"Advanced" Gravitational Waves + QT

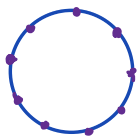
Outline:

Today:
- Introduction GW
- Basics general gravity
- Application to GW's

Tomorrow: not yet completely fixed
- Catching GW @ SRF
- Bridge to further quantum effect
  (Quantum sensing,
  Quantum backreaction
  → Plasma accelration)

What are Gravitational Waves (GW)?

→ oscillations in spacetime



→ amplitudes extremely small

Example:

- "strong" GW:

  for a length of 4 km → $\Delta l \sim 10^{-18}$ m

⇒ in other words:

  ∅ Milky Way galaxy : $\Delta \emptyset_{MW} \hat{=} 1$ m!

⇒ great challenge for detection!

Why are GW important?

→ offers – together with particle collider physics – a view behind the "cosmic curtain",

(~380 000 years after big bang, = "trasparent Universe", time since cosmic microwave background)

up to inflation ~ 10^{-38} s after big bang.

⇒ So, makes sense to study how to calculate properly these phenomena!

"Oscillations in spacetime", what do we learn from Einstein?

gravity is property of spacetime;
$\Rightarrow$ relation to curvature ("metric")

What is the problem?

The metric $g_{\mu\nu}$

$\downarrow$

information about curvature and gravitation

$\uparrow$

What $g_{\mu\nu}$?

In other words: gravity causes the measure of the space! Different to electromagnetic field etc.

Comparison with Electrodynamics:

We have:

current, source

$$\Box A_\mu \sim j_\mu \quad \Rightarrow \text{EOM}: \vec{F}_L \sim m\ddot{\vec{x}}$$

Aside: $\vec{A} \hat{=}$ vector field "Photon", wave equation

Now GR: we need

a) EOM in curved space ...

$\Rightarrow$ particles "live" on <u>geodesics</u>

$$\| \; \ddot{x}^\mu + \Gamma^\mu_{\nu\rho}\dot{x}^\nu \dot{x}^\mu = 0 \; \|$$

$\Rightarrow$ derivatives: <u>Christoffel symbols</u>

What are Christoffel symbols?

$$\Gamma^\mu_{\nu\rho} := \frac{1}{2} g^{\mu\lambda}\left(\frac{\partial g_{\lambda\nu}}{\partial x^\rho} + \frac{\partial g_{\lambda\rho}}{\partial x^\nu} - \frac{\partial g_{\nu\rho}}{\partial x^\lambda}\right)$$

$\rightarrow$ "changes in structure of spacetime"

b) "source" term $\overset{?}{=}$ mass / pressure / momentum

$\downarrow$

effects of gravity

c) field equations (with Newtonian limit in T_{00})

$\Delta \phi \sim G_N \rho$, $-\nabla\phi \sim \ddot{x}$

Einstein tensor $\downarrow$; Ricci tensor $\downarrow$; Ricci scalar $\downarrow$; Energy-Momentum-tensor $\downarrow$

$$\boxed{\Rightarrow \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \sim G_N T_{\mu\nu}}$$

<u>Einstein equations</u>

Ricci-tensor and -scalar :

- involved functions of derivatives and metric tensor

$$R_{\mu\nu} = g^{\rho\sigma} R_{\rho\mu\sigma\nu} = g \cdot (\text{2nd der. of } g_{\mu\nu} + \text{Christoffels})$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

⇒ "Matter tells spacetime how to curve,
spacetime tells matter how to move!"
(John Wheeler)

Problem: In $T_{\mu\nu} = T_{\mu\nu}(g_{\mu\nu})$ $\left(=: \frac{-2}{\sqrt{|\det g|}} \frac{\delta S_{matter}}{\delta g^{\mu\nu}} \right)$

→ solve simultaneously metric and matter...

→ Depending on problem, use different solution strategies (symmetries etc.) and approximations (linear, etc.)

How to derive GW solutions in spacetime?

GW have to fulfill the Einstein equations

We use:

- metric is almost flat
- gravity is weak

$\Rightarrow$ use linear approximations, i.e. start with "known" solution and extend it to further solutions close to the known one.

Ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad |h_{\mu\nu}| \ll 1$

$\uparrow$ Minkowski $\qquad \Downarrow$ neglect $(h_{\mu\nu})^2 \to 0$

$\Rightarrow$ Expand Einstein equations to linear order in the small perturbation $h_{\mu\nu}$.

$\Rightarrow$ Call gravity a symmetric "spin 2" field $h_{\mu\nu}$ that propagates in flat Minkowski space $\eta_{\mu\nu}$.

Derive Christoffels and Riemann/Ricci's in linear approximation (in h):

- $$\Gamma^{\sigma}_{\nu\rho} = \frac{1}{2}\,\eta^{\sigma\lambda}\left(\partial_\nu h_{\lambda\rho} + \partial_\rho h_{\nu\lambda} - \partial_\lambda h_{\nu\rho}\right)$$

- $$R_{\mu\nu} = \frac{1}{2}\left(\partial^\rho \partial_\mu h_{\nu\rho} + \partial^\rho \partial_\nu h_{\mu\rho} - \underbrace{\Box}_{\partial^\alpha\partial_\alpha} h_{\mu\nu} - \partial_\mu \partial_\nu h\right)$$

$\Rightarrow$ Einstein equations in linear approximation:

$$\boxed{\partial^\rho \partial_\mu h_{\nu\rho} + \partial^\rho \partial_\nu h_{\mu\rho} - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h - \left(\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h\right)\eta_{\mu\nu} \sim G_N T_{\mu\nu}}$$

l.s.: 2nd order linear diff. operator on $h_{\mu\nu}$

r.s.: should be small as well

Gauge symmetry in linearized gravity:

$\Rightarrow$ can be seen via infinitesimal coordinate transformation $x^\mu \to x^\mu - \varepsilon^\mu(x)$

:

$\Rightarrow$ also change in "gravity" $h_{\mu\nu}$:

$$h_{\mu\nu} \to h_{\mu\nu} + \underbrace{\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu}_{\hat{=}\ \delta g_{\mu\nu}\ \text{metric change}}$$

similar to – remember electrodynamics –

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

(Reason: linearized electromagnetic field strength $F_{\mu\nu}$ is gauge invariant as well as linearized Riemann tensor $R^\sigma{}_{\mu\nu}$.)

In electromagnetism, one has to pick a gauge, often used Lorentz gauge $\partial^\mu A_\mu = 0$.

$\Rightarrow$ Maxwell equations $\partial^\mu F_{\mu\nu} = j_\nu$ becomes wave equation:

$$\Box A_\nu = j_\nu$$

<u>Now</u>: Impose similar gauge fixing condition in linearized gravity:

$$\boxed{\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h = 0}$$

<u>De Donder gauge</u>

(Exists a de Donder gauge in full non-linear theory as well: $g^{\mu\nu}\Gamma^\rho_{\mu\nu} = 0$)

Einstein equations become:

$$\Box h_{\mu\nu} - \frac{1}{2} \Box h \, \eta_{\mu\nu} \sim T_{\mu\nu}$$

Often used:

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \, \eta_{\mu\nu}$$

Let's now solve the equations and assume a vacuum, where the GW propagate:

$$\Rightarrow \quad \Box \tilde{h}_{\mu\nu} = 0$$

Ansatz:

$$\tilde{h}_{\mu\nu} = \mathrm{Re}\left(H_{\mu\nu} e^{i k_\beta x^\beta}\right)$$

wavevector, real 4-vector

complex, symmetric polarization matrix

$\Rightarrow$ Plane wave ansatz solves Einstein equations if/since we have a null wave vector : $k_\mu k^\mu = 0$

$\downarrow$

speed of light

"graviton" = massless

Set polarization matrix:

$H_{\mu\nu} \hat{=} 10$ components ?

De Donder gauge only fulfilled if:
$(\partial^\mu \bar{h}_{\mu\nu} = 0)$

$$k^\mu H_{\mu\nu} = 0 \qquad (*)$$

$\Rightarrow$ polarization $\hat{=}$ transverse to propagation direction

One further gauge transformation (still # o.f.):

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

fulfills De Donder if $\Box \xi_\nu = 0$, $\xi_\mu = \lambda_\mu e^{ikx}$

Polarization matrix gets shifted to

$$H_{\mu\nu} \to H_{\mu\nu} + i(k_\mu \lambda_\nu + k_\nu \lambda_\mu - k^\rho \lambda_\rho \eta_{\mu\nu}) \quad (**)$$

Use λ_μ to set $H_{0\mu} = 0$ and $H^\mu{}_\mu = 0$ (Δ)

$\Rightarrow$ transverse traceless gauge!

$(\Rightarrow h_{\mu\nu} = \bar{h}_{\mu\nu})$

Counting: # o.f. $\quad H_{\mu\nu} = 10 - \underset{\substack{\uparrow \\ (*)}}{4} - \underset{\substack{\uparrow \\ (**)}}{4} = \underline{\underline{2}}$

(electrom. # o.f. $\quad A_\mu = 4 - \underset{\substack{\uparrow \\ \partial^\mu A_\mu}}{1} - \underset{\substack{\uparrow \\ \text{residual gauge}}}{1} = \underline{\underline{2}}$

If now: wave propagation in z

$$\Rightarrow \quad k^{\mu} = (\omega, 0, 0, \omega)$$

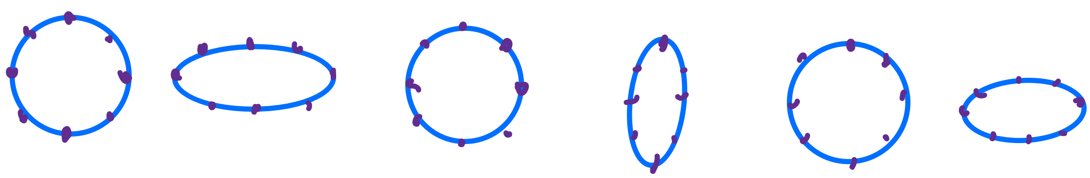
$$\overset{(*)}{\Rightarrow} \quad h_{0\nu} + h_{3\nu} = 0$$

and (Δ):

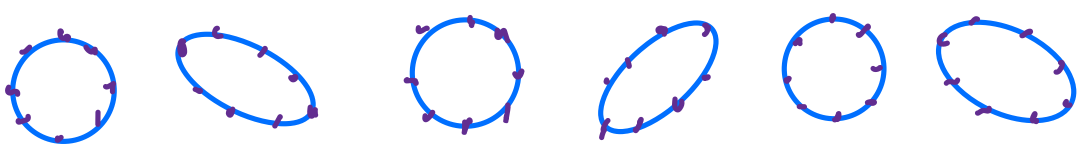
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ 2 polarization states h_+ and $h_\times$

h_+:



$h_\times$:



$\Rightarrow$ Displacement of GW invariant under rotations by π. $\leftarrow$ spin 2 !

<u>Contrary:</u> polarization of light, described via vector

$\downarrow$

only invariant under 2π !

$\downarrow$

spin 1