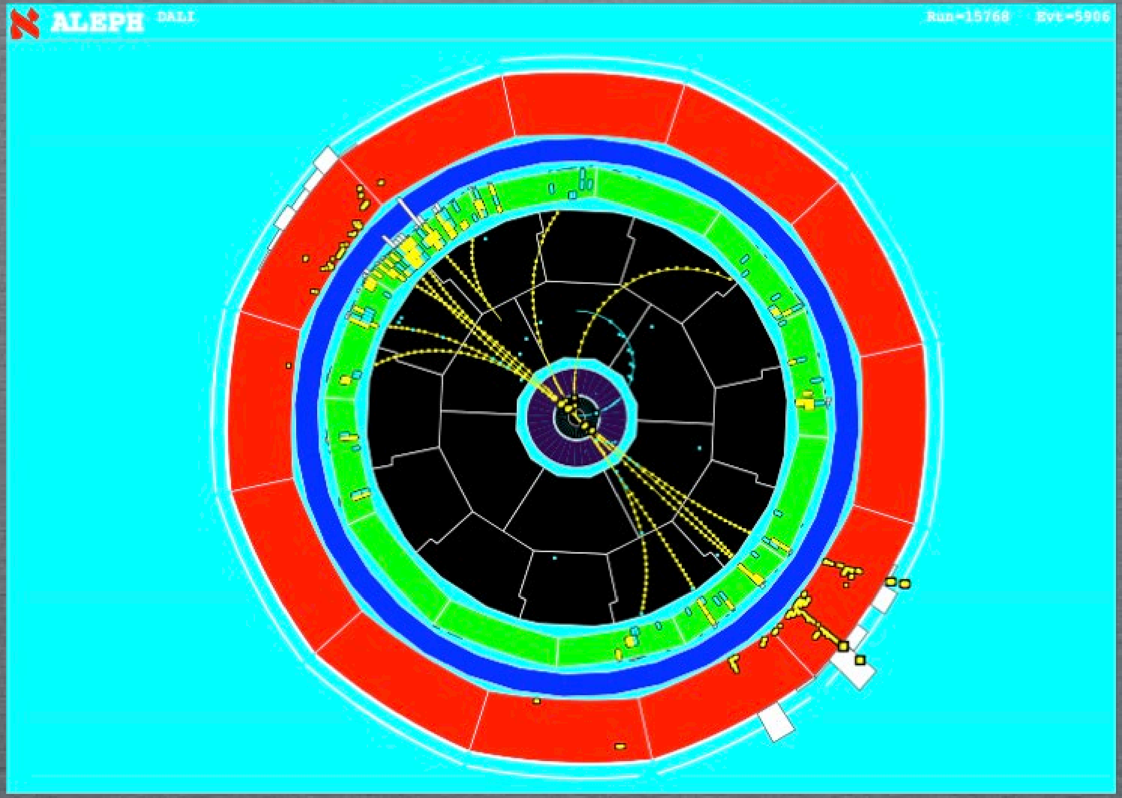
Resummation of jet observables in QCD



Andrea Banfi

University of Sussex

DESY – 26 November 2025 – Hamburg

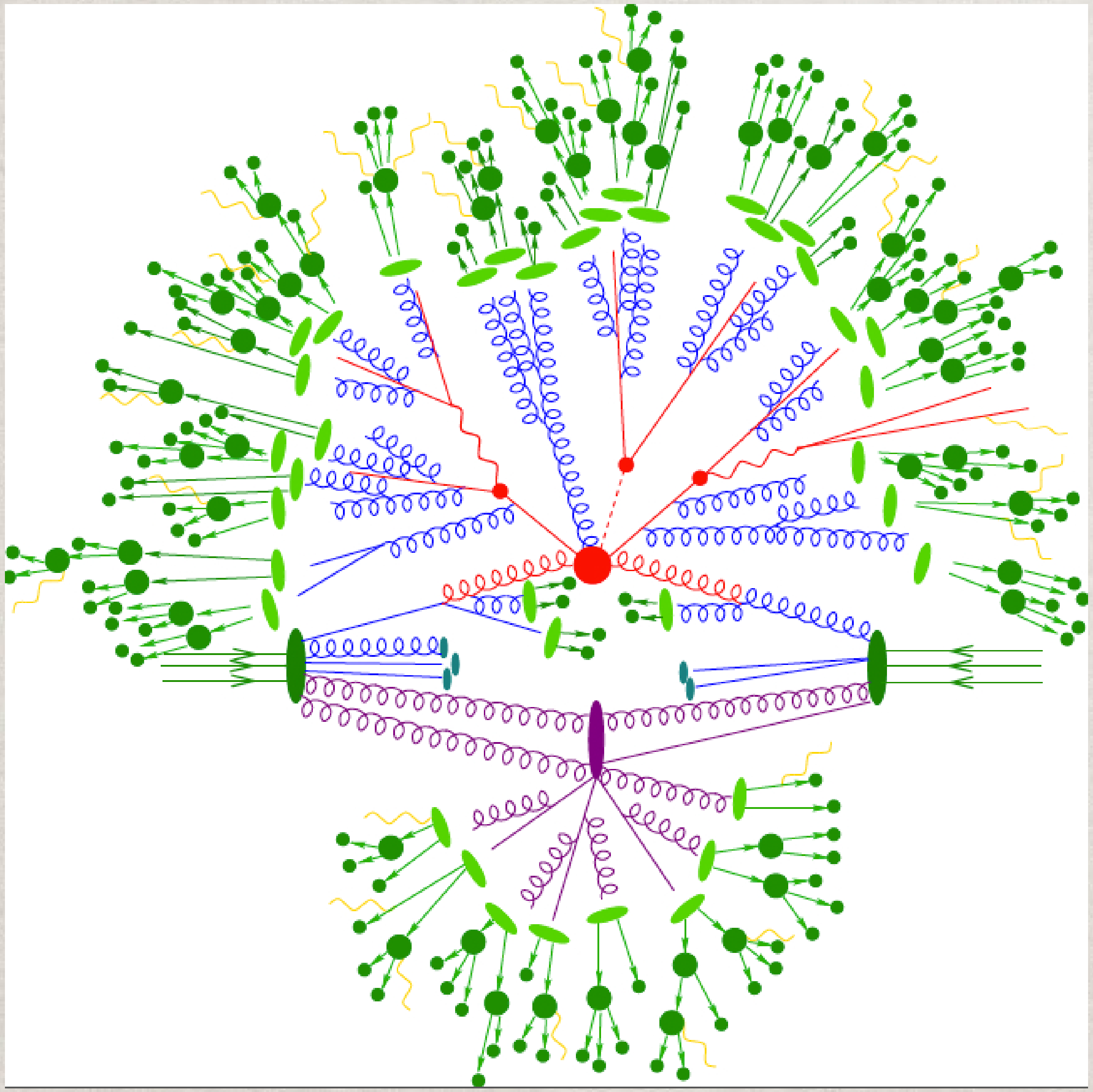
Outline

- **Brief introduction to jets observables**
- **General principles of resummation of jet observables in QCD**
- Non-global logarithms
- Coherence-violating logarithms

Introduction to jet observables

Events at hadron colliders

The description of LHC events involves different levels

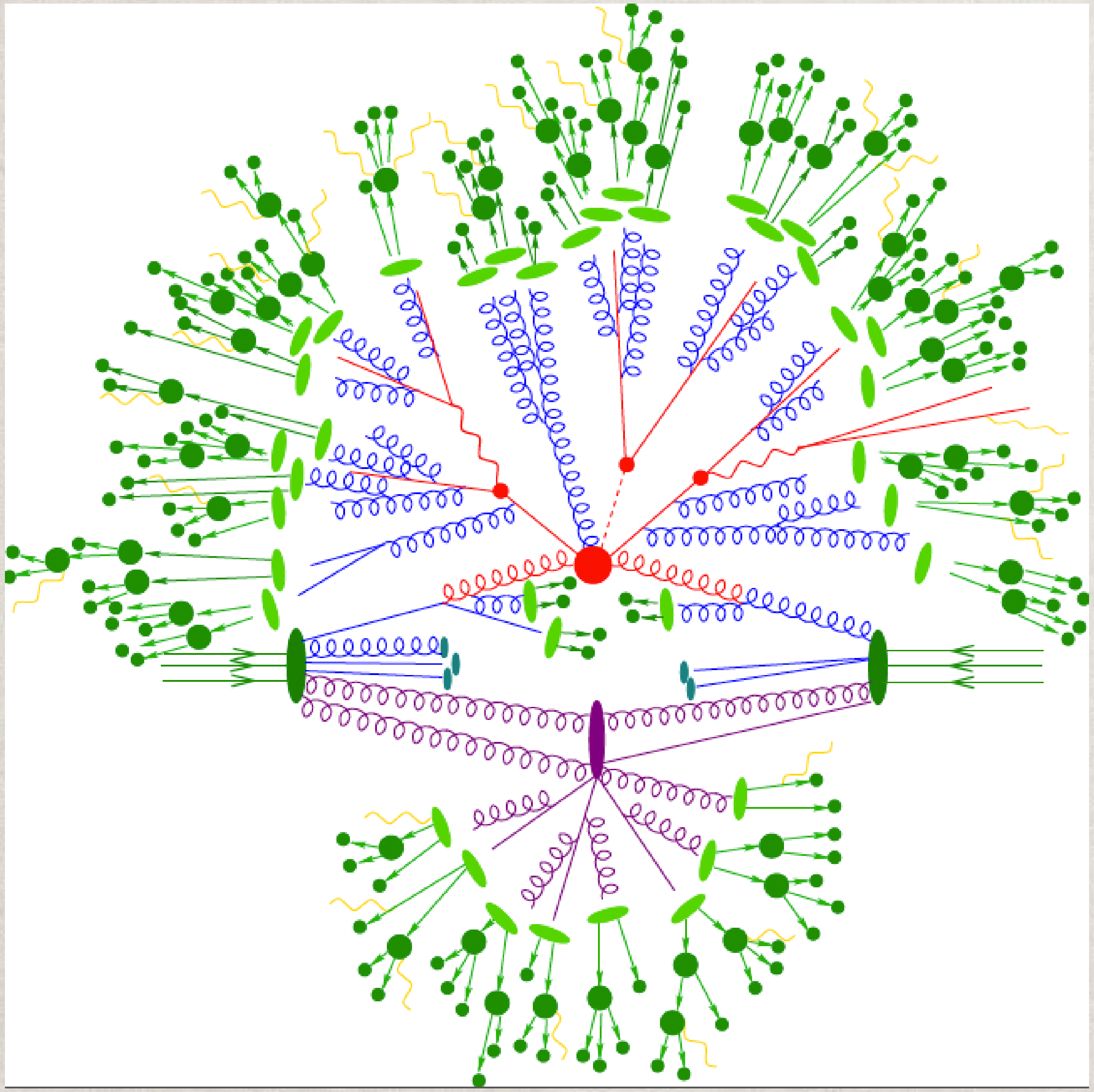


- A hard event, with well separated partons $\Rightarrow$ fixed-order QCD
- Radiation of secondary soft and collinear gluons from the hard partons $\Rightarrow$ Monte Carlo, all-order resummation
- Hadronisation $\Rightarrow$ Monte Carlo or analytical models
- Scattering of proton remnants, underlying event, etc.

The main challenge is to find “good” observables that relate high-multiplicity events to the dynamics of the underlying quarks and gluons

Events at hadron colliders

The description of LHC events involves different levels

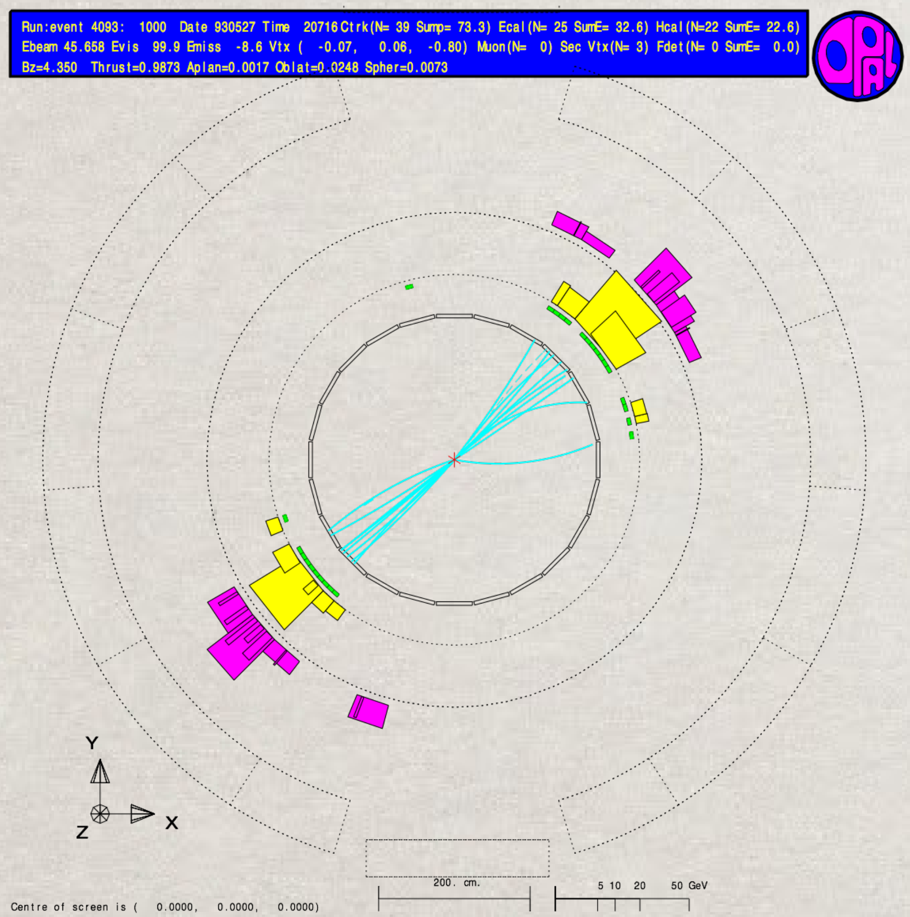


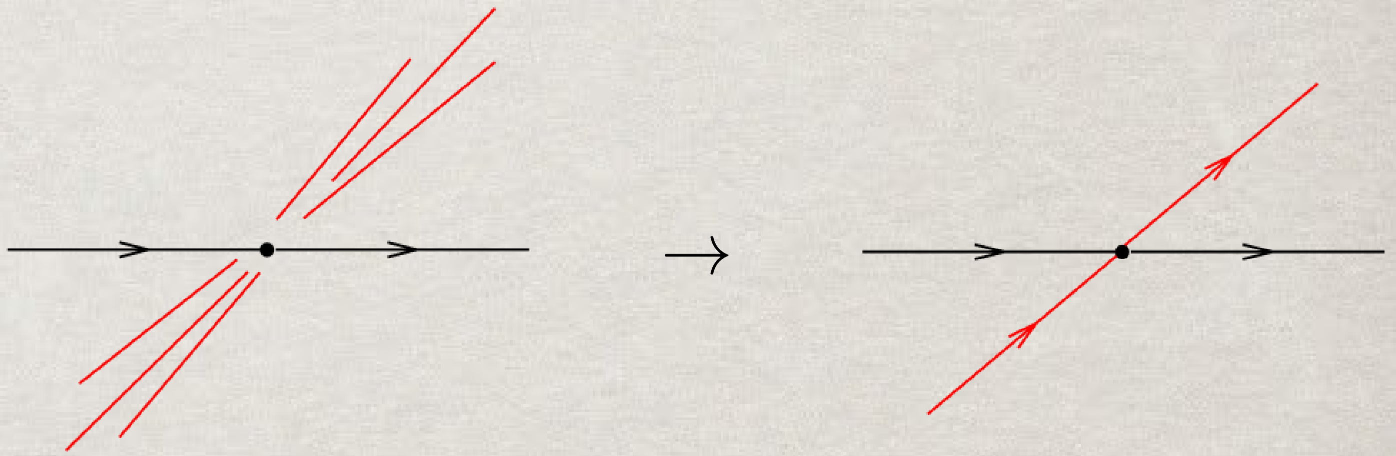
- A hard event, with well separated partons ⇒ fixed-order QCD
- Radiation of secondary soft and collinear gluons from the hard partons ⇒ Monte Carlo, all-order resummation
- Hadronisation ⇒ Monte Carlo or analytical models
- Scattering of proton remnants, underlying event, etc.

The main challenge is to find “good” observables that relate high-multiplicity events to the dynamics of the underlying quarks and gluons

JETTY EVENTS

High-energy hadronic events contain highly collimated jets of hadrons

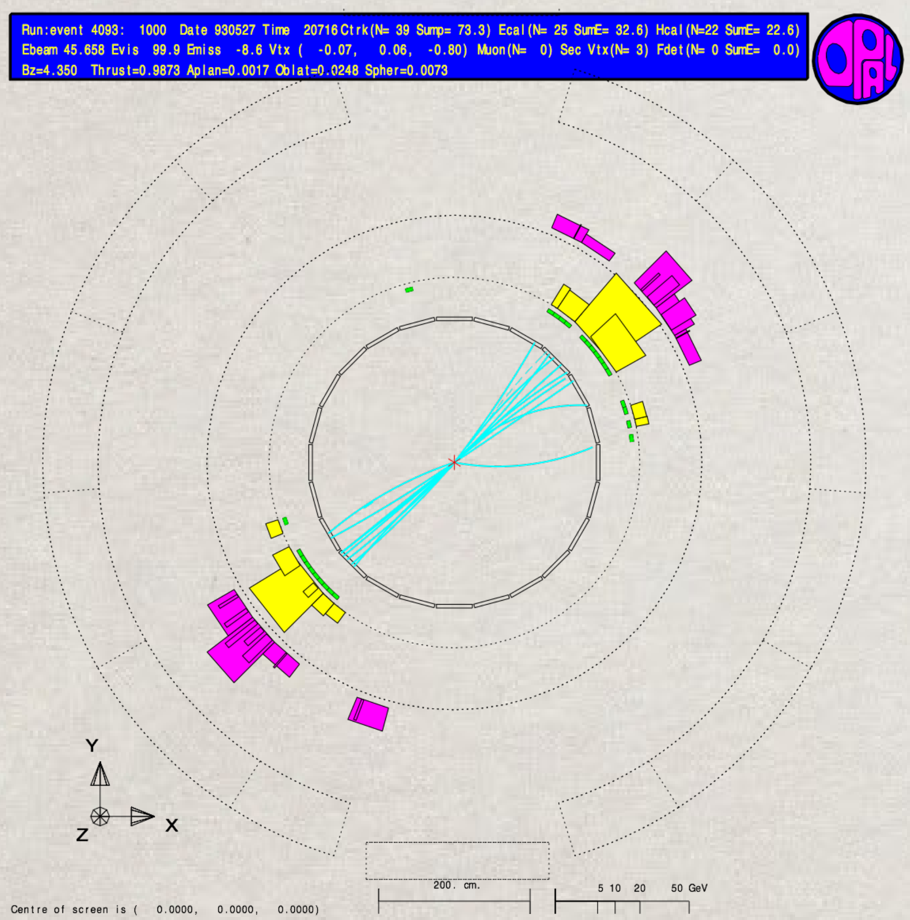


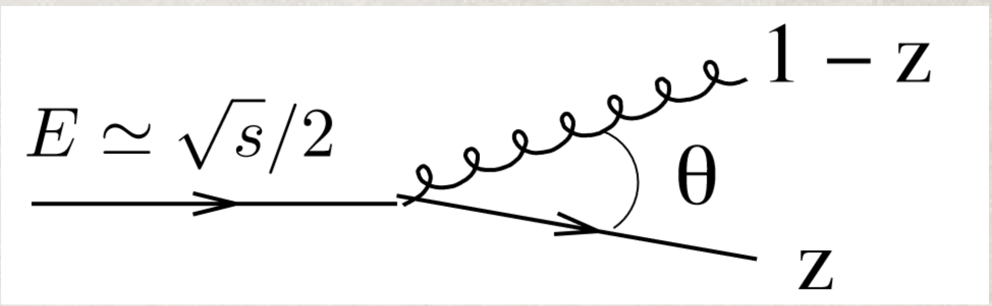


- Jets are the natural objects that we would like to associate with quarks and gluons, using the equality 1 jet = 1 parton
- We need to find a mathematical definition of jets that relate what we observe to what we can compute in QCD

Infrared and collinear safety

- Inside a QCD jet we find a number of soft and collinear splittings





$$E_q \simeq zE \qquad\qquad E_g \simeq (1-z)E$$

$$dP_{q\to qg} = C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2} dz \frac{1+z^2}{1-z}$$

Splitting probabilities are singular when emissions are soft ($z \to 1$) or collinear ($\theta \to 0$). If these singularities cancel with virtual corrections, then jet observables are insensitive to

- the addition of any number of soft partons (IR safety)
- an arbitrary number of collinear splittings (collinear safety)

Sequential algorithms

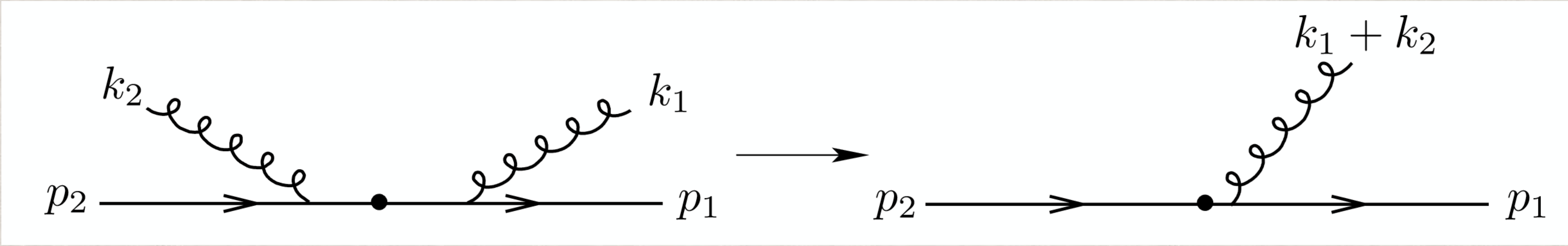
- Sequential algorithms were first introduced in e^+e^- annihilation, where one has full information on hadron momenta
- First-ever sequential algorithm used by experiments is the JADE algorithm: For any pair of particles, p_i, p_j , find the minimum "distance"

$$y_{ij} = \frac{(p_i + p_j)^2}{Q^2}$$

- If $y_{ij} < y_{\rm cut}$, merge particles p_i, p_j into a jet
- Repeat until all pairs have $y_{ij} > y_{\rm cut}$. The number of jets is the number of particles left

Sequential algorithms

Problem: the distance measure of the JADE algorithm is the invariant mass of two partons. The JADE can cluster together two soft particles collinear to different legs, leading to spurious large-angle soft jets



Solution: Durham (a.k.a. k_t) algorithm. The distance measure is the relative transverse momentum of the softer particle with respect to the harder one

[Catani Dokshitzer Olsson Turnock Webber PLB 269 (1991) 432]

$$y_{ij} = \frac{2\min\{E_i^2, E_j^2\}}{Q^2}(1 - \cos\theta_{ij})$$

The Cambridge algorithm is a more sophisticated version that uses angles only to determine the clustering sequence [Dokshitzer Leder Moretti Webber hep-ph/9707323]

Generalised k_T algorithms

In hadron collisions, the most-used algorithms are the generalised-k_t algorithms [Cacciari Salam Soyez 0802.1189]

- One finds the minimum over all particles of the distances

$$d_{ij} = \frac{\min\{k_{ti}^{2p}, k_{tj}^{2p}\}}{R^2}\Delta R_{ij}^2 \qquad d_{iB} = k_{ti}^{2p} \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- If the minimum distance is d_{iB} or d_{jB}, then p_i or p_j is a jet and is removed from the list of particles, otherwise p_i and p_j are merged into a jet
- The procedure is repeated until no particles are left
- The parameter R is the jet “radius”. If there are only two particles, they are in the same jet only if $R_{ij} < R$
- The parameter p identifies the algorithm, and usually assumes values 1 (k_t algorithm), 0 (Cambdridge-Aachen algorithm) and -1 (anti- k_t algorithm)

IRC safe jets

- After a sequential clustering, all jets with $p_t > p_{t,\min}$ are IRC safe
- IRC safe jet cross sections can be safely computed in massless QCD $\Rightarrow$ non-perturbative effects associated to quark masses are power suppressed

$$d\sigma_{pp\to\text{jets}}\left(\alpha_s(p_{t,\min}), \frac{m}{p_{t,\min}}\right) = d\sigma_{pp\to\text{jets}}\left(\alpha_s(p_{t,\min}), 0\right) + \mathcal{O}\left(\left(\frac{m}{p_{t,\min}}\right)^p\right)$$

- Collimation of the jets is naturally explained in terms of the collinear enhancement of QCD matrix elements

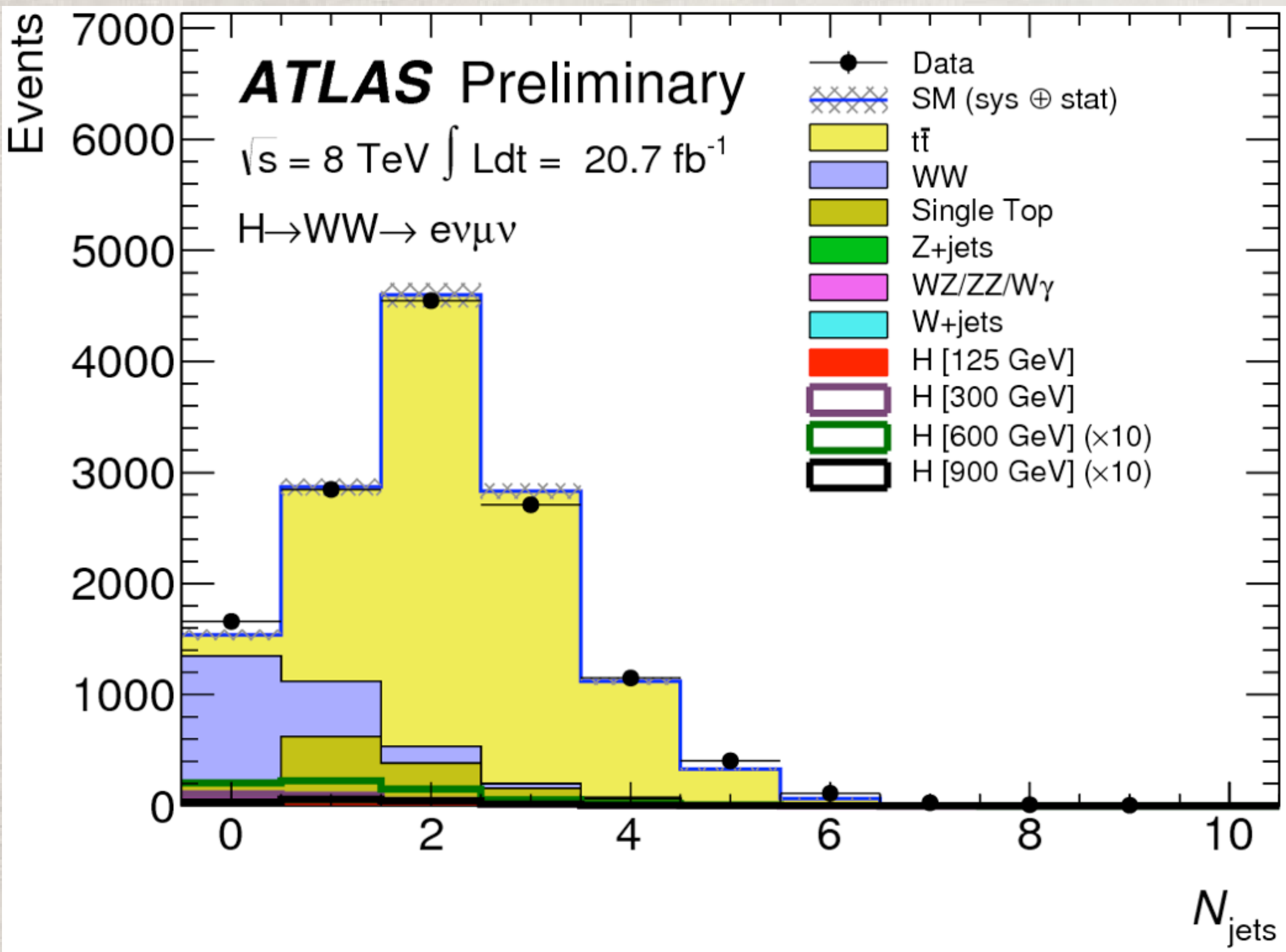
$$dP_{q\to qg} = C_F \frac{\alpha_s[z(1-z)\theta p_{t,\min}]}{2\pi}\frac{d\theta^2}{\theta^2} dz \frac{1+z^2}{1-z}$$

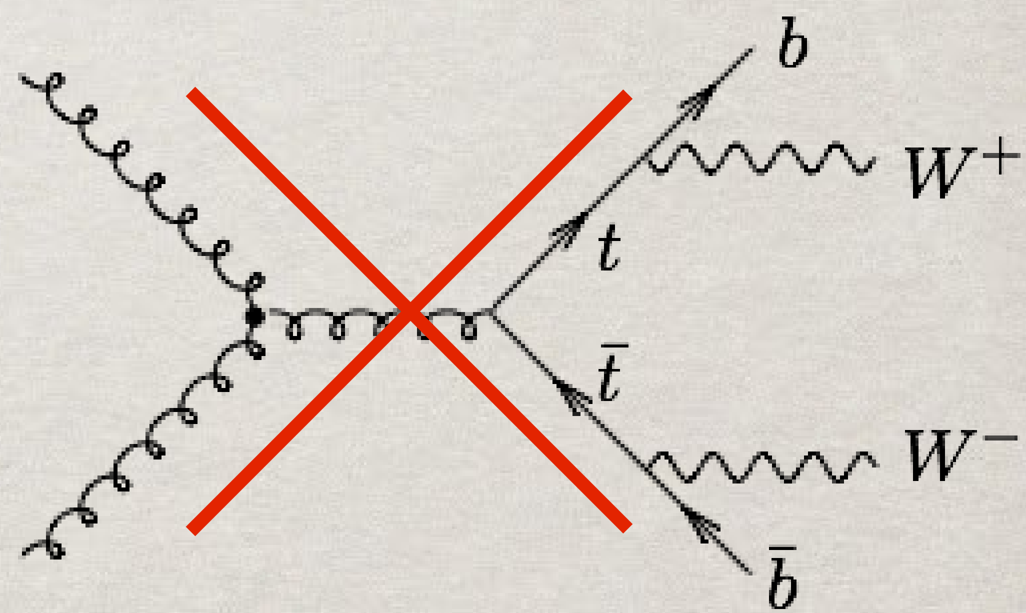
- Due to asymptotic freedom, $\alpha_s(p_{t,\min}) \to 0$ for $p_{t,\min} \to \infty$: the higher the energy, the more collimated the jets $\Rightarrow$recovery of the equality 1jet = 1 parton in the high-energy limit

Resummation

JET VETOES

In many situations (e.g. WW production), one puts a jet-veto to eliminate overwhelming top-antitop background

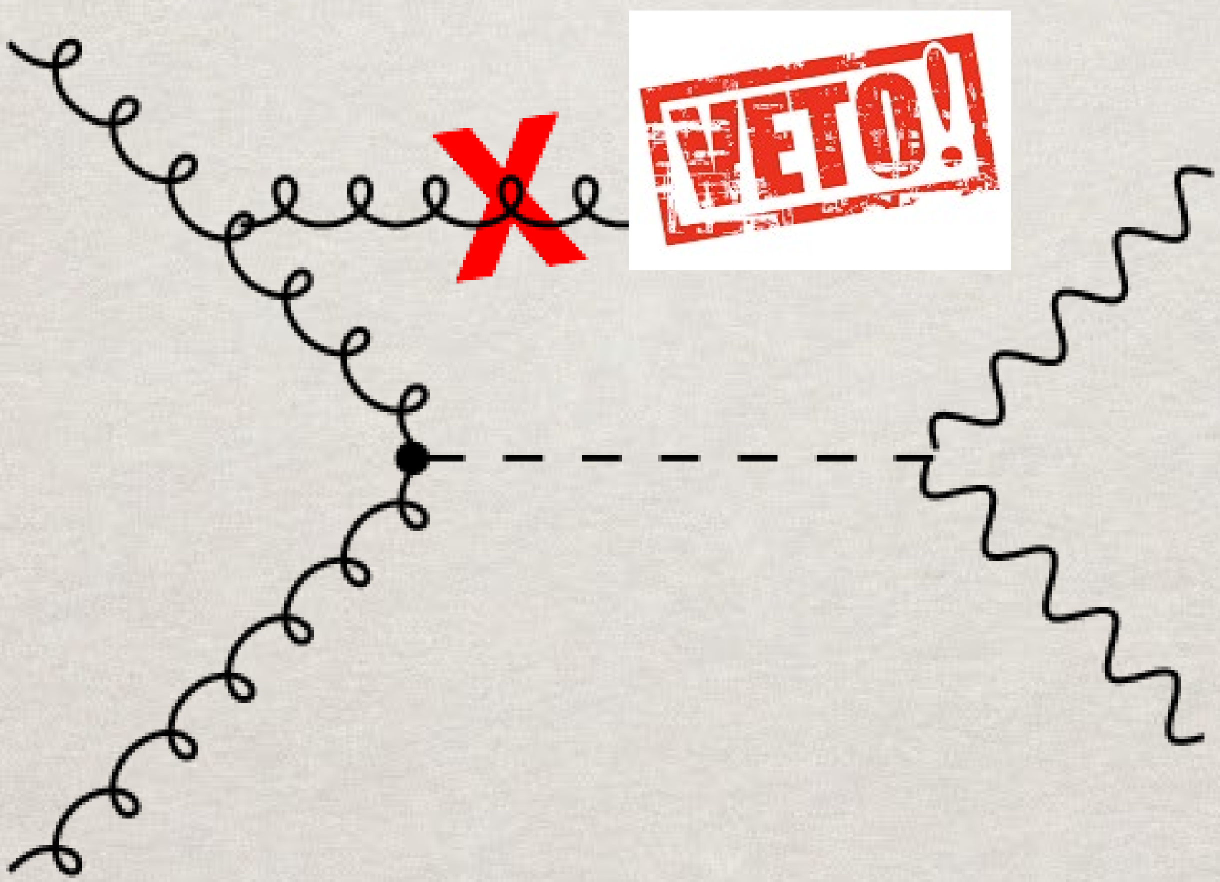




The main object of study is the zero-jet cross section $\sigma_{0\text{-jet}}(p_{\mathrm{t,veto}})$, obtained by imposing that all jets have $p_t < p_{\mathrm{t,veto}}$

Two-scale problems

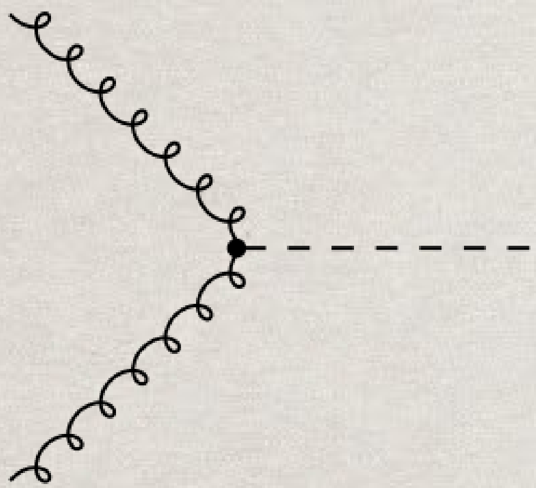
The zero-jet cross section is characterised by two scales, the mass of the produced object M and the jet resolution $p_{\mathrm{t,veto}}$

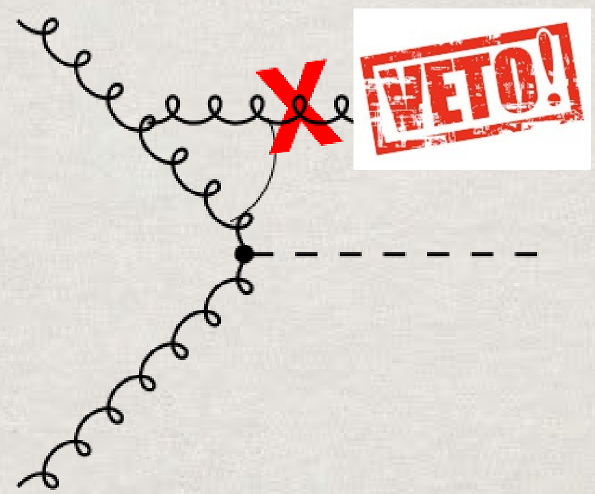


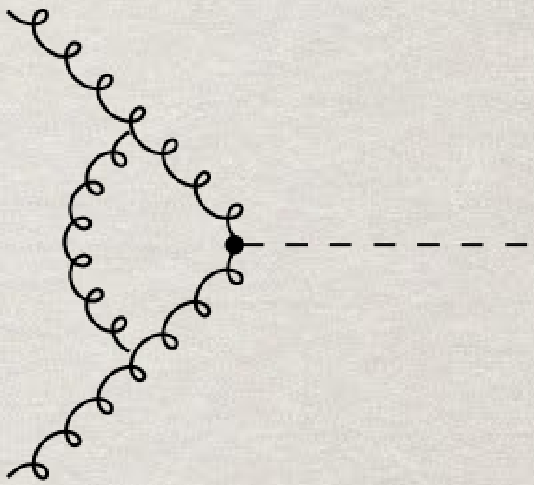
In QCD, large logarithms such as $\ln(M/p_{\mathrm{t,veto}})$ appear whenever the phase space for the emission of soft and/or collinear gluons is restricted

One gluon emission

Example: veto one soft ($E \ll M$) and collinear ($\theta \ll 1$) gluon







BORN **REAL** **VIRTUAL**

soft collinear

$$\sigma_0 \left[1 \quad + \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \Theta\left(p_{\mathrm{t,veto}} - E\theta\right) \quad - \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \right]$$

colour "charge" of a gluon = Casimir of the adjoint representation of SU(N_c)

factorisation of soft radiation

$$\boxed{\sigma_{\text{0-jet}} = \sigma_0 \left[1 - 2 C_A \frac{\alpha_s}{\pi} \ln^2 \left(\frac{M}{p_{\mathrm{t,veto}}} \right) \right]}$$

All-order resummation

All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with e.g. $L \equiv \ln(M/p_{\mathrm{t,veto}})$

$$\sigma_{0-\mathrm{jet}} \sim \sigma_0 \, e^{\overbrace{Lg_1(\alpha_s L)}^{}_{\mathrm{LL}}} \times \Big(\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots \Big)$$

All-order resummation

All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with e.g. $L \equiv \ln(M/p_{\mathrm{t,veto}})$

$$\sigma_{0-\text{jet}} \sim \sigma_0 \, e^{\overbrace{Lg_1(\alpha_s L)}_{\text{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\text{NNLL}} + \dots \right)$$

All-order resummation

All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with e.g. $L \equiv \ln(M/p_{\mathrm{t,veto}})$

$$\sigma_{0-\mathrm{jet}} \sim \sigma_0\, e^{\underbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \dots \right)$$

All-order resummation

All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with e.g. $L \equiv \ln(M/p_{\mathrm{t,veto}})$

$$\sigma_{0-\mathrm{jet}} \sim \sigma_0\, e^{\underbrace{L g_1(\alpha_s L)}_{\mathrm{LL}}} \times \left(\begin{array}{ccc} 1 & + & \alpha_s \quad + \ldots \\ \underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} & + & \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots \end{array} \right)$$

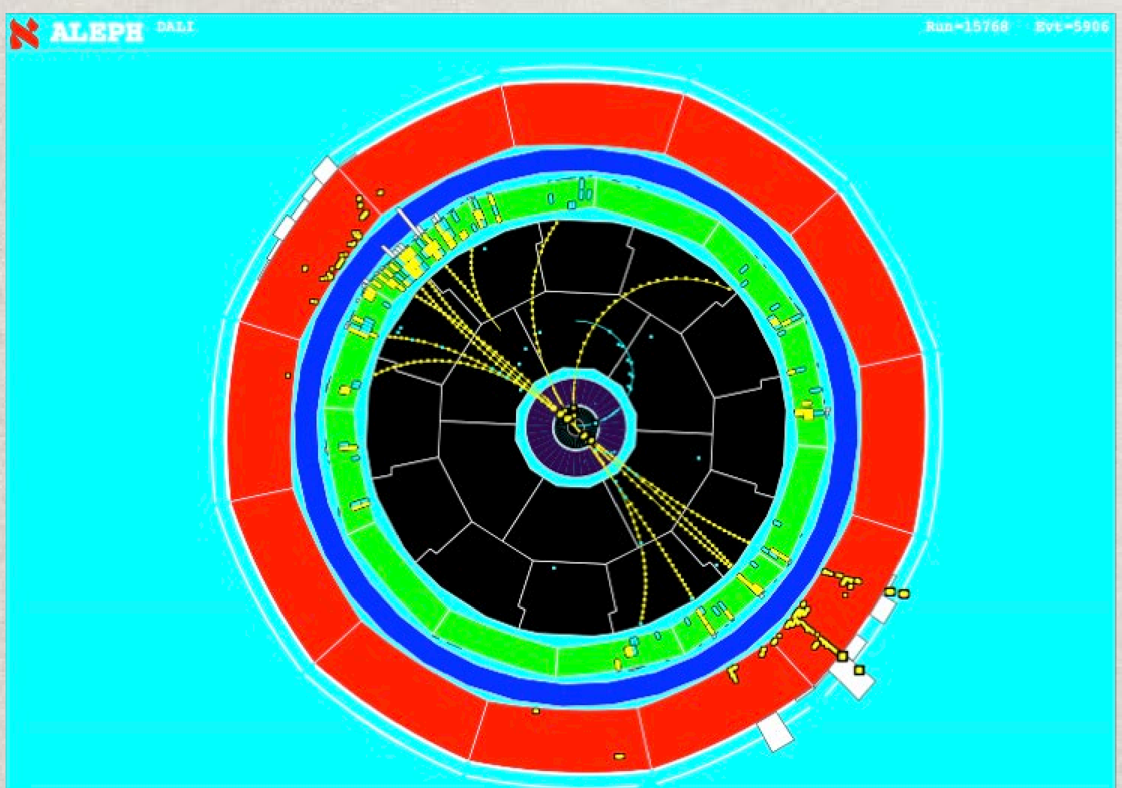
Final-state observables

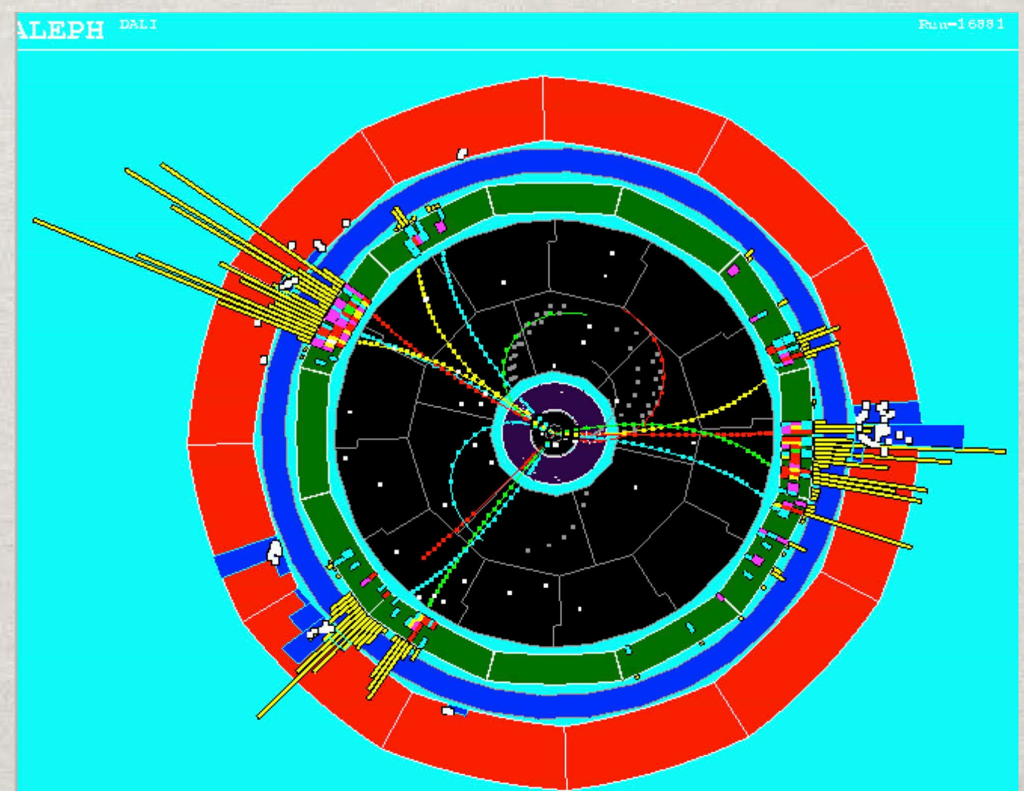
- We consider a generic IRC safe final-state observable, a function $V(p_1, \ldots, p_n)$ of all final-state momenta $p_1, \ldots, p_n$
- Example: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \text{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Pencil-like events $T \lesssim 1$

Planar events $T \gtrsim 2/3$

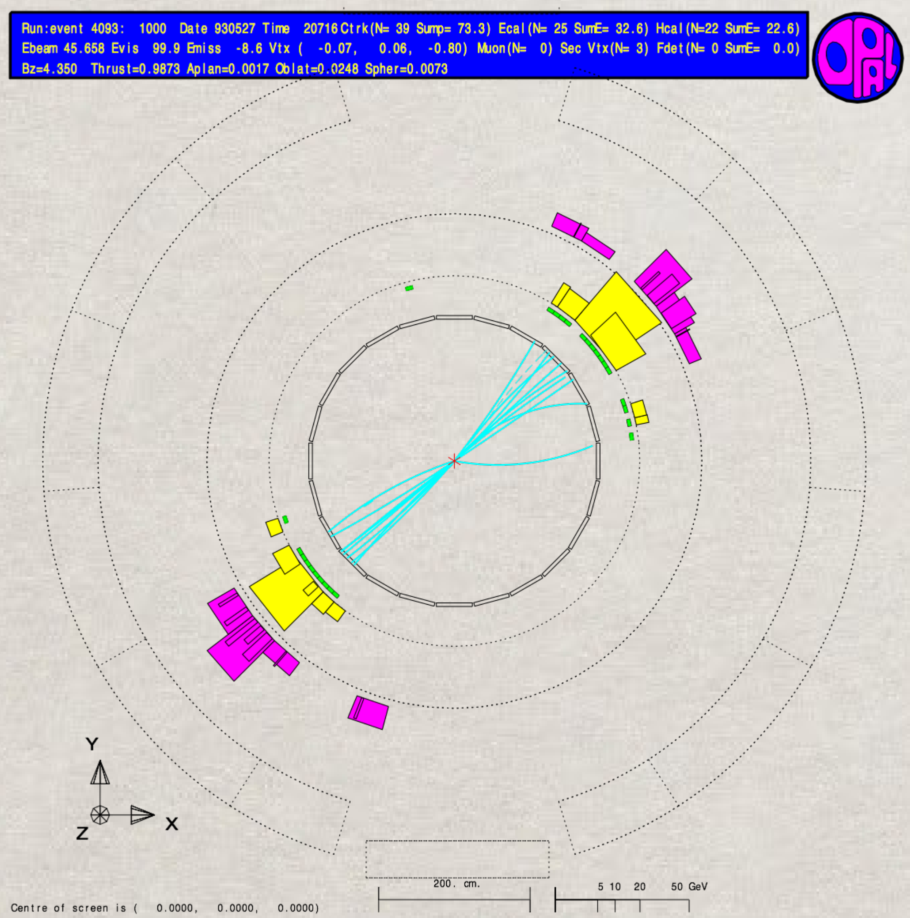




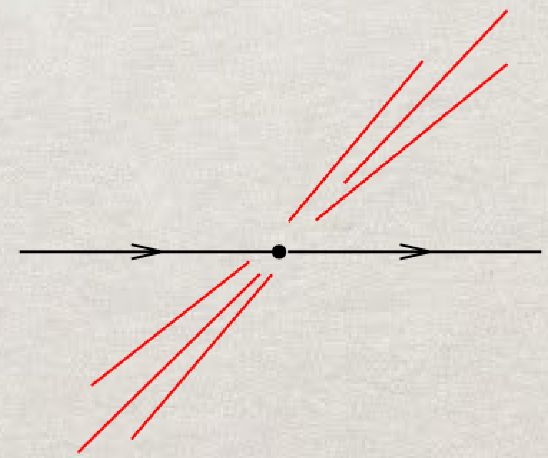
DEPARTURE FROM THE BORN LIMIT

- Final-state observables have the property that, for configurations close to the Born limit (e.g. a back-to-back $q\bar{q}$ pair), their value is close to zero

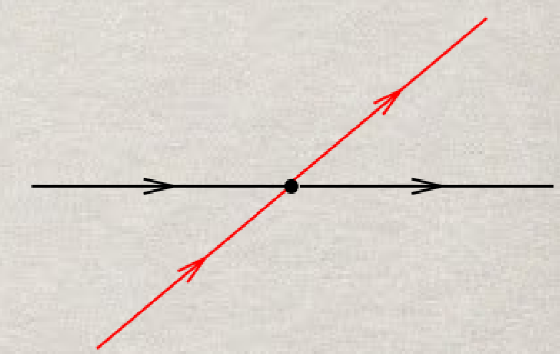
- Example: in two-jet events, one minus the thrust is the sum of the invariant masses of the two jets, which vanishes in the Born limit



$$\Sigma(v) = \text{Prob}[V(p_1, \dots, p_n) < v]$$

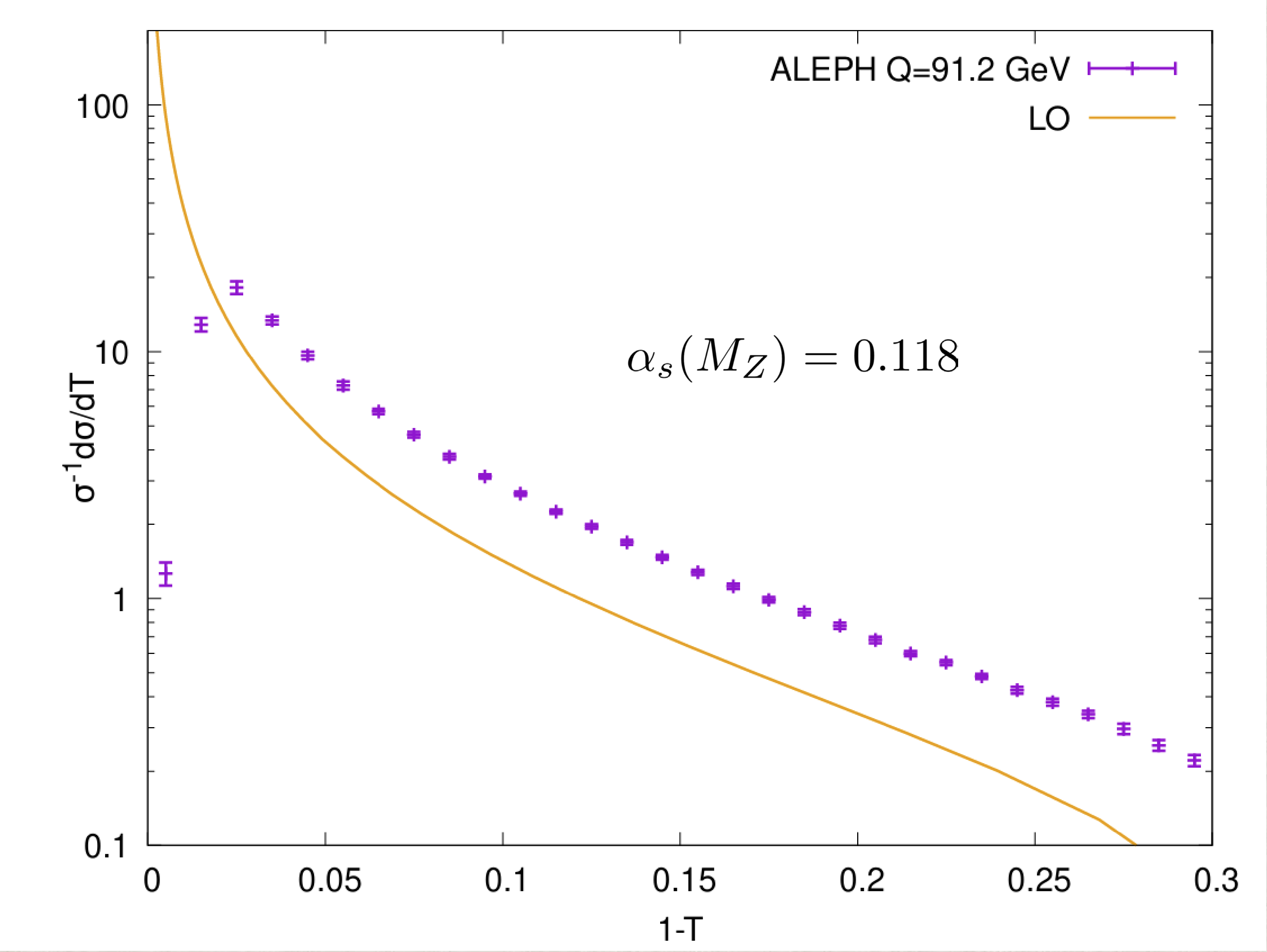


$v \ll 1$



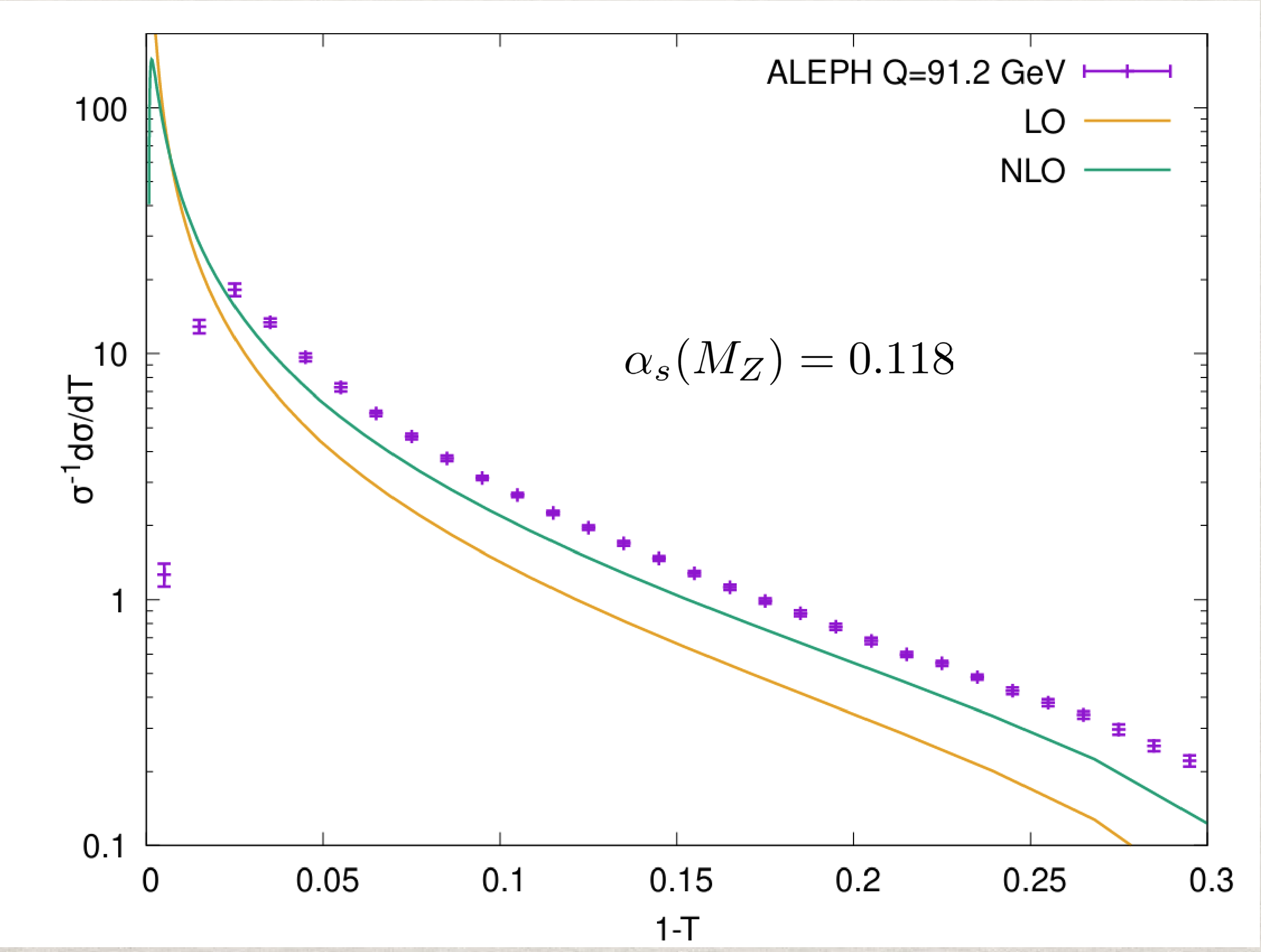
- To quantify the departure from the Born limit, we consider $\Sigma(v)$, the fraction of events such that $V(p_1, \dots, p_n) < v$

Comparision to data



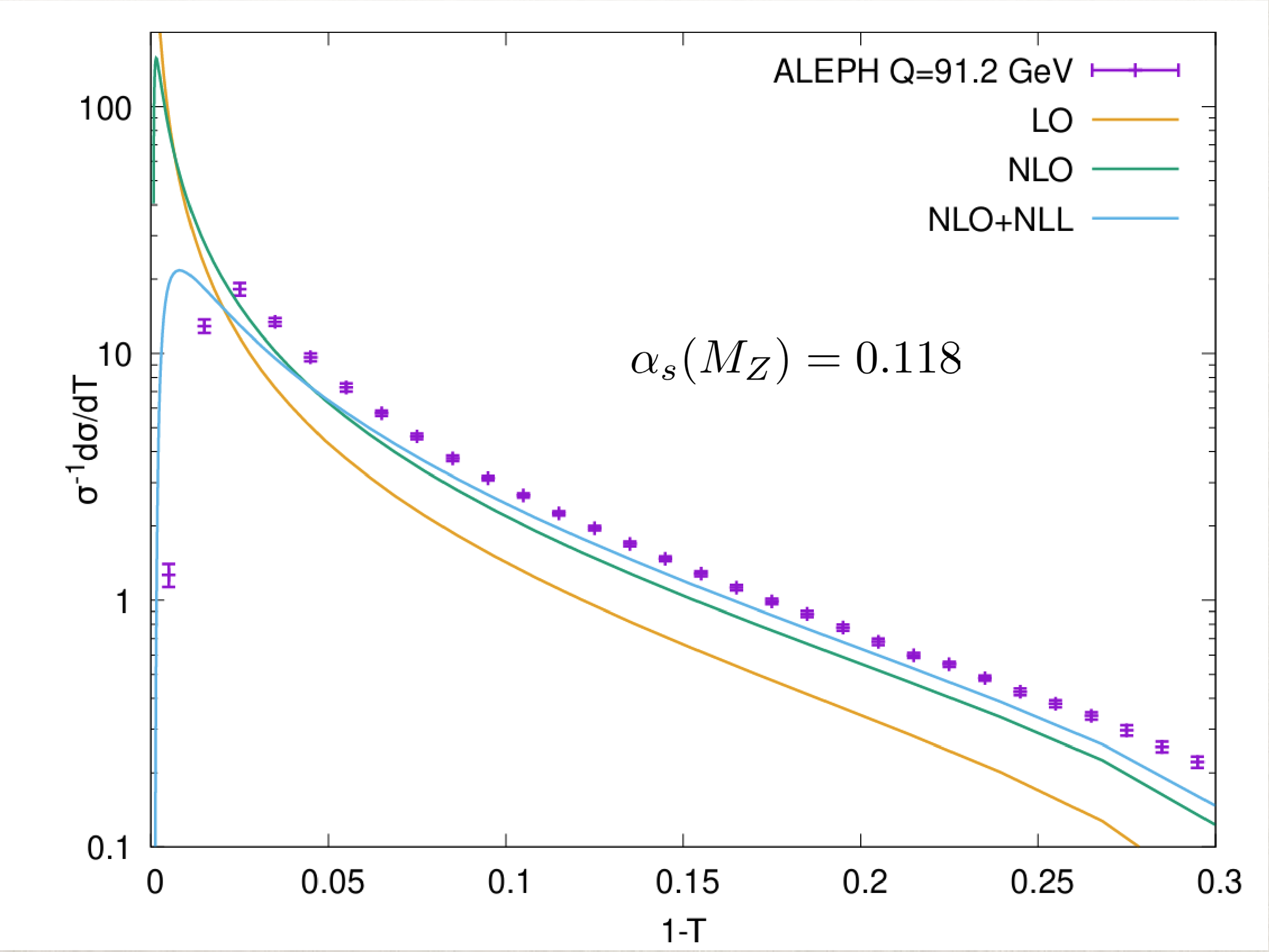
- LO predictions generally undershoot data

Comparision to data



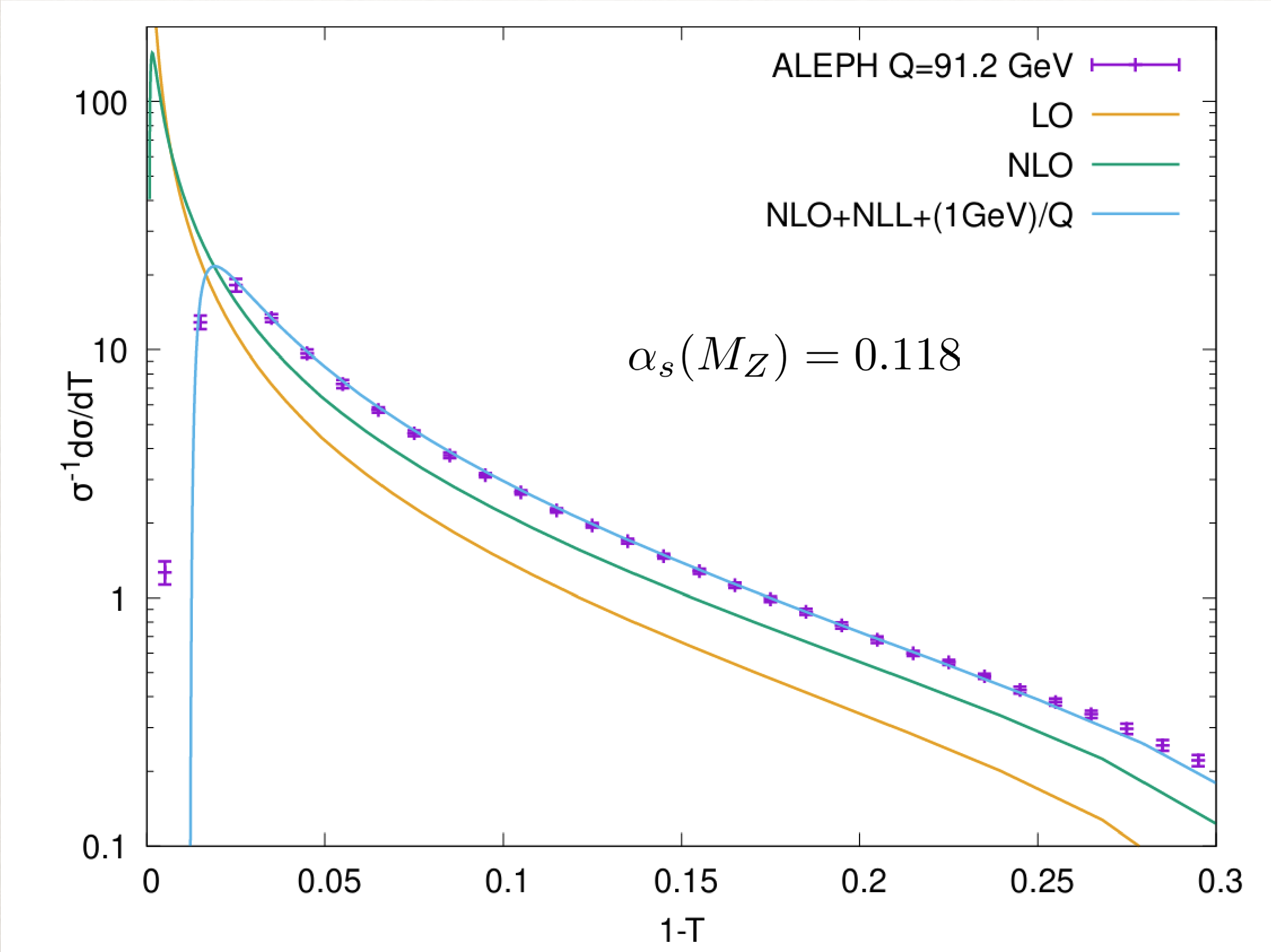
- LO predictions generally undershoot data
- NLO predictions have the right size, but diverge at low values of v

Comparision to data



- LO predictions generally undershoot data
- NLO predictions have the right size, but diverge at low values of v
- Resummation (matched to NLO) has the right shape

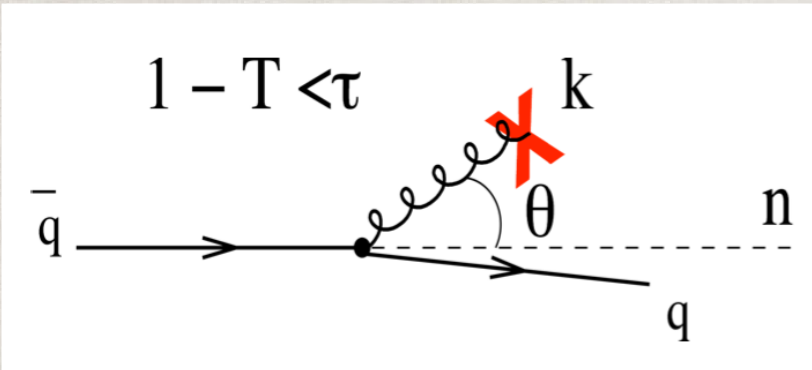
Comparision to data



- LO predictions generally undershoot data
- NLO predictions have the right size, but diverge at low values of v
- Resummation (matched to NLO) has the right shape
- Agreement with data requires including hadronisation corrections

The Lund plane

Soft-collinear emissions can be visualised as points in the Lund plane

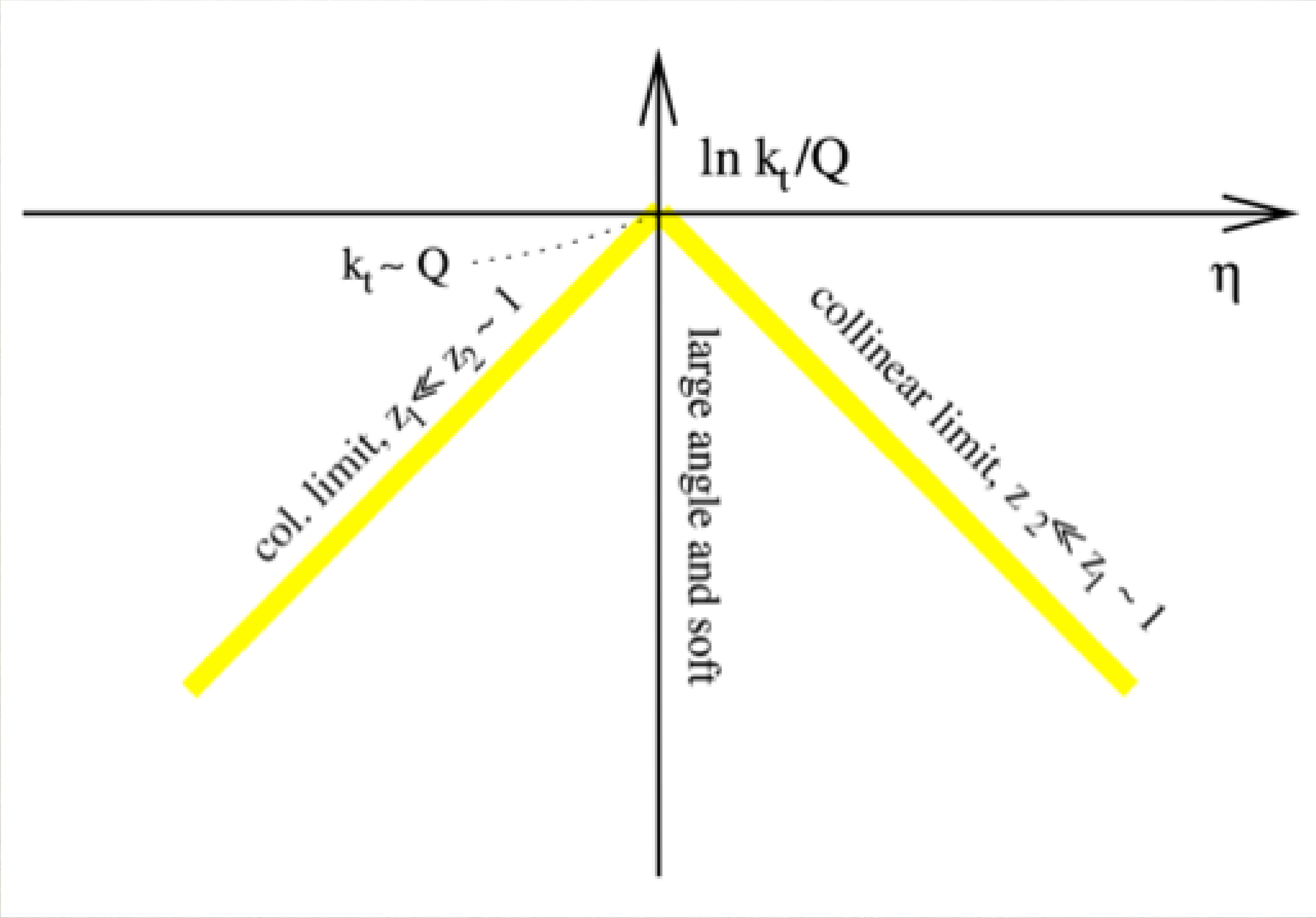


Sudakov decomposition

$$P_1 = \frac{Q}{2}(1, \vec{n}) \qquad P_2 = \frac{Q}{2}(1, -\vec{n})$$

$$k = z^{(1)} P_1 + z^{(2)} P_2 + k_t$$

$$\eta = \frac{1}{2} \ln \left(\frac{z^{(1)}}{z^{(2)}} \right) \simeq \ln \frac{1}{\theta} \qquad z_1 \gg z_2$$



- Collinear limit

$$z_1, z_2 < 1 \Rightarrow |\eta| < \ln \left(\frac{Q}{k_t} \right)$$

- Soft-collinear matrix element

$$[dk] M^2(k) \simeq 2 C_F \frac{\alpha_s}{\pi} \frac{dk_t}{k_t} d\eta \frac{d\phi}{2\pi}$$

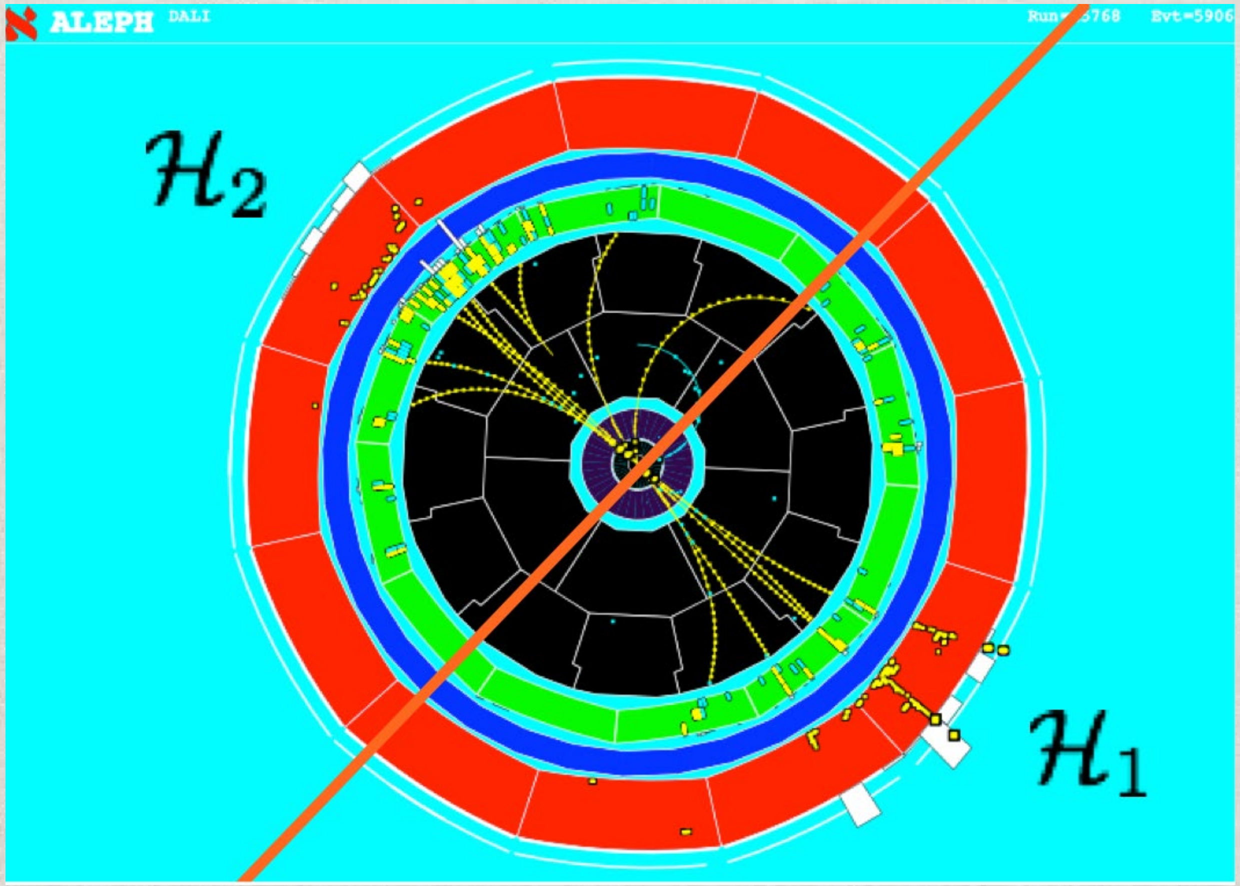
colour “charge” of a quark = Casimir of the fundamental representation of SU(N_c)

The thrust in the Lund plane

Behaviour of the thrust in the soft-collinear limit

Final-state $q\bar{q}$ pair

$$1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|} + \sum_{\ell=1,2} \frac{1}{Q^2} \frac{\left|\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}\right|^2}{1 - \sum_{i\in\mathcal{H}_\ell} z_i^{(\ell_i)}}$$



The thrust in the Lund plane

Behaviour of the thrust in the soft-collinear limit

recoiling $q\bar{q}$ pair

$$1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|} + \sum_{\ell=1,2} \frac{1}{Q^2} \frac{\left|\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}\right|^2}{1 - \sum_{i\in\mathcal{H}_\ell} z_i^{(\ell_i)}}$$

- Soft and collinear

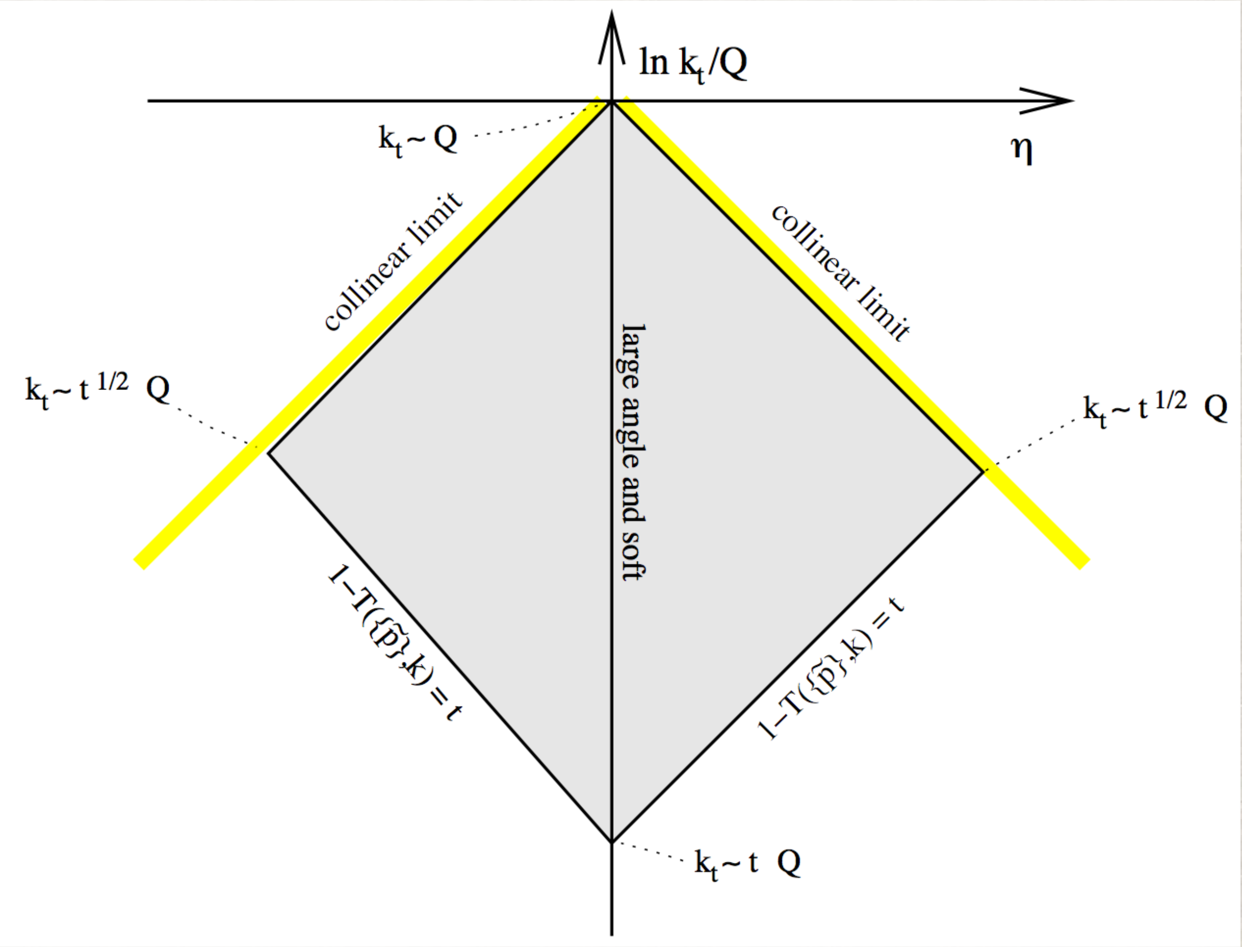
$$1 - T(\{\tilde{p}\}, k) \simeq \frac{k_t}{Q} e^{-|\eta|}$$

- Soft and large angle

$$1 - T(\{\tilde{p}\}, k) \sim k_t$$

- Hard and collinear

$$1 - T(\{\tilde{p}\}, k) \sim k_t^2$$



An observable in the Lund plane

Behaviour of an IRC safe observable in the soft-collinear limits

- Soft and collinear to leg $\ell = 1, 2$

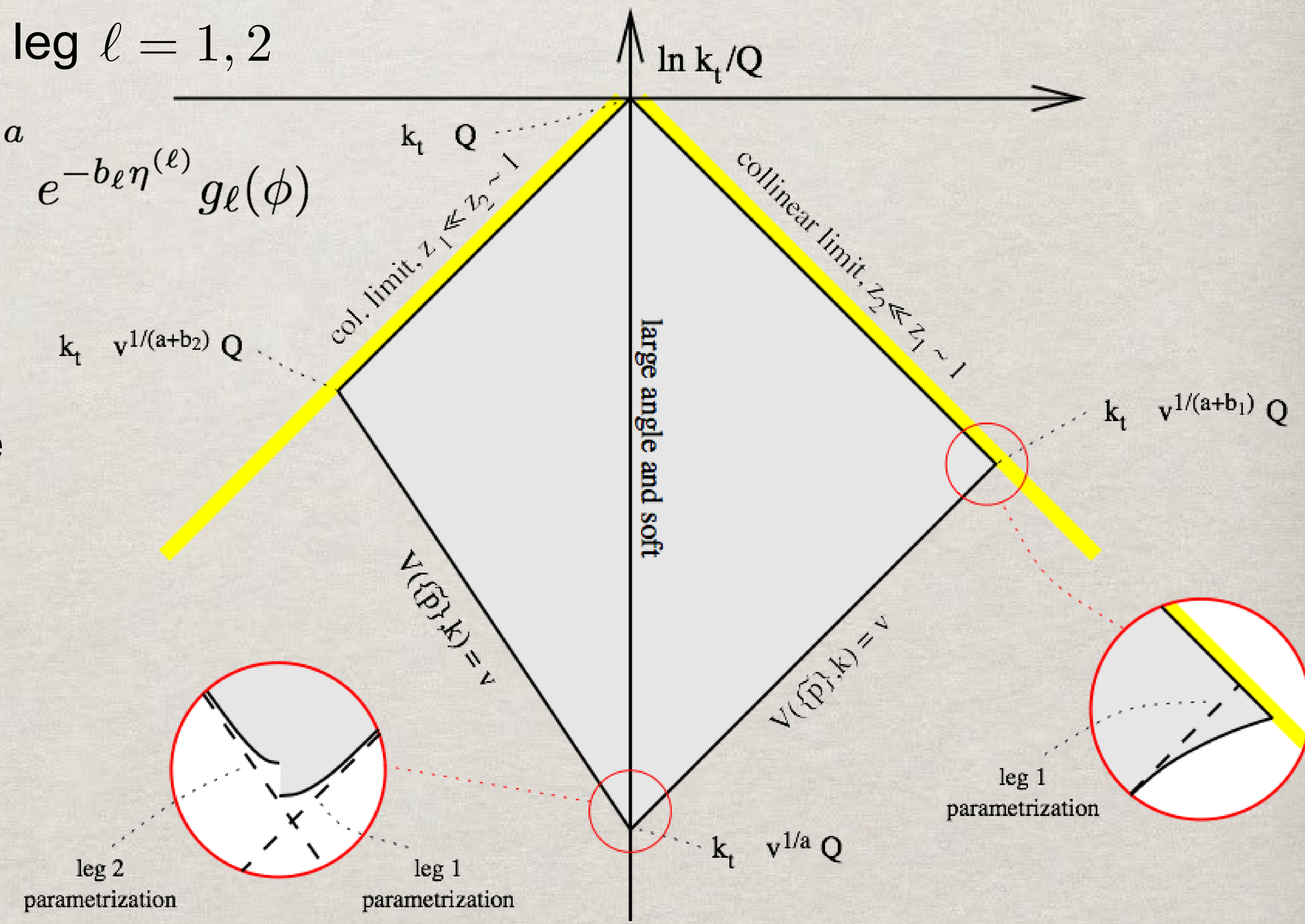
$$V(\{\tilde{p}\}, k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)$$

- Soft and large angle

$$V(\{\tilde{p}\}, k) \sim k_t^a$$

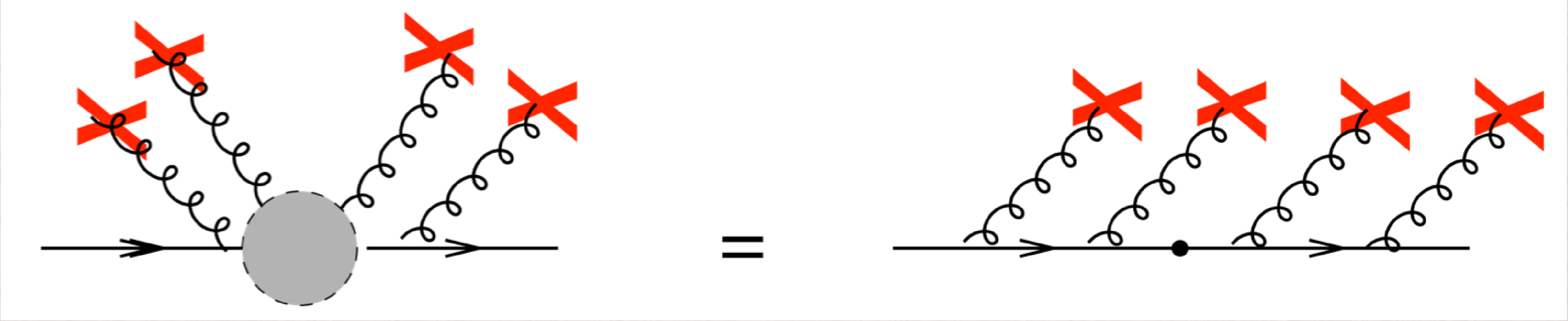
- Hard and collinear

$$V(\{\tilde{p}\}, k) \sim k_t^{a+b_\ell}$$



Multiple soft-collinear emissions

- We first consider an ensemble of soft-collinear emissions widely separated in angle (rapidity)
- Due to QCD coherence, the multi-gluon matrix element factorises into the product of single-emission matrix elements



- Contribution of multiple soft-collinear emissions to $\Sigma(v)$

$$\Sigma(v) = e^{-\int [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_n))$$

virtual corrections, ensure that the inclusive sum over all emissions gives one

Sudakov form factor

- Strategy: split the exponent in two parts

$$\int [dk] M^2(k) = \int_v [dk] M^2(k) + \int^v [dk] M^2(k) \qquad \int_v [dk] M^2(k) \equiv R(v)$$

$$\Sigma(v) = \underbrace{e^{-R(v)}}_{\text{Sudakov form factor}} \underbrace{\left\{ e^{-\int^v [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_n)) \right\}}_{\text{multiple-emission correction}}$$

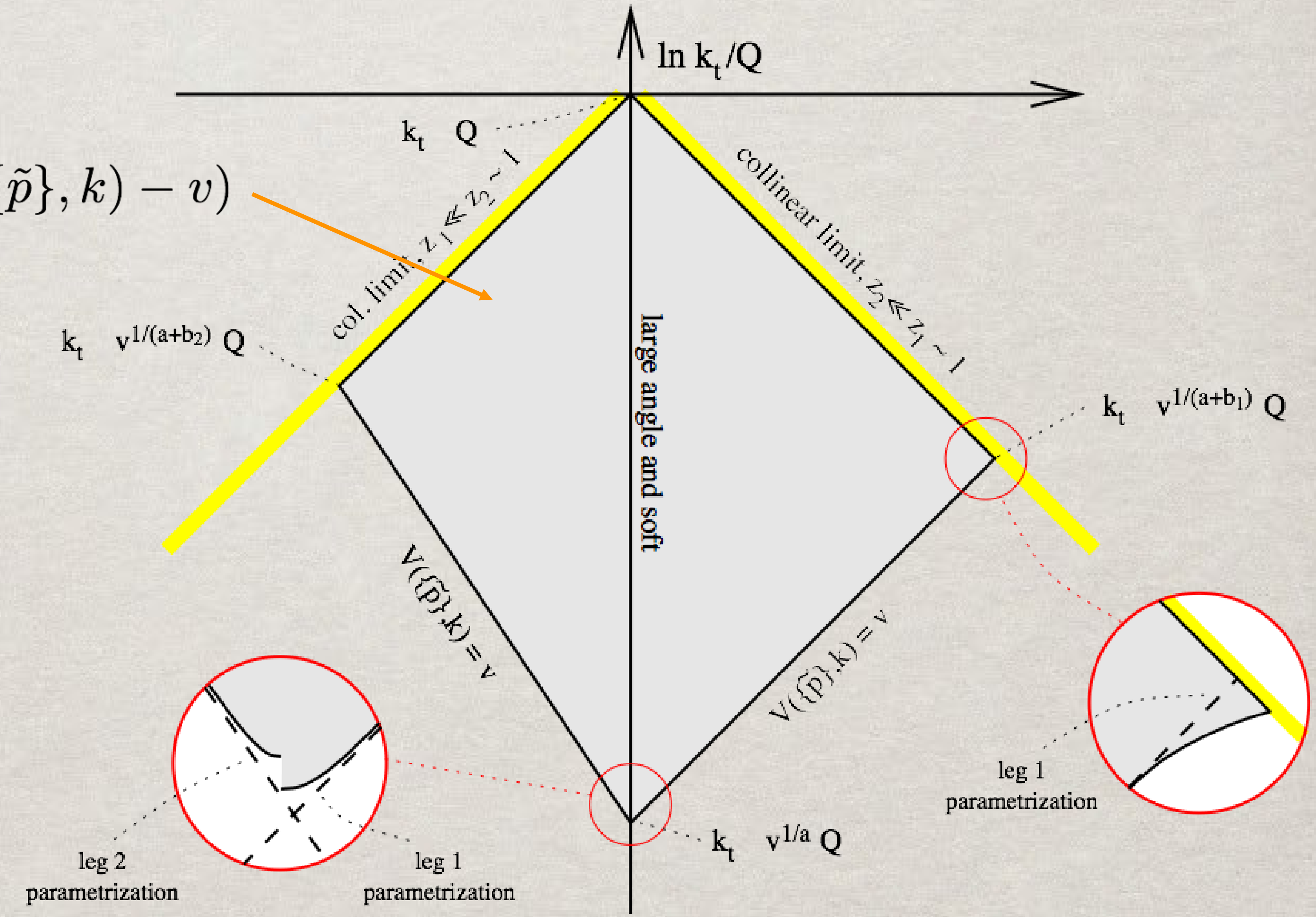
Sudakov form factor

multiple-emission correction

Double logarithmic radiator

The Sudakov exponent, a.k.a. as “radiator”, is just the area of the shaded region in the Lund plane

$$R(v) = \int [dk] M^2(k)\, \Theta(V(\{\tilde{p}\}, k) - v)$$



Since it is an area in the Lund plane, its contribution is double logarithmic

Perfect exponentiation

Consider an observable that takes contribution only from the emission for which $V(\{\tilde{p}\},k)$ is the largest

$$V(\{\tilde{p}\},k_1,\ldots,k_n) = \max_i V(\{\tilde{p}\},k_i) \qquad \text{e.g.} \qquad \frac{p_{\mathrm{t,max}}}{m_H} = \max_{j\in\mathrm{jets}} \frac{p_{t,j}}{m_H}$$

$$\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_n)\right) = \prod_{i=1}^{n} \Theta\left(v - V(\{\tilde{p}\},k_i)\right)$$

$$\Sigma(v) = e^{-R(v)} \left\{ e^{-\int^v [dk] M^2(k)} \underbrace{\sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} [dk_i] M^2(k_i) \Theta\left(v - V(\{\tilde{p}\},k_i)\right)}_{=e^{\int^v [dk] M^2(k)}} \right\} = e^{-R(v)}$$

- The cumulative distribution for such observables is a Sudakov form factor
- Interpretation of the Sudakov form factor: probability that all emissions have $V(\{\tilde{p}\},k_i) < v$

Additive observables

- Consider an observable that, in the soft and collinear limit, is the sum of the contributions of individual emissions

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \sum_i V(\{\tilde{p}\}, k_i) \quad \text{, e.g.} \quad 1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|}$$

$$\Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_n)\right) = \Theta\left(v - \sum_{i=1}^{n} \underbrace{V(\{\tilde{p}\}, k_i)}_{\equiv v\zeta_i}\right) = \Theta\left(1 - \sum_{i=1}^{n} \zeta_i\right)$$

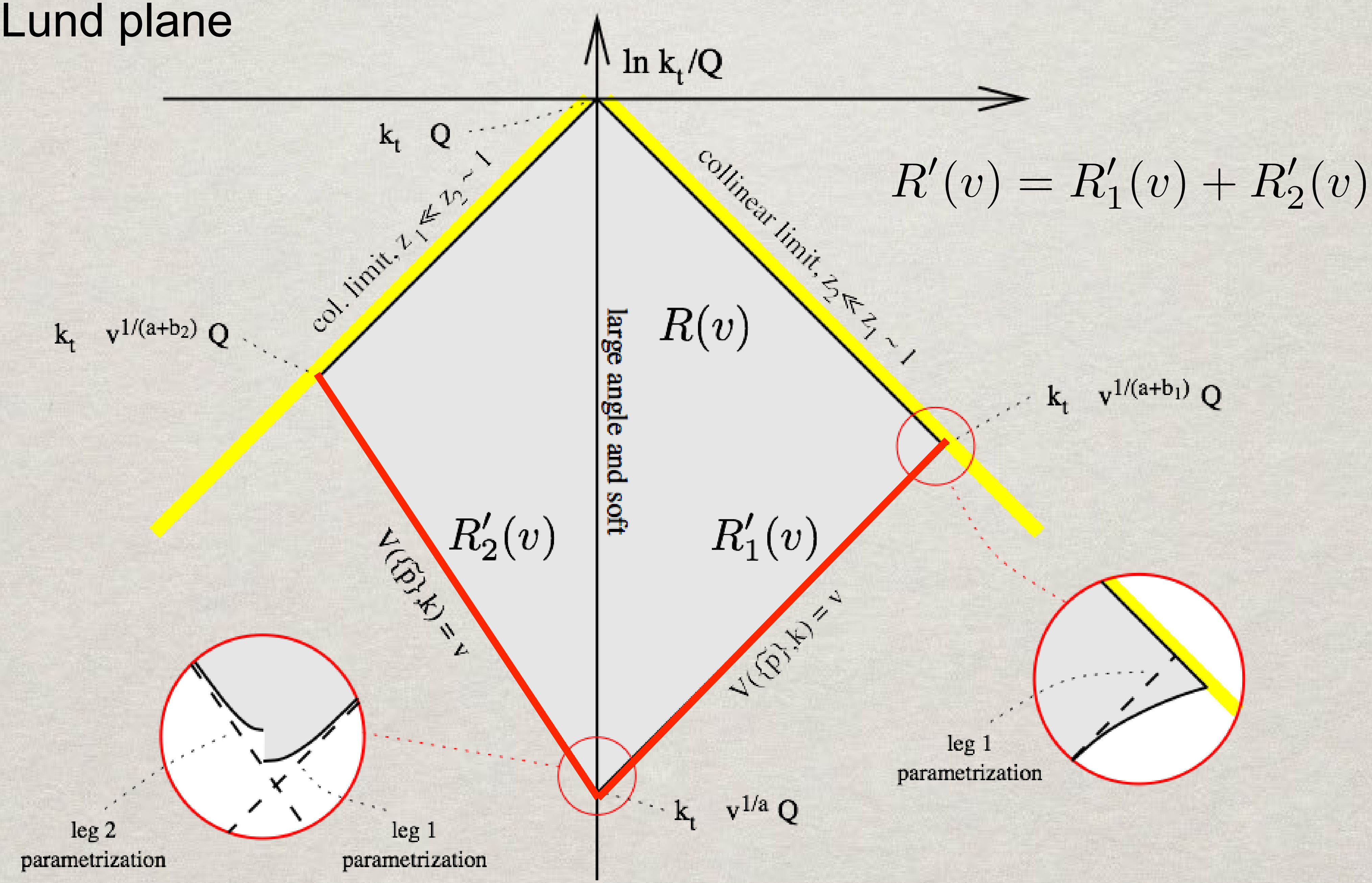
- Change of variable

$$R(v) \simeq \int_v^{\infty} [dk] M^2(k) = \int_0^{\infty} \frac{d\zeta}{\zeta} R'(\zeta v) \Theta(\zeta - 1) \quad \Rightarrow \quad [dk] M^2(k) \to \frac{d\zeta}{\zeta} R'(\zeta v)$$

- The function $R'(v) = -v \dfrac{dR}{dv} \sim \alpha_s L$ is single-logarithmic

Single logarithmic functions

The logarithmic derivative of the radiator is the boundary of the shaded region in the Lund plane



Since it is a line in the Lund plane, its contribution is single logarithmic

Additive observables

- Consider an observable that, in the soft and collinear limit, is the sum of the contributions of individual emissions

$$V(\{\tilde{p}\},k_1,\ldots,k_n)=\sum_i V(\{\tilde{p}\},k_i) \quad \text{, e.g.} \quad 1-T(\{\tilde{p}\},k_1,\ldots,k_n)\simeq\sum_i \frac{k_{ti}}{Q}e^{-|\eta_i|}$$

- Multiple-emission correction

$$\mathcal{F}_{\rm sc}(v)\equiv e^{-\int_{\epsilon}^{1}\frac{d\zeta}{\zeta}R'(\zeta v)}\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{i=1}^{n}\int_{\epsilon}^{\infty}\frac{d\zeta_i}{\zeta_i}R'(\zeta_i v)\Theta\left(1-\sum_{i=1}^{n}\zeta_i\right)$$

cutoff

- Integral over ζ_i is finite $\Rightarrow \zeta_i\sim 1\Rightarrow R'(\zeta v)\simeq R'(v)+\mathcal{O}(\alpha_s)$

$$\mathcal{F}_{\rm sc}(v)\simeq\epsilon^{R'}\sum_{n=0}^{\infty}\frac{(R')^n}{n!}\int\prod_{i=1}^{n}\int_{\epsilon}^{\infty}\frac{d\zeta_i}{\zeta_i}\Theta\left(1-\sum_{i=1}^{n}\zeta_i\right)=\frac{e^{-\gamma_E R'}}{\Gamma(1+R')}$$

NLL function

Examples of NLL resummation

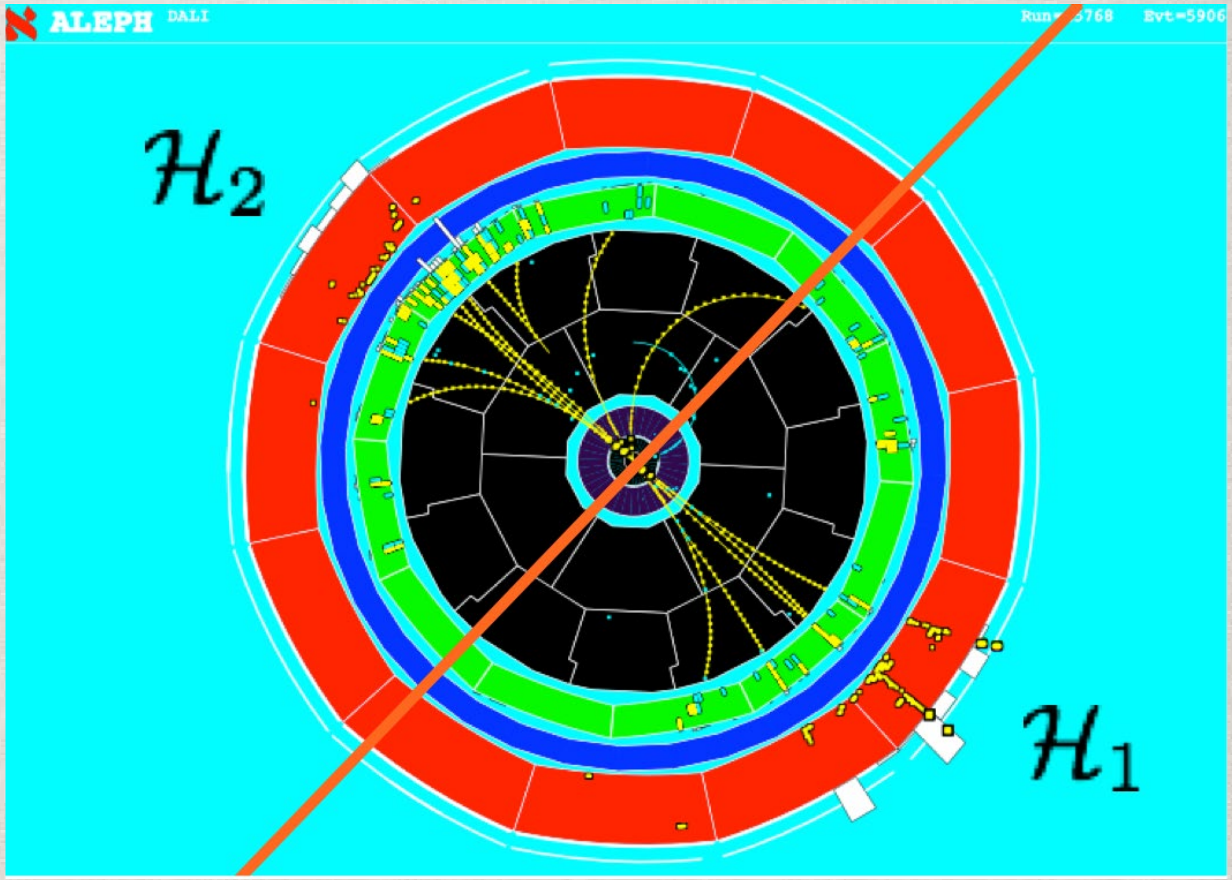
$$V(\{\tilde{p}\},k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)$$

- Heavy-jet mass

$$\rho_H = \max\left(\frac{M_1^2}{Q^2}, \frac{M_2^2}{Q^2}\right)$$

$$a = 1, \quad b_\ell = 1, \quad d_\ell = 1, \quad g_\ell(\phi) = 1$$

$$\mathcal{F}(R') = \frac{e^{-\gamma_E R'}}{\Gamma^2\left(1+\frac{R'}{2}\right)}$$



- Total and wide-jet broadening

$$2B_\ell Q = \sum_{i\in\mathcal{H}_\ell} k_{ti} + \left|\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}\right|$$

$$a = 1, \quad b_\ell = 0, \quad d_\ell = 1, \quad g_\ell(\phi) = 1$$

$$B_T = B_1 + B_2$$

$$\mathcal{F}(R') = \frac{4^{R'} e^{-\gamma_E R'}}{\Gamma(1+R')}\left(\frac{{}_2F_1\left(\frac{R'}{2}, 1+\frac{R'}{2}, 2+\frac{R'}{2}; -1\right)}{1+\frac{R'}{2}}\right)^2$$

$$B_W = \max(B_1, B_2)$$

$$\mathcal{F}(R') = \frac{4^{R'} e^{-\gamma_E R'}}{\Gamma^2\left(1+\frac{R'}{2}\right)}\left(\frac{{}_2F_1\left(\frac{R'}{2}, 1+\frac{R'}{2}, 2+\frac{R'}{2}; -1\right)}{1+\frac{R'}{2}}\right)^2$$

Monte Carlo resummation

The normalisation of independent soft-collinear emission suggests a Markov-chain procedure to compute $\mathcal{F}_{\mathrm{NLL}}(R')$

$$1 = \epsilon^{R'} \sum_{n=0}^{\infty} \frac{(R')^n}{n!} \int_{\epsilon}^{1} \prod_{i=1}^{n} \frac{d\zeta_i}{\zeta_i}$$

$$= \epsilon^{R'} + \int_{\epsilon}^{1} \underbrace{R' \frac{d\zeta_1}{\zeta_1} \zeta_1^{R'}}_{dP(\zeta_1)} \left[\left(\frac{\epsilon}{\zeta_1} \right)^{R'} + \int_{\epsilon}^{\zeta_1} \underbrace{R' \frac{d\zeta_2}{\zeta_2} \left(\frac{\zeta_2}{\zeta_1} \right)^{R'}}_{dP(\zeta_2)} \left[\left(\frac{\epsilon}{\zeta_2} \right)^{R'} + \dots \right. \right.$$

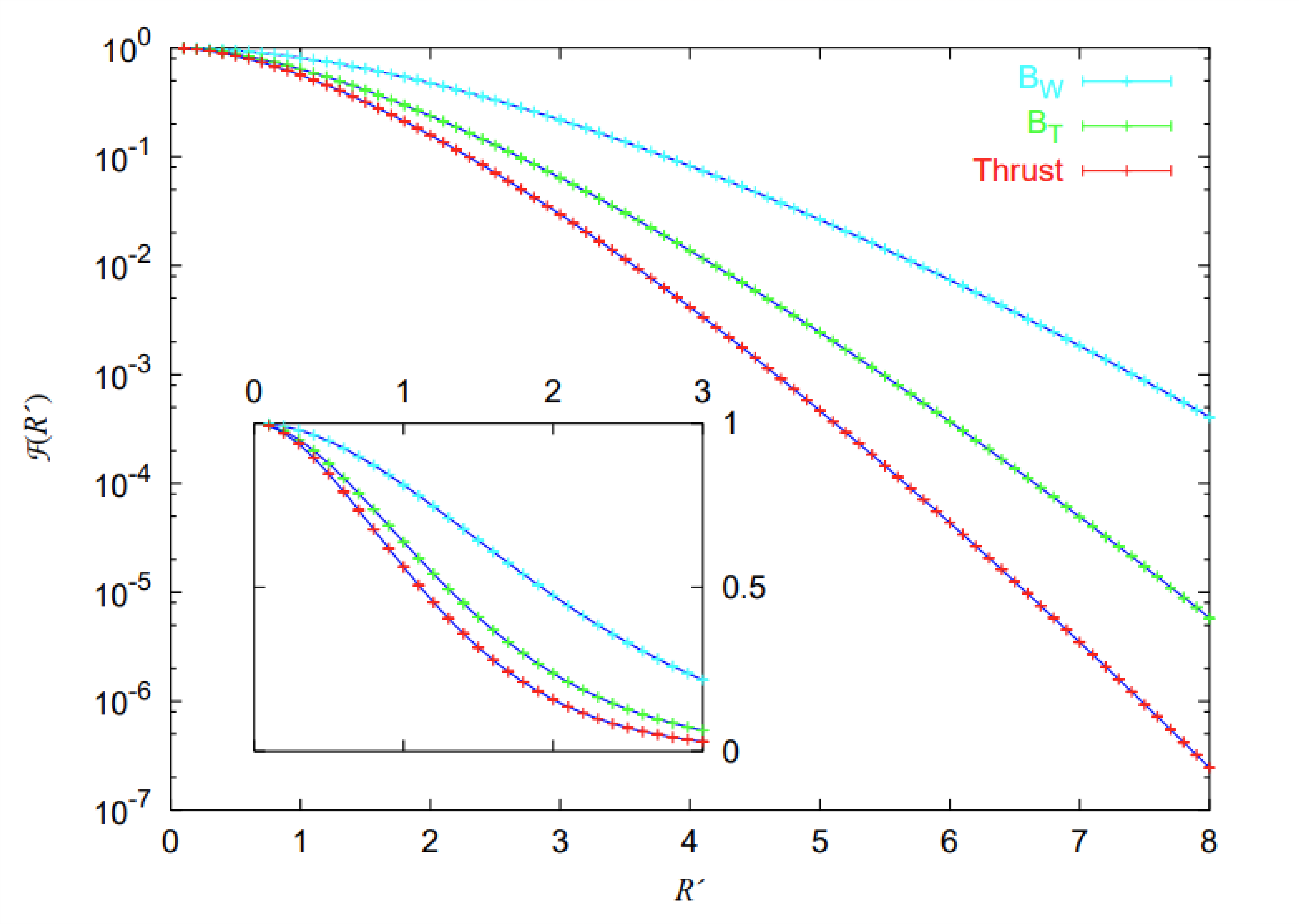
- Generate $\zeta_i < \zeta_{i-1}$ with probability $P(\zeta_i) = \left(\frac{\zeta_i}{\zeta_{i-1}} \right)^{R'}$
- If $\zeta_i < \epsilon$ stop
- Otherwise, generate ζ_{i+1}

Additive observable: $\mathcal{F}_{\mathrm{NLL}}(R') = \epsilon^{R'} \sum_{n=0}^{\infty} \frac{(R')^n}{n!} \prod_{i=1}^{n} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \Theta \left(1 - \sum_{i=1}^{n} \zeta_i \right)$

Numerical resummation

If NLL corrections are well-defined, jet observables can be resummed numerically

[Banfi Salam Zanderighi hep-ph/0112156, hep-ph/0407286]



Recursive IRC safety condition 1

- The first crucial property that ensures that $\mathcal{F}_{\rm sc}(v)$ does not give rise to double logarithms is that $V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$ whenever $V(\{\tilde{p}\}, k_i) \sim v$

$$\lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v} = \sum_{i=1}^{n} \zeta_i \quad \text{(finite and non-zero)}$$

- In general, for each soft emission collinear to leg ℓ , we can perform the change of variables (recall $V(\{\tilde{p}\}, k) \sim k_t^a e^{-b_\ell \eta^{(\ell)}}$)

$$V(\{\tilde{p}\}, k) = \zeta v \qquad \eta^{(\ell)} = \xi^{(\ell)} \eta^{(\ell)}_{\max} \qquad \eta^{(\ell)}_{\max} = \frac{1}{a+b_\ell} \ln \frac{1}{v}$$

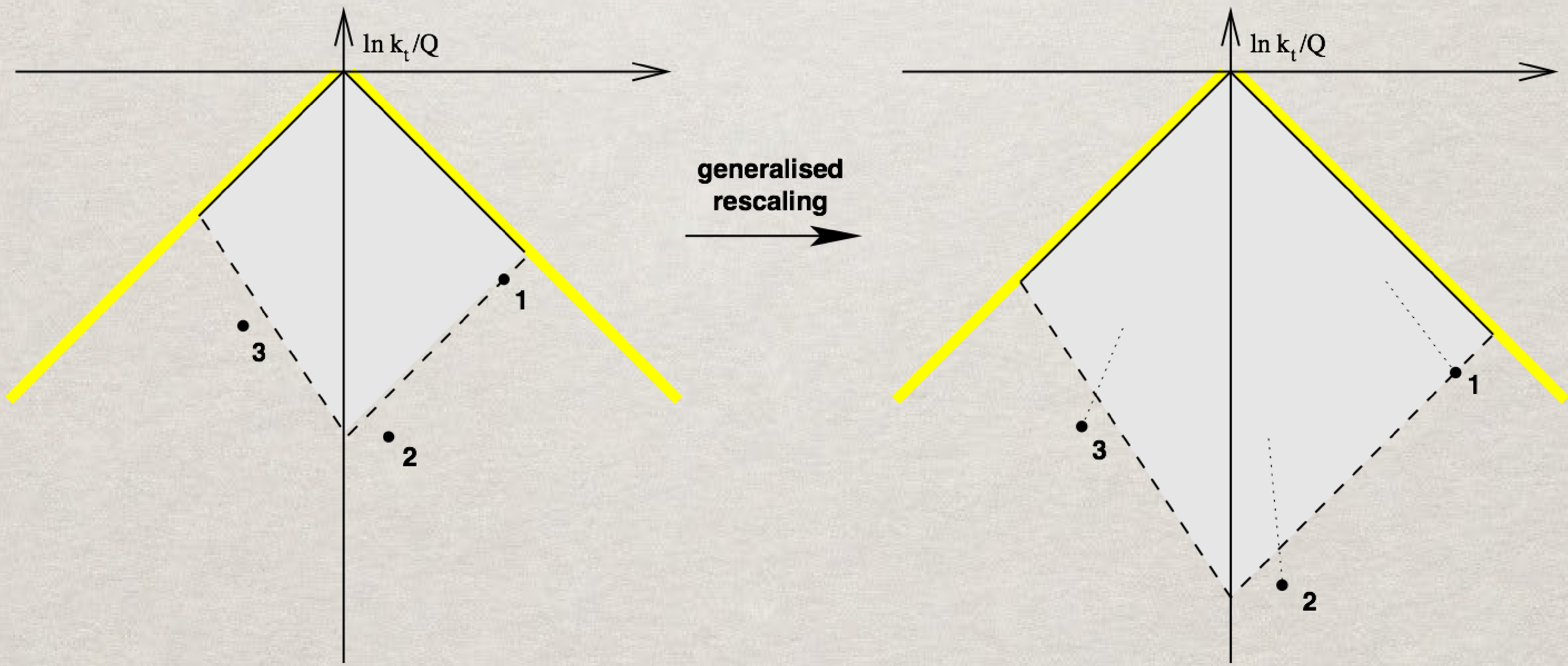
- For a general observable, for fixed $\zeta_i, \ell_i, \xi^{(\ell_i)}, \phi^{(\ell_i)}$, we must have

$$\lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v} = f_V(\{\zeta_1, \ell_1, \xi^{(\ell_1)}, \phi^{(\ell_1)}\}, \ldots, \{\zeta_n, \ell_n, \xi^{(\ell_n)}, \phi^{(\ell_n)}\})$$

Recursive IRC safety condition 1

The requirement that the observable scales in the same way irrespectively of the number of emission is formalised as follows

$$\lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v} = \text{finite and non-zero}$$



- This is the first of the requirements known as “recursive” IRC safety
- rIRC safe observables are the only ones that can be resummed so far

Non-exponentiating observables

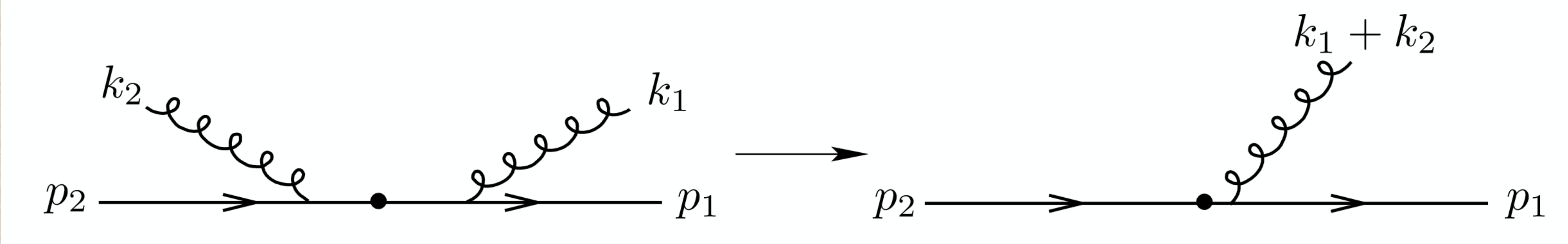
In the case of the two-jet rate in the JADE algorithm, double logarithms do not exponentiate

[Brown Stirling PLB 252 (1990) 657]

$$\Sigma(y_{\rm cut}) = 1 - \frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\rm cut}}\right) + \frac{1}{2!}\times\frac{5}{6}\times\left(\frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\rm cut}}\right)\right)^2 + \ldots$$

This is due to the peculiar way JADE performs sequential recombinations

$$y_{ij} = \frac{(p_i+p_j)^2}{Q^2}$$



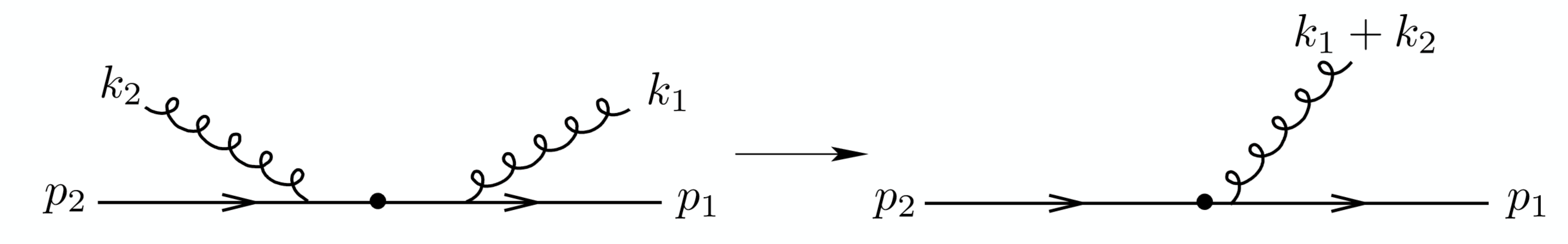
The JADE algorithm is able to recombine together two soft emissions collinear to two different legs $\Rightarrow$ violation of rIRC safety

Failure of rIRC safety condition 1

The way in which the JADE algorithm performs sequential recombinations changes the scaling properties of the three-jet resolution

$$y_{k_1 p_1} = y_3(\{\tilde{p}\}, k_1) = \frac{(k_1 + p_1)^2}{Q^2} \simeq \frac{k_{t1}}{Q} e^{-\eta_1} = y_{\rm cut}$$

$$y_{k_2 p_2} = y_3(\{\tilde{p}\}, k_2) = \frac{(k_2 + p_2)^2}{Q^2} \simeq \frac{k_{t2}}{Q} e^{+\eta_2} = y_{\rm cut}$$



$$y_{k_1 k_2} = \frac{(k_1 + k_2)^2}{Q^2} \simeq y_{\rm cut}^{2-\xi_1-\xi_2} < y_{\rm cut} \quad \Leftrightarrow \quad \xi_1 + \xi_2 < 1$$

$$\frac{y_3(\{\tilde{p}\}, k_1, k_2)}{y_{\rm cut}} = y_{\rm cut}^{1-\xi_1-\xi_2} \Rightarrow \text{depends on } y_{\rm cut} \Rightarrow \mathcal{F}_{\rm sc}(y_{\rm cut}) \text{ gives double logs}$$

Recursive IRC safety condition 2a

- The second crucial property that ensures that $\mathcal{F}_{\rm sc}(v)$ does not give double logarithms is that the integral over ζ_i is finite
- This means that we can neglect all emissions with $V(\{\tilde{p}\}, k_i) < \epsilon v$, with the cutoff $\epsilon \gg v$, independent of v

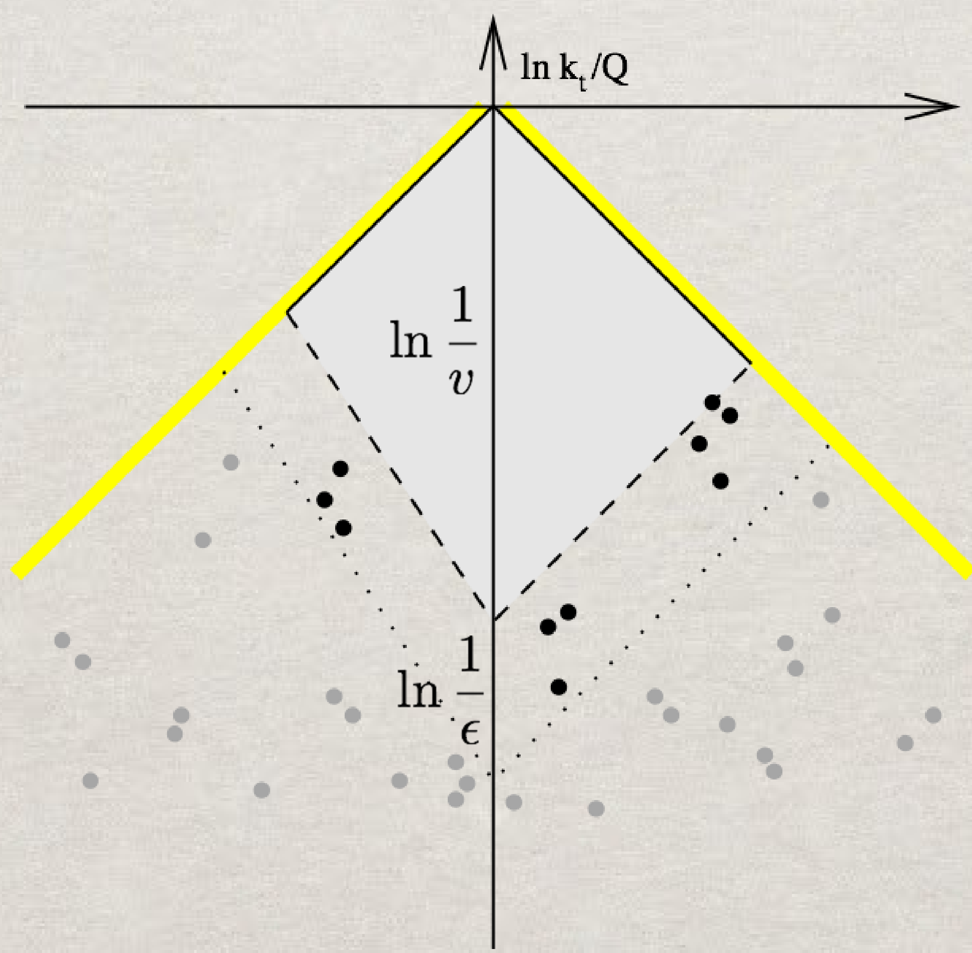
$$[dk]M^2(k) \simeq \sum_\ell \underbrace{R'_\ell(v)}_{\sim\alpha_s L} \frac{d\zeta}{\zeta} d\xi^{(\ell)} \frac{d\phi}{2\pi} \qquad\qquad R' = \sum_\ell R'_\ell = -v\frac{dR}{dv}$$

$$\mathcal{F}_{\rm sc}(v) \simeq \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left(\int_\epsilon^\infty \frac{d\zeta_i}{\zeta_i} \sum_{\ell_i} R'_{\ell_i} \int_0^1 d\xi^{(\ell_i)} \int_0^{2\pi} \frac{d\phi_i^{(\ell_i)}}{2\pi} \right) \times$$
$$\times\, \Theta\left(1 - \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) = \mathcal{F}_{\rm NLL}(R')$$

Recursive IRC safety condition 2a

The fact that we can neglect emissions with $\zeta_i < \epsilon$ is expressed formally by rIRC safety condition 2a

$$\lim_{\zeta_{n+1}\to 0} \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n, k_{n+1})}{v} = \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}$$

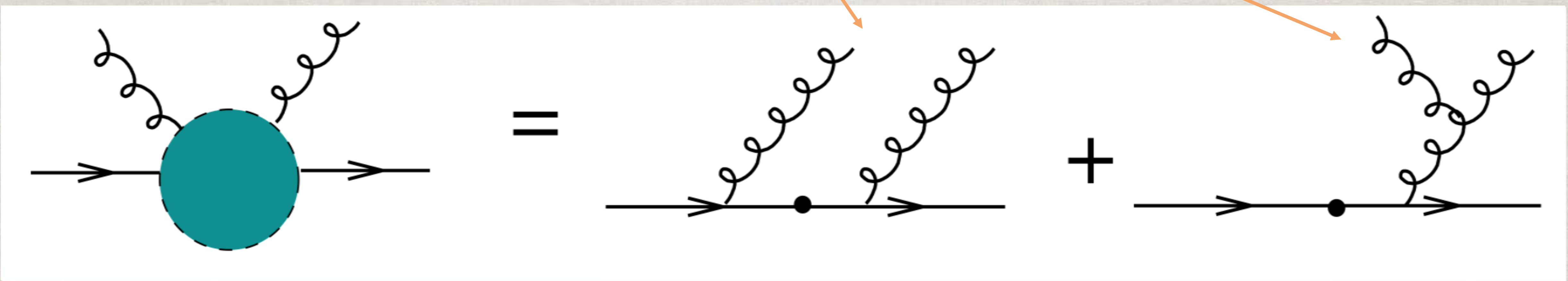


Note the order of the limits: the reversed limit trivially holds because the observable is IRC safe!

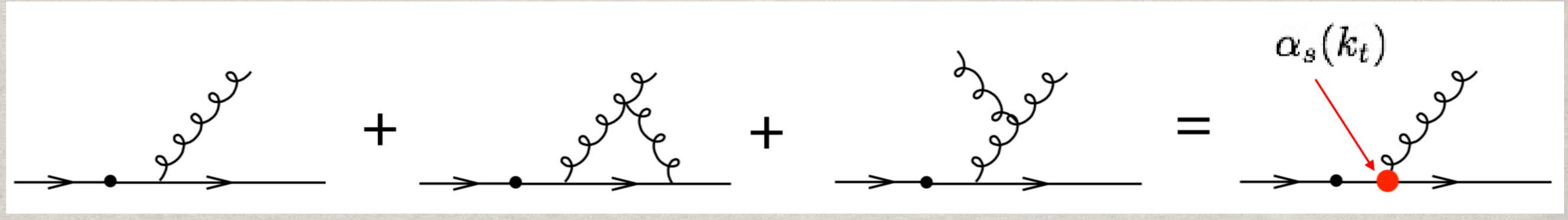
Two-gluon correlated emission

The matrix element for two soft-collinear gluons can always be written as the sum of an independent and correlated emission part

$$M^2(k_1,k_2) = M^2(k_1)M^2(k_2) + \tilde{M}^2(k_1,k_2)$$



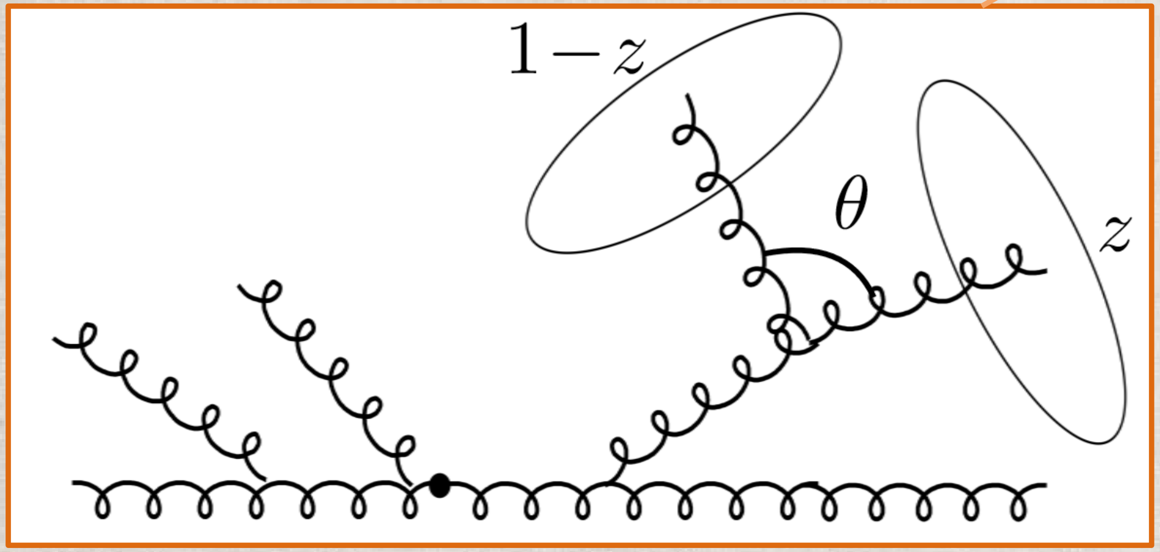
The correlated emission part, if integrated inclusively, is combined with the one-loop one-gluon matrix element to give the running coupling in a physical renormalisation scheme

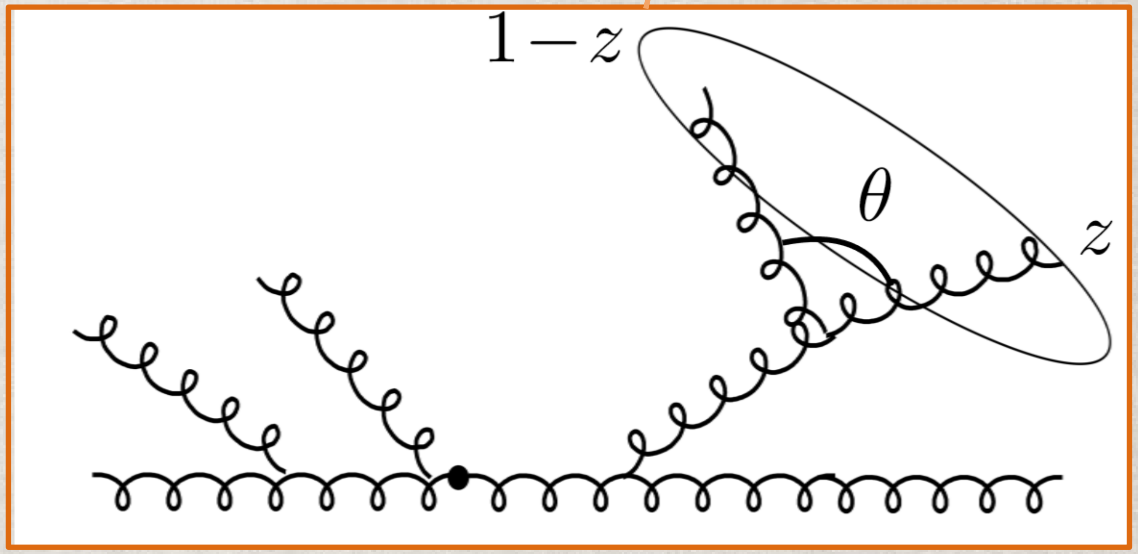


Two-gluon correlated emission

The remainder after the extraction of the coupling

$$\int_{\epsilon v}[dk_1]\int_{\epsilon v}[dk_2]\tilde{M}^2(k_1,k_2)\left[\Theta\left(v-V(\{\tilde{p}\},k_1,k_2)\right)-\Theta\left(v-V(\{\tilde{p}\},k_1+k_2)\right)\right]$$





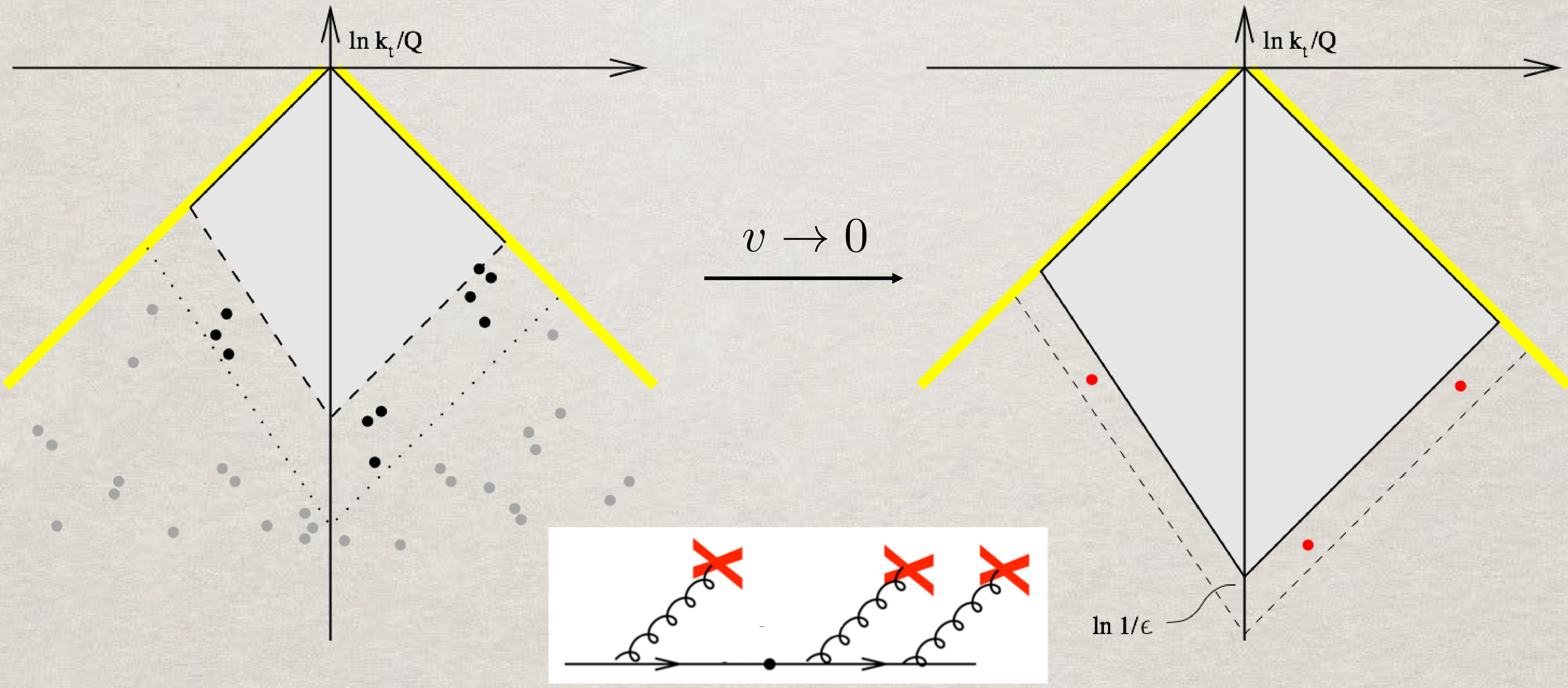
- Example: in a jet-rate, the two gluons can be clustered into different jets

$$\int_{\epsilon v}[dk]M^2(k)\times\left[C_A\int\frac{d\theta^2}{\theta^2}\int\frac{dz}{z(1-z)}\frac{\alpha_s[z(1-z)\theta k_t]}{2\pi}\right]$$

- Potential source of double logarithms, which are however absent for a rIRC safe observable

Recursive IRC safety condition 2b

This condition ensures that the contribution of correlated gluon emissions, hard collinear and soft large-angle emissions is beyond NLL

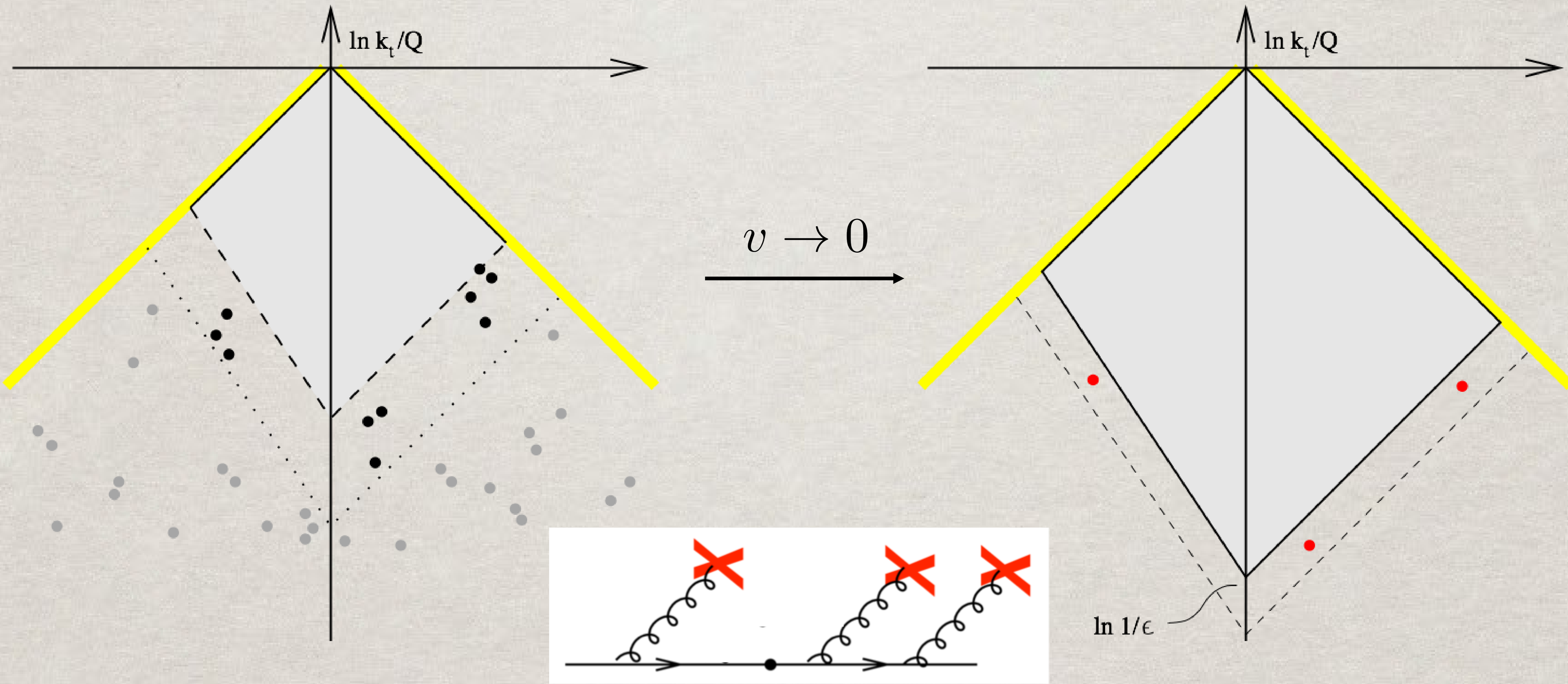


- At NLL accuracy, relevant emissions are soft and collinear, widely separated in angle, and in a strip of size $\ln v \times \ln \epsilon$
- The strip is a line in the Lund plane, hence a single logarithmic contribution

General NLL resummation

NLL resummation of rIRC safe observables can be performed with a universal master formula

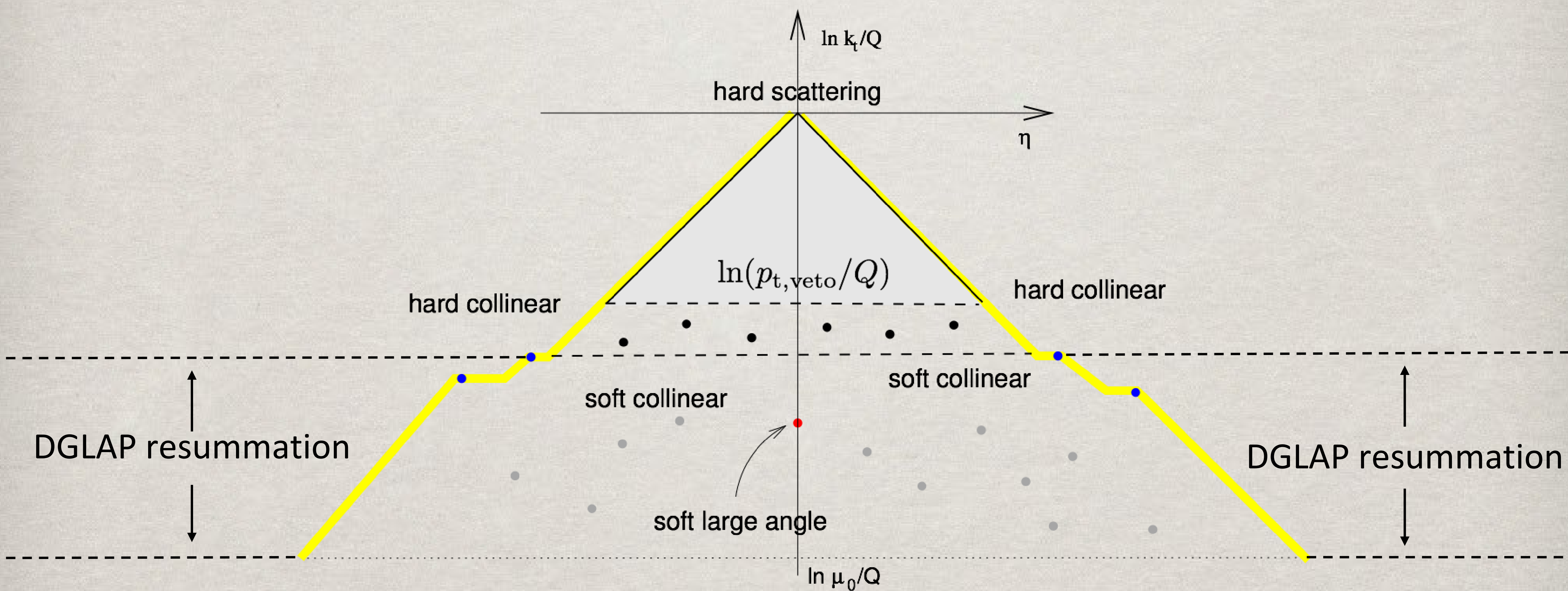
[Banfi Salam Zanderighi hep-ph/0407286]



$$\Sigma(v) = e^{-R(v)} \underbrace{\left\{ \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left(\sum_{\ell_i} R'_{\ell_i} \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \int_0^1 d\xi_i^{(\ell_i)} \int_0^{2\pi} \frac{d\phi}{2\pi} \right) \Theta\left(1 - \lim_{v \to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) \right\}}_{\text{single-logarithmic correction } \mathcal{F}_{\rm NLL}(R')}$$

Higgs plus zero jets at NLL

In the presence of initial state radiation, the zero-jet cross section inclusive with respect to hard-collinear emission up to the scale $p_{\mathrm{t,veto}}$



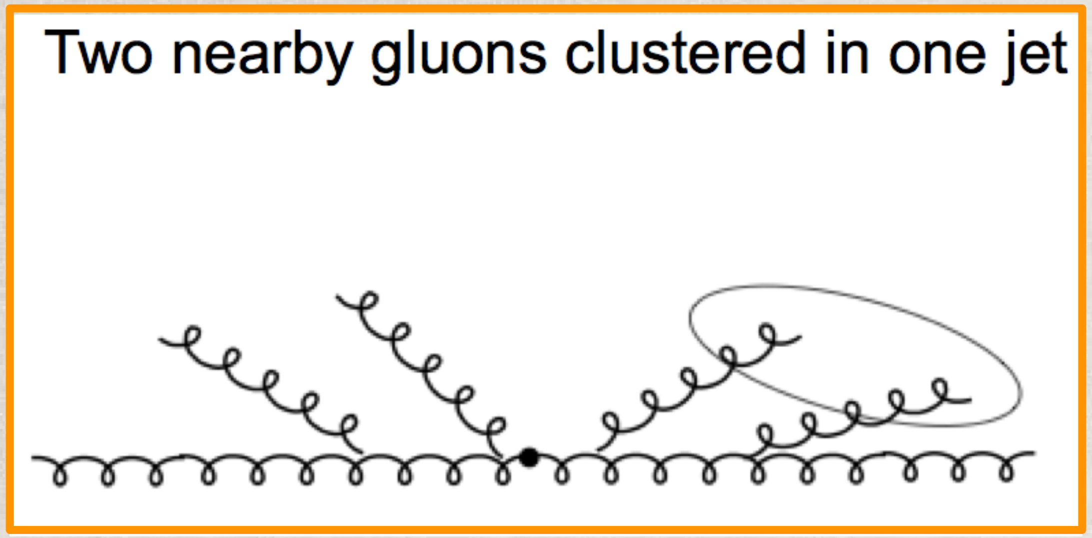
No k_t-type algorithm can recombine gluons that are widely separated in angle: perfectly exponentiating observable

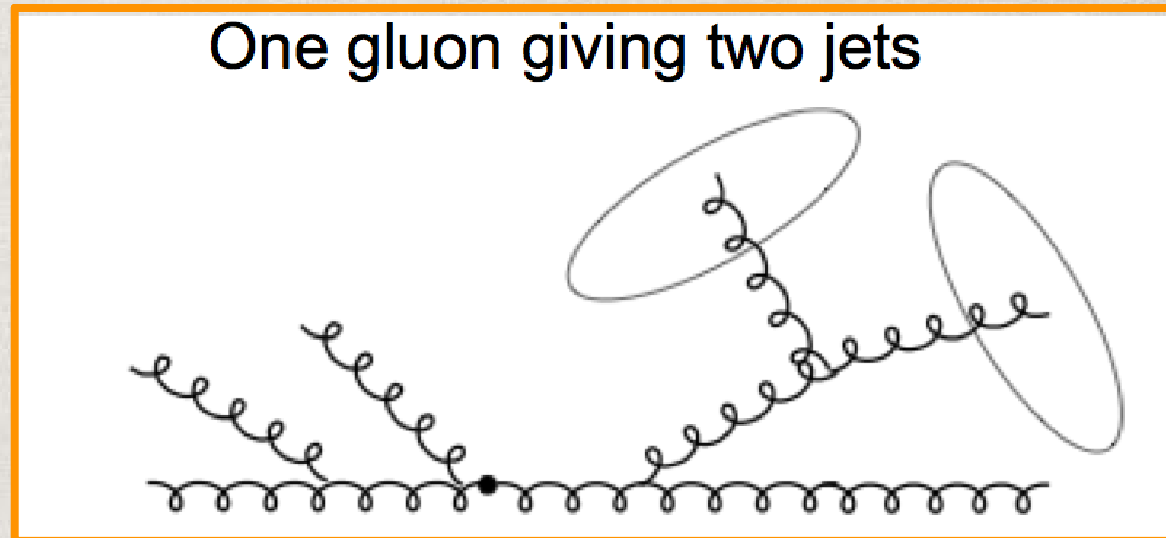
$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}})\, e^{-R(p_{\mathrm{t,veto}})}$$

Higgs plus zero jets at NNLL

Jet recombination effects start to matter at NNLL accuracy

[Banfi Monni Salam Zanderighi 1206.4998]





$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}}) \left(1 + \underbrace{\alpha_s(p_{\mathrm{t,veto}})\, R'(p_{\mathrm{t,veto}})\, f(R)}_{\mathrm{NNLL}} \right) e^{-R(p_{\mathrm{t,veto}})}$$

The function $f(R) \sim \ln R$ since the jet radius provides an effective cutoff to the collinear singularity in gluon splitting. Leading logarithm of the jet radius can be also resummed at all orders

[Dasgupta Dreyer Salam Soyez 1411.5182]