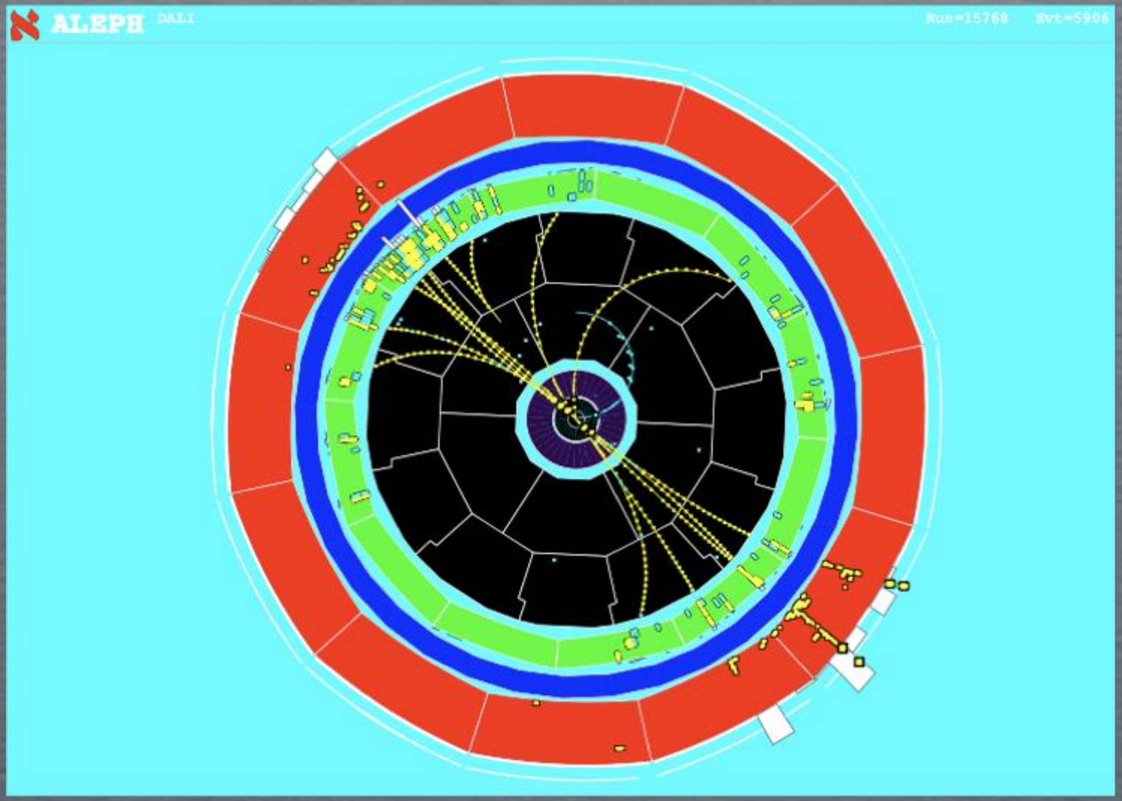
Resummation of jet observables in QCD



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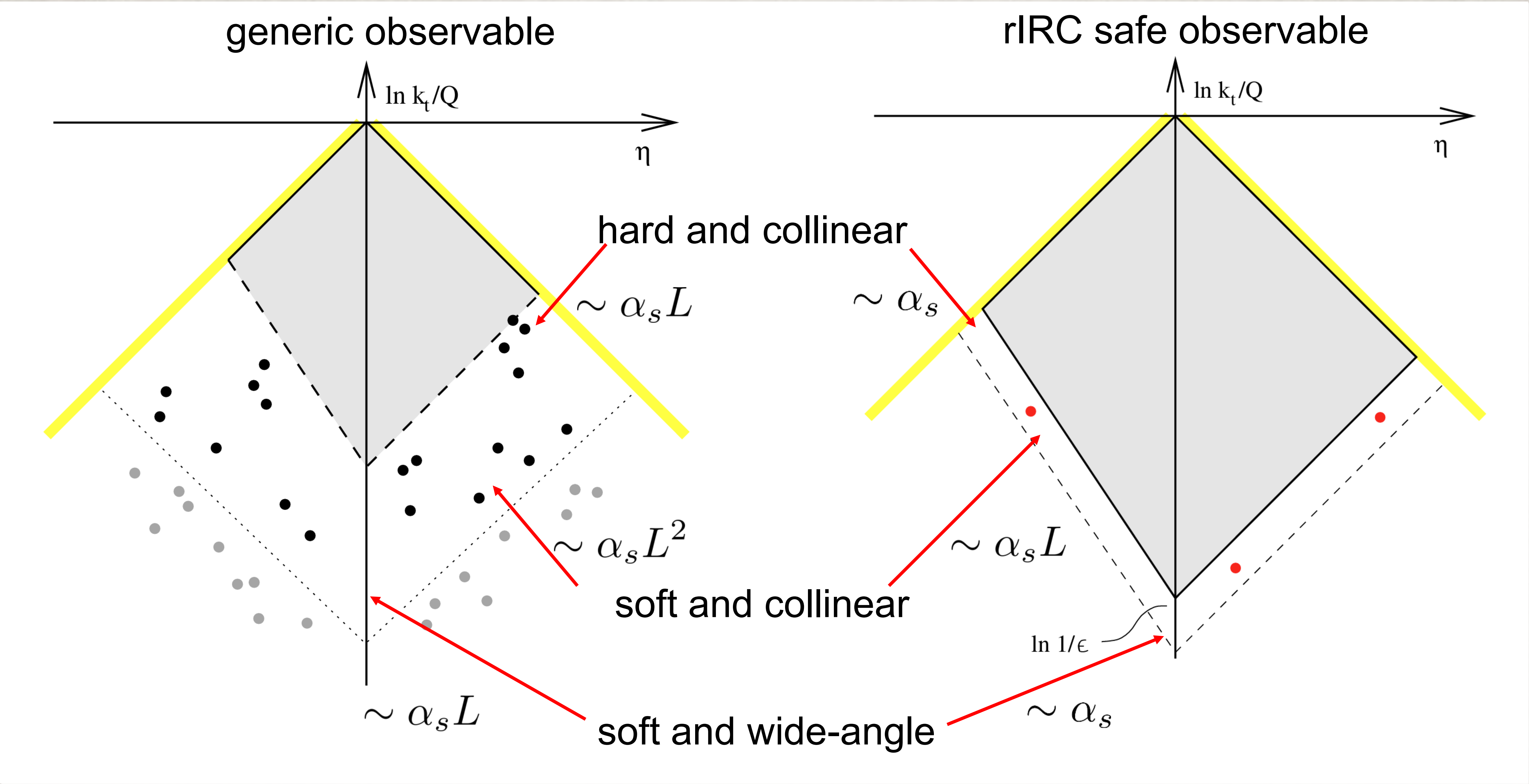
Outline

- Brief introduction to jets observables
- General principles of resummation of jet observables in QCD
- **Non-global logarithms**
- **Coherence-violating logarithms**

Recursive IRC safety

The counting of logarithms arising from real emissions for rIRC safe observables is different from naïve counting based on QCD singularities



Only the effect of soft and collinear emissions needs to be resummed to all orders

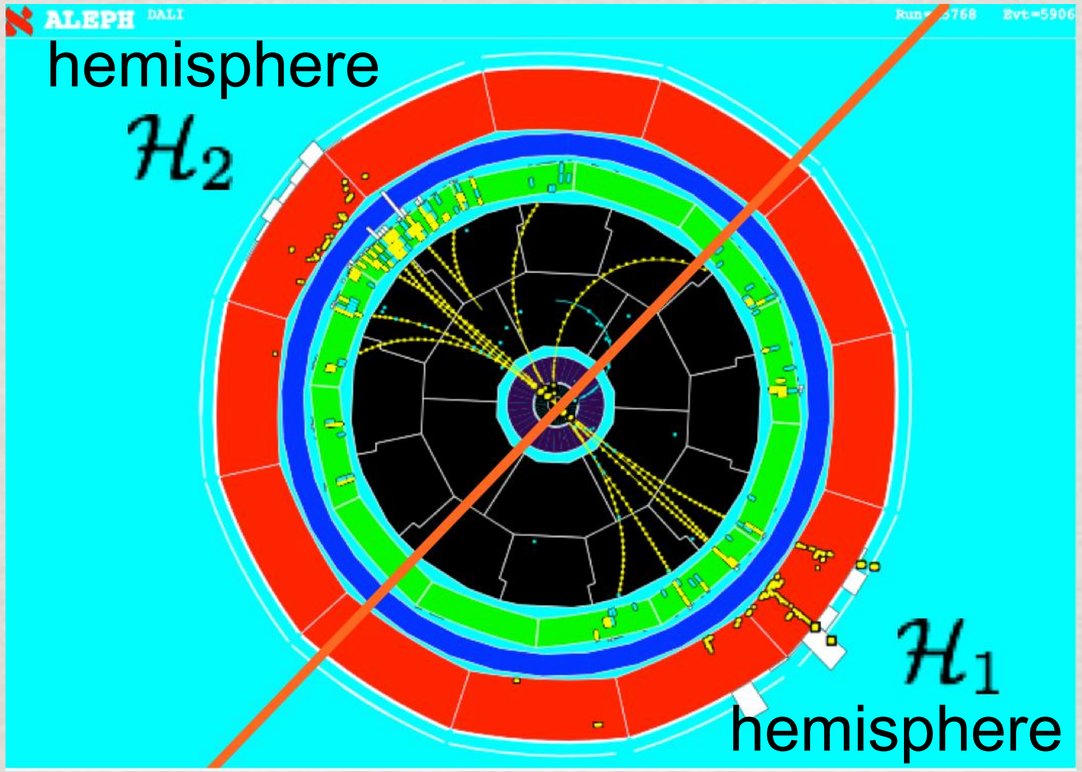
Non-global logarithms

Global vs non-global

Global observables are those whose measurement is sensitive to emissions everywhere in the phase space

global

$$\rho_H = \max\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$



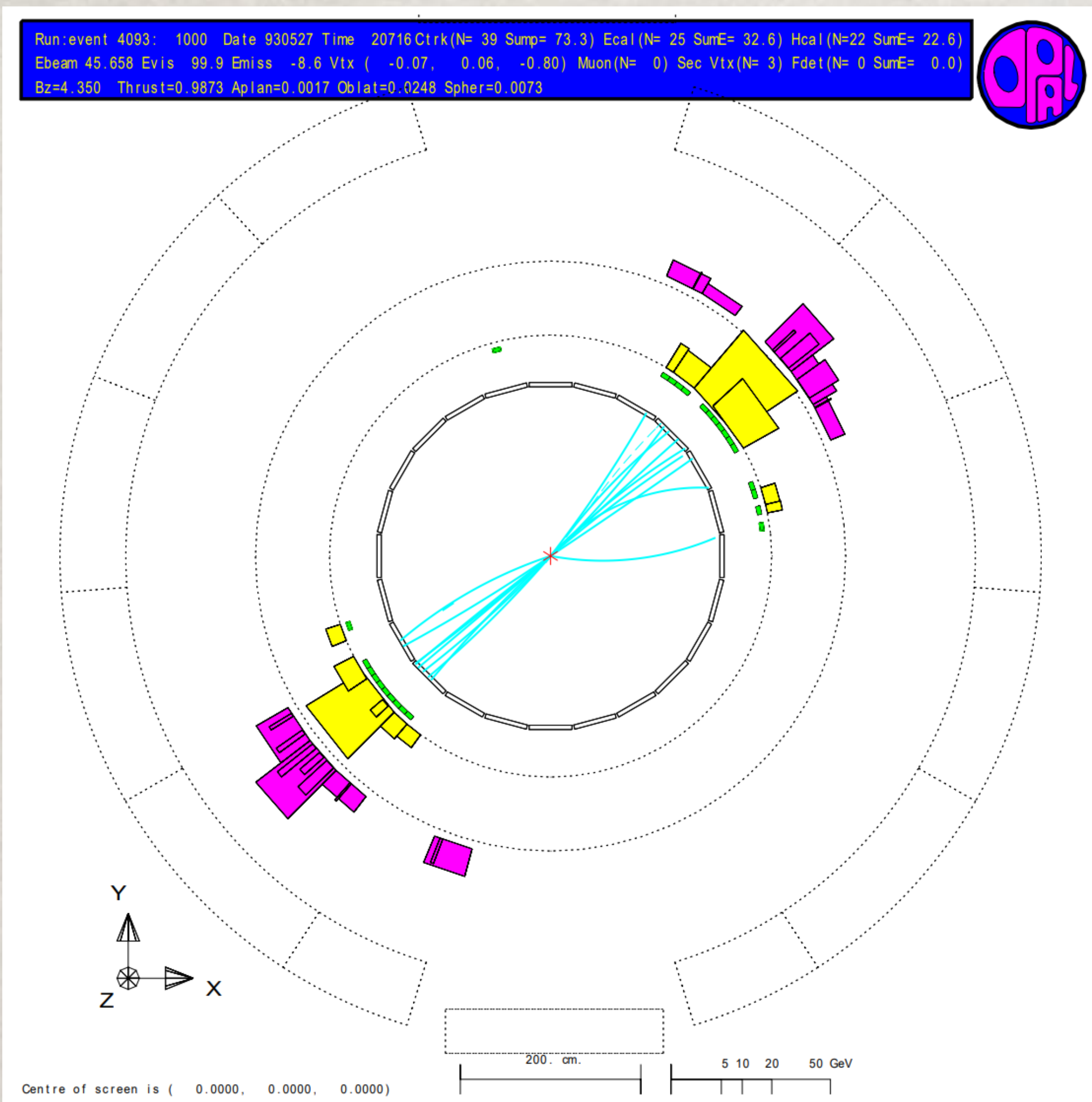
non-global

$$\rho_L = \min\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$

Non-global observables are most common in hadron collisions, where the definition of jet observables often imposes restrictions on hadron phase space (e.g. only measure hadrons inside the central tracker region)

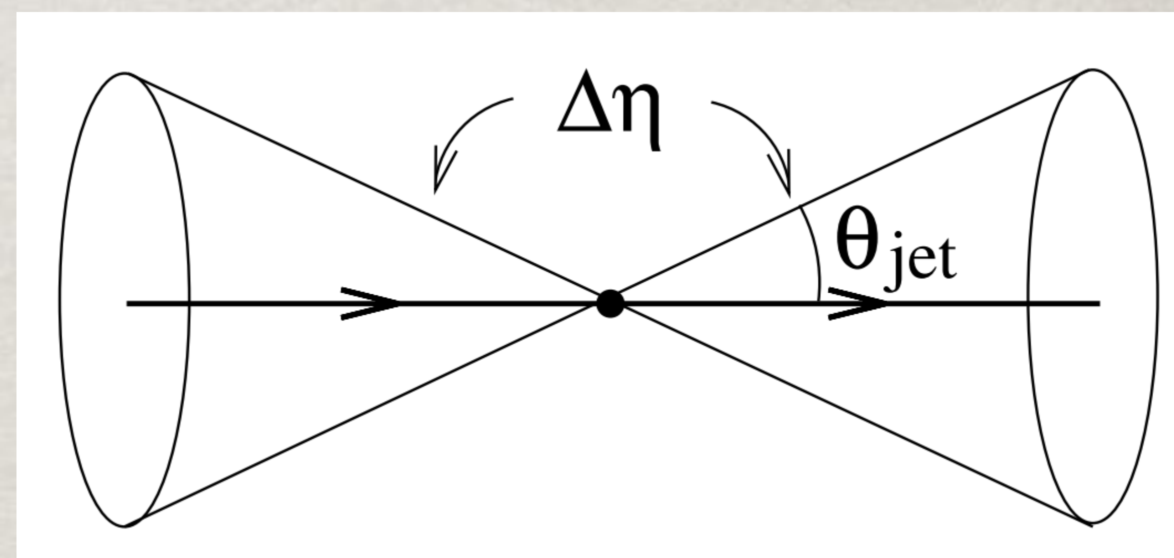
INTERJET ENERGY FLOW

Case study: fraction of events $\Sigma(Q, Q_0)$ where the total energy of all particles outside two narrow cones around two jets of energy $\sim Q$ is less than $Q_0 \ll Q$



(gap) survival probability

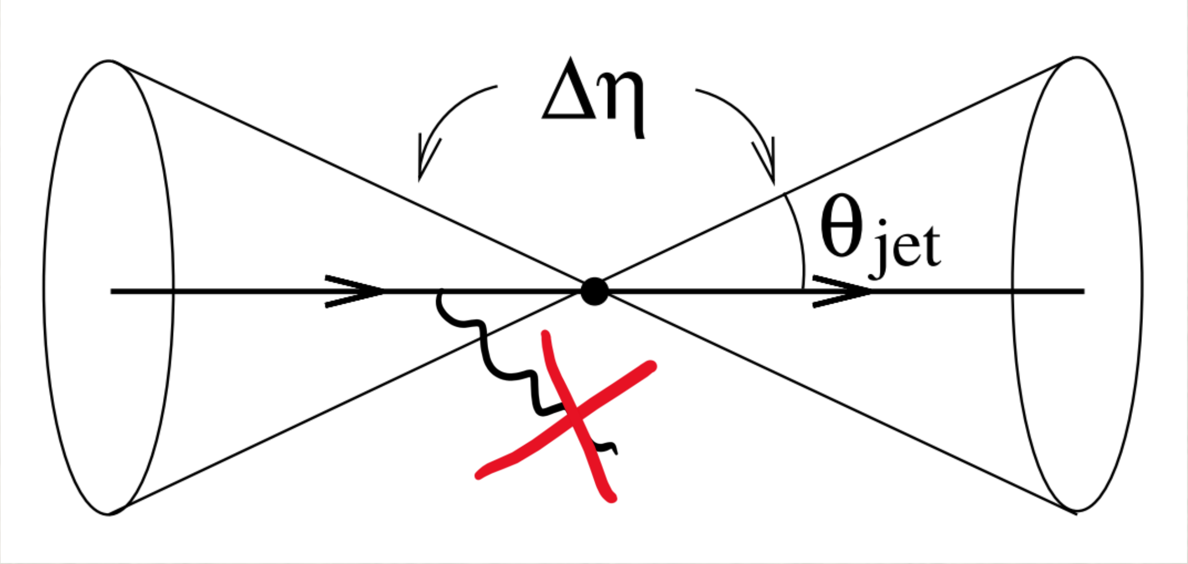
$$\Sigma(Q, Q_0) = \text{Prob}\left[\sum_{i\in\text{out}} E_i < Q_0\right]$$



$$\Delta\eta = \ln\frac{1+c}{1-c} \qquad c \equiv \cos\theta_{\text{jet}}$$

Computing the survival probability

With one soft gluon only, the survival probability is one minus the probability that that gluon is inside the gap and has energy larger than Q_0



$c \equiv \cos\theta_{\rm jet}$

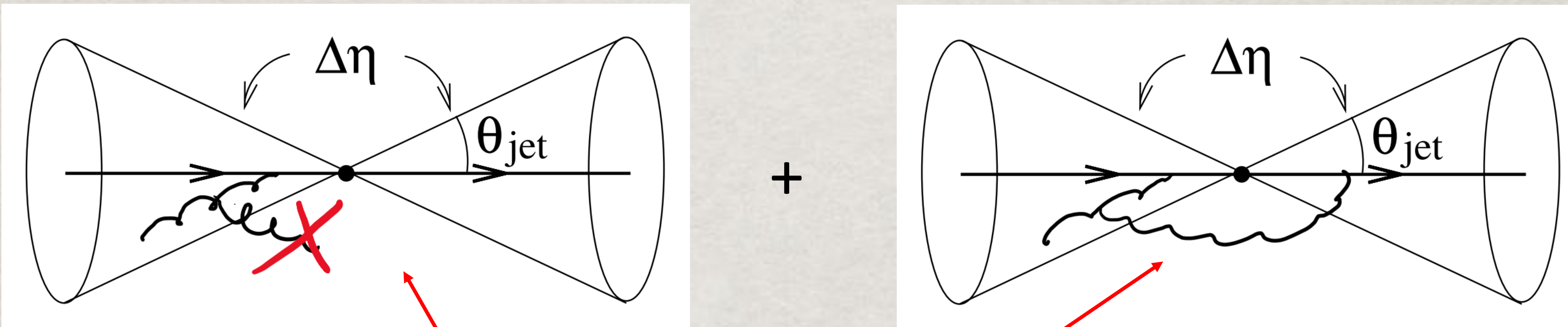
$$\Sigma(Q,Q_0) = 1 - 2C_F\frac{\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dE}{E}\int_{-c}^{c}\frac{d\cos\theta}{\sin^2\theta} = 1 - 2C_F\frac{\alpha_s}{\pi}\Delta\eta\ln\frac{Q}{Q_0}$$

Naïve exponentiation of one-gluon result $L \equiv \ln(Q/Q_0)$

$$\Sigma(Q,Q_0) = \exp\left[-2C_F\frac{\alpha_s}{\pi}\Delta\eta L\right]\left(1+\mathcal{O}(\alpha_s^2 L)\right) \text{ ?}$$

Non-global logarithms

Additional logarithmic contributions arise when harder emissions in the jets coherently emit a softer gluon inside the gap



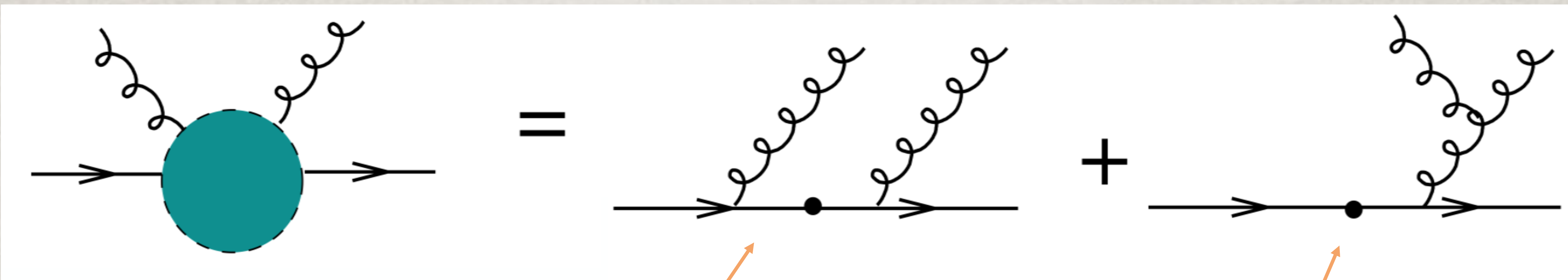
$$C_A C_F \alpha_s^2 \left[\int_0^{Q_0} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} - \int_0^{Q} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} \right] \sim -C_A C_F \alpha_s^2 L^2$$

- Such contributions, universally known as "non-global" logarithms, arise whenever experimental cuts put different constraints on emissions in separate phase-space regions [Dasgupta Salam hep-ph/0104277]
- Sensitivity to correlated emissions $\Longrightarrow$ non-linear dynamics

[AB Marchesini Smye hep-ph/0206076]

Two-gluon correlated emission

Consider the most singular case of two soft gluons strongly ordered in energy



Independent emission

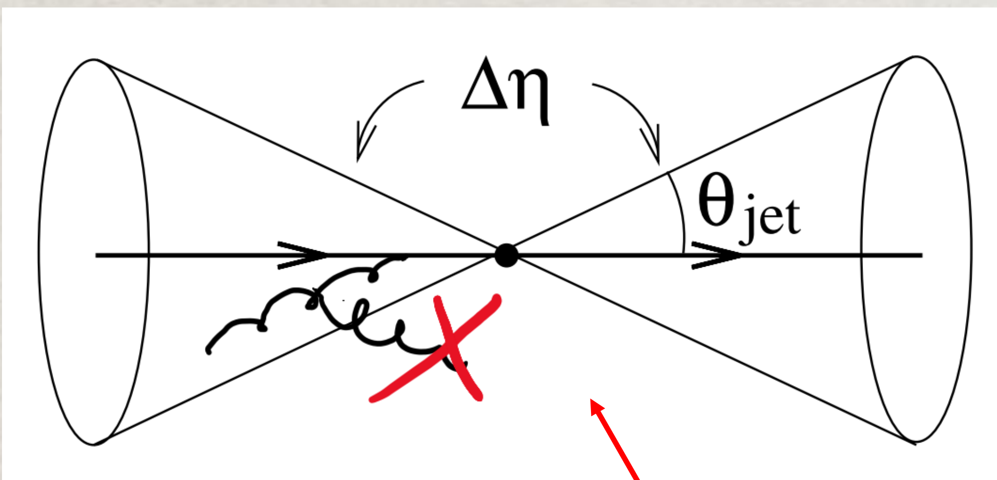
Correlated emission, singular only when the gluons are close in angle

$$M^2(k_1,k_2) = M^2(k_1)M^2(k_2) + C_A C_F \boxed{\left(\frac{(p_1p_2)}{(p_1k_1)(k_1k_2)(k_2p_2)} + \frac{(p_1p_2)}{(p_2k_1)(k_1k_2)(k_2p_1)} - \frac{(p_1p_2)}{(pk_1)(k_1p_2)}\frac{(p_1p_2)}{(pk_2)(k_2p_2)} \right)}$$

For gluons widely separated in angle, only independent emission survives

NGLs at fixed order

Additional logarithmic contributions arise when harder emissions in the jets coherently emit a softer gluon inside the gap



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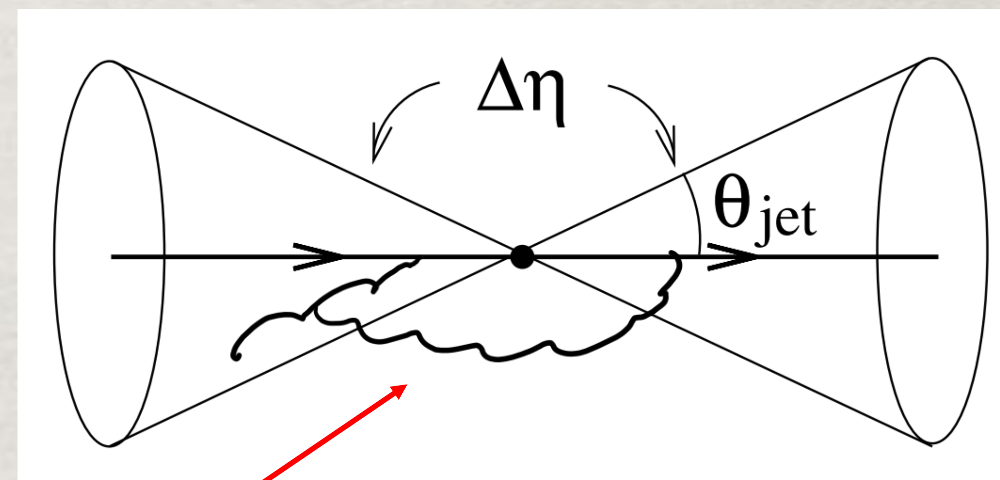


$$C_A C_F \alpha_s^2 \left[\int_0^{Q_0} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} - \int_0^{Q} \frac{d\omega_2}{\omega_2} \int_{\omega_2}^{Q} \frac{d\omega_1}{\omega_1} \right] \int_{-1}^{-c} d\cos\theta_2 \int_{-c}^{c} d\cos\theta_1 \int_0^{2\pi} \frac{d\phi}{2\pi} \times$$

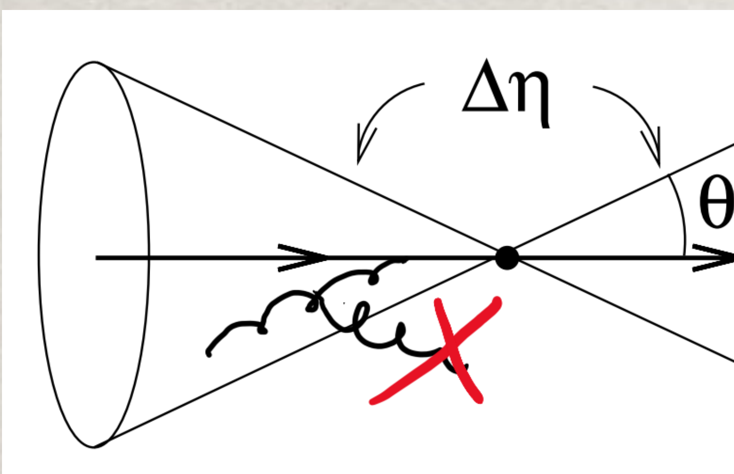
$$\times \left[\frac{1}{(1-\cos\theta_1)(1-\cos\theta_{12})(1+\cos\theta_2)} + \frac{1}{(1+\cos\theta_1)(1-\cos\theta_{12})(1-\cos\theta_2)} - \frac{2}{\sin^2\theta_1 \sin^2\theta_2} \right]$$

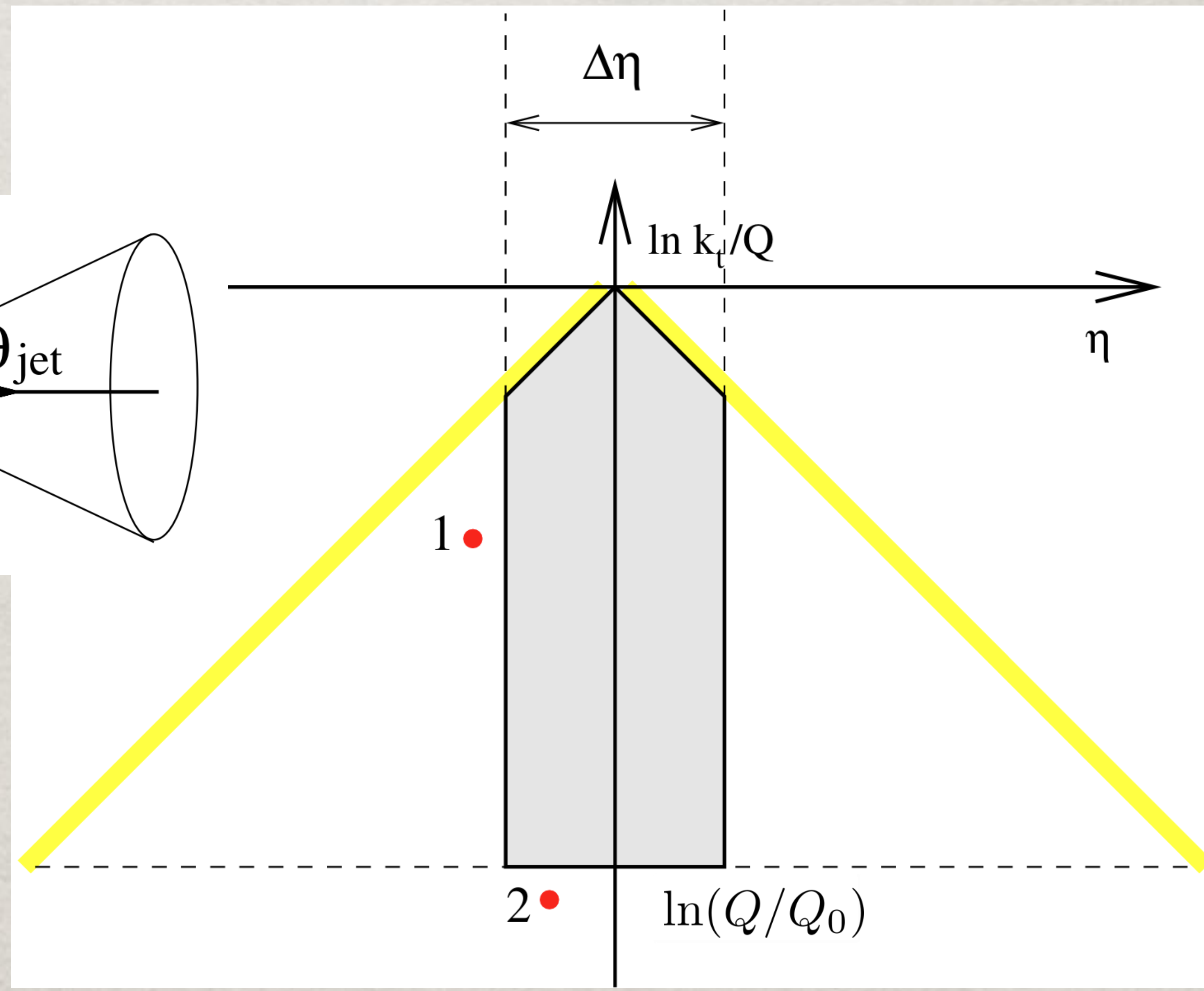
$$= -\frac{1}{2} C_A C_F \alpha_s^2 L^2 \left(\frac{\pi^2}{6} - \mathrm{Li}_2\left[e^{-2\Delta\eta}\right] \right)$$

Note that the singularity when the two gluons become collinear is regularised by the fact that they fall into different phase space regions

NGLs in the Lund plane

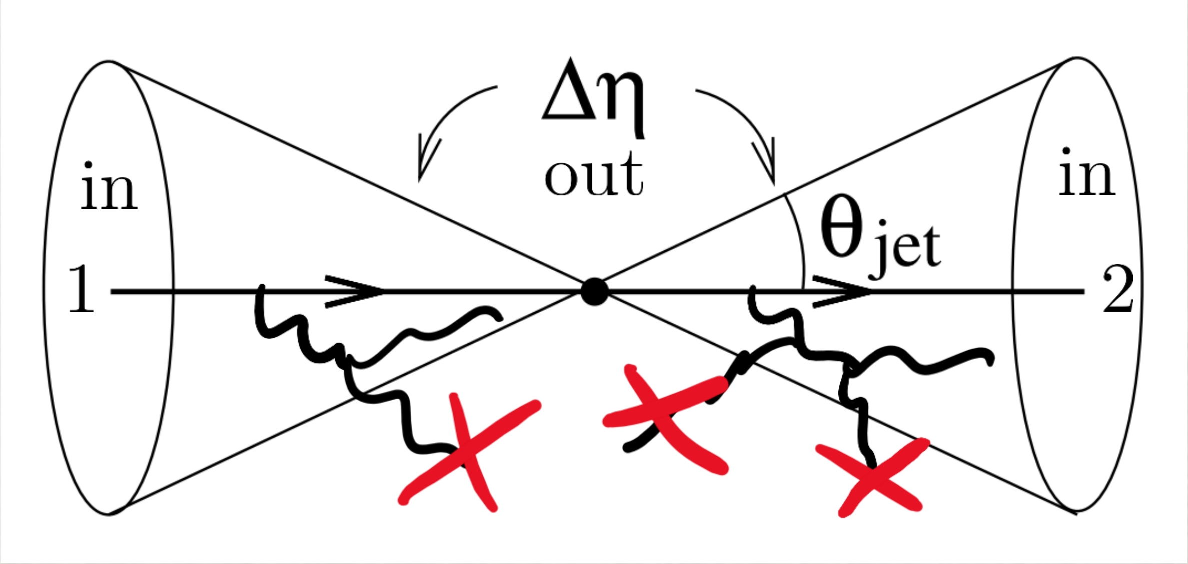
- The energy of the harder gluon (real) spans a single-logarithmic region of size $\ln(Q/Q_0)$
- The softer gluon contribution cancels with virtual corrections below the veto scale Q_0





Non-global resummation

With an arbitrary number of gluons, one recasts the gap event fraction in terms of a Laplace transform



$$\Sigma(Q_0) \simeq \int \frac{d\nu}{2\pi i\nu} e^{\nu Q_0} G_{12}[Q,u]$$

$$u(k) = \Theta_{\rm in}(k) + e^{-\nu V(k)}\Theta_{\rm out}(k)$$

$$V(k) = E, E_t, \ldots$$

- Similarly, one can define a Laplace transform $G_{ij}[Q,u]$ for any pair of unit vectors $\hat{n}_i$ and $\hat{n}_j$ on the celestial sphere

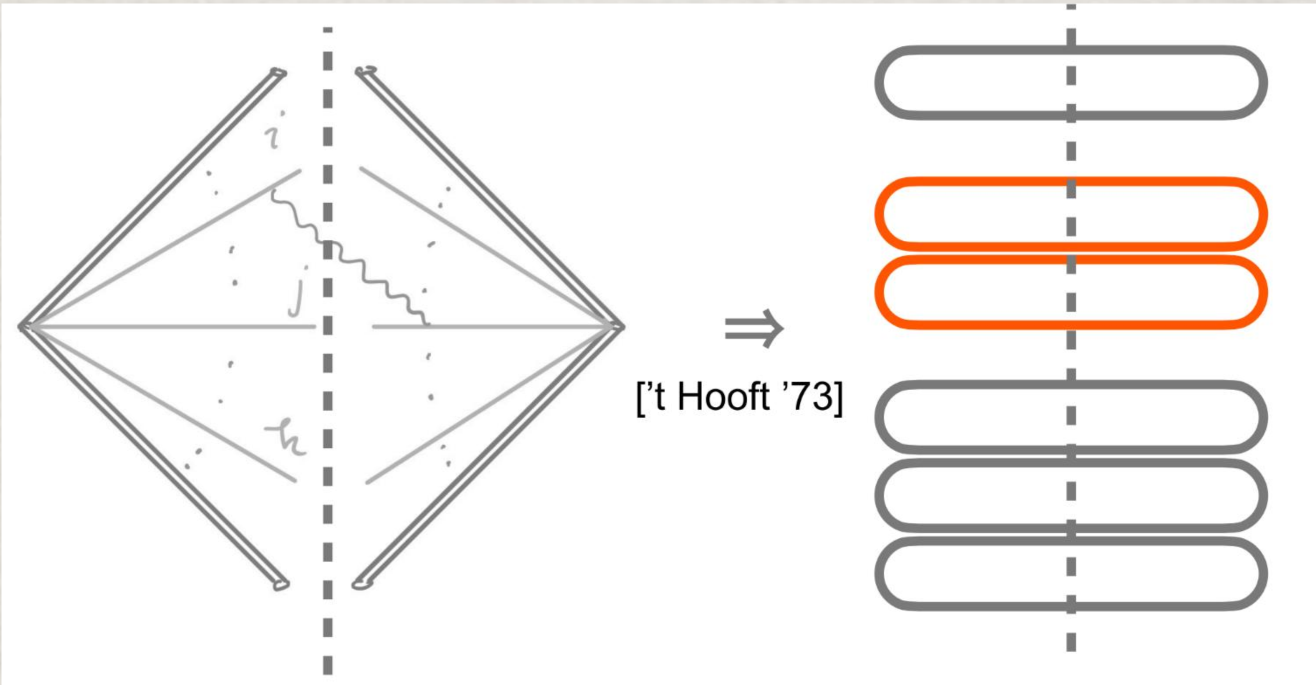
$$G_{ij}[Q,u] \simeq \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} [dk_i] \, \underbrace{A_{ij}^2(p_i, k_1, \ldots, k_n, p_j)} \prod_{i=1}^{n} u(k_i)\, \Theta(Q - \omega_i)$$

soft-gluon emission amplitude squared

The planar limit

The colour structure of squared amplitudes is involved $\Longrightarrow$ large-N_c limit

$$\mathcal{A}_{12}^2 = \bar{\alpha}^n(\mu)(2\pi)^n(\mu^{2\epsilon})^n \sum_{\pi_n} \frac{(p_1 p_2)}{(p_1 k_{i_1})(k_{i_1} k_{i_2})\dots(k_{i_n} p_2)} \qquad \bar{\alpha} \equiv N_c \frac{\alpha_s}{\pi}$$



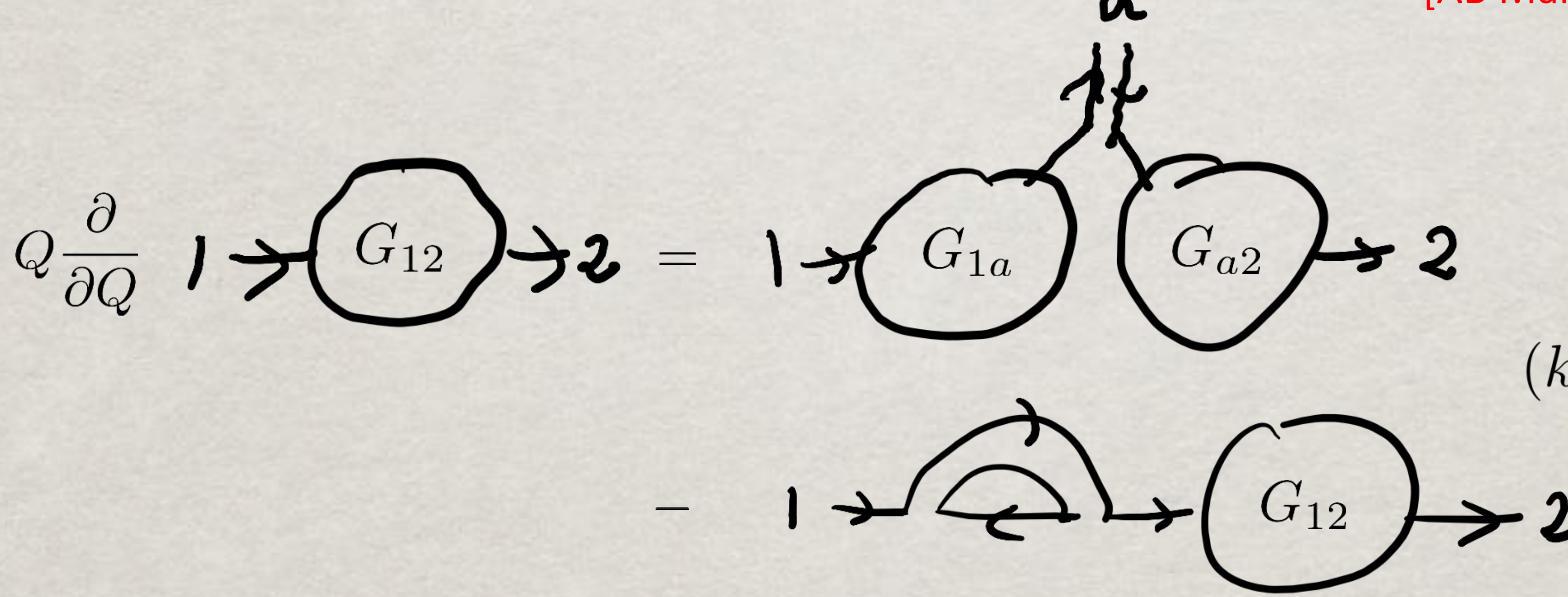
$$\frac{(p_1 p_2)}{(p_1 k_1)(k_1 k_2)\dots(k_n p_2)} = \overbrace{\frac{(p_1 k_i)}{(p_1 k_1)(k_1 k_2)\dots(k_{i-1} k_i)}}^{\sim \mathcal{A}_{1k_i}^2} \times$$

$$\times \underbrace{\frac{(p_1 p_2)}{(p_1 k_i)(k_i p_2)}}_{w_{12}(k_i)} \underbrace{\frac{(k_i p_2)}{(k_i k_{i+1})(k_1 k_2)\dots(k_n p_2)}}_{\mathcal{A}_{k_i 2}^2}$$

The BMS equation

- The factorisation properties of the amplitude squared in the planar limit makes it possible to write closed differential equations for $G_{ij}[Q,u]$
- For $G_{ij}[Q;u]$ we obtain the Banfi-Marchesini-Smye (BMS) equation

[AB Marchesini Smye hep-ph/0206076]



$$(k_{ta}^{(12)})^2 \equiv 2\frac{(p_1 k_a)(k_a p_2)}{(p_1 p_2)}$$

$$k_{ta}^{(12)} \simeq \omega_a$$

$$Q\partial_Q G_{12}[Q,u] = \int [dk_a]\,\bar{\alpha}\left(k_{ta}^{(12)}\right) w_{12}(k_a)\times$$

ordering variable

$$\times\left\{u(k_a)G_{1a}[Q,u]G_{a2}[Q,u] - G_{12}[Q,u]\right\} Q\delta\left(Q - k_{ta}^{(12)}\right)$$

The solution of the BMS equation gives the LL resummation of NGLs

Monte-Carlo solution

The LL evolution equation can be solved with a Monte-Carlo procedure:

1. Introduce a regulator for all angular integrations

$$Q\partial_Q G_{12}[Q,u] = \int [dk_a]\bar{\alpha}(k_{ta})\, w_{12}(k_a)u(k_a)G_{1a}[k_{ta},u]G_{a2}[k_{ta},u]Q\delta(Q-k_{ta})$$
$$- \left[\int [dk_a]\bar{\alpha}(k_{ta})\, w_{12}(k_a)Q\delta(Q-k_{ta})\right] G_{12}[Q,u]$$

2. Define a “Sudakov form factor”

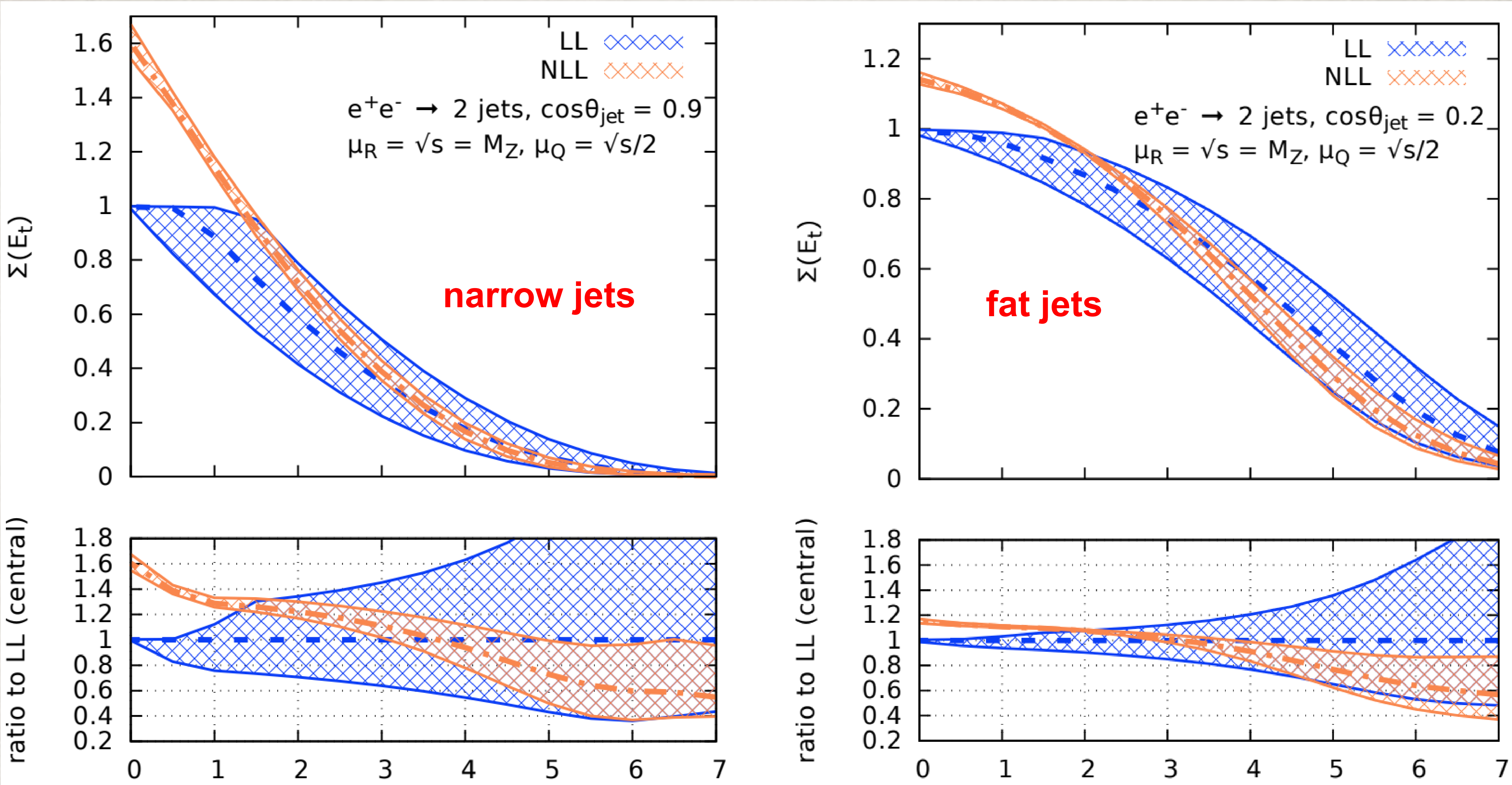
$$\Delta_{12}(Q) = \exp\left[-\int^{Q} [dk]\bar{\alpha}(k_t)w_{12}(k)\right] \Rightarrow \frac{\partial \ln \Delta_{12}}{\ln Q} = -\int [dk]\bar{\alpha}(k_t)w_{12}(k)Q\delta(Q-k_t)$$

3. Integral equation for $G_{12}[Q,u]$ with $\Delta_{12}(Q)$ the no-emission probability

$$G_{12}[Q,u] = \Delta_{12}(Q) + \int^{Q} [dk_a]\,\bar{\alpha}(k_{ta})\,\underbrace{w_{12}(k_a)\frac{\Delta_{12}(Q)}{\Delta_{12}(k_{ta})}}_{\text{soft-gluon emission probability}}\,u(k_a)G_{1a}[k_{ta},u]G_{a2}[k_{ta},u]$$

Next-to-leading NGLs

A modification of the previous algorithm makes it possible to resum non-global logarithms to NLL accuracy [AB Dreyer Monni 2104.06416]



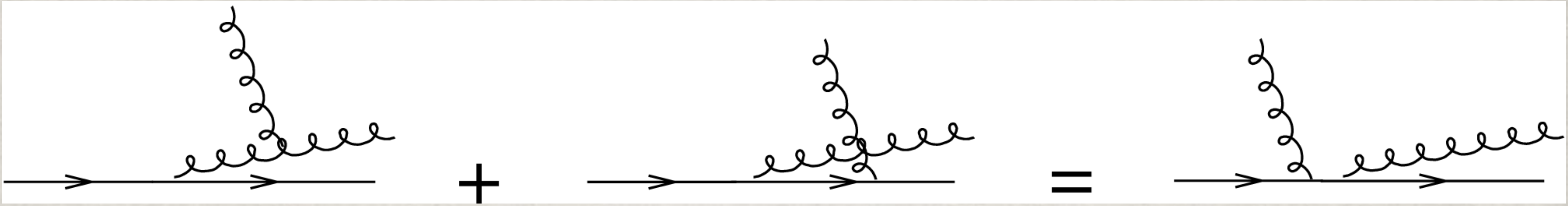
[AB Dreyer Monni 2111.02413]

Note the massive reduction of theoretical uncertainties from LL to NLL

Coherence violating logarithms

- The basis of resummation of rIRC safe observables is that soft emissions widely separated in angle are emitted independently off the hard legs
- This property is a consequence of “coherence” of QCD radiation: gluons at large angles feel only the total colour charge of emitters

[Bassetto Ciafaloni Marchesini Phys. Rept. 100 (1983) 201]



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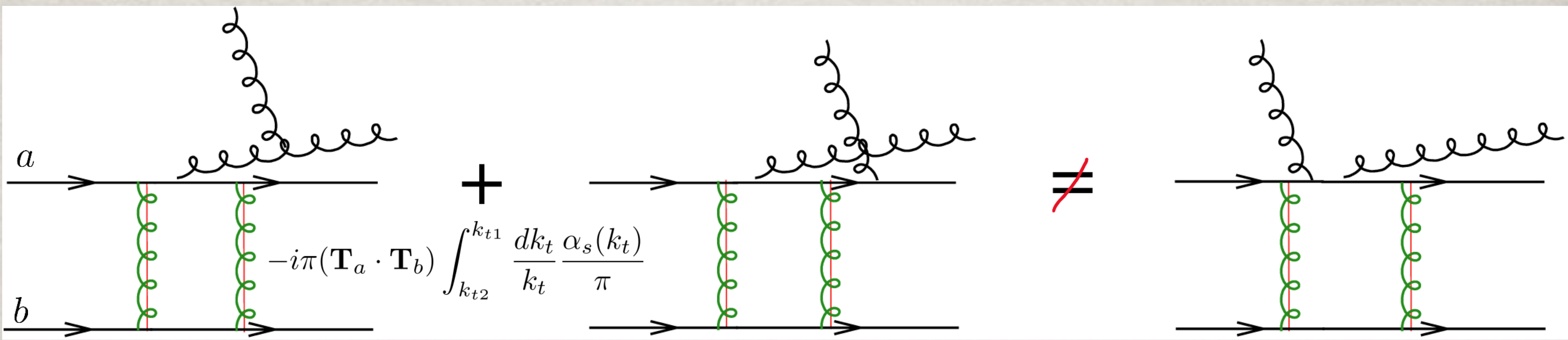


- For initial-state radiation, coherence is violated in the presence of Coulomb gluons

[Forshaw Kyrieleis Seymour hep-ph/0604094]

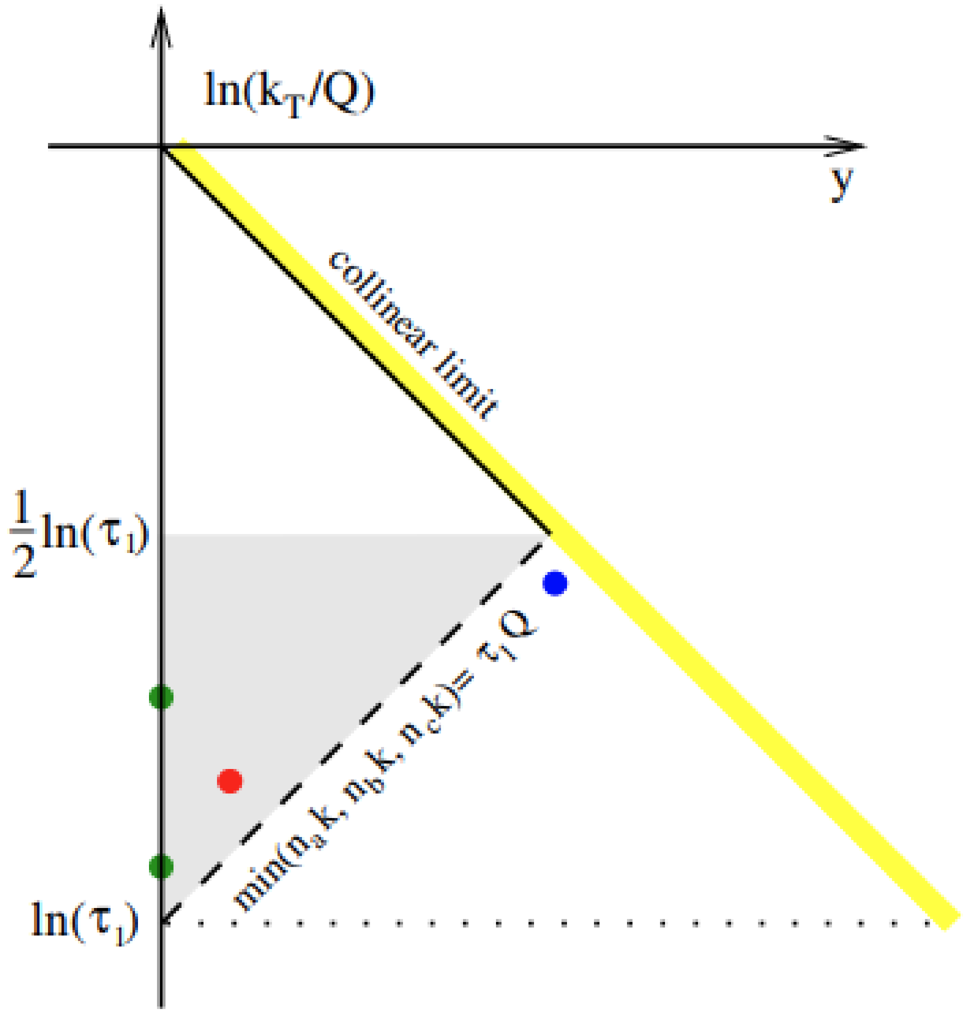
- These virtual corrections give **super-leading logarithms**, starting at $\sim \alpha_s^4 L^5$, that have been resummed in the case of the transverse energy flow between jets

[Becher Neubert Shao 2107.01212]

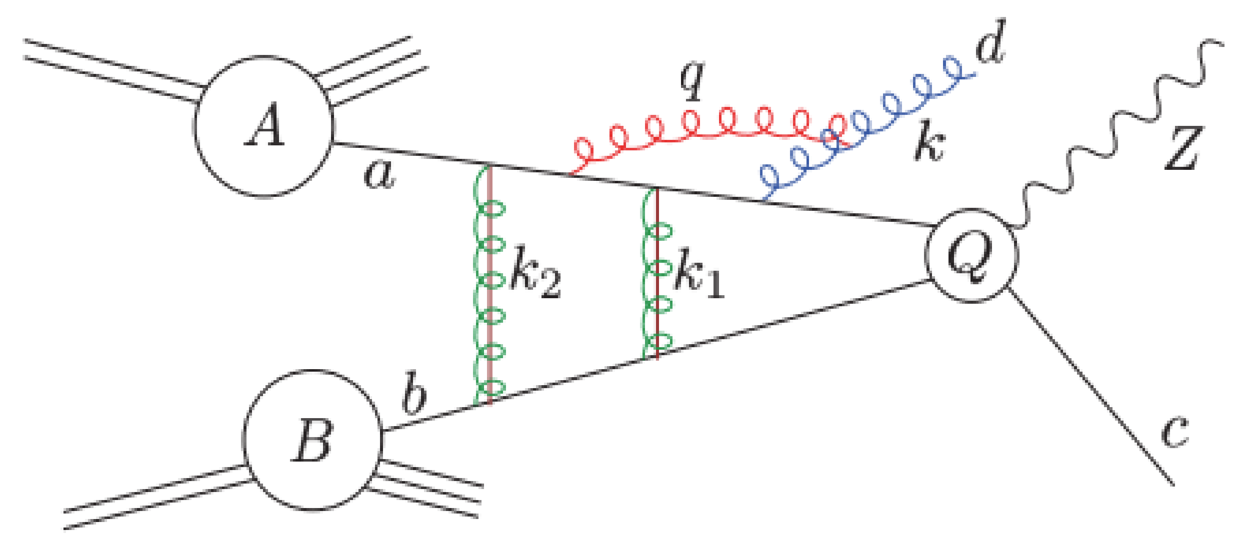
- Coherence-violating logarithms are subleading in N_c

SUPER-SUPER LEADING LOGARITHMS?

Coherence violating logarithms can appear also in rIRC safe observables, e.g. one-jettiness [AB Forshaw Holguin 2511.11799]



$$\tau_1 Q \equiv \sum_i \min\{n_a \cdot k_i, n_b \cdot k_i, n_J \cdot k_i\}$$



$$A_a \sim \mathrm{Tr}\left([\mathbf{T}_a \cdot \mathbf{T}_b, [\mathbf{T}_a \cdot \mathbf{T}_b, \mathbf{T}_a \cdot \mathbf{T}_d]]\mathbf{t}_a \mathbf{H}_{ab} \mathbf{t}_a^\dagger\right)$$

$$\frac{d\sigma_{\mathrm{CVLs}}}{dx_a dx_b d\mathcal{B}} = \sum_{a=q,g} A_a \left(\frac{\alpha_s}{\pi}\right)^4 \frac{(-i\pi)^2}{480} \left(\ln\frac{1}{\tau_1}\right)^6 \sim \alpha_s^4 L^6$$

Learning outcomes

At the end of these lectures, you should be able to

- Give examples of jet observables, and state their properties
- Carry out simple resummations for global observables at NLL accuracy
- Derive the BSM equation for non-global logarithms
- Understand the origin of coherence violation

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Thank you for your attention!