

# Negative Event Weights

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- Where do negative weights come from?
- Why are negative weights a problem?
- How to handle negative weights?

[References at the end]

Where do negative weights come from?

# Where do negative weights come from?

## Monte Carlo events

Predict (integrated) differential cross section for  $PP \rightarrow X$ :

$$\int_{\mathcal{D}} d\sigma_{PP \rightarrow X} = \frac{1}{F} \int_{\mathcal{D}} d\phi |\mathcal{M}_{PP \rightarrow X}(p)|^2$$

phase space selection  
e.g. histogram bin

flux factor

scattering amplitude

# Where do negative weights come from?

## Monte Carlo events

Predict (integrated) differential cross section for  $PP \rightarrow X$ :

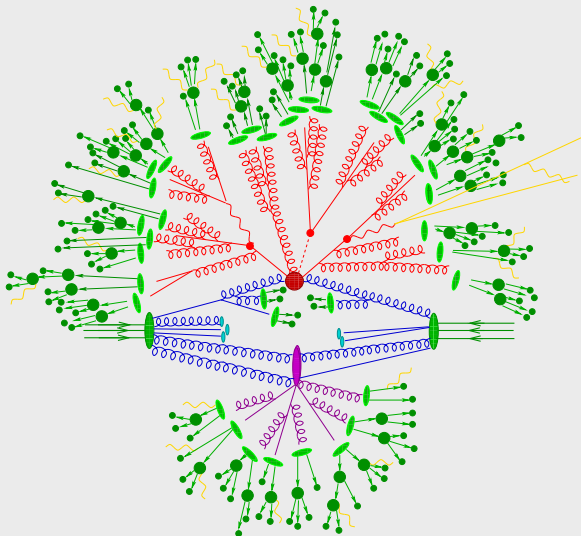
$$\int_{\mathcal{D}} d\sigma_{PP \rightarrow X} = \int_{\mathcal{D}} d\phi |\mathcal{M}_{PP \rightarrow X}(p)|^2 \xrightarrow{\text{Monte Carlo}} \sum_{i=1}^N \underbrace{\frac{\mathcal{D}}{FN} |\mathcal{M}_{PP \rightarrow X}(p_i)|^2}_{\text{weight } w_i}$$

Exact weights  $w_i$

- are non-negative: proportional to modulus square
- indicate relative probability of event

# Where do negative weights come from?

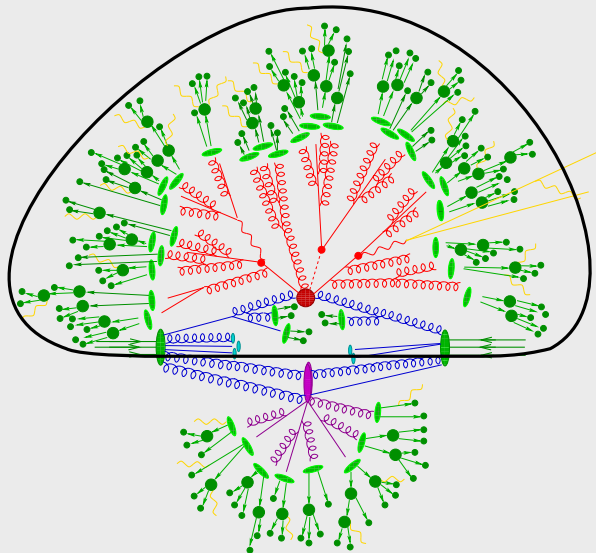
Standard theory picture



[Sherpa collaboration]

# Where do negative weights come from?

Standard theory picture



[Sherpa collaboration]

# Where do negative weights come from?

## PDFs

$$\int_{\mathcal{D}} d\sigma_{PP \rightarrow X} \approx \int dx_a dx_b \int_{\mathcal{D}} d\sigma_{ab \rightarrow X} f_a(x_a) f_b(x_b)$$

- Factorisation not unique, PDFs are scheme dependent beyond leading order  
→ physical interpretation not obvious
- PDFs can be negative, but [References]
  - ▶ **Universality:** same PDFs have to reproduce many positive cross sections
  - ▶ **Naturalness:** leading-order PDFs are non-negative, higher-order corrections should be small
- Can impose PDF positivity, e.g. NNPDF4.0MC

Partonic cross section generally main source of negative weights



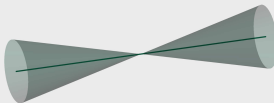
# Where do negative weights come from?

## Partonic cross section

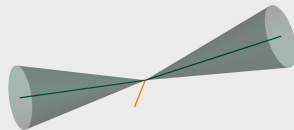
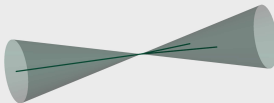
Consider simple scenario:

- exclusive dijet production
- no parton shower  $\leftarrow$  similar discussion, but more involved

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow 2j} = d\sigma_{ab \rightarrow 2j} (2 \text{ partons})$$



$$+ d\sigma_{ab \rightarrow 2j} (3 \text{ partons})$$



+ ...

Separation **unphysical**, not **InfraRed** & **Collinear** safe

# Where do negative weights come from?

## NLO partonic cross section

Truncate at NLO:

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow 2j} \Big|_{\text{NLO}} = \int d\phi_2 (B + V) + \int d\phi_3 R$$

The diagram illustrates the components of the NLO cross-section formula. Three green arrows point from the terms in the equation to their corresponding Feynman diagrams below:

- Born**: An arrow from  $B$  in  $\int d\phi_2 (B + V)$  points to a diagram of two horizontal lines connected by a vertical line, representing a tree-level process.
- Virtual**: An arrow from  $V$  in  $\int d\phi_2 (B + V)$  points to a diagram of two horizontal lines connected by a loop, representing a one-loop virtual correction.
- Real**: An arrow from  $R$  in  $\int d\phi_3 R$  points to a diagram of two horizontal lines with an additional line branching off, representing a real emission process.

- Terms on right-hand side diverge individually
- Introduce subtraction (Catani-Seymour, FKS, ...)

# Where do negative weights come from?

## NLO partonic cross section

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow 2j} \Big|_{\text{NLO}} = \int d\phi_2 (B + V + C_{\text{int}}) + \int d\phi_3 (R - C)$$
$$\xrightarrow[\text{up to normalisation}]{\text{Monte Carlo}} \sum_{i=1}^{N_2} [B(p_i) + V(p_i) + C_{\text{int}}(p_i)] + \sum_{i=1}^{N_3} [R(p_i) - C(p_i)]$$

# Where do negative weights come from?

## NLO partonic cross section

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Sources of negative weights:

- 1  $R(p_i) - C(p_i)$  negative
- 2  $B(p_i) + V(p_i) + C_{\text{int}}(p_i)$  negative
- 3 Split up

$$\sum_{i=1}^{N_2} [B(p_i) + V(p_i) + C_{\text{int}}(p_i)] \rightarrow \sum_{i=1}^{N_B} B(p_i) + \sum_{i=1}^{N_V} [V(p_i) + C_{\text{int}}(p_i)]$$

and  $V(p_i) + C_{\text{int}}(p_i)$  negative

# Where do negative weights come from?

Negative Born + Virtual

$$\underbrace{B(p_i)}_{\geq 0} + \underbrace{V(p_i) + C_{\text{int}}(p_i)}_{\mathcal{O}(\alpha_s \times B)} < 0$$

# Where do negative weights come from?

## Negative Born + Virtual

$$\underbrace{B(p_i)}_{\geq 0} + \underbrace{V(p_i) + C_{\text{int}}(p_i)}_{\mathcal{O}(\alpha_s \times B)} < 0$$

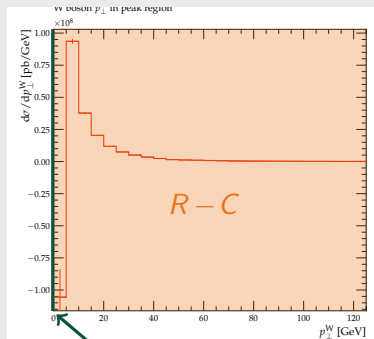
- Suggests oversubtraction or **local breakdown of perturbation theory**
  - Expect physical result when including (all) higher orders
  - Possible pragmatic solution:
    - 1 Carefully identify phase-space region where prediction breaks down
    - 2 Discarding **complete** “bad” phase-space region is formally  $>$ NLO change

Just discarding all negative weights is **wrong**

# Where do negative weights come from?

## Negative Born + Virtual

Example: W boson production at vanishing transverse momentum



$B + V + C^{\text{int}}$

- Cause of breakdown: Sudakov logarithms  $\ln^2 \frac{m_W}{p_{\perp}^W}$
- Discard  $B + V + C^{\text{int}}$ ?
  - ▶ **Wrong** prediction for small  $p_{\perp}^W > 0$
  - ▶ **Loose NLO accuracy** in total cross section
- Discard region  $[0, p_{\perp}^{\text{min}}]$  with  $\int_0^{p_{\perp}^{\text{min}}} dp_{\perp} \frac{d\sigma}{dp_{\perp}} = 0$ 
  - ▶ Preserves total NLO cross section
  - ▶ Differential cross section suddenly drops to 0

Why are negative weights a problem?



# Why are negative weights a problem?

Negative weights are

- “unphysical”, but needed to cancel “unphysical” positive weights!
  - ▶ Wrong cross section prediction  $\int_{\mathcal{D}} d\sigma_{PP \rightarrow X}$  for small enough  $\mathcal{D}$
  - ▶ At odds with  $|\mathcal{M}_{PP \rightarrow X}(p_i)|^2 \geq 0$
  - ▶ No clear interpretation as probabilities
- problematic for machine learning
- bad for statistical convergence: state-of-the-art predictions become computationally extremely expensive or even infeasible

# Why are negative weights a problem?

## Statistical convergence

Assume **unweighted** sample of  $N \gg 1$  events:

- $N_-$  events with weights  $w_i = -W < 0$
- $N_+ = N - N_-$  events with weights  $w_i = W > 0$

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$$\sigma = \sum_{i=1}^N w_i = -N_-W + (N - N_-)W = \left(1 - 2 \underbrace{\frac{N_-}{N}}_{\text{negative weight fraction } r_-}\right) NW$$

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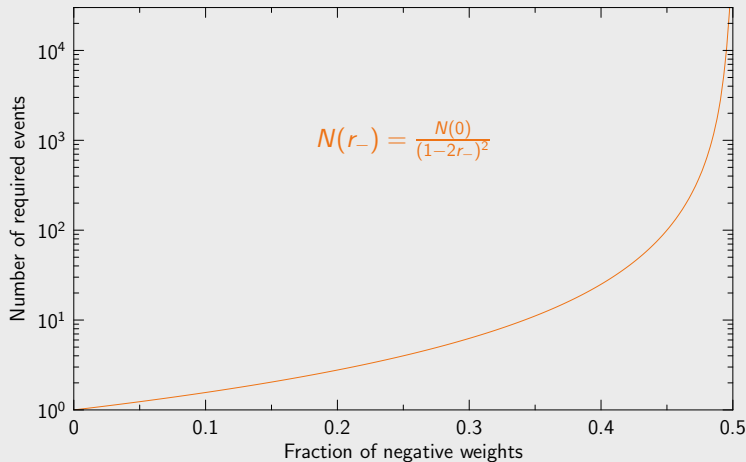
For **uncorrelated** events:  $\Delta\sigma = \sqrt{\sum_{i=1}^N w_i^2} = \sqrt{NW}$

$$\frac{\Delta\sigma}{\sigma} = \frac{1}{(1 - 2r_-)\sqrt{N}}$$

# Why are negative weights a problem?

## Statistical convergence

Number of required events to reach given statistical accuracy:



# Summary I

Negative weights come from

- Splitting the cross section into **unphysical parts** & introducing **subtraction terms**
  - ▶ Example:  $R(p_i) - C(p_i)$
  - ▶ Details depend on formalism
- Local breakdown of perturbation theory
- Potentially negative PDFs
- ...

They are a problem because they

- hinder statistical convergence
- are bad for machine learning
- do not have a physical interpretation

# How to handle negative weights?

# How to handle negative weights?

Many approaches for cancelling negative weights:

- Independent of event generation method
  - Cell resampling
  - Neural reweighting/refining
  - Folding
- Origin in additive parton shower matching
  - Born spreading
  - MC@NLO- $\Delta$
  - ARCANE reweighting
- Origin in multiplicative parton shower matching
  - ESME
  - KrkNLO matching
- ...

+ combinations of complementary methods



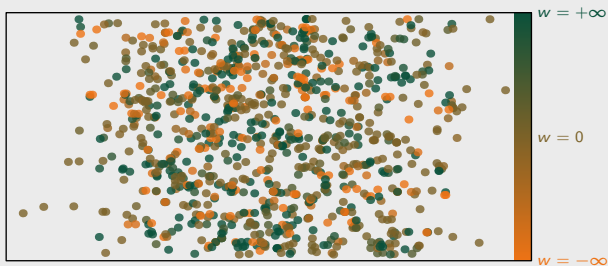
# How to handle negative weights?

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*Cell resampling*

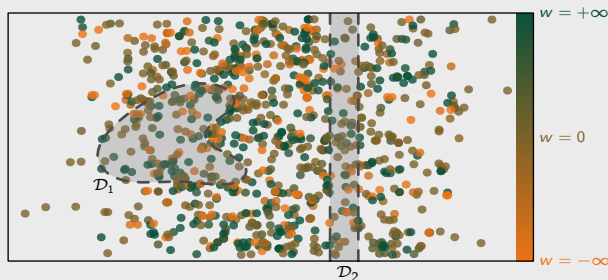
# Cell resampling

Events in 2D projection of phase space:



# Cell resampling

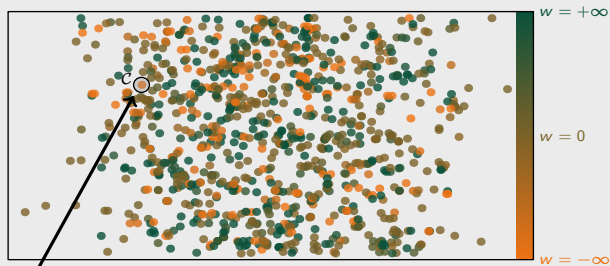
Integrated differential cross sections:  $\int_{\mathcal{D}} d\sigma = \sum_{i \in \mathcal{D}} w_i$



- $\int_{\mathcal{D}} d\sigma = \sum_{i \in \mathcal{D}} w_i \geq 0$ , provided
  - Theory can be trusted
  - Enough statistics
- Analysis has finite resolution  $\Rightarrow$  minimum size for  $\mathcal{D}$

Idea: redistribute weights over small distances

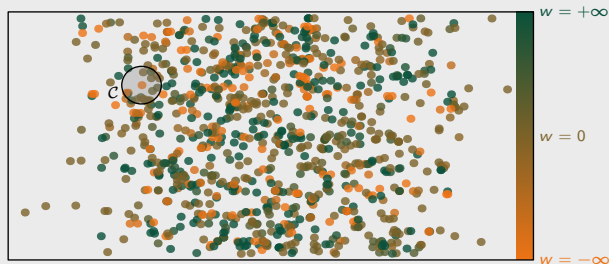
# Cell resampling



Cell resampling:

- 1 Choose seed event with negative weight for cell  $c$

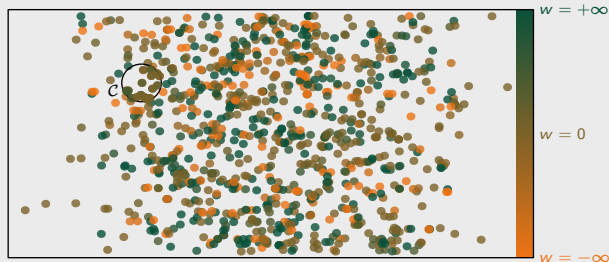
# Cell resampling



Cell resampling:

- 1 Choose seed event with negative weight for cell  $C$
- 2 Iteratively add nearest event to cell until  $\sum_{i \in C} w_i \geq 0$  or radius exceeds  $r_{\max}$

# Cell resampling



Cell resampling:

- 1 Choose seed event with negative weight for cell  $\mathcal{C}$
- 2 Iteratively add nearest event to cell until  $\sum_{i \in \mathcal{C}} w_i \geq 0$  or radius exceeds  $r_{\max}$
- 3 Redistribute weights, e. g. average over cell:  $w_i \rightarrow w = \frac{\sum_{j \in \mathcal{C}} w_j}{\# \text{ events in } \mathcal{C}}$
- 4 Repeat

# Cell resampling

## Subsampling

Generate more events inside cells with incomplete cancellation:

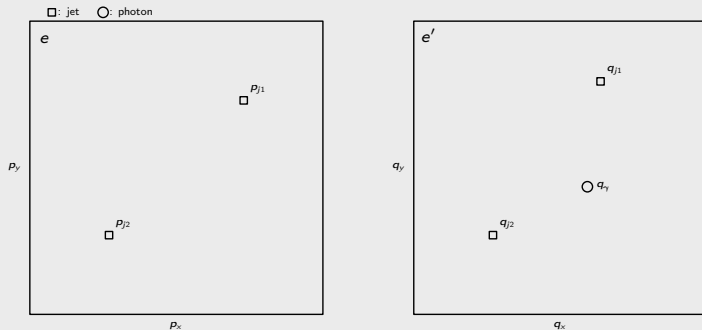


# Cell resampling

## Distance in phase space

Need distance that measures similarity between events

Example:

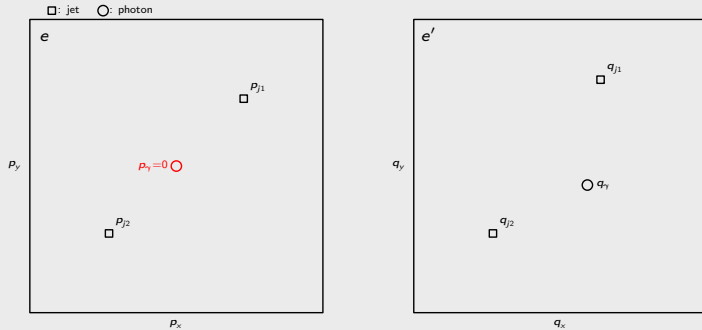




# Cell resampling

## Distance in phase space

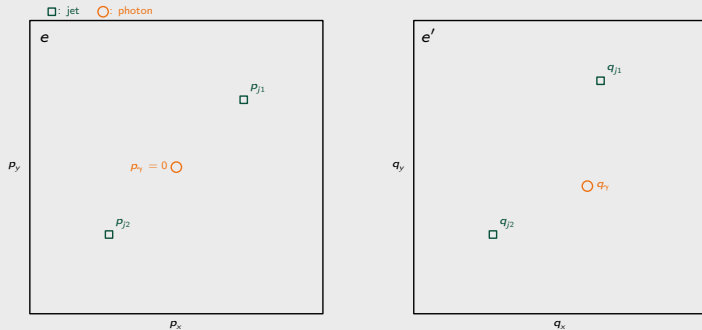
Ensure same multiplicities



# Cell resampling

## Distance in phase space

Compare physics objects of same type

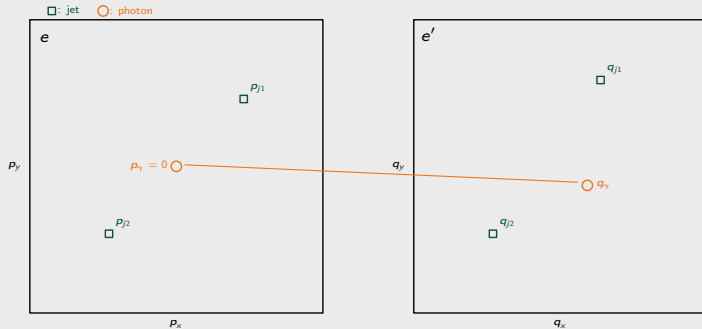


$$d(e, e') = d(s_j, s'_j) + d(s_\gamma, s'_\gamma)$$

# Cell resampling

## Distance in phase space

### Compare photons

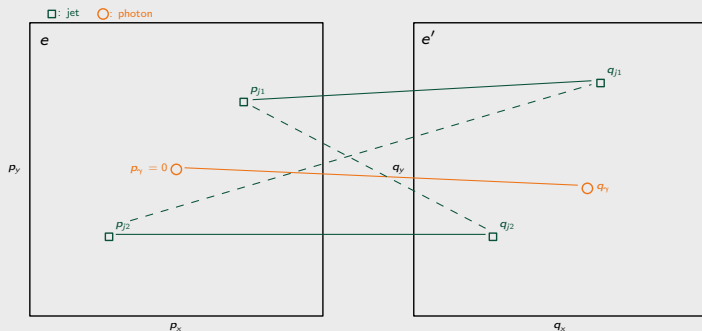


$$\begin{aligned} d(e, e') &= d(s_j, s'_j) + d(s_\gamma, s'_\gamma) \\ &= d(s_j, s'_j) + d(p_\gamma, q_\gamma) \end{aligned}$$

# Cell resampling

## Distance in phase space

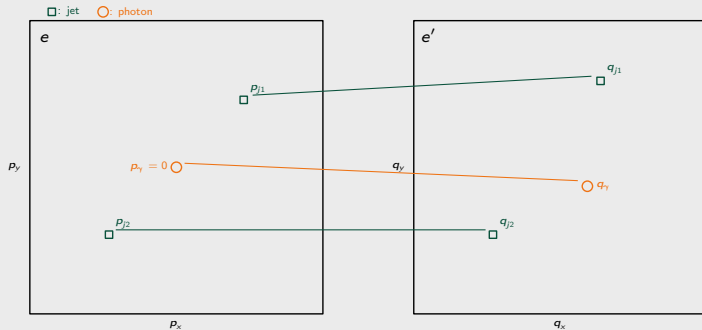
Compare jets: find pairs of most similar jets



$$\begin{aligned} d(e, e') &= d(s_j, s'_j) + d(s_\gamma, s'_\gamma) \\ &= \min [d(p_{j1}, q_{j1}) + d(p_{j2}, q_{j2}), d(p_{j1}, q_{j2}) + d(p_{j2}, q_{j1})] + d(p_\gamma, q_\gamma) \end{aligned}$$

# Cell resampling

## Distance in phase space

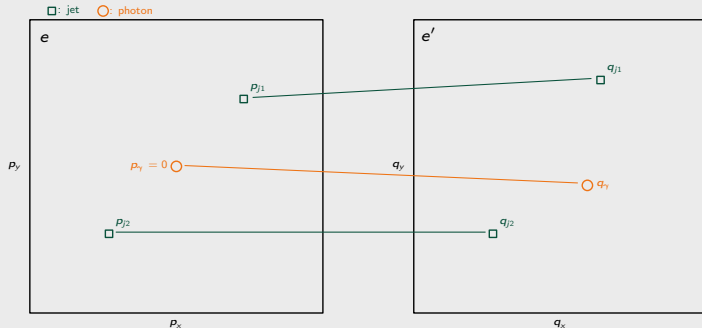


$$\begin{aligned} d(e, e') &= d(s_j, s'_j) + d(s_\gamma, s'_\gamma) \\ &= d(p_{j1}, q_{j1}) + d(p_{j2}, q_{j2}) + d(p_\gamma, q_\gamma) \end{aligned}$$

# Cell resampling

## Distance in phase space

Compare momenta



$$\begin{aligned} d(e, e') &= d(s_j, s'_j) + d(s_\gamma, s'_\gamma) \\ &= |\vec{p}_{j1} - \vec{q}_{j1}| + |\vec{p}_{j2} - \vec{q}_{j2}| + |\vec{p}_\gamma - \vec{q}_\gamma| \end{aligned}$$

# How to handle negative weights?

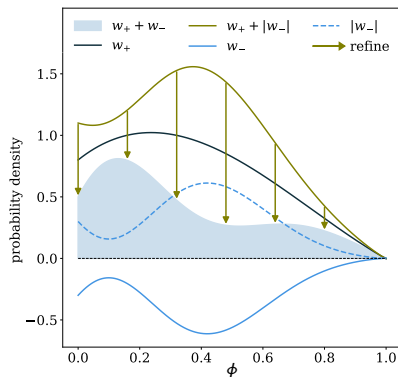
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*Neural reweighting/refining*

# Neural reweighting/refining

Idea: train neural network to replace weight  $w_i$  at phase space point  $\phi_i$

- **Neural reweighting**: replace by average weight  $w'(\phi_i)$
- **Neural refining**: replace by rescaled weight  $r(\phi_i)|w_i|$ 
  - Preserves weights in purely positive samples
  - Better behaviour in negative phase space regions





# How to handle negative weights?

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*Folding*

Assume  $\int d\phi I(p) \geq 0$

$$\sum_i I(p_i) \longrightarrow \sum_i \underbrace{\frac{1}{K} \sum_k I(p_i + k\Delta p)}_{w_i}$$

- +  $w_i$  is better estimate of integral  $\Rightarrow w_i \geq 0$  more likely
- + general technique, straightforward to use as ingredient in others
- each  $w_i$  requires multiple evaluations of  $I \longrightarrow$  more computing time

# Additive parton shower matching

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NLO

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{NLO}} = \int d\phi_B \left[ B + V + \int d\phi_r C \right] + \int d\phi_B d\phi_r [R - C]$$

Evaluate analytically



# Additive parton shower matching

MC@NLO

Modify to subtract shower approximation of real emission

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{MC@NLO}} = \int d\phi_B \left[ B + V + \int d\phi_r C_{\text{PS}} \right] \mathcal{F}_{\text{PS}}^{(B)} + \int d\phi_B d\phi_r [R - C_{\text{PS}}] \mathcal{F}_{\text{PS}}^{(R)}$$

Evaluate numerically  
→ negative weights

Shower evolution

# How to handle negative weights?

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*Born spreading*

# Born spreading

Standard evolution events:

$$\int d\phi_B d\phi_r \left[ \frac{B(\phi_B)}{\int d\phi_r} + \frac{V(\phi_B)}{\int d\phi_r} + C_{\text{PS}}(\phi_B, \phi_r) \right] \mathcal{F}_{\text{PS}}^{(B)}$$

# Born spreading

Standard evolution events:

$$\int d\phi_B d\phi_r \left[ \frac{B(\phi_B)}{\int d\phi_r} + \frac{V(\phi_B)}{\int d\phi_r} + C_{\text{PS}}(\phi_B, \phi_r) \right] \mathcal{F}_{\text{PS}}^{(B)}$$

Idea: “spread out” large + positive Born contribution to cancel negative weights:

$$\frac{B(\phi_B)}{\int d\phi_r} \rightarrow \frac{B(\phi_B)F(\phi_r)}{\int d\phi_r F(\phi_r)}$$

- 1 Sample integrand with original Born function  $B(\phi_B)$
- 2 Define spreading function  $F(\phi_r)$  via grid in  $\phi_r$  to make integrand non-negative



# How to handle negative weights?

---

*MC@NLO- $\Delta$*

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{MC@NLO}} = \int d\phi_B \underbrace{\left[ B + V + \int d\phi_r C_{\text{PS}} \right]}_{\text{Standard}} \mathcal{F}_{\text{PS}}^{(B)} + \int d\phi_B d\phi_r \underbrace{[R - C_{\text{PS}}]}_{\text{Hard}} \mathcal{F}_{\text{PS}}^{(R)}$$

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Idea:

- Born term  $B$  in  $\mathbb{S}$  good for cancelling negative weights
- Move contributions from  $\mathbb{H}$  to  $\mathbb{S}$
- Preserve shower accuracy:  
can only move **Born-like** (soft, collinear) contributions

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{MC@NLO-}\Delta} = \int d\phi_B \left[ B + V + \int d\phi_r [C_{\text{PS}} - (1 - \Delta)(R - C_{\text{PS}})] \right] \mathcal{F}_{\text{PS}}^{(B)} \\ + \int d\phi_B d\phi_r \Delta [R - C_{\text{PS}}] \mathcal{F}_{\text{PS}}^{(R)}$$

- $0 \leq \Delta \leq 1$
- Must not move hard wide-angle emissions:  $\Delta \rightarrow 1$
- Move all soft and collinear contributions:  $\Delta \rightarrow 0$

$\Rightarrow$  construct  $\Delta$  from Sudakov form factors

# How to handle negative weights?

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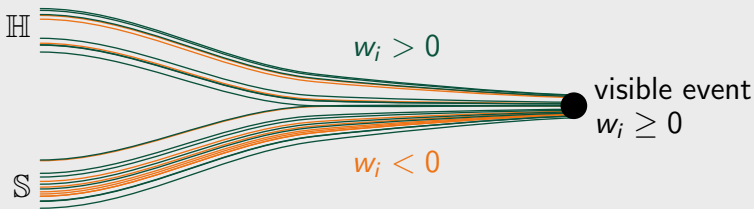
*ARCANE reweighting*

# ARCANE reweighting

- Negative weights come from separating cross section into unphysical parts
- Adding up all parts would result in non-negative weight

# ARCANE reweighting

- Negative weights come from separating cross section into unphysical parts
- Adding up all parts would result in non-negative weight
- In parton shower Monte Carlo: sum over all possibilities to generate event (impractical)



Idea: redistribute weights between history strands

# How to handle negative weights?

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*Multiplicative shower matching*



# Multiplicative shower matching

POWHEG

Starting point: leading order + parton shower

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{LOPS}} = \int d\phi_B B \mathcal{F}_{\text{PS}}$$

# Multiplicative shower matching

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Needed for NLO + PS:

- 1 NLO normalisation
- 2 Hardest emission distribution must be NLO-accurate

# Multiplicative shower matching

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Needed for NLO + PS:

### 1 NLO normalisation

$$B \rightarrow B + V + C_{\text{int}} + \int d\phi_r (R - C)$$

- ▶ Parton-shower evolution  $\mathcal{F}_{\text{PS}}$  unitary  $\Rightarrow$  correct NLO normalisation
- ▶ Only Born kinematics passed on, **so far all real emissions from parton shower**

### 2 Hardest emission distribution must be NLO-accurate

# Multiplicative shower matching

## POWHEG

Starting point: leading order + parton shower

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{LOPS}} = \int d\phi_B B \mathcal{F}_{\text{PS}}$$

Needed for NLO + PS:

- 1 NLO **normalisation**
- 2 **Hardest emission distribution** must be NLO-accurate

$$\int_{\mathcal{D}} d\sigma_{ab \rightarrow X} \Big|_{\text{NLOPS}} = \int d\phi_B \left[ B + V + C_{\text{int}} + \int d\phi_r (R - C) \right] \mathcal{G}_{\text{PS}}$$

With modified shower  $\mathcal{G}_{\text{PS}}$ :

- ▶ **unitary**  $\Rightarrow$  preserves NLO normalisation
- ▶ generates **hardest emission according to  $R$**  (or no emission)

# Multiplicative shower matching

## POWHEG

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Monte Carlo estimate of  $\int d\phi_r (R - C)$  only source of negative weights

assuming perturbation theory holds

# How to handle negative weights?

---

*ESME*

# Exponentiated Subtraction for Matching Events

Goal: better way to evaluate  $\int d\phi_r (R - C)$

Sudakov algorithm to compute  $n_B$  with  $\langle n_B \rangle = 1 + \int d\phi_r \frac{R-C}{B}$ :

- Start with  $n_B = 1, p_\perp = p_\perp^{\max}$
- While  $p_\perp > p_\perp^{\min}$ 
  - 1 Sample next  $p_\perp$  from  $e^{-\frac{M}{B} \ln p_\perp}, M = \max(R, C)$
  - 2 Generate random number  $0 < r < 1$
  - 3 If  $r > \frac{|R-C|}{M}$  keep current  $n_B$
  - 4 otherwise, if  $R > C$  set  $n_B = n_B + 1$
  - 5 otherwise, if  $R < C$  set  $n_B = n_B - 1$

Each step away from  $n_B = 1$  suppressed by  $\mathcal{O}(\alpha_s)$   
 $\Rightarrow$  negative  $n_B$  beyond NLO, can be discarded

# How to handle negative weights?

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*KrkNLO*



- PDFs have to absorb collinear divergences in partonic cross section
- Shift additional **finite** collinear remnant into PDFs: **Krk PDF scheme**
- PDFs in Krk scheme remain **positive** in most regions

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- Shift additional **finite** collinear remnant into PDFs: **Krk PDF scheme**
- PDFs in Krk scheme remain **positive** in most regions

## KrkNLO algorithm

For each parton-showered Born-level event

- 1 If there is a first emission, reweight by **positive factor** for NLO accuracy
- 2 Reweight by factor  $1 + \frac{V}{B} + \frac{C_{\text{int}}}{B} + \Delta^{\text{FS}}$   
 $\Delta^{\text{FS}}$ : from change of PDF scheme, **positive** and **large**

# Summary II

Various approaches for cancelling negative weights:

- Redistribute weights between similar/indistinguishable events:  
local cancellations between negative and excess positive weights
- Modified formulations to facilitate internal cancellations
- Alternative formalisms to avoid unphysical separations and Monte Carlo outliers

Many better methods than just discarding negative weights

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