Neutral meson mixing beyond the Standard Model from lattice QCD

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HU / DESY Lattice Seminar

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OVERVIEW

1) short-distance contribution to neutral meson mixing on the lattice
   - 1a) $\mathrm{K}-\bar{\mathrm{K}}$ mixing
   - 1b) status of $\mathrm{B_q}$ mixing by RBC/UKQCD and JLQCD
2) long-distance contribution
   - 2a) ϵ_{K}
   - 2b) outlook: $\mathrm{D}-\bar{\mathrm{D}}$ mixing

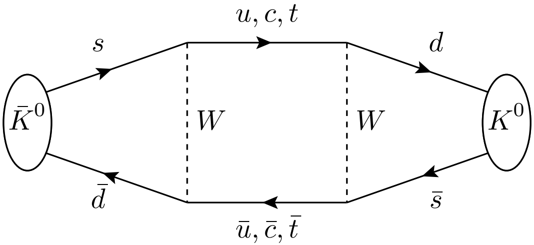
K MESON MIXING

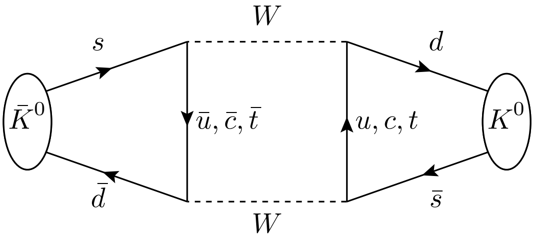
With CP symmetry, neutral kaons have eigenstates

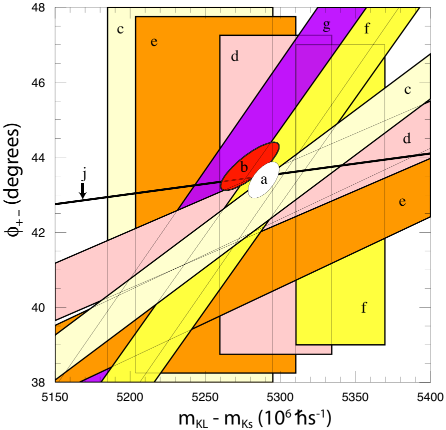
$$|K_L\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$
$$|K_S\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

with a long-lived $|K_L\rangle$ and a short-lived $|K_S\rangle$.
Indirect CP violation parameter ϵ_K can be parameterized by mass and widths splittings

$$\Delta M_K = M_{K_L} - M_{K_S}, \qquad \Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$$
$$\phi_\epsilon = \frac{\Delta M_K}{\Delta\Gamma_K/2}$$
$$\epsilon_K = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}\right)$$







(a)–[PDG, PRD 24] $\chi^2 = 1$ contour of fit to experimental data:
(b)–FNAL KTeV '11, (c)–CERN CPLEAR '99, (d)–FNAL E773 '95,
(e)–FNAL E731 '93, (f)–CERN '74, (g)–CERN NA31 '90

- second-order weak transition
- sensitive to new physics
- precisely measured experimentally
  - $\Delta M_K = 3.484(6) \times 10^{-12}$ MeV
  - $\phi_\epsilon = 43.52(5)^\circ$

$$\epsilon_K = e^{i\phi_\epsilon} \sin(\phi_\epsilon) \left(\frac{-\mathrm{Im} M_{12}}{\Delta M_K} + \frac{\mathrm{Re} A_0}{\mathrm{Im} A_0} \right)$$

A_0 is the $K \to (\pi\pi)_{I=0}$ decay amplitude
M_{12} splits into

$$\begin{aligned} M_{12} = \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle &= \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{SD} + \langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{LD} \\ &= \langle K^0 | \mathcal{H}_W^{\Delta S=2} | \bar{K}^0 \rangle + \sum_n \frac{\langle K^0 | \mathcal{H}_W^{\Delta S=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta S=1} | \bar{K}^0 \rangle}{M_K - E_n} \end{aligned}$$

On the lattice, we can compute both:

- $\langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{SD}$ [Kaon mixing beyond the standard model with physical masses; FE et al., PRD 24]
- $\langle K^0 | \mathcal{H}_W^{eff} | \bar{K}^0 \rangle_{LD}$ [Long-distance contribution to ϵ_K from lattice QCD; Bai et al., PRD 24]

Short-distance contribution of $\mathrm{K} - \bar{\mathrm{K}}$ mixing

- hadronic contribution to ϵ_K conventionally described by bag parameters $\mathcal{B}$

$$\mathcal{B}_i = \frac{\langle \bar{K}^0 | \mathcal{O}_i | K \rangle}{\langle \bar{K}^0 | \mathcal{O}_i | K \rangle_{\text{VSA}}}$$

- can be computed from ratios of three-point and two-point functions
- knowledge of **ground states** suffices to extract them

$\Rightarrow$ less involved computation than for long-distance contribution

$\Rightarrow$ more rigourous extraction in terms of chiral-continuum limit & physical quark masses

- SM parameter $B_K = \mathcal{B}_1$
- 4 extra parameters encoding BSM physics $\mathcal{B}_{2/3/4/5}$
- BSM physics involve heavy, unobserved particles $\Rightarrow$ short-distance dominated

SHORT-DISTANCE CALCULATION [KAON MIXING BEYOND THE STANDARD MODEL WITH PHYSICAL MASSES; FE ET AL., PRD 24]

2pt-functions

$$\langle K(t)K^{\dagger}(0)\rangle_{L,a,m_l} \Rightarrow M_K(L,a,m_l), f_K(L,a,m_l)$$

3pt-functions

$$\langle K(\Delta T)\mathcal{O}_i(t)K^{\dagger}(0)\rangle_{L,a,m_l} \Rightarrow M_K(L,a,m_l), f_K(L,a,m_l), \mathcal{B}_i(L,a,m_l)$$

Leading to

$$\mathcal{B}_i = \lim_{a\to 0}\lim_{L\to\infty}\lim_{m_l\to m_l^p}\mathcal{B}_i(L,a,m_l)$$

or more precise values for

$$R_i = \lim_{a\to 0}\lim_{L\to\infty}\lim_{m_l\to m_l^p}\frac{\langle \bar{K}^0|O_i|K^0\rangle}{\langle \bar{K}^0|O_1|K^0\rangle}(L,a,m_l)$$

Non-perturbative renormalization

- bare lattice operators need to be properly renormalized
- We use the *Rome-Southampton method*, which completely avoids the use of lattice perturbation theory

⇒ Called ***Non-Perturbative Renormalization*** or ***NPR***

- First done for 2-fermion operators, which cannot be renormalized by solving the Ward identity [Martinelli et al., 1995]
- Idea is to fix renormalization conditions via tree-level matrix elements like

$$Z_\Gamma \langle p | O_\Gamma | p \rangle \big|_{p^2 = -\mu^2} = \langle p | O_\Gamma | p \rangle \big|_{\text{tree}}$$

⇒ Renormalization constants can be computed on the lattice

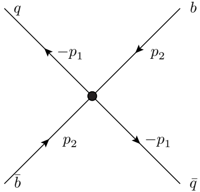
- ***NPR calculations of this project lead by Rajnandini Mukherjee***

$$\langle \mathcal{O} \rangle_i^{S}(\mu) = \lim_{a^2 \to 0} \sum_{j=1}^{5} [Z_{\mathcal{O}}^{S}(a, \mu)]_{ij} \langle \mathcal{O} \rangle_j^{\text{bare}}(a)$$

for some regularisation independent scheme S at mass scale μ. Continuum perturbation theory can then match

$$\langle \mathcal{O} \rangle_i^{\overline{\text{MS}}}(\mu) = R^{\overline{\text{MS}} \leftarrow S} \langle \mathcal{O} \rangle_i^{S}(\mu)$$

We use the "RI-SMOM" scheme. For mixing we need to compute four-quark vertices for $(\bar{b}q) \to (\bar{q}b)$. [Boyle et al., JHEP 10 (2017) 054]



RI-SMOM

Kinematics for fermion bilinears:

Original Rome-Southampton method **RI-MOM** [Martinelli et al., 1995]

$$p_1^2 = p_2^2 = -\mu^2, \quad p_1 = p_2 \Rightarrow q = 0$$

which has *exceptional kinematics* $q^2 = 0 \ll \mu^2$, chiral symmetry breaking effects vanish with $1/p^2$

Non-exceptional kinematics **RI-SMOM** [Sturm et al., 2009]

$$p_1^2 = p_2^2 = q^2 = -\mu^2, \quad q = p_1 - p_2$$

chiral symmetry breaking and infrared effects vanish with $1/p^6$

Domain-Wall Fermions

- we use "Domain-Wall Fermions"
  - automatic $O(a)$ improvement in absence of odd powers in a

  $\Rightarrow$ reduced discretisation effects
  - chirally symmetric formulation

  $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}$$

$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{22} & \mathcal{O}_{23} \\ \mathcal{O}_{32} & \mathcal{O}_{33} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{44} & \mathcal{O}_{45} \\ \mathcal{O}_{54} & \mathcal{O}_{55} \end{pmatrix} \end{pmatrix}$$

Block-structure:

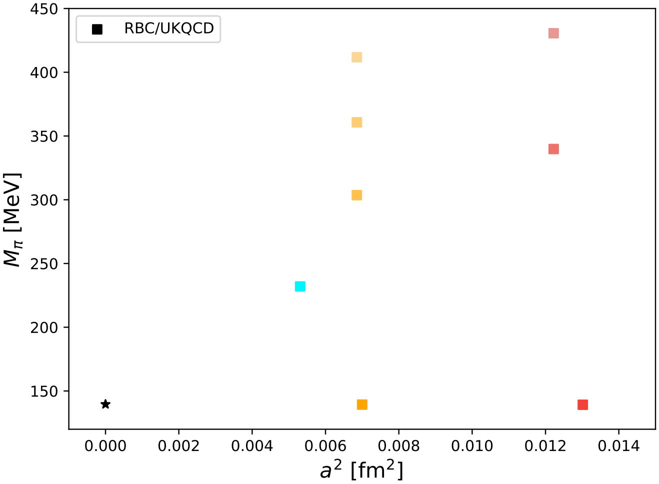
- $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix
- linearly independent from each other and from $\mathcal{O}_1$
- more complicated mixing pattern for other lattice fermions

[Boyle et al., JHEP 10 (2017) 054]

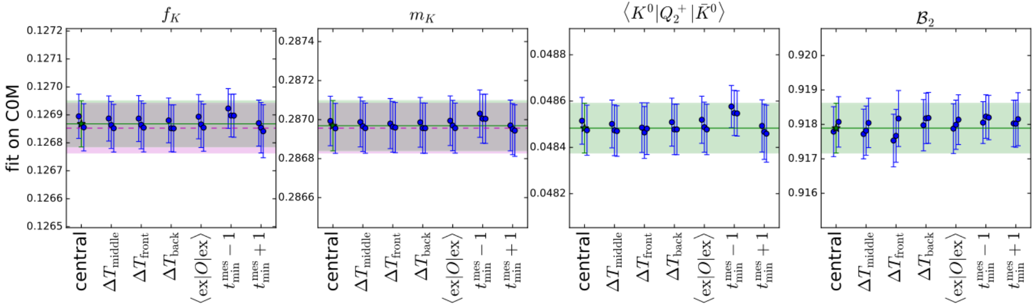
RBC/UKQCD Domain-Wall Fermion ensembles

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

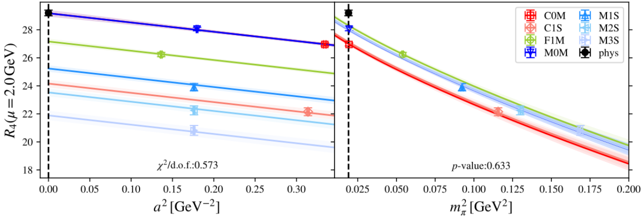


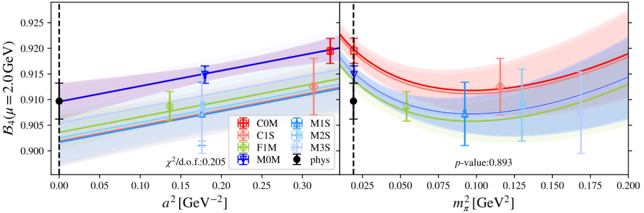
We perform combined, correlated fits to 2pt and 3pt functions



stability of correlation function fit variations on C0M ensemble for O_2

- chiral continuum fits of ratios $\mathcal{R}_i$ (top) and bag parameters $\mathcal{B}_i$ (bottom)
- two precise data points at M_π^{phys} render mass extrapolation very benign
- discretization effects $O(a), O(a^3), \ldots$ suppressed by DWF
- discretization effects $O(a^2)$ controlled by 3 lattice spacings
- remaining $O(a^4)$ estimated via variety of NPR estimators



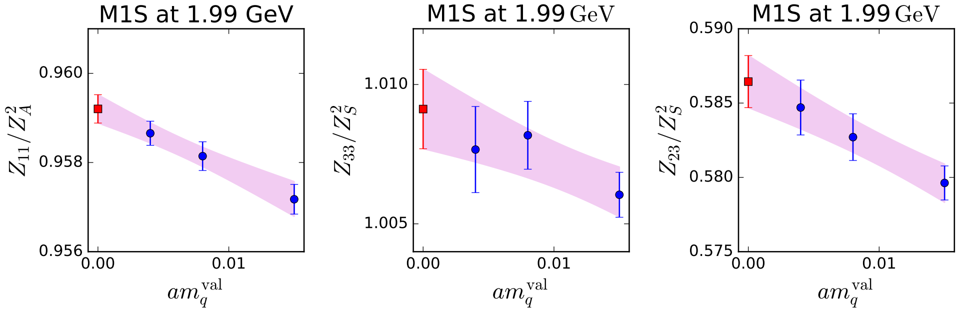


$$Y_i(a^2, m_\pi^2, m_s^{sea}) = Y_i^{phys}\left(1 + \alpha_i (a\Lambda)^2 + \beta_i \frac{m_\pi^2 - (m_\pi^{phys})^2}{(m_\pi^{phys})^2} + \gamma_i \delta_{m_s}^{sea} + L_i^Y(m_\pi) - L_i^Y(m_\pi^{phys})\right).$$

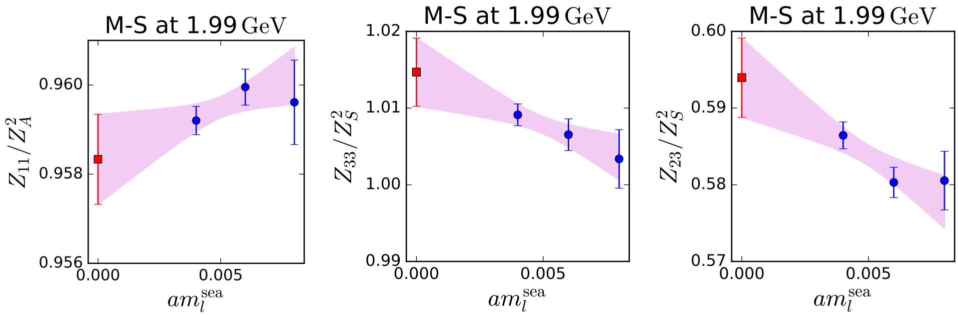
TABLE VII. Chiral-continuum limit fit systematics depending on choice of ansatz at $\mu = 2.0$ GeV in RI-SMOM$^{(\gamma_\mu,\gamma_\mu)}$ in the SUSY basis. The first column shows the central value with statistical uncertainty, while the remaining columns quantify variations arising from different choices in the data that enters the fit as well as the model to which the chiral dependences is fitted. The last column illustrates the effect of using the alternative choice of correlation function fits underlying the analysis.

	Central fit	No δ_{m_s} (%)	No chiral logs (%)	$m_\pi < 440$ MeV (%)	$m_\pi < 370$ MeV (%)	$m_\pi < 350$ MeV (%)	Alternate fit (%)
R_2	−15.106(87)	0.22	···	0.46	0.05	0.08	0.45
R_3	4.643(41)	0.42	···	0.28	0.10	0.11	0.21
R_4	29.22(19)	0.51	0.59	0.58	0.04	0.06	0.49
R_5	7.965(62)	0.10	0.47	0.50	0.00	0.05	0.13
$\mathcal{B}_1$	0.5268(13)	0.10	0.21	0.49	0.02	0.06	0.40
$\mathcal{B}_2$	0.5596(23)	0.05	0.17	0.06	0.01	0.08	0.02
$\mathcal{B}_3$	0.856(11)	0.06	0.28	0.02	0.07	0.06	0.00
$\mathcal{B}_4$	0.9097(35)	0.03	0.17	0.08	0.01	0.02	0.06
$\mathcal{B}_5$	0.750(19)	0.02	0.24	0.18	0.02	0.03	0.13

valence light-quark mass dependence on single ensemble
renormalization constants formally defined in zero-mass limit



sea light-quark mass dependence on single lattice spacing
renormalization constants formally defined in zero-mass limit

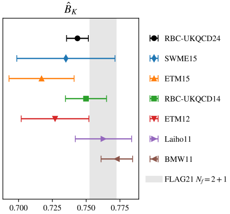


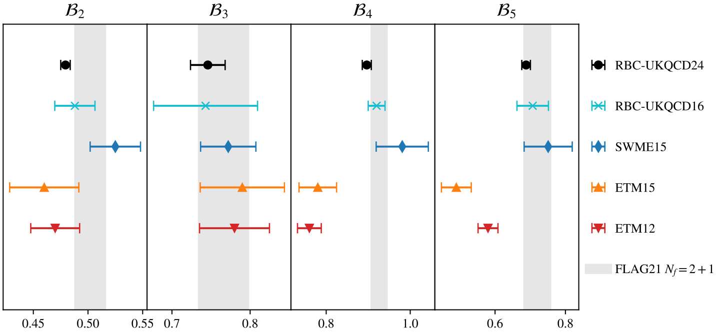
NPR (SYSTEMATIC EFFECTS)

TABLE VIII. Bag and ratio parameters at $\mu = 3$ GeV in RI-SMOM$^{(\gamma_\mu,\gamma_\mu)}$ in the SUSY basis. Central value comes from performing the chiral-continuum limit fit at $\mu = 2$ GeV and nonperturbative scaling the result to $\mu = 3$ GeV using $\sigma(3\text{ GeV}, 2\text{ GeV})$. We also list variations where the continuum step-scaling is obtained in steps, or the data is renormalized directly at 3 GeV. The central value uses Z-factors with chirally vanishing elements removed (masked) from $(P\Lambda)^T$ before the inversion $Z = F((P\Lambda)^T)^{-1}$. We list the percent shift in the result in foregoing this step, labeled residual chiral symmetry breaking (rcsb). We also compare with performing the entire analysis in the NPR basis and then rotating to the SUSY basis.

	$\sigma(3\text{ GeV}, 2\text{ GeV})$	$\sigma(3\text{ GeV} \xleftarrow{\Delta=0.5} 2\text{ GeV})$ (%)	$\sigma(3\text{ GeV} \xleftarrow{\Delta=0.33} 2\text{ GeV})$ (%)	NPR at 3 GeV (%)	rcsb (%)	SUSY ← NPR (%)
R_2	−18.37(10)	0.12	0.13	0.17	0.11	0.02
R_3	5.485(36)	0.18	0.50	0.37	0.14	0.15
R_4	38.60(27)	0.02	0.02	0.48	0.09	0.01
R_5	10.932(47)	0.11	0.01	0.97	0.03	1.22
$\mathcal{B}_1$	0.5164(14)	0.00	0.01	0.01	0.04	0.01
$\mathcal{B}_2$	0.5150(12)	0.04	0.20	0.45	0.03	0.05
$\mathcal{B}_3$	0.7624(52)	0.32	0.24	1.51	0.06	0.15
$\mathcal{B}_4$	0.9107(19)	0.02	0.16	0.02	0.01	0.02
$\mathcal{B}_5$	0.7792(79)	0.11	0.24	0.38	0.00	0.26

- BSM bag parameters $\mathcal{B}_4$, $\mathcal{B}_5$ are in tension between results using RI-MOM (with manually removed pion poles) and RI-SMOM
- tension confirmed by our calculation [FE et al., PRD 24]





B_q-meson mixing

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle = \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} + \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{LD}$$
$$= \langle B_q^0|\mathcal{H}_W^{\Delta B=2}|\bar{B}_q^0\rangle + \sum_n \frac{\langle B_q^0|\mathcal{H}_W^{\Delta B=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta B=1}|\bar{B}_q^0\rangle}{M_{B_q} - E_n}$$

short-distance contribution:

- t-loop enhancement (like for kaons)
- additional CKM hierachy enhancement

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} \sim \Big(\sum_{q'=u,c,t} V_{q'q}^* V_{q'b} S_0(m_{q'}^2/M_W^2)\Big)^2$$

long-distance contribution:

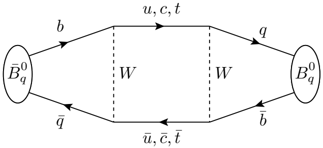
- CKM-suppressed

B_q**-mixing dominated by short-distance contribution**

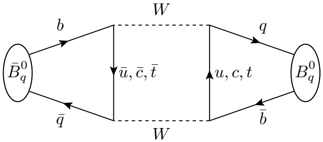
B_q-MESON MIXING

B-mesons B_d, B_s have mass eigenstates

$$|B^0_{qL}\rangle = p_q|B^0_q\rangle + q_q|\bar{B}^0_q\rangle$$
$$|B^0_{qH}\rangle = p_q|B^0_q\rangle - q_q|\bar{B}^0_q\rangle$$



with mass m_{qL} and total decay width Γ_{qL} for the lighter eigenstate.
Splittings:

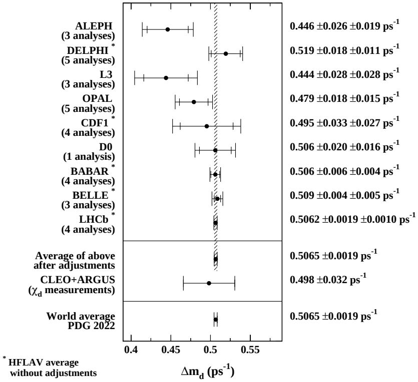


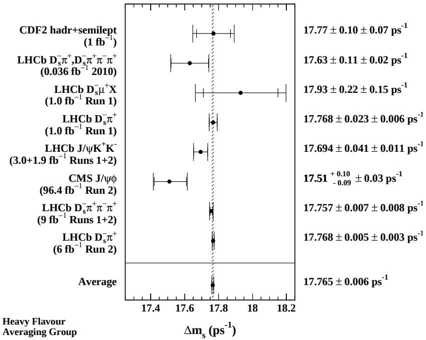
$$\Delta m_q = m_{qH} - m_{qL}$$
$$\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$$

Experimentally, time dependent probabilities give access to the splittings, e.g.

$$\mathcal{P}(B^0_q \to \bar{B}^0_q) = \frac{1}{2} e^{-\Gamma_q t}[\cosh(\frac{1}{2}\Delta\Gamma_q t) - \cos(\Delta m_q t)]|q_q/p_q|^2$$

Experimental results, HFLAV 2021 [Phys.Rev.D 107 (2023) 5]

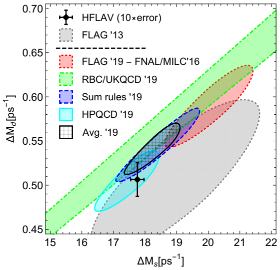




$$\Delta m_d = 0.5065(19)\text{ps}^{-1} \qquad \Delta m_s = 17.765(6)\text{ps}^{-1}$$

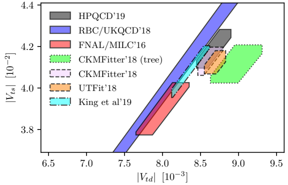
B_q MIXING - LATTICE

- current tension between Δm_d, Δm_s lattice determinations
  - FNAL/MILC [Bazavov et al., PRD 16] is in tension with experiment
  - HPQCD [Dowdall et al., PRD 19] is compatible with experiment
  - RBC/UKQCD [Boyle et al., arxiv 1812.08791] result still missing renormalization factors
  - theory uncertainty dominates experimental one



[Di Luzio et al., JHEP 19]

- similar picture in $|V_{td}|$, $|V_{ts}|$
  - lattice results in slight tension, but all compatible with sum-rules [King et al., JHEP 19]
  - unitarity-triangle fits favour HPQCD '19 result



[HPQCD, PRD 19]

CONTINUUM LIMIT (B-MIXING)

We need to control on each ensemble

- light-quark discretisation effects $\Rightarrow M_\pi L \gtrapprox 4$
- heavy-quark discretisation effects am_h

Two approaches for heavy quark:

effective theories

- allow expansion in $1/am_b$
- truncation at some order
- not easily improvable

method:

- Relativistic action (HQET, RHQ, Fermilab method)
- Nonrelativistic QCD (NRQCD)

fully relativistic

- $am_h \ll 1$ needed

$\Rightarrow$ fine lattice spacing for am_b^{phys}

- improvable with finer, larger boxes

method:

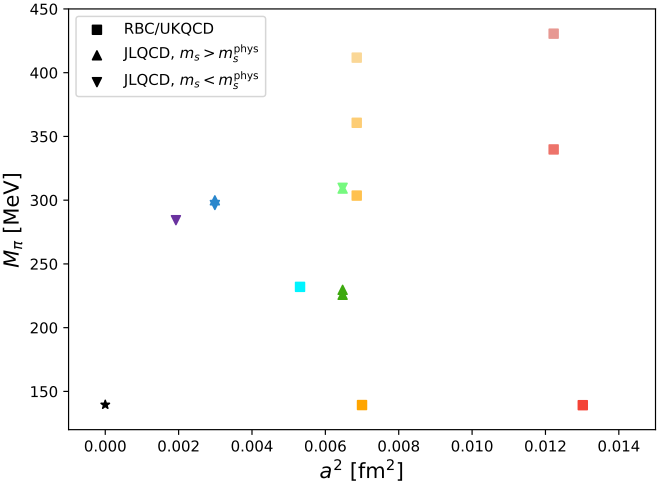
- extrapolation $am_h \to am_b$ for multiple $am_h < am_b$
- today impossible to reach am_l^{phys}, am_b^{phys} simultaneously

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

JLQCD:

- 7 ensembles
- 3 lattice spacings $a = 0.044 - 0.081\text{fm}$
- one pair of ensembles with $M_\pi L \sim 3$ and $M_\pi L \sim 4$



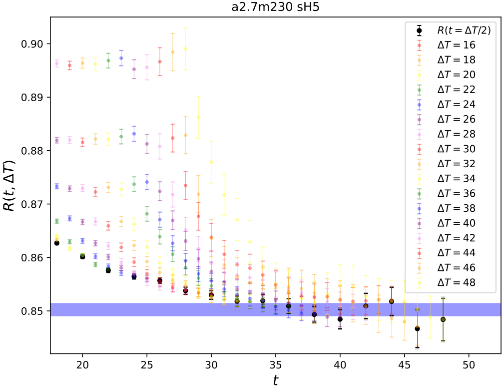
[PoS LATTICE2021 (2022) 224]

Fits to lattice correlation functions

- similar to K mixing, but much larger dataset:
- 15 ensembles
- 4-6 heavy-quark masses per ensemble
- heavy-light and heavy-strange sector
- 5 operators

$\Rightarrow$ over 700 combined fits

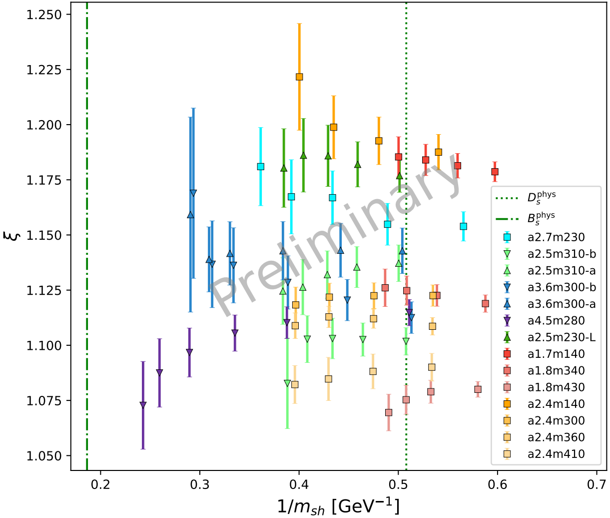
- multiple values for ΔT to control fits better
- two independent analyses by FE and J.T. Tsang
  - Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble



Mixing ratios ξ

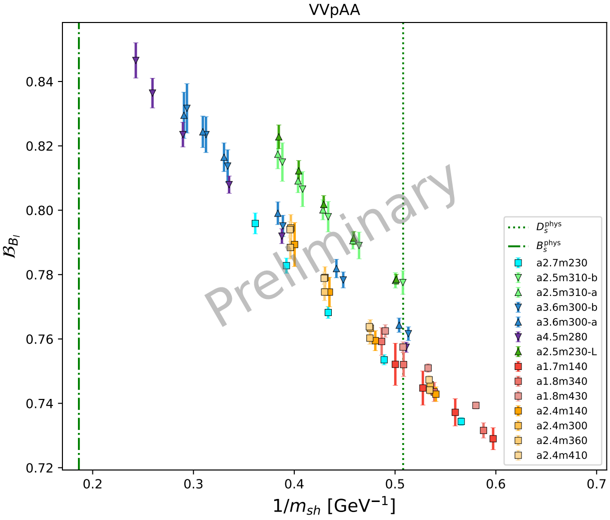
- update of RBC/UKQCD work

  [Boyle et al., arxiv 1812.08791]
- includes JLQCD ensembles
- completely new, fully correlated fitting strategy
- cancellation of renormalisation constants
- relatively flat $1/m_{sh}$ dependence with improved reach towards m_b^{phys}
- global fits on the data are investigated



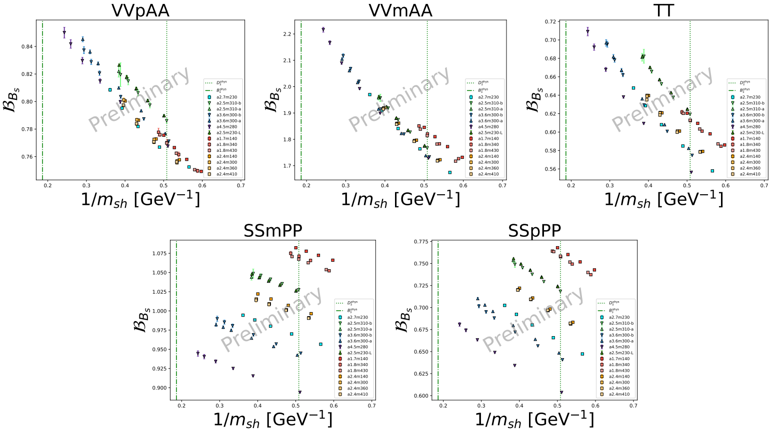
Bag parameter $\mathcal{B}_{hl}$ - $VV + AA$

- heavy-light bag parameters, renormalised at mass scale μ
- not yet matched to continuum scheme
- discretisation effects for O_1 are small
- global fits to renormalised bag parameters are investigated



Bag parameter $\mathcal{B}_{hs}$ - all 5 operators

- heavy-strange bag parameters, renormalised at mass scale μ
- O_1, O_2: mild a^2 dependence
- O_3, O_4: strong a^2 dependence
- O_5: medium a^2 dependence and curvature in $1/m_{sh}$
- very similar for heavy-light sector



LD CONTRIBUTIONS

1) short-distance contribution to neutral meson mixing on the lattice
   1a) $K - \bar{K}$ mixing
   1b) status of B_q mixing by RBC/UKQCD and JLQCD
2) long-distance contribution
   2a) ϵ_K
   2b) outlook: $D - \bar{D}$ mixing

Long-distance contribution to ϵ_K

EXTRACTING THE LONG-DISTANCE AMPLITUDE [CHRIST ET AL., PRD 13]

extracting the $K - \bar{K}$ mixing amplitude from finite-volume correlators [Christ et al., PRD 13]

- closest Euclidean correlation function: integrated 4pt correlator
  $\int dt_1 dt_2 \, \langle 0|T[\bar{K}^0(t_f) H_W(t_2) H_W(t_1) \bar{K}^0(t_i)]|0\rangle$
- on-shell intermediate states $|n\rangle\langle n|$ between H_W complicate calculation:

growing exponentials

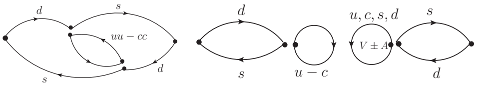
- FV states E_n with mass $M_n < M_K$ lead to unphysical growing exponentials
- these must be removed explicity and then added back in later

finite-volume effects

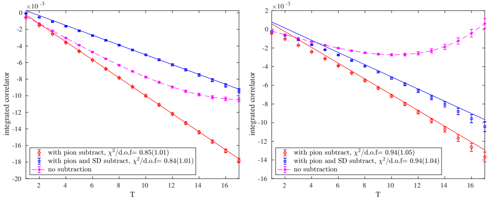
- consequently, FV estimator has poles at removed energies
- power-like volume effects are understood and described by $K \to \pi\pi$ and $\pi\pi \to \pi\pi$ scattering amplitudes

$\Rightarrow$ Precise knowledge of **excited-state spectrum** needed to extract long-distance amplitude from Euclidean finite-volume correlators

- RBC/UKQCD Domain-Wall Fermion ensembles
- one coarse lattice spacing $a^{-1} = 1.78$ GeV
- 2 pion masses 339 MeV and 592 MeV
- non-perturbative renormalization
- result: $\epsilon_K^{LD} = 0.195(77)e^{i\phi_\epsilon} \times 10^{-3}$
- comparison: $\epsilon_K^{SD} = 1.360(154)e^{i\phi_\epsilon} \times 10^{-3}$
- smaller than experimental value: $|\epsilon_K| = 2.228(11) \times 10^{-3}$
- discrepancy not understood, but $|V_{cb}|$ contributes to ϵ_K determination, present uncertainty in incl. vs excl.



a selection of topologies to be computed



integrated 4pt-correlator, with subtractions

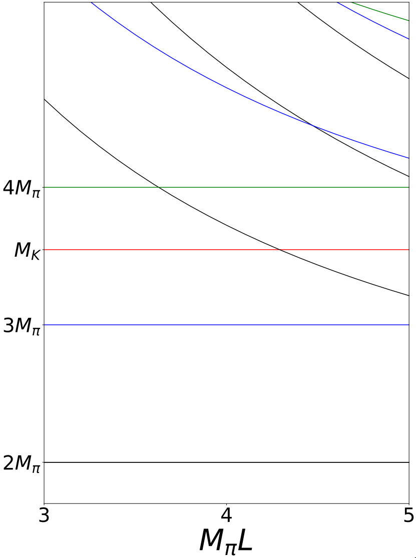
Calculation at physical pion mass underway, **progress report at this year's lattice conference** [Yikai Huo, Lattice 24]

Long-distance contribution to $\mathrm{D}-\bar{\mathrm{D}}$ mixing

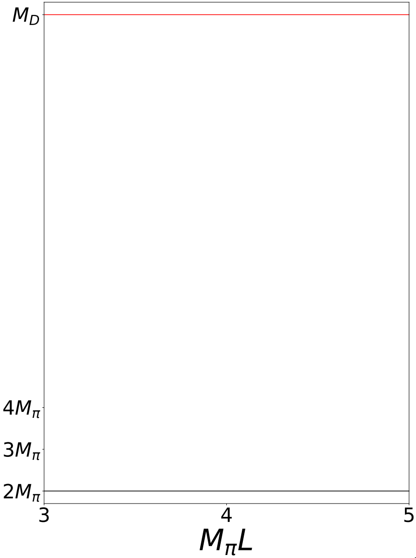
- can the same be done for D mixing?
- phenomenological estimates range 4 orders of magnitude [Lenz, Piscopo, Vlahos; PRD 20]

- essential part of ϵ_K formalism: removal of intermediate states $E_n < M_K$
- Kaon decay spectrum on lattices $M_\pi L \sim 4$
- removal of $2 - 3$ states $\rightarrow$ conceptually clear
- 3-pion state kinematically suppressed, not removed in RBC/UKQCD work
- formalism for explicit removal known

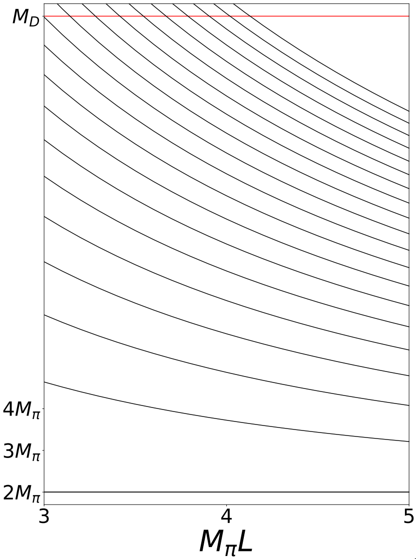
[Jackura, Briceño, Hansen; PoS Lattice22]



- D-meson decay spectrum on lattices $M_\pi L \sim 4$



- D-meson decay spectrum on lattices $M_\pi L \sim 4$
- 17 interacting states below M_D at $M_\pi L \sim 4$



- D-meson decay spectrum on lattices $M_\pi L \sim 4$
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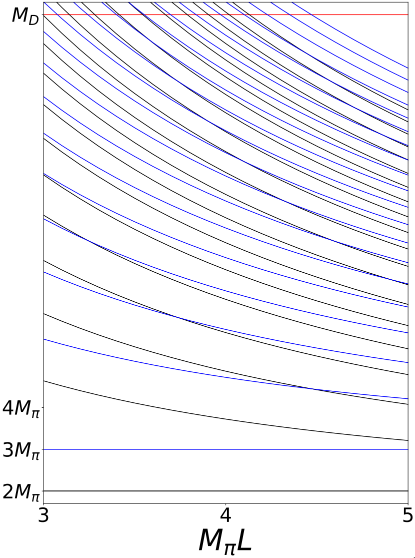
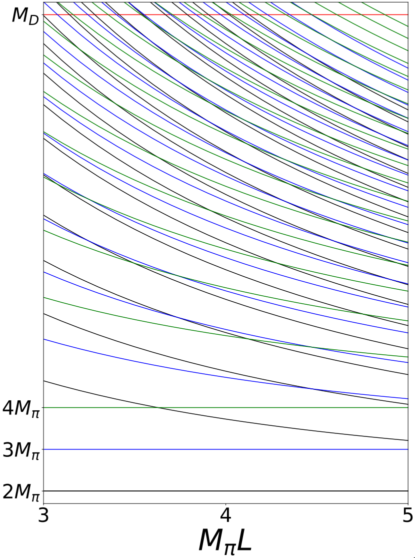


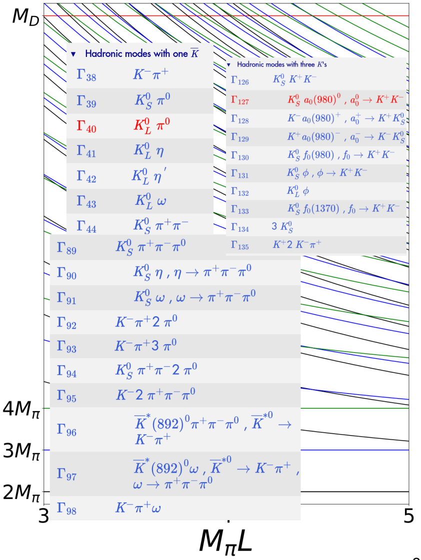
$D - \bar{D}$ MIXING

- D-meson decay spectrum on lattices $\mathrm{M}_\pi \mathrm{L} \sim 4$
- 17 interacting states below M_D at $\mathrm{M}_\pi \mathrm{L} \sim 4$
- 3π states $\to$ conceptually clear
- 4π states $\to$ no formalism yet



$D - \bar{D}$ MIXING

- D-meson decay spectrum on lattices $M_\pi L \sim 4$
- 17 interacting states below M_D at $M_\pi L \sim 4$
- 3π states $\rightarrow$ conceptually clear
- 4π states $\rightarrow$ no formalism yet
- $K\pi$, $K3\pi$, $3K$, 6π, . . . $\rightarrow$ 🤯



$\mathrm{D}-\bar{\mathrm{D}}$ MIXING

- no hope to extract full D-meson decay spectrum from lattice QCD with current formalisms and techniques
- exciting recent developments in **spectral-function methods** in lattice QCD
  - $\mathrm{O}(3)$ non-linear σ model [Bulava et al.; JHEP 22]
  - R-Ratio [Alexandrou et al.; PRL 23], can help constrain $(g-2)_{\mu}^{\mathrm{HVP}}$
  - inclusive decays [Hansen et al.; PRD 17] [Gambino, Hashimoto; PRL 21] [Barone, Lattice@CERN 24] [De Santis, Lattice24] [Groß, Lattice24] [Kellermann, Lattice24]
- a whole week was dedicated to these problems at our Lattice Theory Institute last year [Lattice@CERN 24, week 1]

The finite-volume correlator

$$C_L(\tau) = \int d^3\mathbf{x}\, e^{-E_D \tau} \langle \bar{D}^0 | \mathcal{H}_W(\tau, \mathbf{x}) \mathcal{H}_W(0) | D^0 \rangle_L .$$

can be rewritten

$$C_L(\tau) = \int \frac{d\omega}{2\pi}\, e^{-\omega\tau} \rho_L(\omega)$$

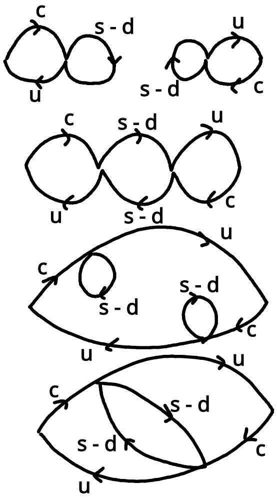
with the finite-volume spectral density

$$\rho_L(\omega) = \langle \bar{D}^0 | \mathcal{H}_W\, (2\pi)\delta(\hat{H} - \omega)\, L^3 \delta_{\hat{\mathbf{P}},\mathbf{p}_D}\, \mathcal{H}_W | D^0 \rangle_L .$$

- formalism is being developed in collaboration with ***Matteo di Carlo*** and ***Max Hansen***
- similar weak Hamiltonian to the ϵ_K case, but without QCD penguins
- U-spin symmetry leads to $s - d$ differences in loop diagrams

$\Rightarrow$ Variance reduction methods could lead to acceptable signal [Giusti et al.; EPJC 19]

- renormalization needs revisiting as well
- D mixing vanishes at $SU(3)_F$ point $\Rightarrow$ physical quark masses might be necessary



Conclusions: Neutral meson mixing

SD contributions

- recent result for $K - \bar{K}$, controlling all limits [FE et al., PRD 24]
- extension to $B_q - \bar{B}_q$ mixing
- 15 ensembles, 6 lattice spacings from 2 collaborations, including two ensembles at M_π^{phys}
- fully relativistic treatment of heavy-quark
- all limits will be under control

LD contributions

- ϵ_K exploratory calculation by RBC/UKQCD with 40% errors, physical-point in progress [Yikai Huo, Lattice 24]
- spectral-reconstruction techniques can give access to $D - \bar{D}$ mixing
- challenging, but phenomenologically minimally constrained
- U-spin symmetry provides grounds for cautious optimism

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BACKUP

LATTICE SETUP

- RBC-UKQCD's 2+1 flavour domain wall fermions [Blum et al. Phys.Rev.D 93 (2016) 7]
  - pion masses from $M_\pi = 139$ MeV to $M_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5m_b$, using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ [Boyle et al. arxiv:1812.08791]
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at M_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions [Kaneko et al. EPJ Web Conf. 175 (2018) 13007]
  - pion masses from $M_\pi = 226$ MeV to $M_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

Lattice setup

	L/a	T/a	a^{-1} [GeV]	M_π [MeV]	$M_\pi L$	hits $\times N_{conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48×90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32×100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32×101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64×82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32×83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32×76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32×81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24×100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16×100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16×100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16×100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48×72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24×50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24×50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32×50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last

LD/SD NEUTRAL MESON MIXINGS

For other neutral mesons $M^0 \in \{K, D, B_q\}$

$$\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle = \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} + \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD}$$
$$= \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle + \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}$$

short-distance contribution:

- t enhancement for $K, B_{(s)}$
- additional CKM hierachy enhancement for $B_{(s)}$
- sub-dominant for D, but ok to describe CP-violating contributions

long-distance contribution:

- relevant but smaller than short-distance for K
- dominant for D
- CKM-suppressed for $B_{(s)}$