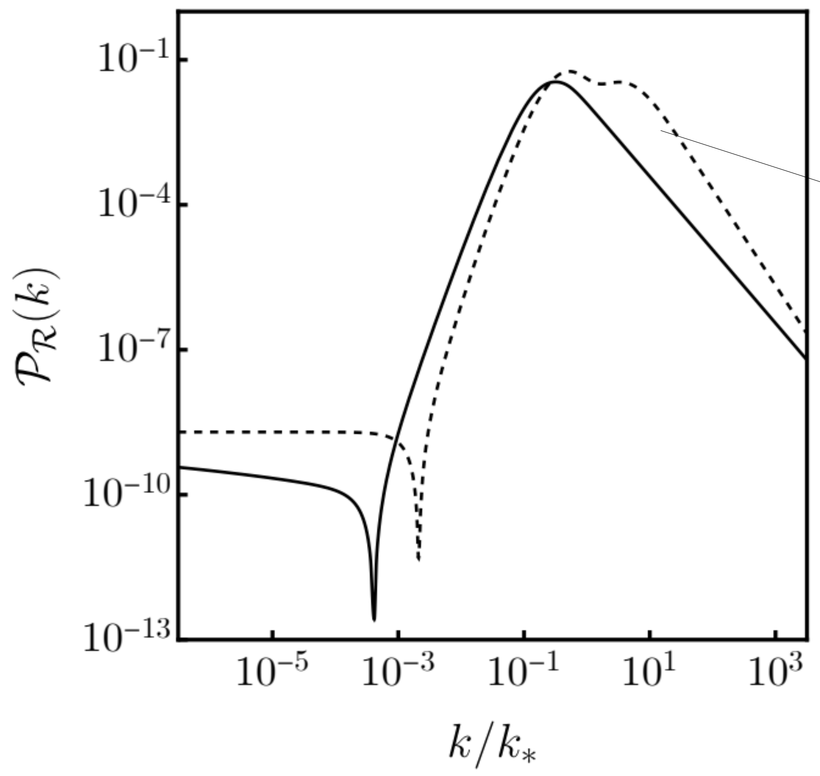
1 **Inflexion point** in inflaton potential leads to an **Ultra Slow-Roll (USR)** phase

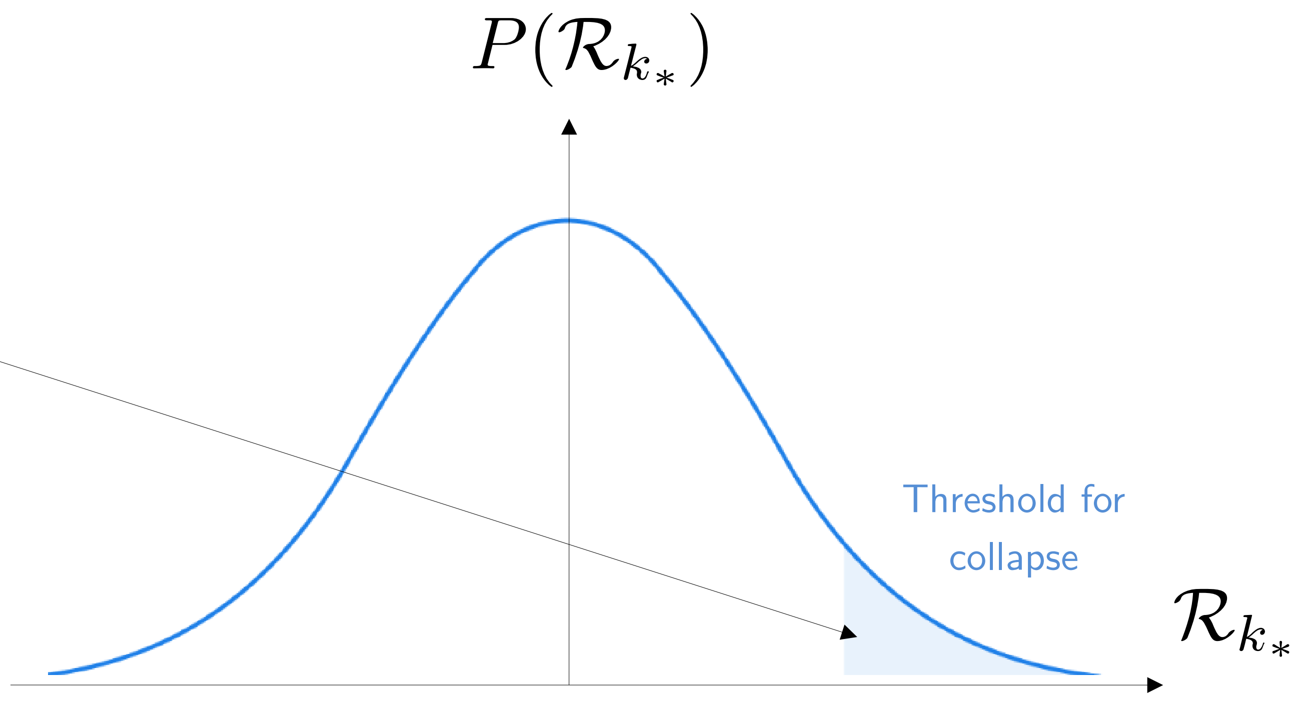
$\rightarrow$ affect scales $k \gg k_{\rm CMB}$

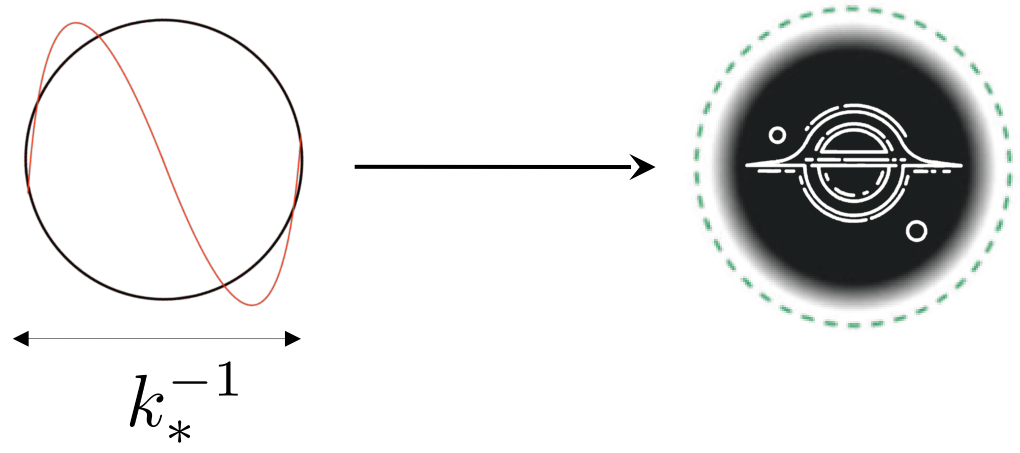
2 Consider **Constant Roll (CR)** $\eta = \text{constant}$ phases prior and posterior to USR

A phase of Ultra-Slow Roll during inflation

$\mathcal{P}_{\mathcal{R}} \propto H^4/\dot{\phi}^2$







→ A large spectrum **favors** large overdensities

→ Overdensities **above** critical threshold **collapse**

A phase of Ultra-Slow Roll during inflation

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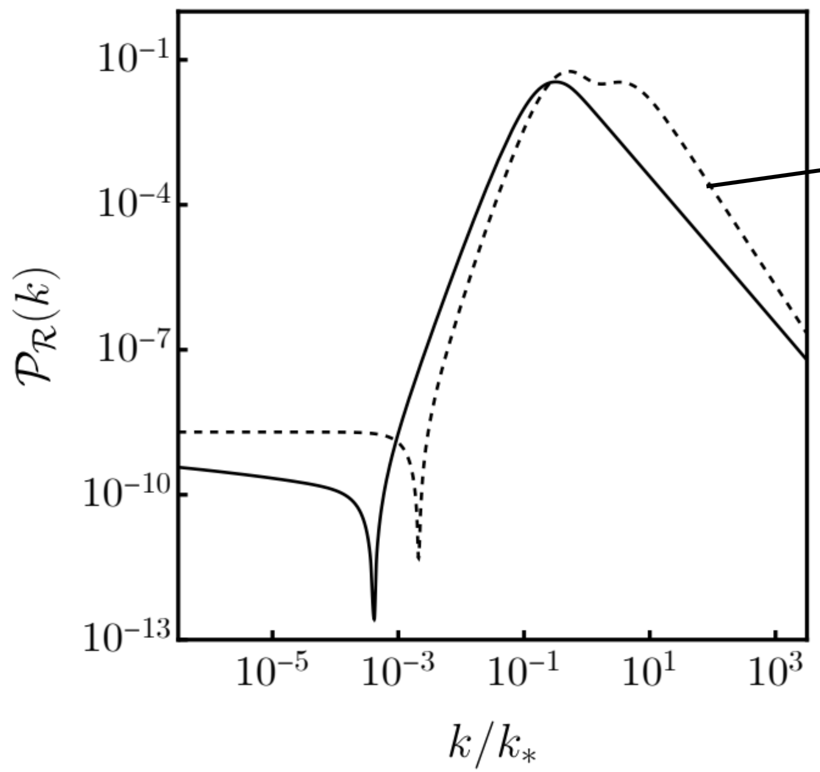

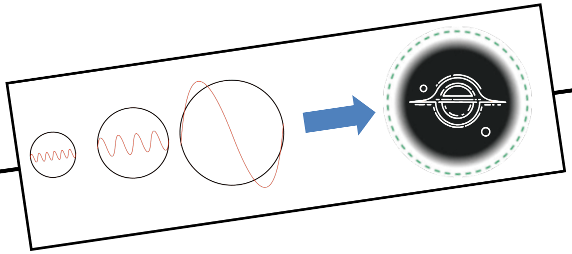

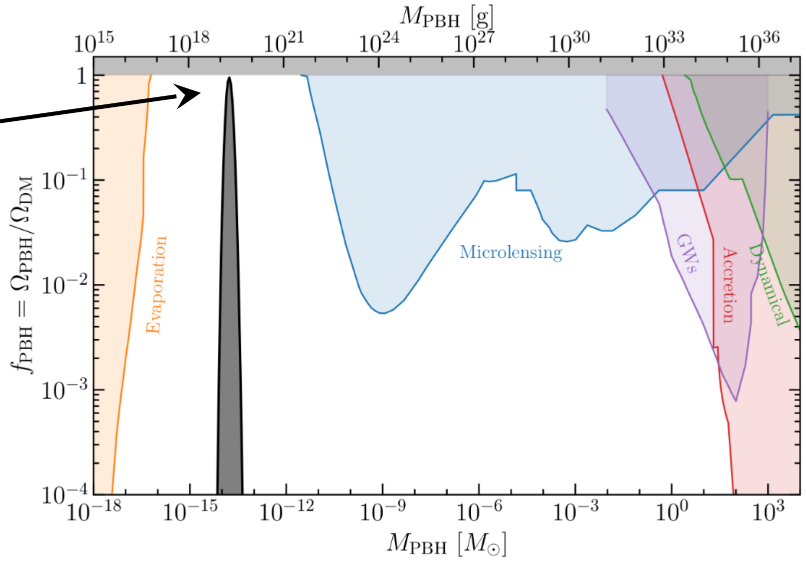

1 Hubble-size overdensities can form **Primordial Black Holes (PBH)**

$\rightarrow$ **can explain 100% of dark matter**

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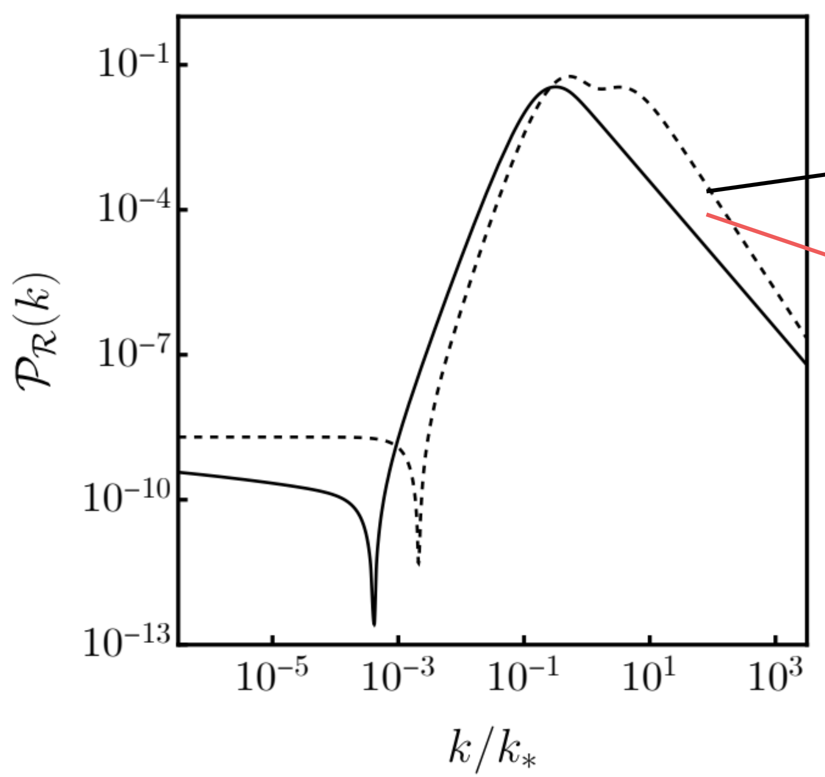
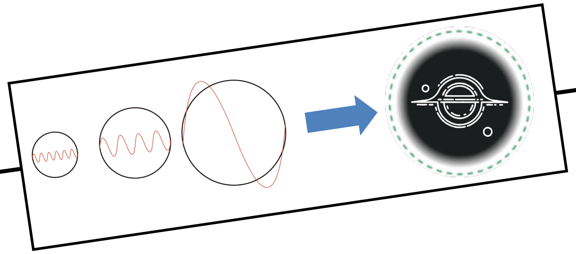
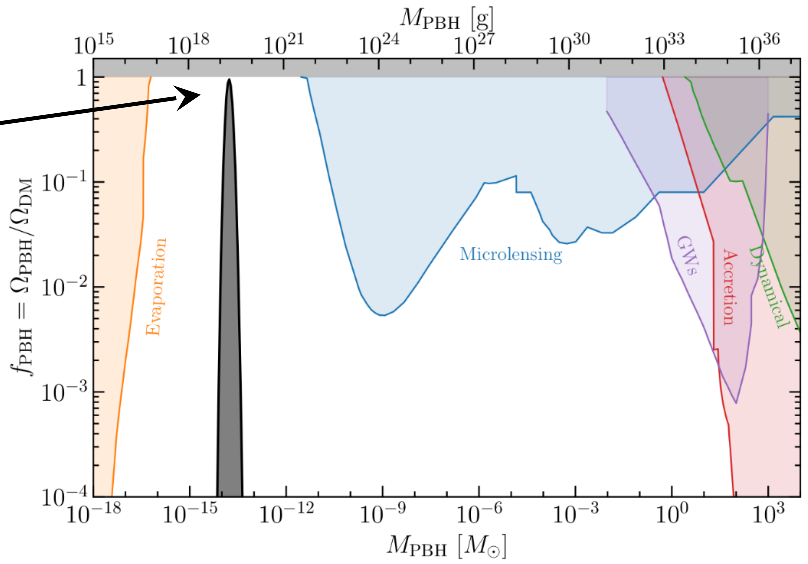
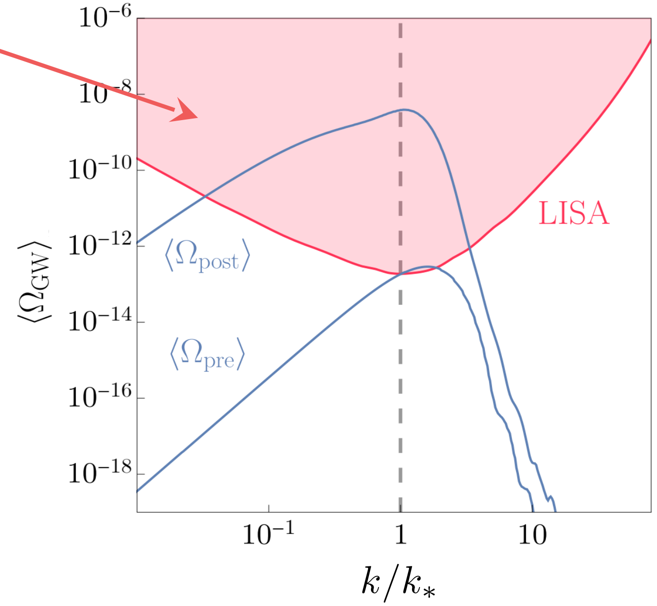






$$\partial^2 h_{ij} \sim \left[\partial_i \mathcal{R} \partial_j \mathcal{R}\right]^{\rm TT}$$

[Domenech 2109.01398]



1 Hubble-size overdensities can form **Primordial Black Holes (PBH)**

$\rightarrow$ **can explain 100% of dark matter**

2 Source stochastic **Gravitational Wave (GW)** background

$\rightarrow$ **will be probed by LISA**

A phase of Ultra-Slow Roll during inflation

Challenge: PBH production is sensitive to the tail of the PDF

Goal: estimate the PDF of ζ induced by USR phase

A phase of Ultra-Slow Roll during inflation

Challenge: PBH production is sensitive to the tail of the PDF

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Spoiler: it might not be a Gaussian

Non-linearities & non-Gaussianities

Perturbations beyond leading order

→ to go **beyond** linear order, consider **ADM parametrization** of the metric

$$\mathrm{d}s^2 = -\mathcal{N}^2\,\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\,\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\,\mathrm{d}t\right),$$

1 Express the field as $\Phi(t, \boldsymbol{x}) = \phi(t) + \delta\phi(t, \boldsymbol{x})$

→ 2 gauges $\delta\phi(t, \boldsymbol{x}) = 0, \quad h_{ij} = a^2 e^{2\zeta}\delta_{ij}$ $\zeta(t, \boldsymbol{x}) = 0, \quad h_{ij} = a^2\delta_{ij}$

2 Perform **gradient** expansion, on **super-horizon** scales one finds a **non-linear** relation [Lyth, Malik, Sasaki '05]

$$\zeta = \delta N = N\left[\{\phi(N_i) + \delta\phi, \pi(N_i) + \delta\pi\} \to \phi(N_{\mathrm{end}})\right] - N\left[\{\phi(N_i), \pi(N_i)\} \to \phi(N_{\mathrm{end}})\right] \qquad \pi \equiv \phi'$$

→ Compare **e-folds** of expansion between **perturbed** and **unpertubed** trajectories

→ Can be computed by simply **solving background EOM**

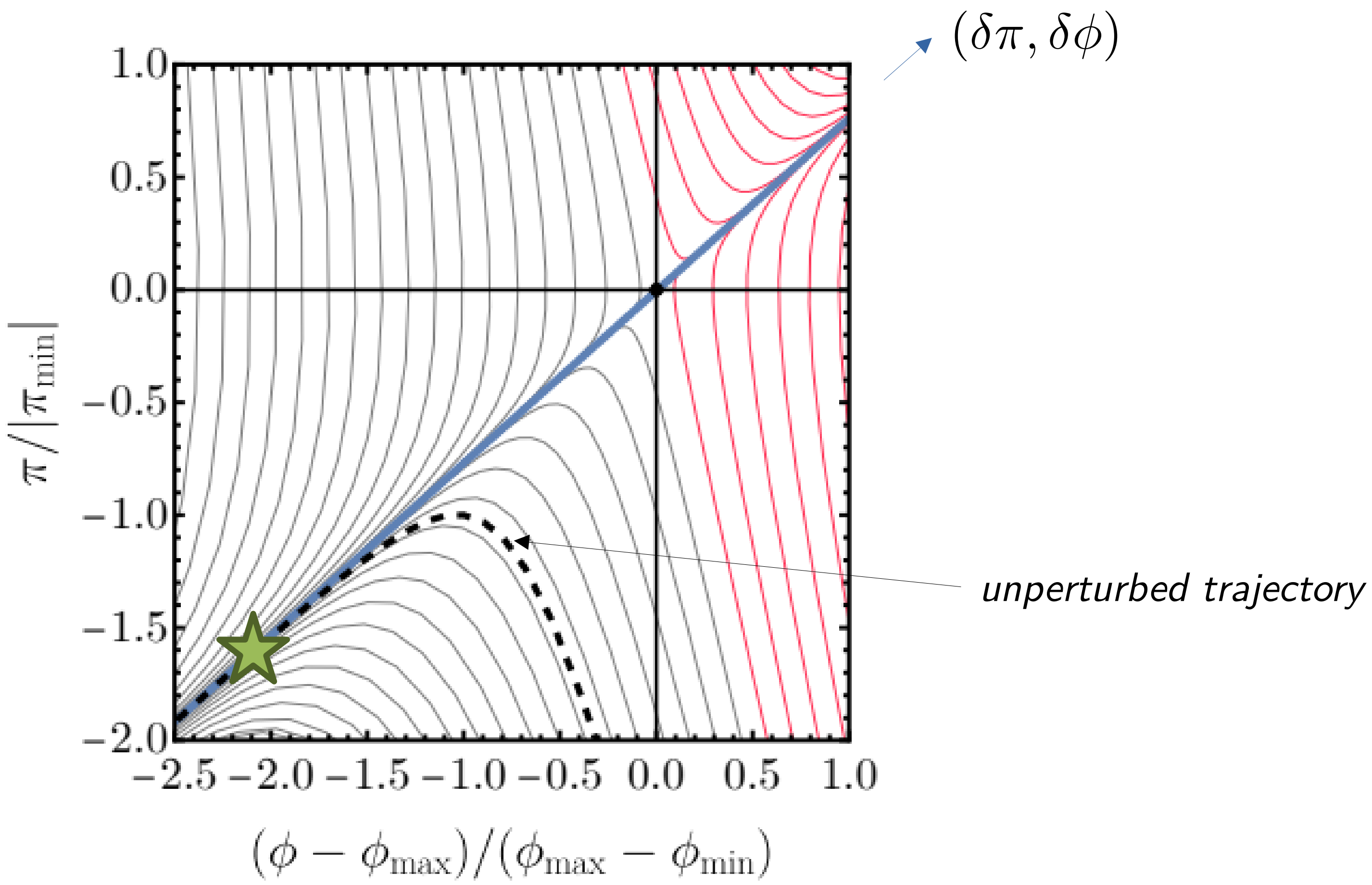
3 Should be applied at N_i when relevant modes (at **linear** order) are **frozen** $\dot{\zeta} = 0$

→ **Perturbed** quantities reduce to a **single DOF** $\delta\pi = (\pi'/\pi)|_{N_i}\delta\phi$

Perturbations beyond leading order

1 Take N_i (★) on the **constant-roll (CR) attractor** $\mathrm{d}\pi/\mathrm{d}\phi \simeq -\eta_{\mathrm{CR}}$

$\rightarrow$ During CR, **EOM** for the field becomes a **linear ODE** $\phi(N) \sim e^{-\eta_{\mathrm{CR}} N}$



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1 Take N_i (★) on the **constant-roll (CR) attractor** $\mathrm{d}\pi/\mathrm{d}\phi \simeq -\eta_{\mathrm{CR}}$

→ During CR, **EOM** for the field becomes a **linear ODE** $\phi(N) \sim e^{-\eta_{\mathrm{CR}} N}$

→ Deduce $\delta\phi = \phi(N_i)\left(e^{\eta_{\mathrm{CR}}\delta N} - 1\right)$

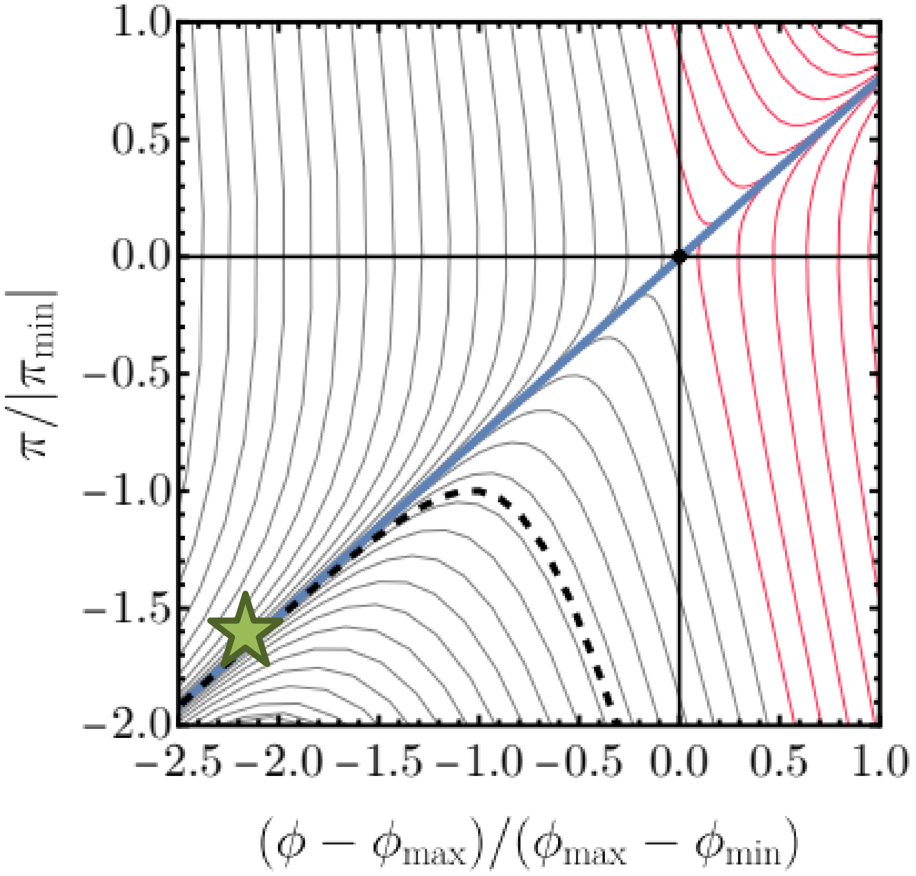
2 Extract **full non-linear relation**

$$\eta_{\mathrm{CR}}\zeta = \log\left(1 - \eta_{\mathrm{CR}} H \frac{\delta\phi}{\dot\phi}\right)$$

3 **No information** about **statistics of** $\delta\phi$ **!**

→ If one **assumes** $\delta\phi$ **Gaussian** $\sigma^2_{\delta\phi} = \int \mathcal{P}_{\delta\phi}(k)\,\mathrm{d}\log k$

$$P(\zeta) = P(\delta\phi)\left|\frac{\mathrm{d}\delta\phi}{\mathrm{d}\zeta}\right| \xrightarrow{\text{tail}} P(\zeta) \sim e^{\eta_{\mathrm{CR}}\zeta}$$



The stochastic approach

1 Consider **backreaction of short modes on background** (large wavelengths) [Starobinsky '86]

$k \ll \sigma a H$ | $k \gg \sigma a H$

$$\phi' - \pi = \xi_\phi$$

$$\pi' + (3 - \pi'^2/2)(\pi + \log V_\phi) = \xi_\pi$$

→ Gaussian **white noise** with variance $\mathrm{Var}[\xi_\phi(N)] = \mathcal{P}_{\delta\phi}(k = \sigma a(N) H(N))$

→ **Coarse graining** parameter $\sigma \lesssim 0.1$: delay from horizon crossing $\dot{\zeta} \simeq 0$
classicalization → $\xi_\pi = (\pi'/\pi)_N \xi_\phi$ [Ballesteros et al. '20]

2 Use the δN **formalim** with **stochastic** approach [Ezquiaga, Garcıa-Bellido, Vennin '20]

→ Solve **stochastic** EOMs, determine δN until **end of inflation**

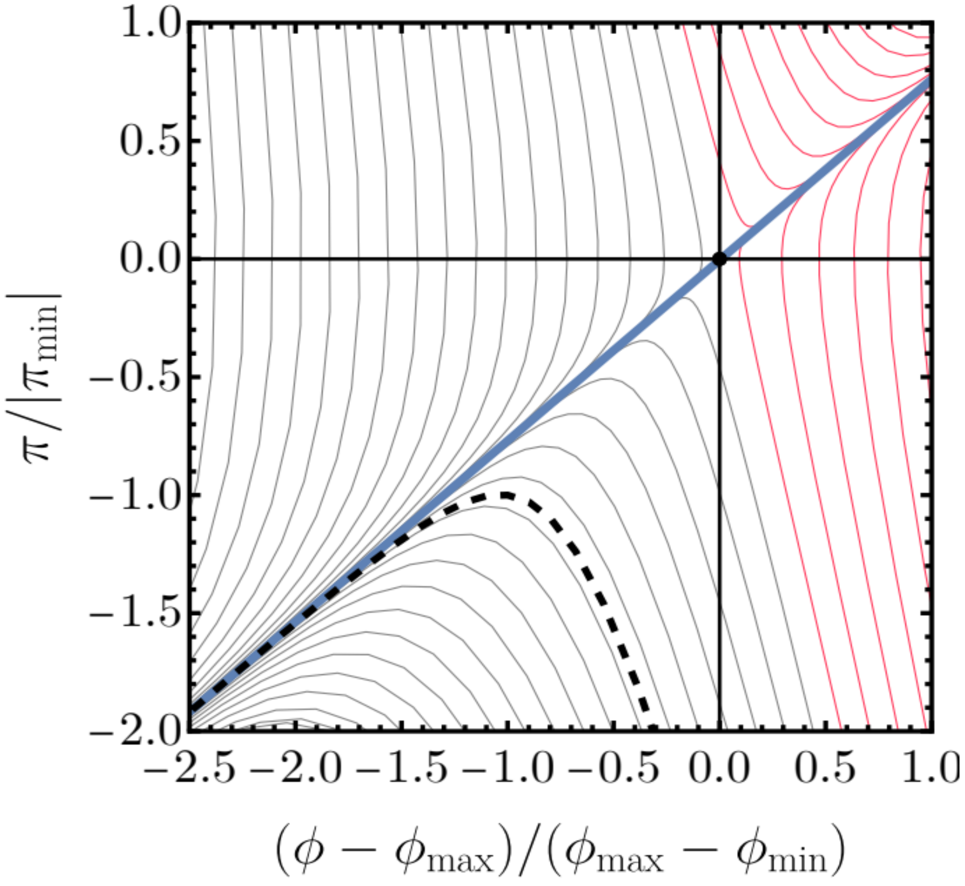
→ Extract **PDF** for curvature perturbation $\zeta \leftrightarrow \delta N$

The classical vs stochastic approaches

→ **Classical** kick along (late-time) **CR attractor**

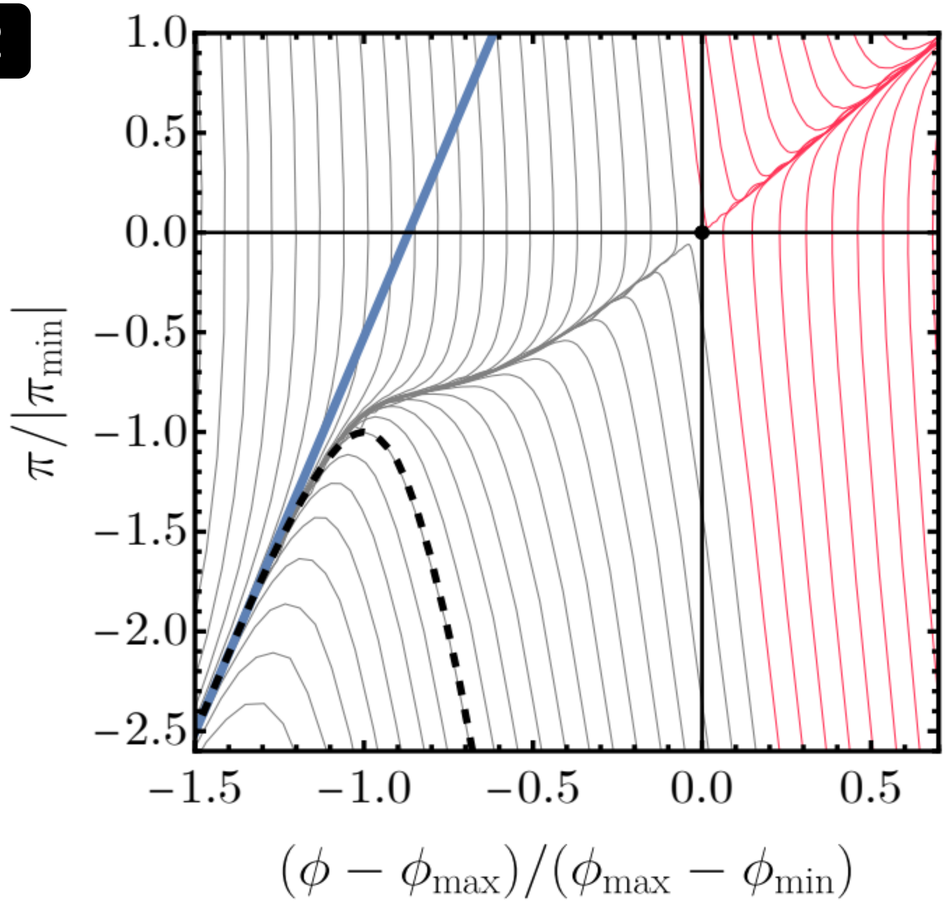
→ **Stochastic** kicks along **local attractor**

1



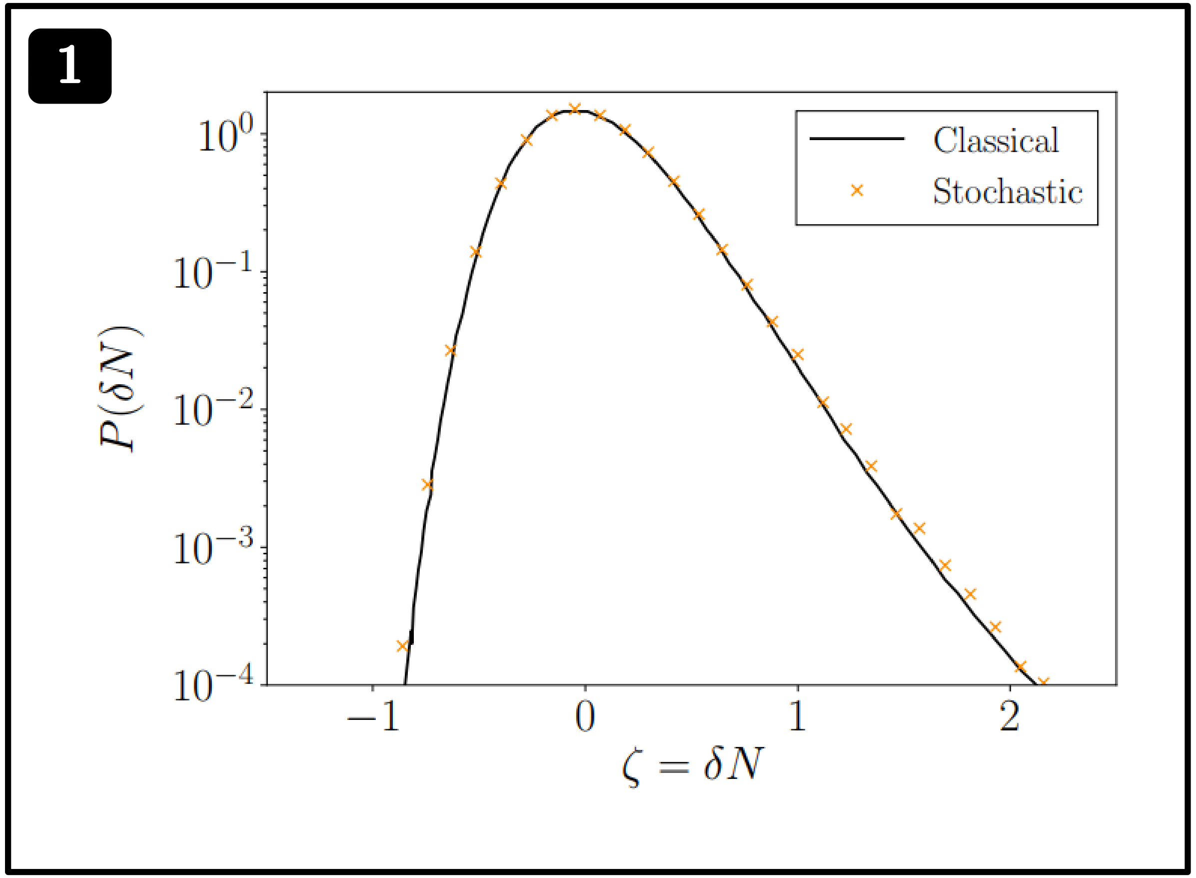
→ Equivalence **garanteed** by CR attractor

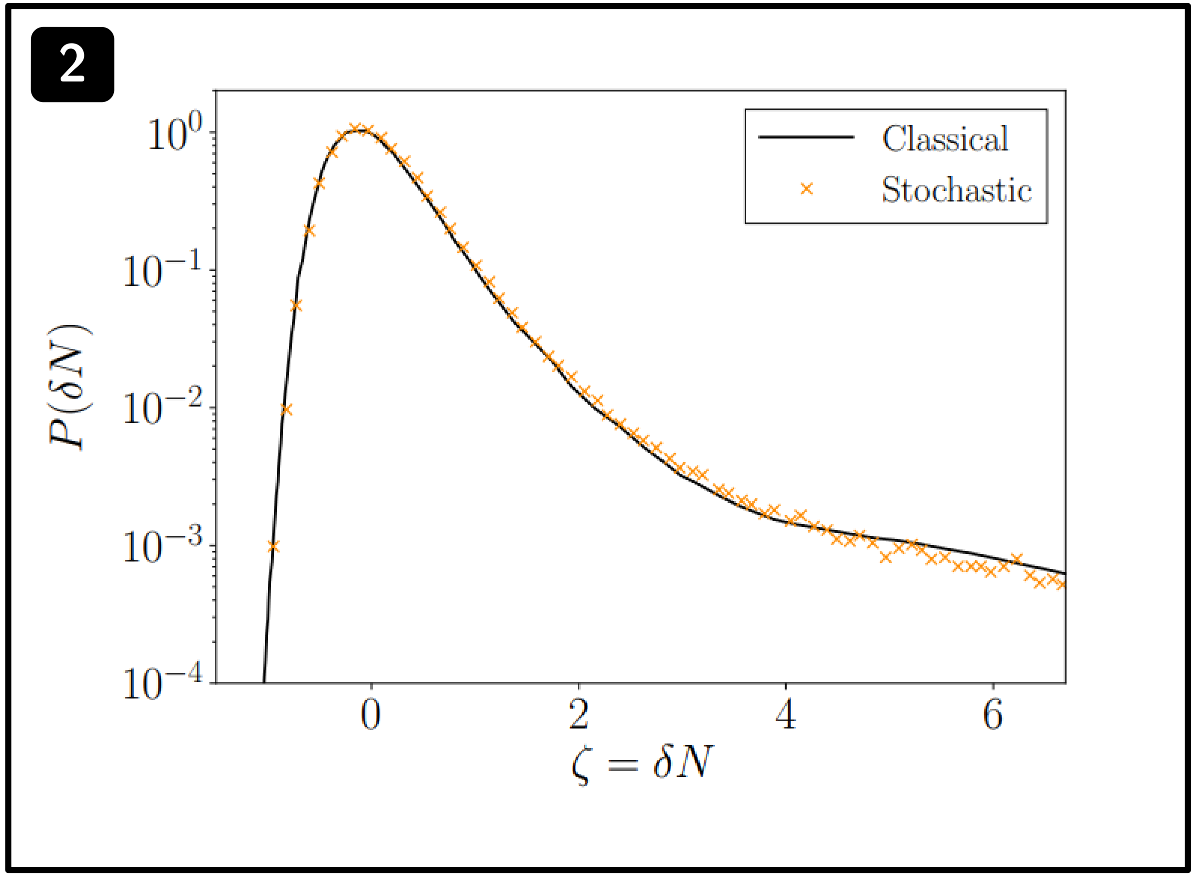
2



→ Equivalence *a priori* **not** garanteed

The classical vs stochastic approaches





$\rightarrow$ Both approaches agree to an **excellent** degree

$\rightarrow$ Tails are **non-Gaussian as a consequence of non-linearities**

Intrinsic non-Gaussianities

The bispectrum

1 **Expand** $$\zeta = \varphi - \frac{\eta_{\mathrm{CR}}}{2}\varphi^2 + \frac{\eta_{\mathrm{CR}}^2}{3}\varphi^3 + \cdots .$$

$$\boxed{\varphi \equiv -(H/\dot{\phi})\delta\phi}$$

2 **Deviations** to Gaussianity appear in the **bispectrum**:

$$\begin{aligned}\langle\hat{\zeta}_{\boldsymbol{p}}\hat{\zeta}_{\boldsymbol{q}}\hat{\zeta}_{\boldsymbol{k}}\rangle_c &= B(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k})(2\pi)^3\delta^{(3)}(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{k}) \\ &= \left[\boxed{\mathcal{I}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k})} + \boxed{\mathcal{N}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k})}\right](2\pi)^3\delta^{(3)}(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{k}) .\end{aligned}$$

Intrinsic

$$\langle\hat{\varphi}_{\boldsymbol{p}}\hat{\varphi}_{\boldsymbol{q}}\hat{\varphi}_{\boldsymbol{k}}\rangle_c$$

→ vanishes in **absence** of interactions

e.g. for Gaussian φ

Non-linearities

$$-\frac{\eta_{\mathrm{CR}}}{2}\left(\int\frac{\mathrm{d}^3\ell}{(2\pi)^3}\langle\hat{\varphi}_{\boldsymbol{k}-\boldsymbol{\ell}}\hat{\varphi}_{\boldsymbol{\ell}}\hat{\varphi}_{\boldsymbol{q}}\hat{\varphi}_{\boldsymbol{p}}\rangle_c + \mathrm{perm.}\right)$$

→ gauge transformation, local type

→ in absence of interactions, **non-vanishing** $\mathcal{N}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{k}) = -\eta_{\mathrm{CR}}\left(|\varphi_q|^2|\varphi_k|^2 + |\varphi_p|^2|\varphi_k|^2 + |\varphi_q|^2|\varphi_p|^2\right)$

→ decomposition **not so obvious** at higher order in interaction Hamiltonian

Computing intrinsic contribution to bispectrum

In cosmology we are interested in the **time-evolution** of correlation functions: **the *in-in* formalism**

$$\langle \hat{\mathcal{O}}(t) \rangle = \langle 0 | \hat{F}^{-1}(t, -\infty_+) \hat{\mathcal{O}}_I(t) \hat{F}(t, -\infty_-) | 0 \rangle \qquad \hat{F}(t, t_0) = T \exp\left(-i \int_{t_0}^{t} \mathrm{d}t' \hat{H}_I(t') \right)$$

$$\langle \hat{\mathcal{O}}(t) \rangle = \langle 0 | \hat{\mathcal{O}}_I(t) | 0 \rangle + 2\,\mathrm{Im} \left\{ \int_{-\infty_-}^{t} \mathrm{d}t' \; \langle 0 | \hat{\mathcal{O}}_I(t) \hat{H}_I(t') | 0 \rangle \right\} + \mathcal{O}\left(\hat{H}_I^2 \right) .$$

1 **To compute** $\langle \hat{\varphi}_{\boldsymbol{p}} \hat{\varphi}_{\boldsymbol{q}} \hat{\varphi}_{\boldsymbol{k}} \rangle_c$ **,** expand the action at **cubic order in** $\boxed{\varphi \equiv -(H/\dot{\phi})\delta\phi}$

[Maldacena '02]

$$\mathcal{L}_I^{(3)} = -c_0\, a^3 H^2 \varphi^3 - c_1\, a^3 \varphi\, \dot{\varphi}^2 - c_2\, a^3 \dot{\varphi}\, \partial_i \varphi\, \partial^i \left(\partial^{-2} \dot{\varphi} \right) - c_3\, a\, \varphi\, \left(\partial_i \varphi \right)^2 - c_4\, a^3 \varphi \left(\partial_i \partial_j \left(\partial^{-2} \dot{\varphi} \right) \right)^2 + \frac{\mathrm{d}}{\mathrm{d}t} \left(c_5\, a^3 H \varphi^3 \right),$$

2 **Most relevant term is** $c_0 = \frac{1}{3} \epsilon\, \eta\, \epsilon_3 (\epsilon + 2\eta - \epsilon_3 - \epsilon_4 - 3) \sim \partial_\phi^3 V \qquad \epsilon_n \equiv \dot{\epsilon}_{n-1}/(H \epsilon_{n-1})$

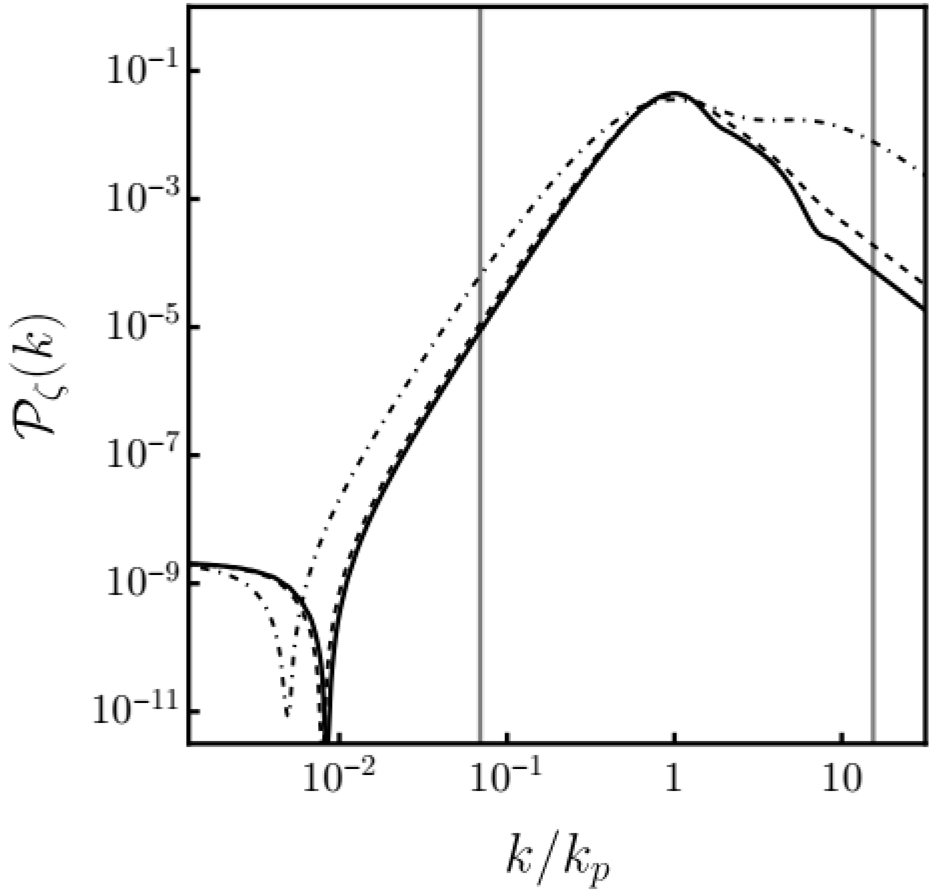
[Ballesteros, Gambín 2404.07196]

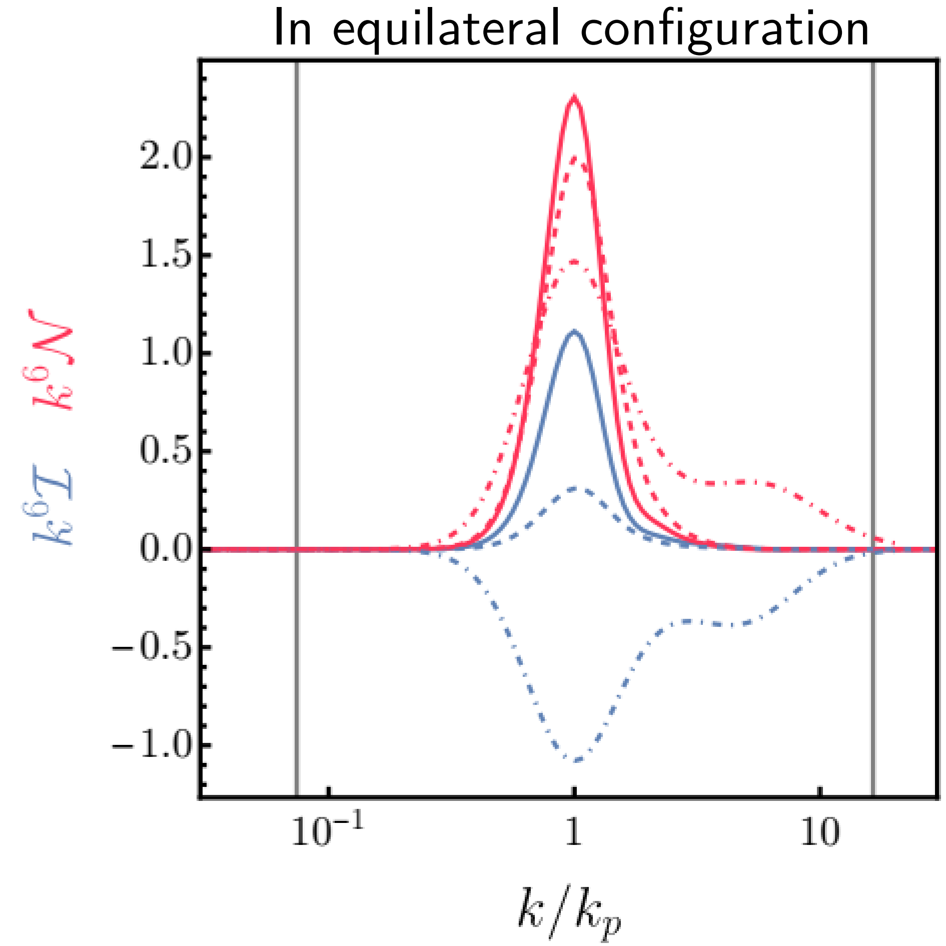
3 **Intrinsic** piece is thus $\mathcal{I}(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{k}) = 3! \int_{-\infty_-}^{t} \mathrm{d}t' a(t')^3 c_0(t') H(t')^2 \; 2\,\mathrm{Im} \left[\varphi_p(t) \varphi_p^*(t') \varphi_q(t) \varphi_q^*(t') \varphi_k(t) \varphi_k^*(t') \right],$

solution to the free mode-equation $\varphi_k'' + (3 + \epsilon - 2\eta) \varphi_k' + \left(\frac{k}{aH} \right)^2 \varphi_k = 0 \,,$

Intrinsic and non-linear contributions to bispectrum

CR-USR-CR transitions: —— **sharp** - - - - - **intermediate** -·-·-·- **smooth**





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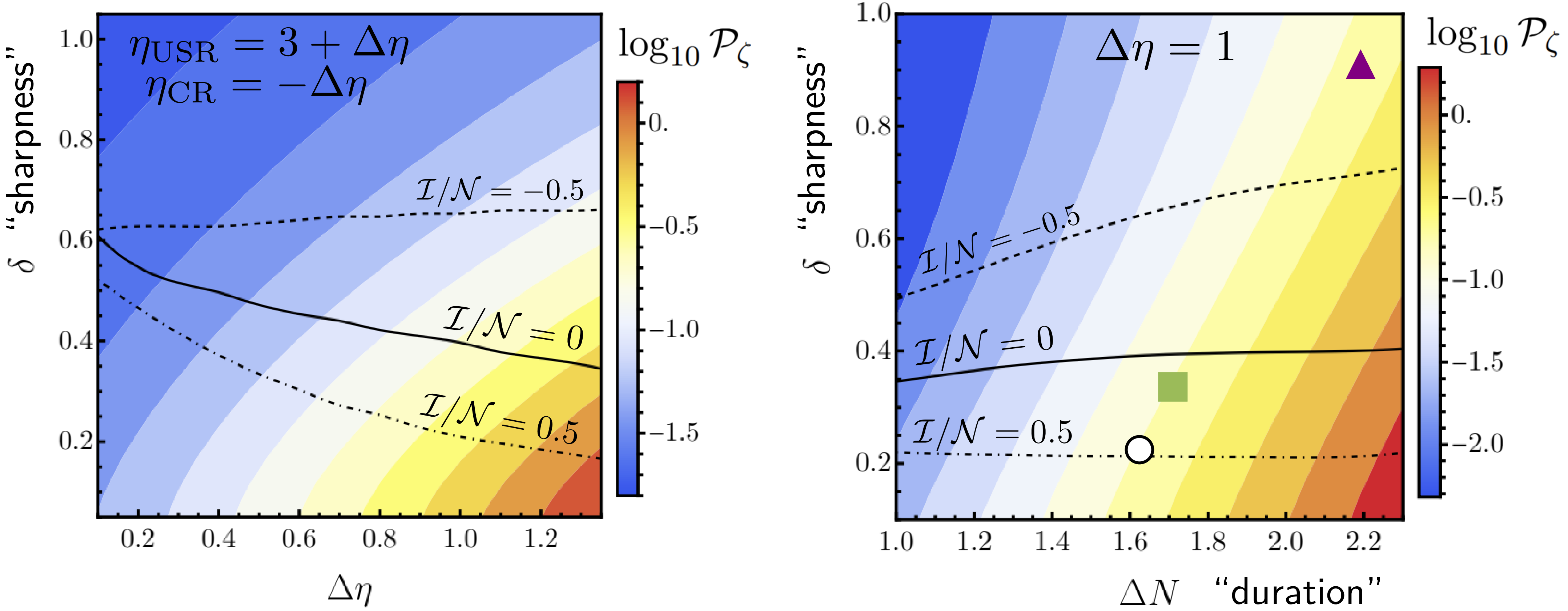
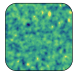


$\rightarrow$ Intrinsic non-Gaussianities **comparable** to non-linear contribution except for $\delta \sim 0.4$

Non-Gaussianities on the lattice 

The lattice idea

→ **Goal:** extract PDF of a stochastic variable, built on the lattice, that reproduce the statistical properties from *in-in* evaluated at the end of inflation

$$\langle \tilde{\varphi}_{\boldsymbol{k}_1} \tilde{\varphi}_{\boldsymbol{k}_2} \cdots \tilde{\varphi}_{\boldsymbol{k}_n} \rangle_c = \langle \hat{\varphi}_{\boldsymbol{k}_1} \hat{\varphi}_{\boldsymbol{k}_2} \cdots \hat{\varphi}_{\boldsymbol{k}_n} \rangle_c$$

stochastic variable on the lattice

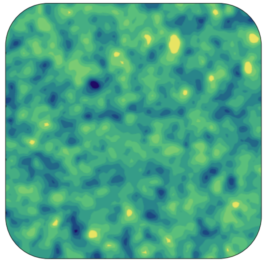
quantum field, input from *in-in*

1 Assume $\tilde{\varphi}_{\boldsymbol{k}} = \sum_n \tilde{\varphi}^{(n)}_{\boldsymbol{k}}$: expansion in powers of $\tilde{\varphi}^{(n)}_{\boldsymbol{k}} \sim \mathcal{G}_n |\varphi_k|^{-(n-1)} \sim (H/M_P)^{n-1}$

$$\tilde{\varphi}^{(n)}_{\boldsymbol{k}} \equiv \frac{1}{n!} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \cdots \frac{\mathrm{d}^3 k_{n-1}}{(2\pi)^3} \boxed{\mathcal{G}_n(\boldsymbol{k}, -\boldsymbol{k}_1, \cdots, -\boldsymbol{k}_{n-1})} \boxed{\frac{\tilde{\psi}_{\boldsymbol{k}_1}}{|\varphi_{k_1}|} \cdots \frac{\tilde{\psi}_{\boldsymbol{k}_{n-1}}}{|\varphi_{k_{n-1}}|}} (2\pi)^3 \delta^{(3)}(-\boldsymbol{k} + \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_{n-1}) \,.$$

2 Determine $\mathcal{G}_n$ order by order

3 Sample random variable $\tilde{\psi}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$



4 Use non-linear relation $\varphi \to \zeta$

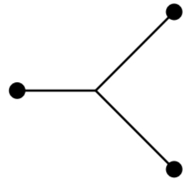
5 Extract PDF for $\tilde{\zeta}$

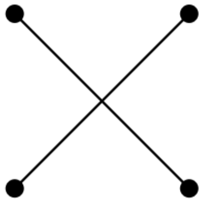
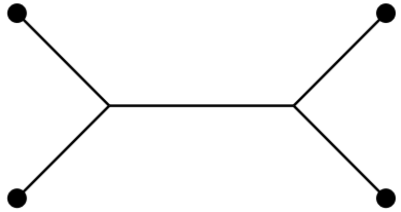
The lattice idea

1 Generalize **intrinsic correlators** at **higher orders**

$$\langle \hat{\varphi}_{\boldsymbol{k}_1} \cdots \hat{\varphi}_{\boldsymbol{k}_n} \rangle_c \equiv \mathcal{I}_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n)(2\pi)^3 \delta^3(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n)\,.$$

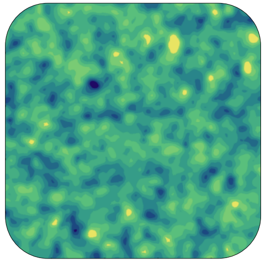
2 Use **matching condition** to get

- $\mathcal{G}_2(\boldsymbol{k}, \boldsymbol{p}) = 2|\varphi_k||\varphi_p|$
- $\mathcal{G}_3(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q}) = \mathcal{I}_3(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q})\,, \quad =$ 
- $$\mathcal{G}_4(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\ell}) = \mathcal{I}_4(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{\ell}) - \frac{4}{9}\left[\frac{\mathcal{I}_3(\boldsymbol{k}, -\boldsymbol{p}-\boldsymbol{k}, \boldsymbol{p})\mathcal{I}_3(\boldsymbol{q}, -\boldsymbol{\ell}-\boldsymbol{q}, \boldsymbol{\ell})}{|\varphi_{|\boldsymbol{p}+\boldsymbol{k}|}|^2} + (\boldsymbol{p} \to \boldsymbol{q}, \boldsymbol{\ell})\right].$$

$=$  $+$  $+ \quad (t, u)$

The lattice pipeline

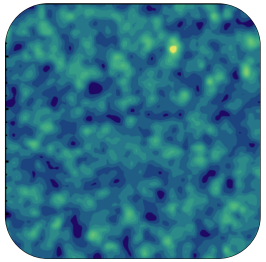
$\tilde{\psi}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$



$$\longrightarrow \boxed{\tilde{\varphi}^{(3)}(\boldsymbol{x})} + \boxed{\tilde{\varphi}^{(2)}(\boldsymbol{x})} \longrightarrow \boxed{\tilde{\varphi}(\boldsymbol{x})}$$

$$\downarrow$$

$\tilde{\zeta}(\boldsymbol{x})$



$$\downarrow$$

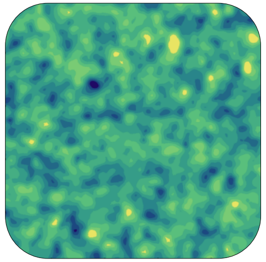
PDF

$P(\tilde{\zeta})$

The lattice pipeline

$\tilde{\psi}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$

Real space
$\boldsymbol{x}$



$\downarrow$ $\mathcal{F}$ourier transform

Fourier space
$\boldsymbol{k}$

$\tilde{\psi}_{\boldsymbol{k}}$

$\downarrow$ $\times |\varphi_k|$

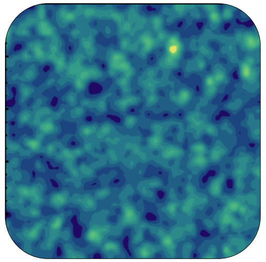
$\tilde{\varphi}^{(2)}_{\boldsymbol{k}}$

$\xrightarrow{\mathcal{F}^{-1}}$

$\tilde{\varphi}^{(3)}(\boldsymbol{x}) + \tilde{\varphi}^{(2)}(\boldsymbol{x}) \longrightarrow \tilde{\varphi}(\boldsymbol{x})$

$\downarrow$

$\tilde{\zeta}(\boldsymbol{x})$



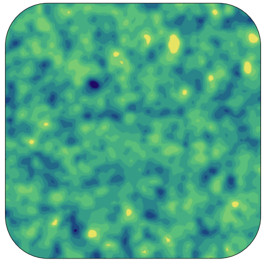
$\downarrow$

PDF
$P(\tilde{\zeta})$

The lattice pipeline

$\tilde{\psi}(\boldsymbol{x}) \sim \mathcal{N}(0,1)$

Real space $\boldsymbol{x}$



$\downarrow \mathcal{F}$

Fourier space $\boldsymbol{k}$

$\tilde{\psi}_{\boldsymbol{k}}$

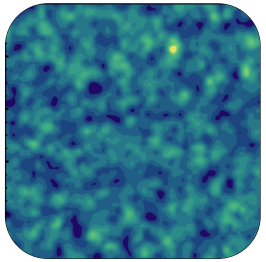
$\tilde{\varphi}^{(3)}_{\boldsymbol{k}}$

$\uparrow \mathcal{F}^{-1}$

$\tilde{\varphi}^{(3)}(\boldsymbol{x}) + \tilde{\varphi}^{(2)}(\boldsymbol{x}) \longrightarrow \tilde{\varphi}(\boldsymbol{x})$

$\downarrow$

$\tilde{\zeta}(\boldsymbol{x})$



$\downarrow$

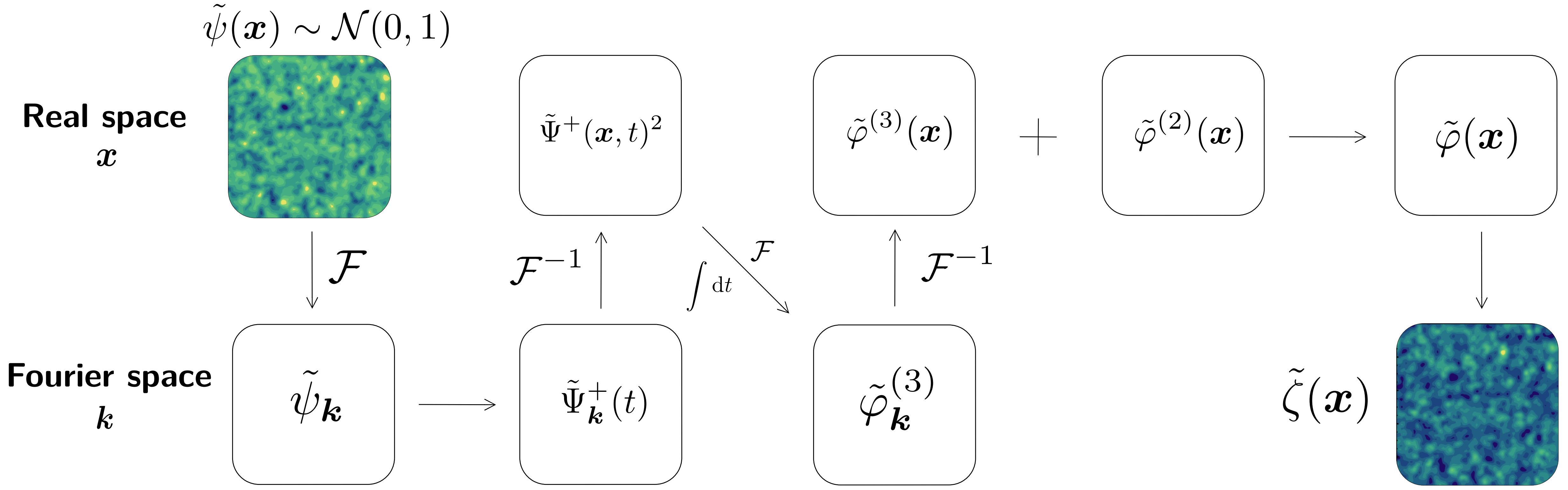
PDF

$P(\tilde{\zeta})$

$$\tilde{\varphi}^{(3)}_{\boldsymbol{k}} = (-i)\int_{-\infty}^{t_e} \mathrm{d}t\; a(t)^3 c_0(t) H(t)^2 \left\{ \lambda_k(t)\left[\tilde{\Psi}^+(t)^2\right]_{\boldsymbol{k}} - \lambda_k^*(t)\left[\tilde{\Psi}^-(t)^2\right]_{\boldsymbol{k}} \right\}$$

$$\tilde{\Psi}^+_{\boldsymbol{k}}(t) \equiv \frac{\lambda_k(t)}{|\varphi_k|}\tilde{\psi}_{\boldsymbol{k}} \qquad \lambda_k(t) \equiv \varphi_k(t_e)\varphi_k^*(t)$$

The lattice pipeline



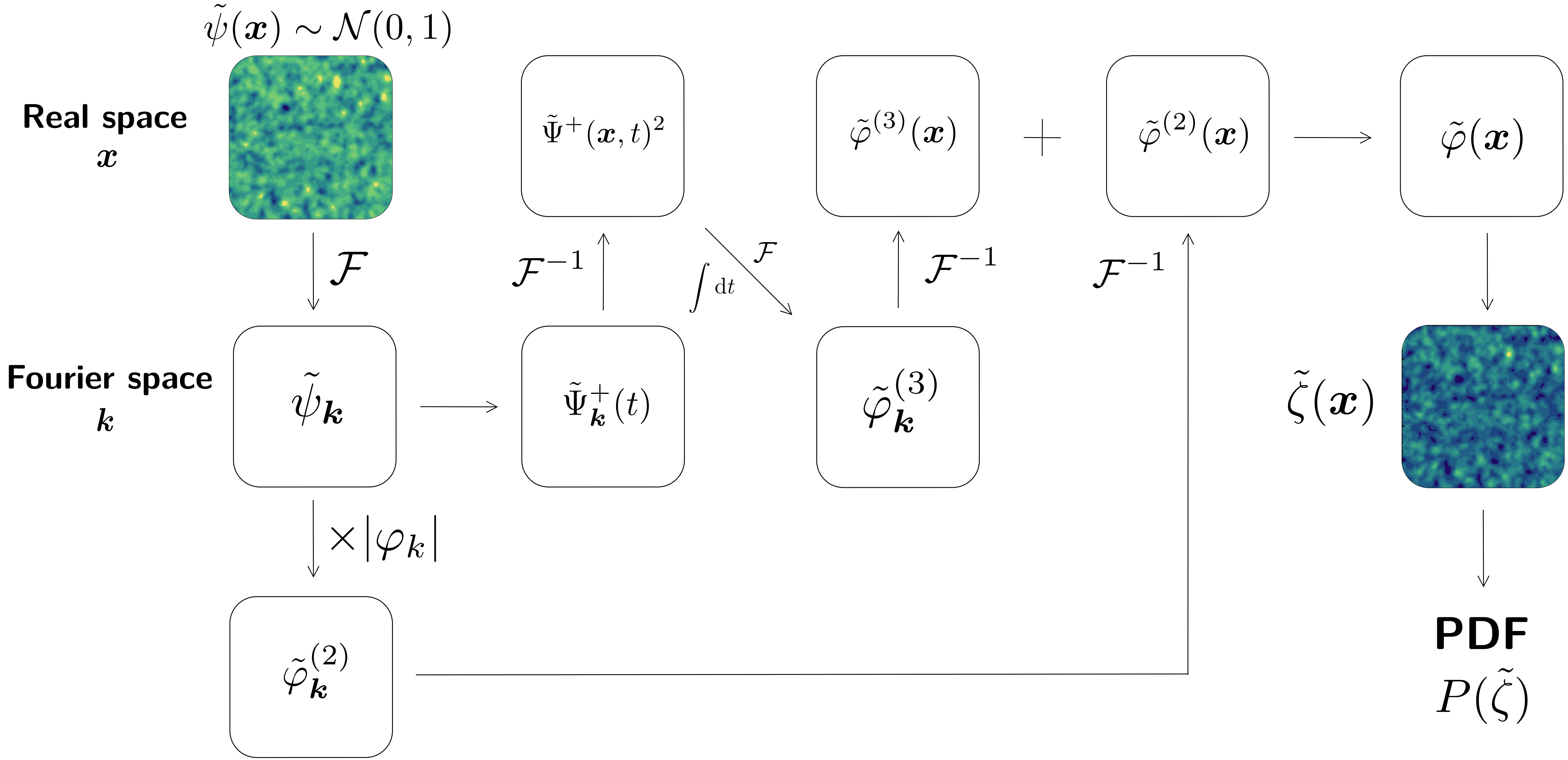
$$\tilde\varphi^{(3)}_{\boldsymbol{k}} = (-i)\int_{-\infty}^{t_e} \mathrm{d}t\; a(t)^3 c_0(t) H(t)^2 \Big\{ \lambda_k(t)\big[\tilde\Psi^+(t)^2\big]_{\boldsymbol{k}} - \lambda_k^*(t)\big[\tilde\Psi^-(t)^2\big]_{\boldsymbol{k}} \Big\}$$

$$\tilde\Psi^+_{\boldsymbol{k}}(t) \equiv \frac{\lambda_k(t)}{|\varphi_k|}\tilde\psi_{\boldsymbol{k}} \qquad \lambda_k(t) \equiv \varphi_k(t_e)\varphi_k^*(t)$$

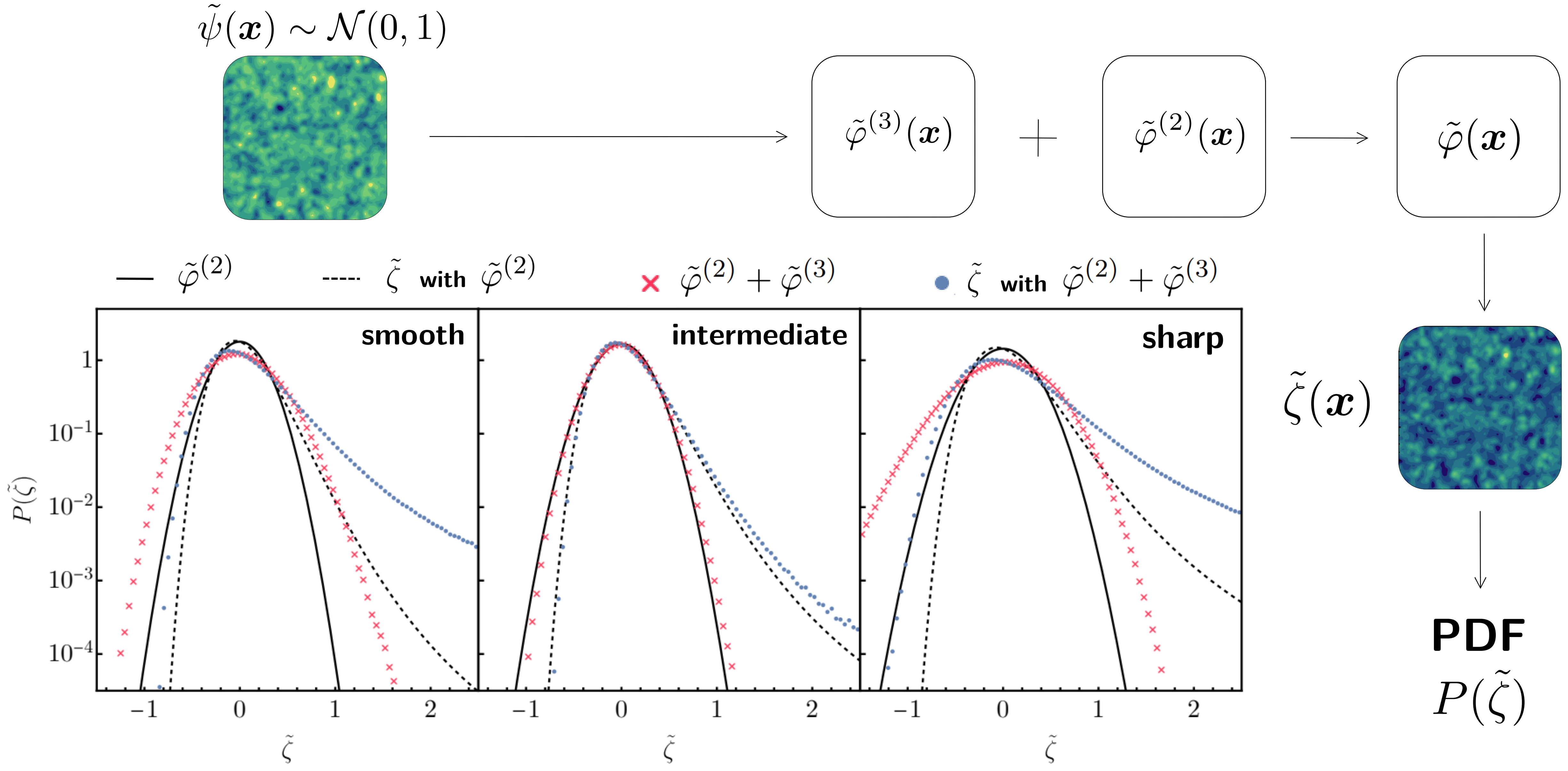
PDF

$P(\tilde\zeta)$

The lattice pipeline



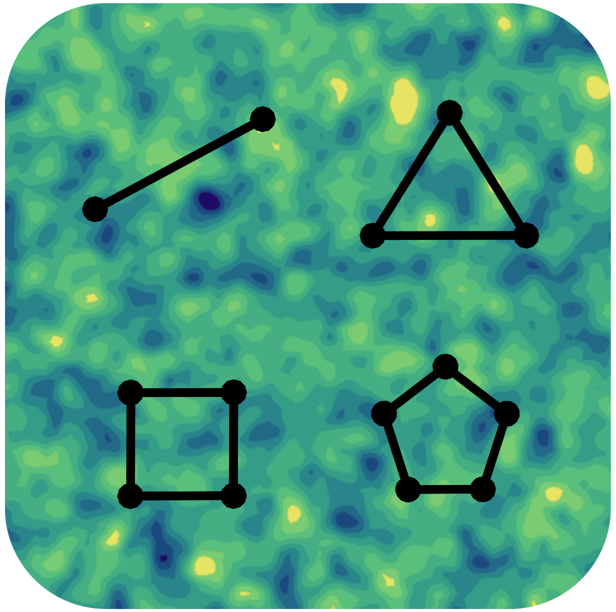
The lattice results



Take home message

1. Large perturbations generated during USR inflation are relevant for **PBH** and **GW**

2. Non-Gaussian tails in the PDF of curvature perturbations are a consequence of **non-linearities** and **intrinsic** inflaton-fluctuations non-Gaussianities



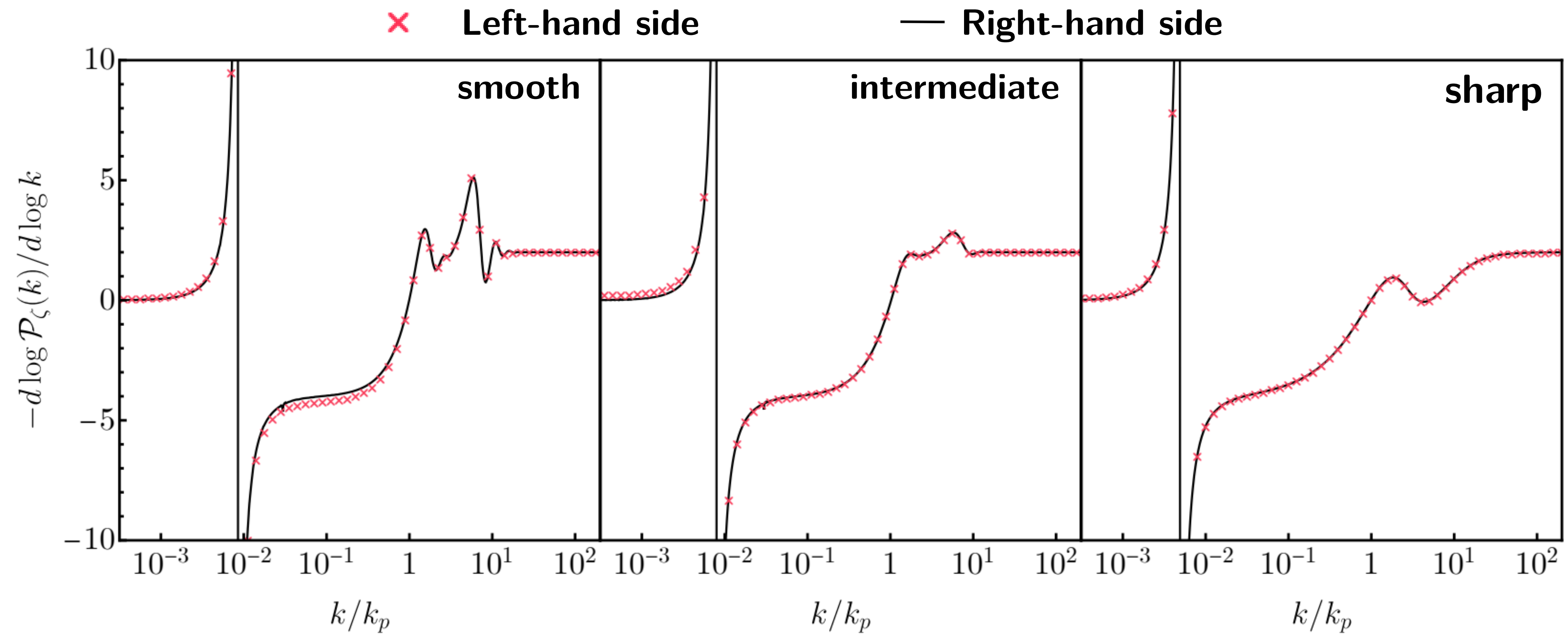
Thank you for your attention

Back up slides

Consistency check

Maldacena's consistency relation

$$\frac{\mathcal{I}(k,p)+\mathcal{N}(k,p)}{|\varphi_k(t_e)|^2|\varphi_p(t_e)|^2} = -\frac{\mathrm{d}\log\mathcal{P}_\zeta(p)}{\mathrm{d}\log p},$$



The reconstructed potential

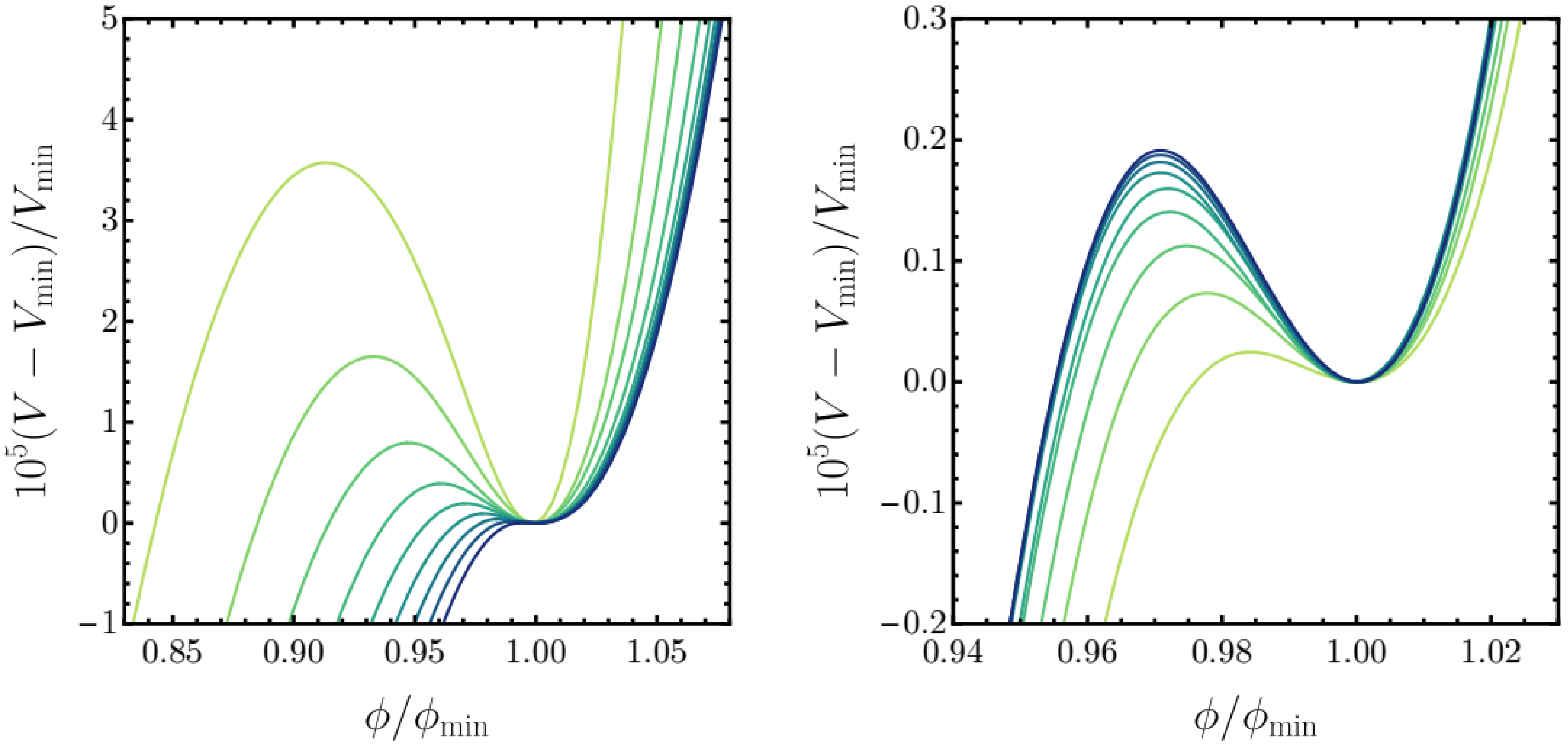


Figure 2: *Left panel: Potentials obtained by varying* δ *between* 0.1 *(green) and* 0.9 *(blue), for* $\Delta N = 2$. *Right panel: Potentials obtained by varying* ΔN *between* 1.1 *(green) and* 2 *(blue), for* $\delta = 0.5$. *We have set* $\eta_{\rm USR} = 4$ *and* $\eta_{\rm CR} = -1$ *for both panels.*