We have 2 polarizations of gravitational waves: H_+ and $H_\times$

→ also linear combinations allowed!

→ $H_+ + H_\times$ → "elliptic displacements"

→ similar to circular polarization of photons!

But ... what is a polarized GW?

What do you feel when a GW is passing (assuming you are pointlike particle, moving on a geodesics ...)?

$\Rightarrow$ Nothing! $\quad \rightarrow$ Equivalence principle!

$\Rightarrow$ "Relative" position counts

Assume: several geodesics, close together

start in rest frame: $u^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

+ gravitational wave: $\tilde{u}^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + O(h)$

Derive displacement s^μ between the geodesics (in linear order of h):

$$\Rightarrow \frac{d^2 s^\mu}{dt^2} \sim \frac{d^2 h^\mu{}_\nu}{dt^2} s^\nu$$

$$H_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & H_+ & H_\times & 0 \\ 0 & H_\times & -H_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{\mu\nu} \sim H_{\mu\nu} e^{ik_\alpha x^\alpha}$$

$$\left(H_{+,\times} = f_{+,\times}(t-z) \right)$$

$\Rightarrow$ if GW moves in z-direction (w.l.o.g.):
<u>no</u> effect in s^z (and s^t) !

$\Rightarrow$ GW are transverse (like photons!)

Effect only in $x-y$ direction:

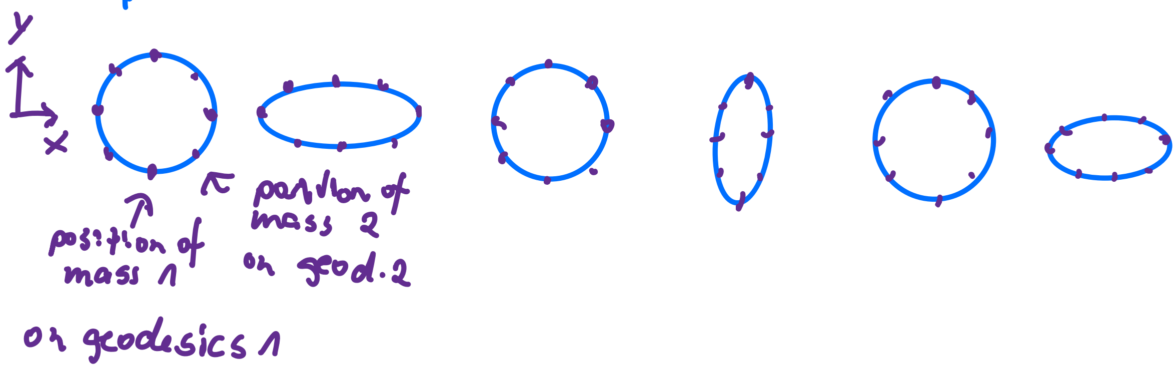
Set $H_\times = 0$:

$\rightarrow$ $\frac{d^2 s^x}{dt^2} \sim -H_+ e^{i\omega t} s^x \qquad \frac{d^2 s^y}{dt^2} \sim H_+ e^{i\omega t} s^y$

i.e., $s^{x,y}(t) = s^{x,y}(0)\left(1 \pm \frac{1}{2} H_\times e^{i\omega t} + \dots\right)$

$\rightarrow$ start, e.g. on a circle $s^x(0)^2 + s^y(0)^2 = R^2$

H_+:



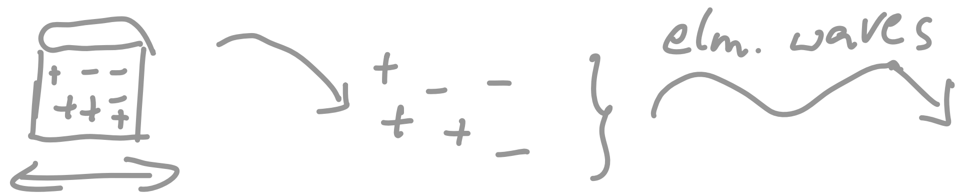
Solving Einstein equations for GWs

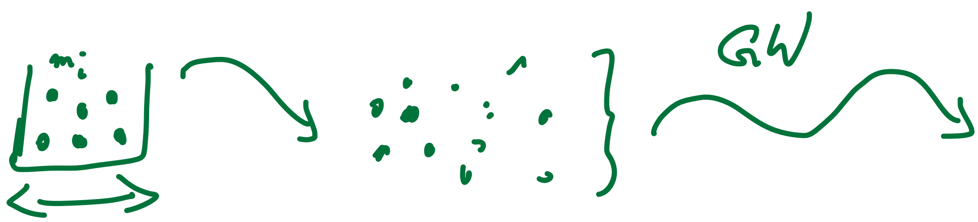
Previous example: GW's behave like plane waves

i.e. $-\infty \xrightarrow{GW} \infty$

But: GW's should start somewhere and then propagate / radiate out.

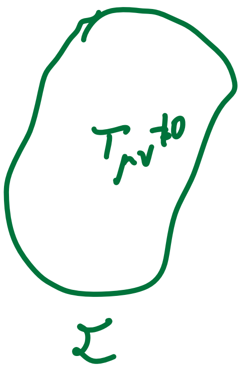
... analogous to electromagnetic waves





Linearized Einstein equation:

$$\Box \tilde{h}_{\mu\nu} \sim -T_{\mu\nu}$$



$T_{\mu\nu}(x',t)=0 \,, \; x'\notin \Sigma$

$\tilde{h}_{\mu\nu} = ?$

??? .... $\Rightarrow$ retarded Green function!

$$\tilde{h}_{\mu\nu}(x,t) \sim \int_{\Sigma} d^3x' \, \frac{T_{\mu\nu}(x', \overset{t-|\vec{x}-\vec{x}'|}{t_{ret}})}{|\vec{x}-\vec{x}'|}$$

$\rightarrow$ As before in our life:

take series of
$$\begin{cases} |\vec{x}-\vec{x}'| = r - \dfrac{\vec{x}\cdot\vec{x}'}{r} + \dots \\ \dfrac{1}{|\vec{x}-\vec{x}'|} = \dfrac{1}{r} + \dfrac{\vec{x}\cdot\vec{x}'}{r^3} + \dots \end{cases}$$

also in $T_{\mu\nu}(\vec{x}', t - r + \frac{\vec{x}\cdot\vec{x}'}{r} + \dots)$.

→ leading order, i.e. 1st term of each expansion:

$$\tilde{h}_{\mu\nu}(\vec{x},t) \sim \frac{1}{r}\int_{\Sigma} d^3x'\, T_{\mu\nu}(x', t-r)$$

- Newton limit for 00-component:

$$\tilde{h}_{00} \sim \frac{4G}{r} E \quad \text{with } E = \int_{\Sigma} d^3x'\, T_{00}(\vec{x}', t-r)$$

$\hat{=}$ Potential $\frac{1}{r}$

- Momentum P_i inside Σ (momentum conservation) for $0i$-components etc.

- Spatial component:

$$\tilde{h}_{ij}(\vec{x},t) \sim \underbrace{\int_{\Sigma} d^3x'\, T_{ij}(x', t-r)}_{\text{one can show: } \sim I_{ij}(t-r)}$$

"quadrupole momentum"

⇒ effects on $\tilde{h}_{0i}$ and $\tilde{h}_{00}$ (due to gauge conditions $\partial_0 \tilde{h}_{0i} = \partial_j \tilde{h}_{ji}$, $\partial_0 \tilde{h}_{00} = \partial_i \tilde{h}_{i0}$

In far-field zone ($\hat{=}$ radiation zone):

$$\tilde{h}_{0i} \sim \underline{\ddot{I}_{ij}(t-r)} \quad \text{and} \quad \tilde{h}_{00} \sim E + \underline{\ddot{I}_{ij}(t-r)}$$

Comparison with Electromagnetism (EM)

GW + EM : multipole expansion $\downarrow$ important for long distance

EM : Due to $\dot{Q} = 0$ charge conservation

$\rightarrow$ no monopole contribution

$\rightarrow$ leading order from dipole!

GW: multipoles: monopol : $\cancel{E = \int_\Sigma d^3x\, T_{00}(\vec{x})}$

due to $\dot{E} = 0$ energy conservation!

dipole : $\cancel{\vec{X} \sim \int_\Sigma d^3x\, T_{00}(x)\vec{x}}$

$\left(E\dot{X}_i = \int_\Sigma d^3x (\partial_0 T_{00}) x_i = \int_\Sigma d^3x (\partial_j T_{j0}) x_i = -\int_\Sigma d^3x\, T_{i0} = P_i \right)$

due to momentum conservation, $\dot{P} = 0 = E\ddot{X}$!

quadrupole : $I_{ij} \sim \int d^3x\, T_{00}(\vec{x}, t)\, x_i x_j$

(Aside : analogon to "magnetic" dipole contribution vanishes $\hat{=}$ angular momentum conservation)

⇒ Remaining leading order contribution for GW radiation : quadrupole

(Difference to Axion Searches : "photon radiation")

Gravitational Wave Detection via SRF

→ use coupled system of two cavities

→ sensitive two interferences of cavities, induced by GW

→ since GW signal very weak (!), high Q-factor (stored versus lost energy, measure for "purity")

→ But additional to "cosmo-noise" also control of elm. noise required!

here : use prototype MAGO-cavities!

→ "hot" news : Tom's poster tomorrow!!!