Making Dark Matter Waves
Cosmic Web & Wavelike Dark Matter

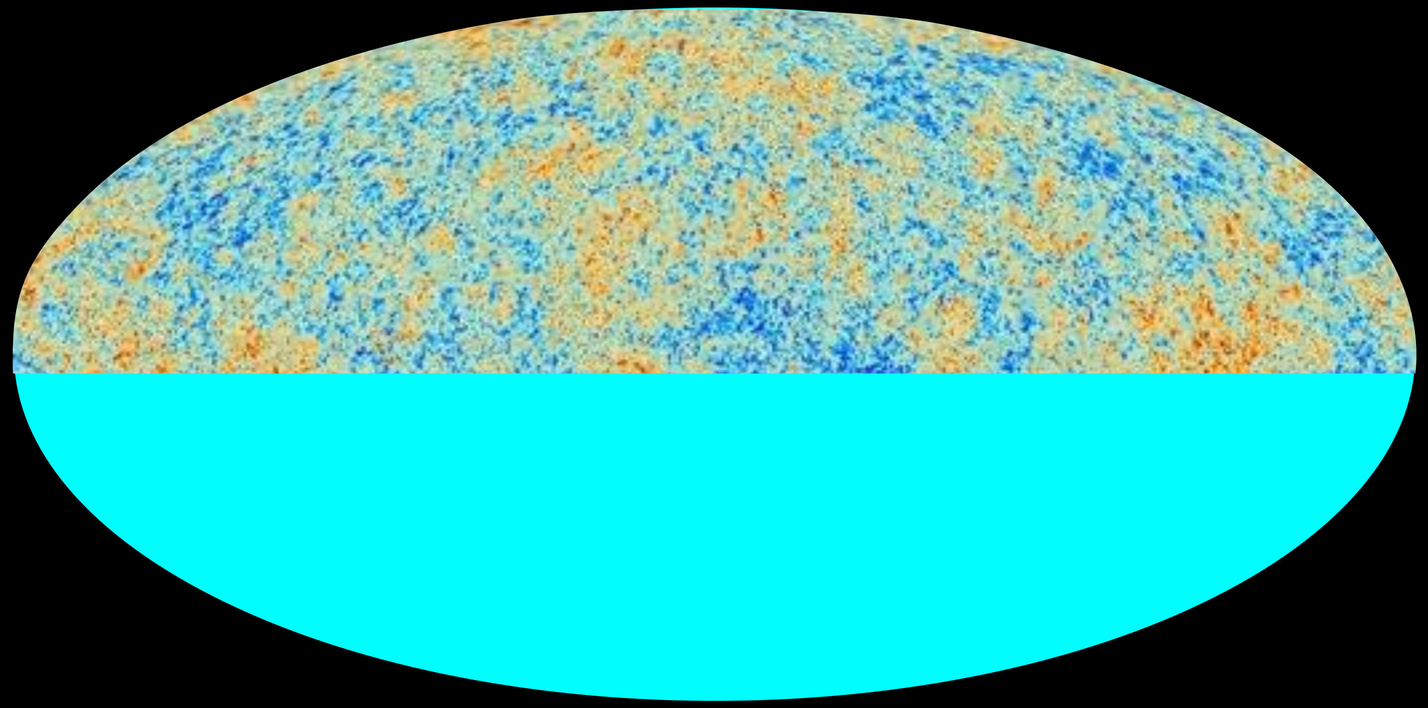
Cora Uhlemann

DESY Theory Group Seminar Feb 2025

BIG QUESTIONS

Cosmic Microwave Background

Planck



nearly uniform

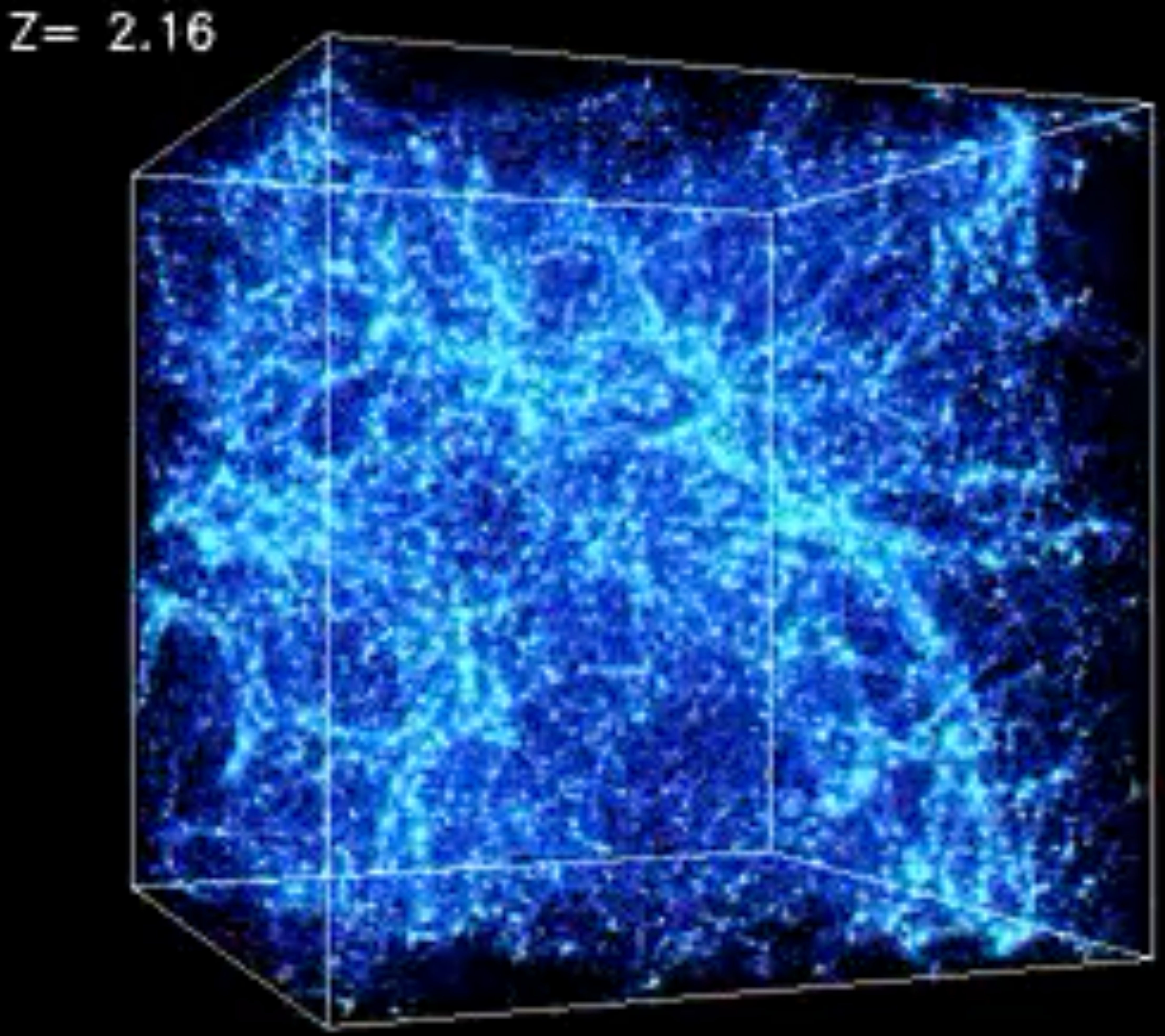
?

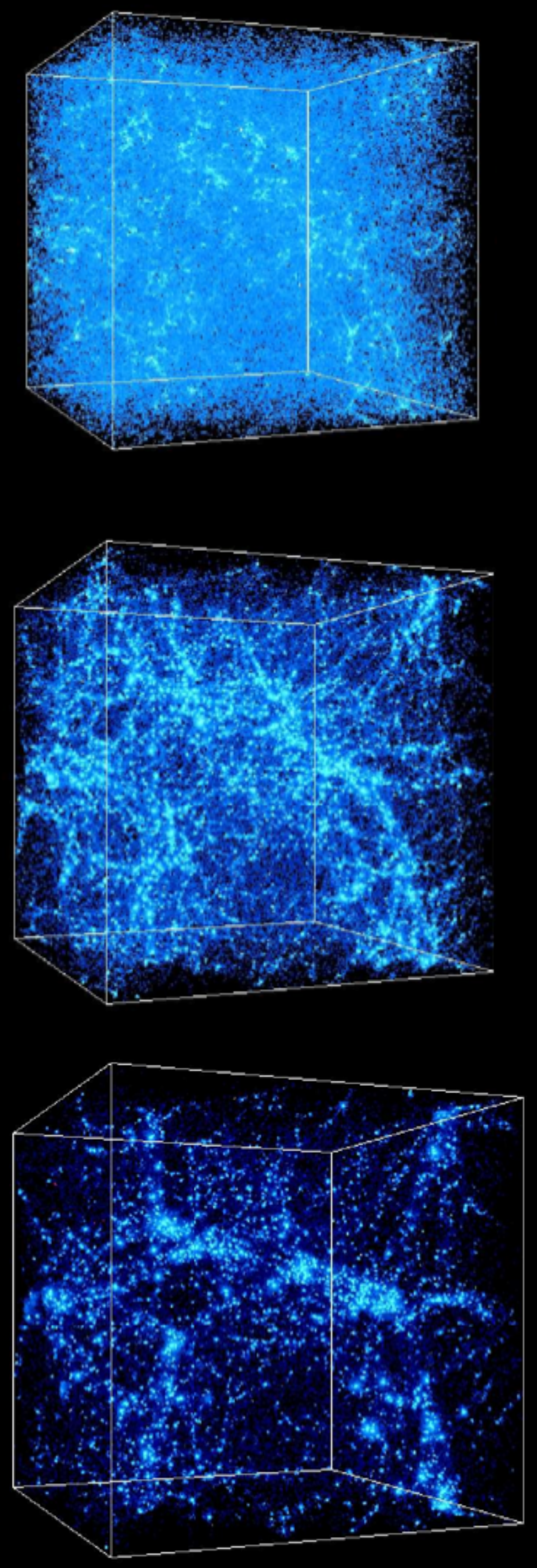
COSMIC WEB of galaxies

2MASS XSXz

rich structure

PIECE OF THE PUZZLE





National Center for Supercomputer Applications (A. Kravtsov & A. Klypin)

PIECE OF THE PUZZLE

Cosmic Microwave Background

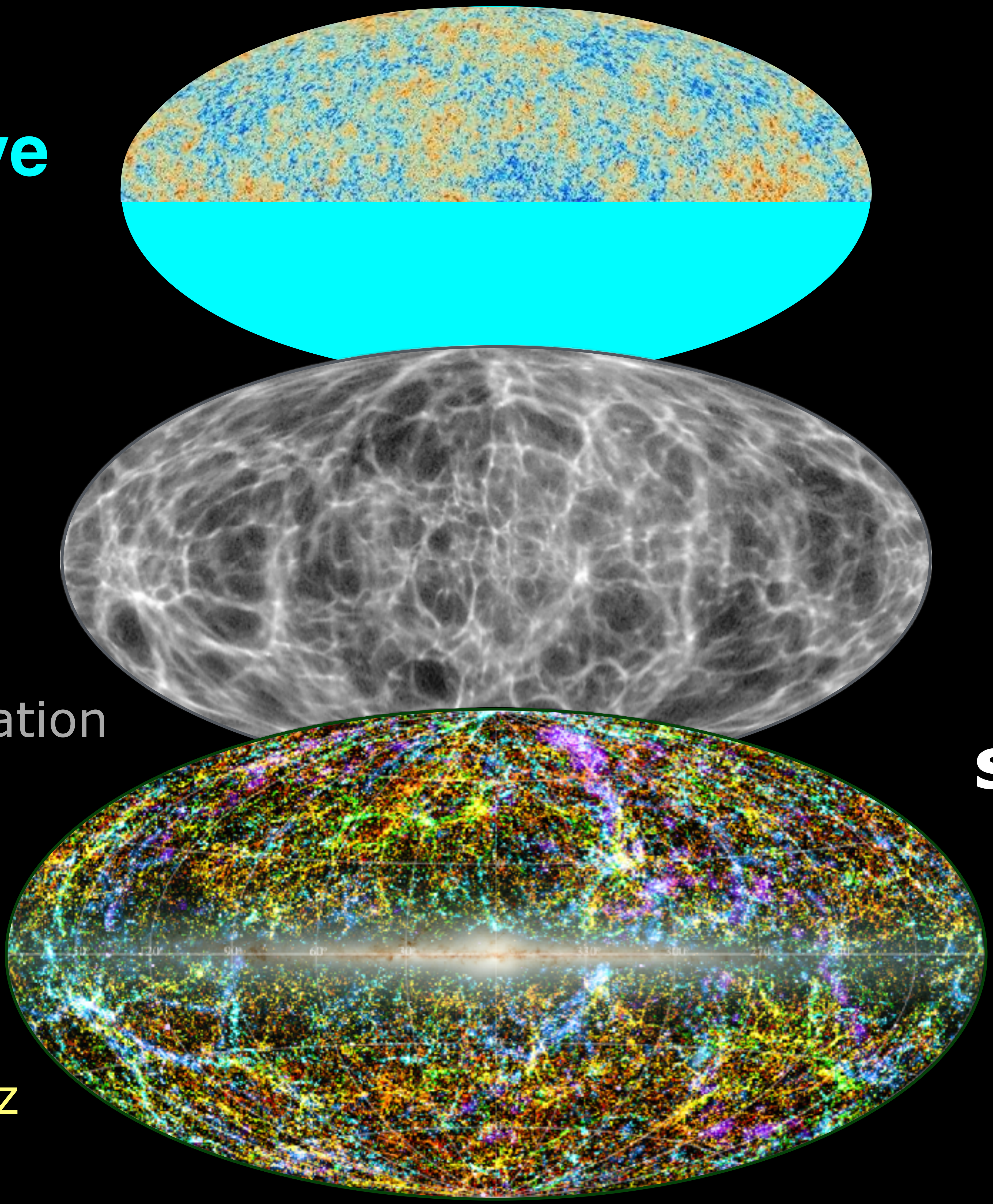
Planck

SKELETON of dark matter

KIGEN simulation

COSMIC WEB of galaxies

2MASS XSXz



nearly uniform

rich structure

Cosmic Large Scale Structure

Cosmic Web

Skeleton

filamentary
network

structural
hierarchy

Halos

Galaxy Hosts

universal
density profiles

mass
distribution

Dark Matter Mass

one of the least constrained physical parameters

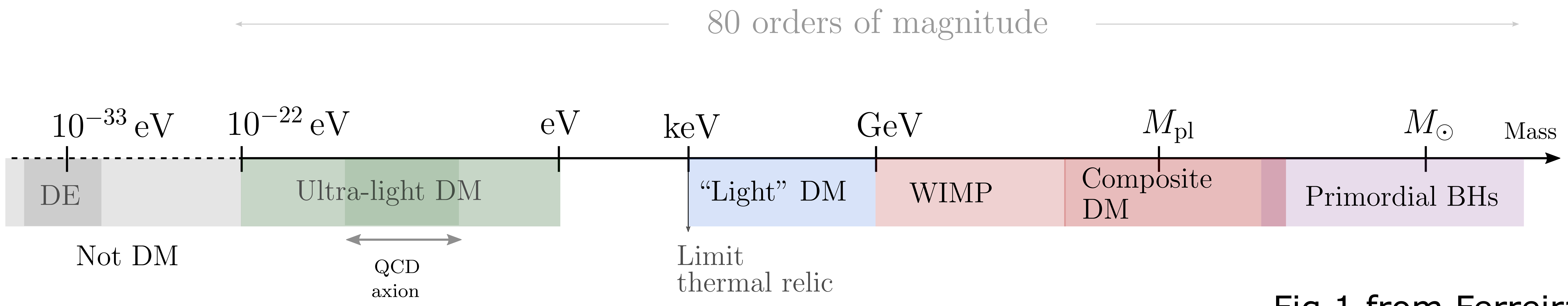


Fig 1 from Ferreira 2021

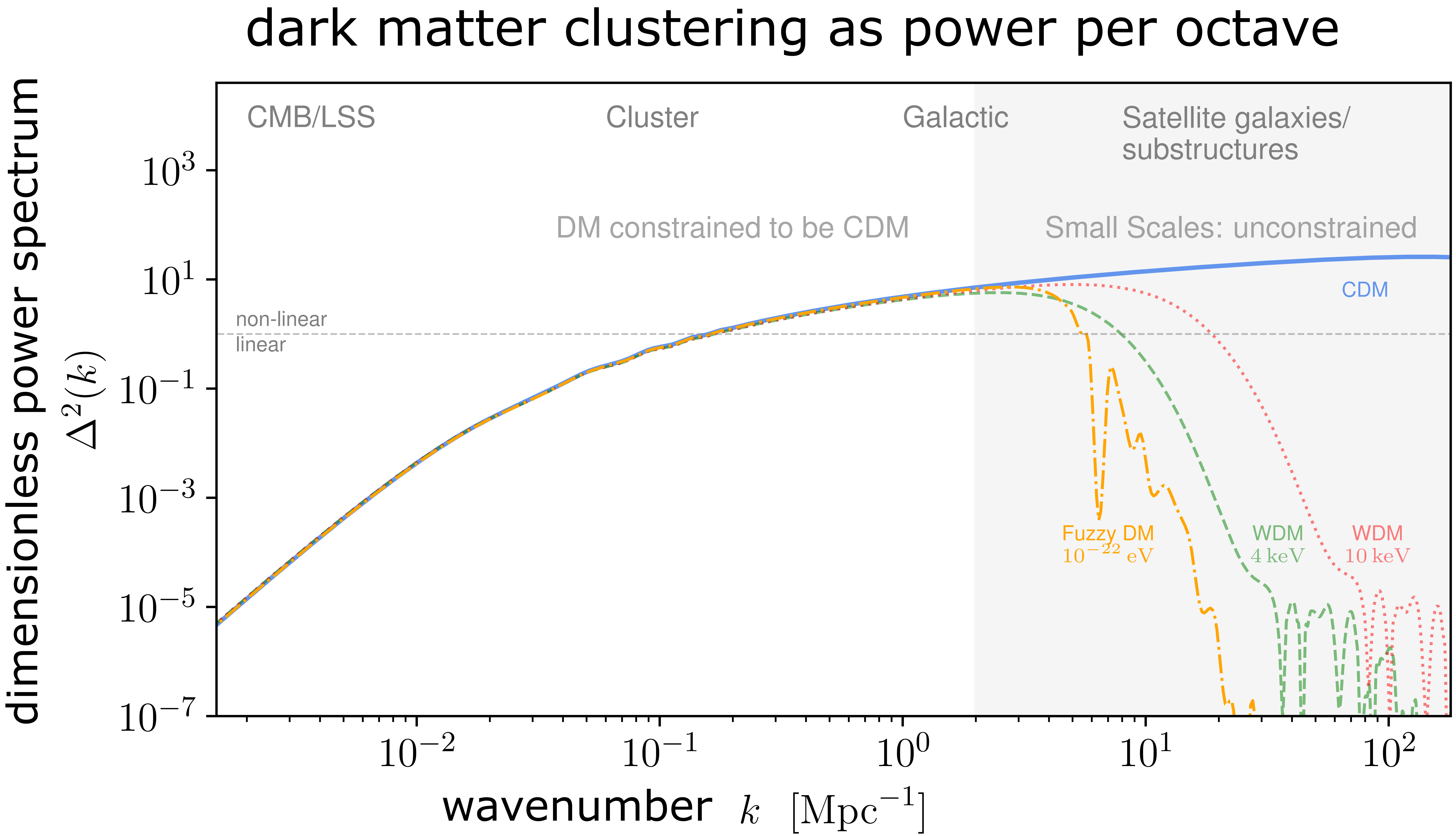
Known particles: ν e⁻ p Higgs

thermal production: hot warm cold

wave dark matter vs. cold dark matter

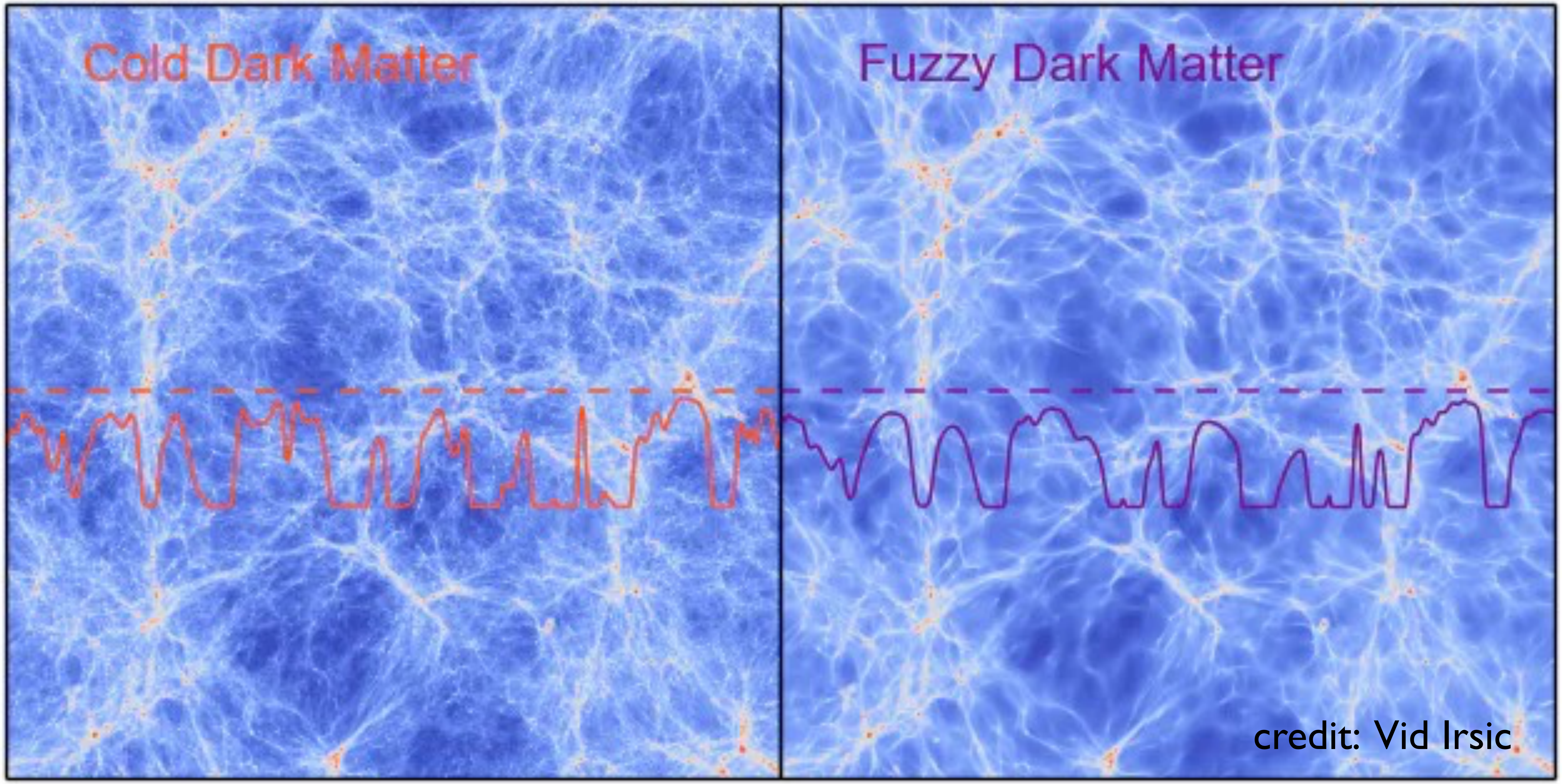
Challenges

Dark Matter Mass -> Clustering



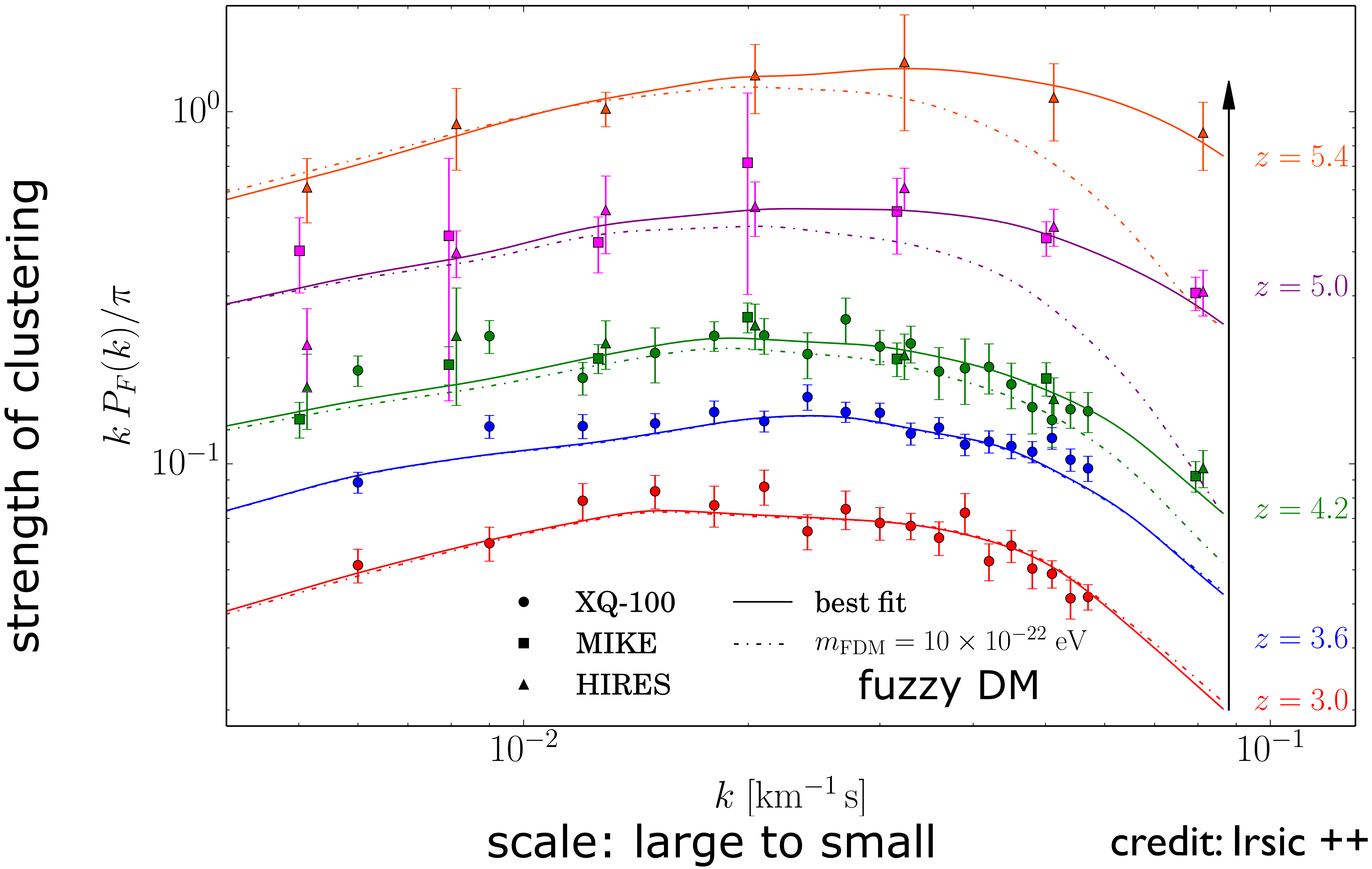
Fuzzy vs. Cold Dark Matter

Lyman-alpha forest: light absorption by hydrogen gas within the intergalactic medium at high redshifts



Fuzzy vs. Cold Dark Matter

lower mass limit by Lyman-alpha forest



credit: Irsic ++ ‘17

Key Problem

Cold Dark Matter Dynamics

Vlasov-Poisson equation (collisionless Boltzmann, long range force)

$$\partial_t f(\boldsymbol{x}, \boldsymbol{p}, t) = \{H, f\} \qquad \Delta V(\boldsymbol{x}, t) \propto \int f(\boldsymbol{x}, \boldsymbol{p}, t)\, \mathrm{d}^3 p - 1$$

3+3 dim

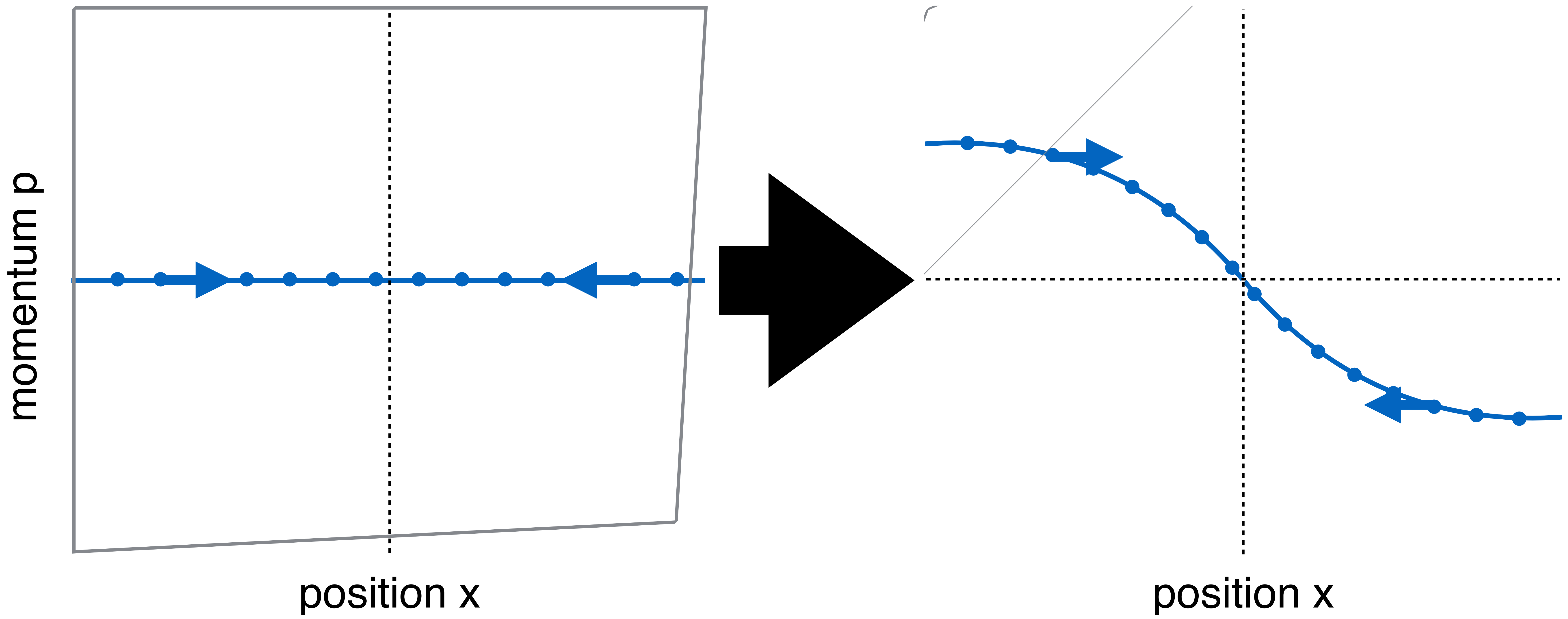
nonlinear

simple “cold” initial conditions: flat sheet

Cold Dark Matter Dynamics

perfect fluid: single stream

$$f_{\mathrm{fl}}(\boldsymbol{x}, \boldsymbol{p}) = \rho(\boldsymbol{x})\delta_D(\boldsymbol{p} - m\boldsymbol{\nabla}\phi(\boldsymbol{x}))$$



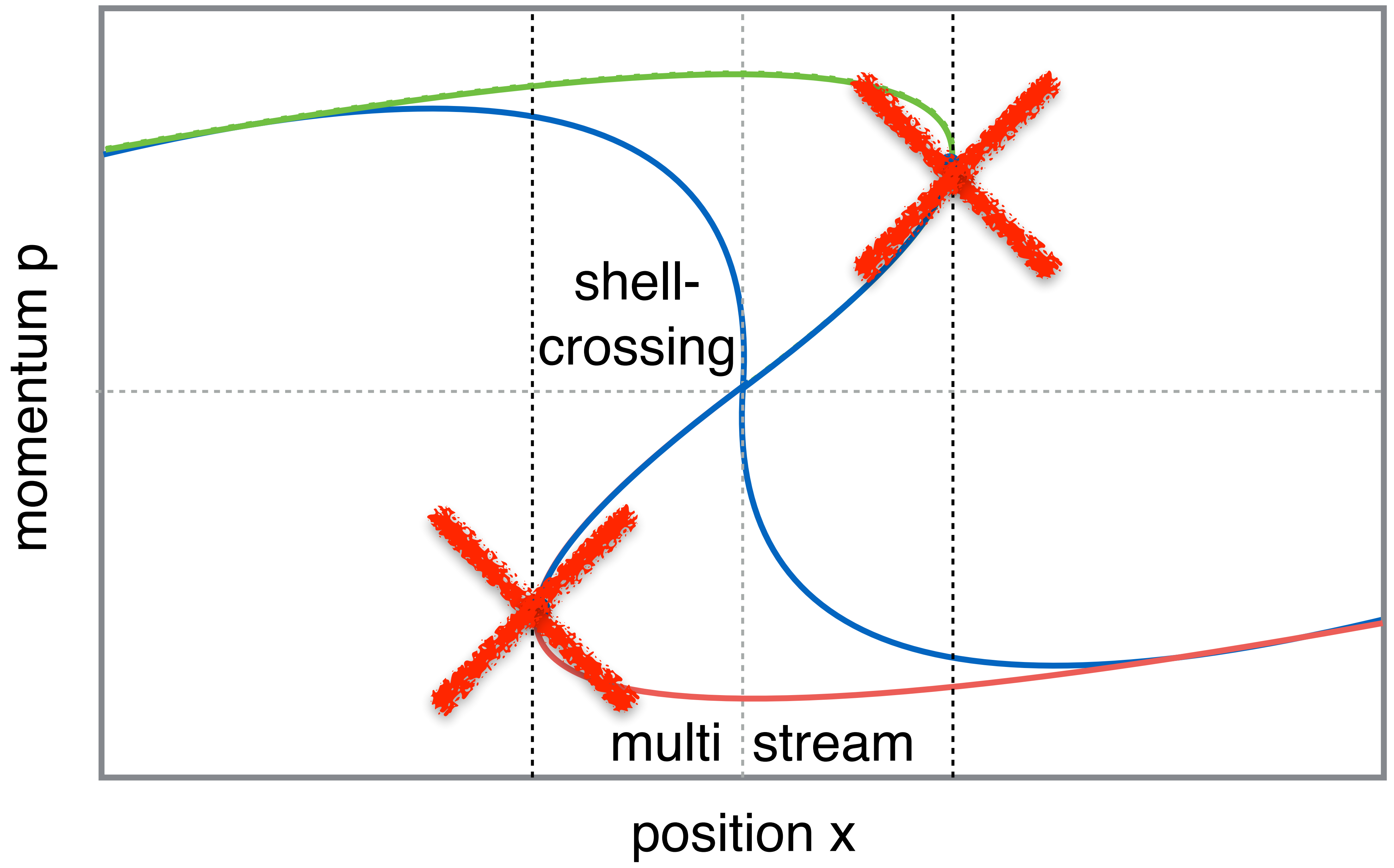
Phenomenology

Cold Dark Matter Dynamics

perfect fluid: single stream

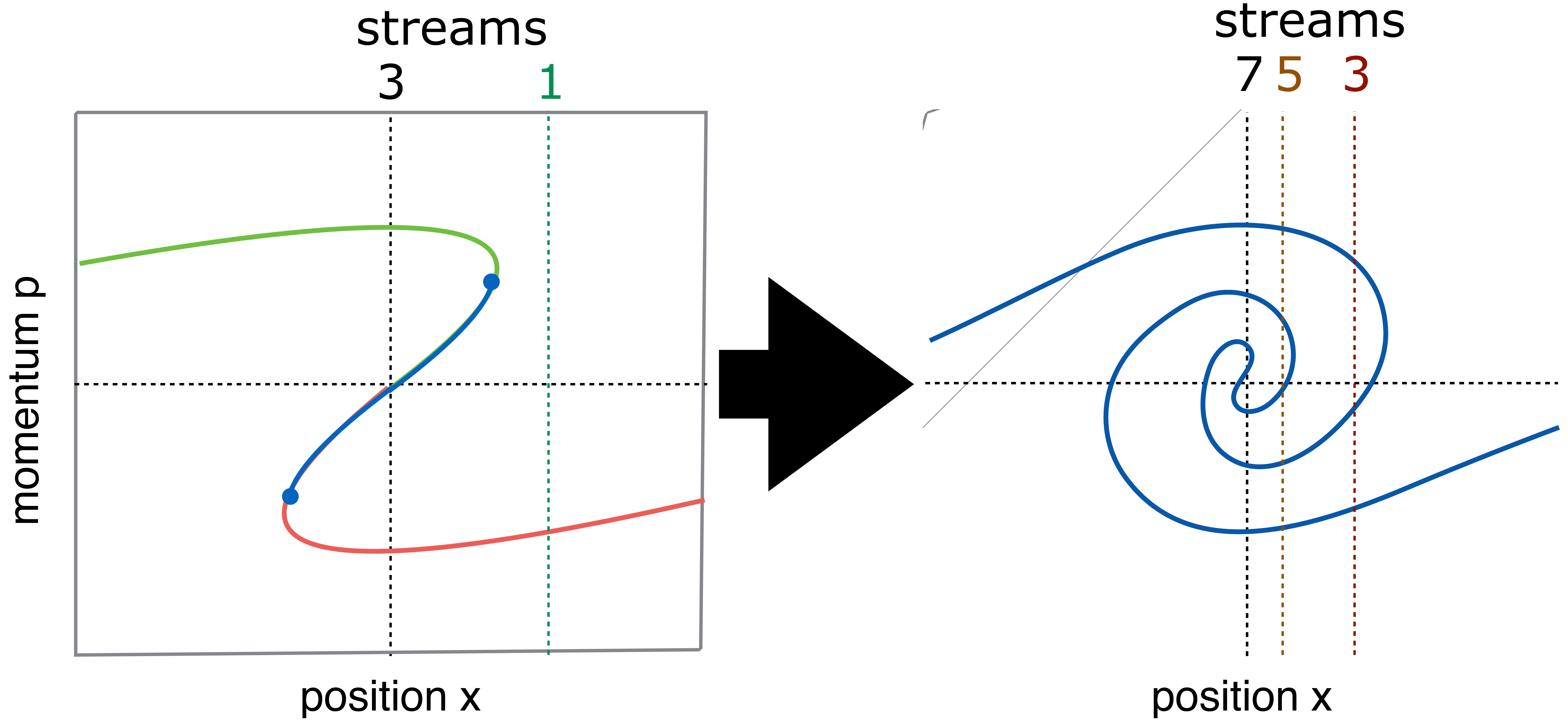
$$f_{\mathrm{fl}}(\boldsymbol{x},\boldsymbol{p}) = \rho(\boldsymbol{x})\delta_D(\boldsymbol{p} - m\boldsymbol{\nabla}\phi(\boldsymbol{x}))$$

fails at shell-crossing



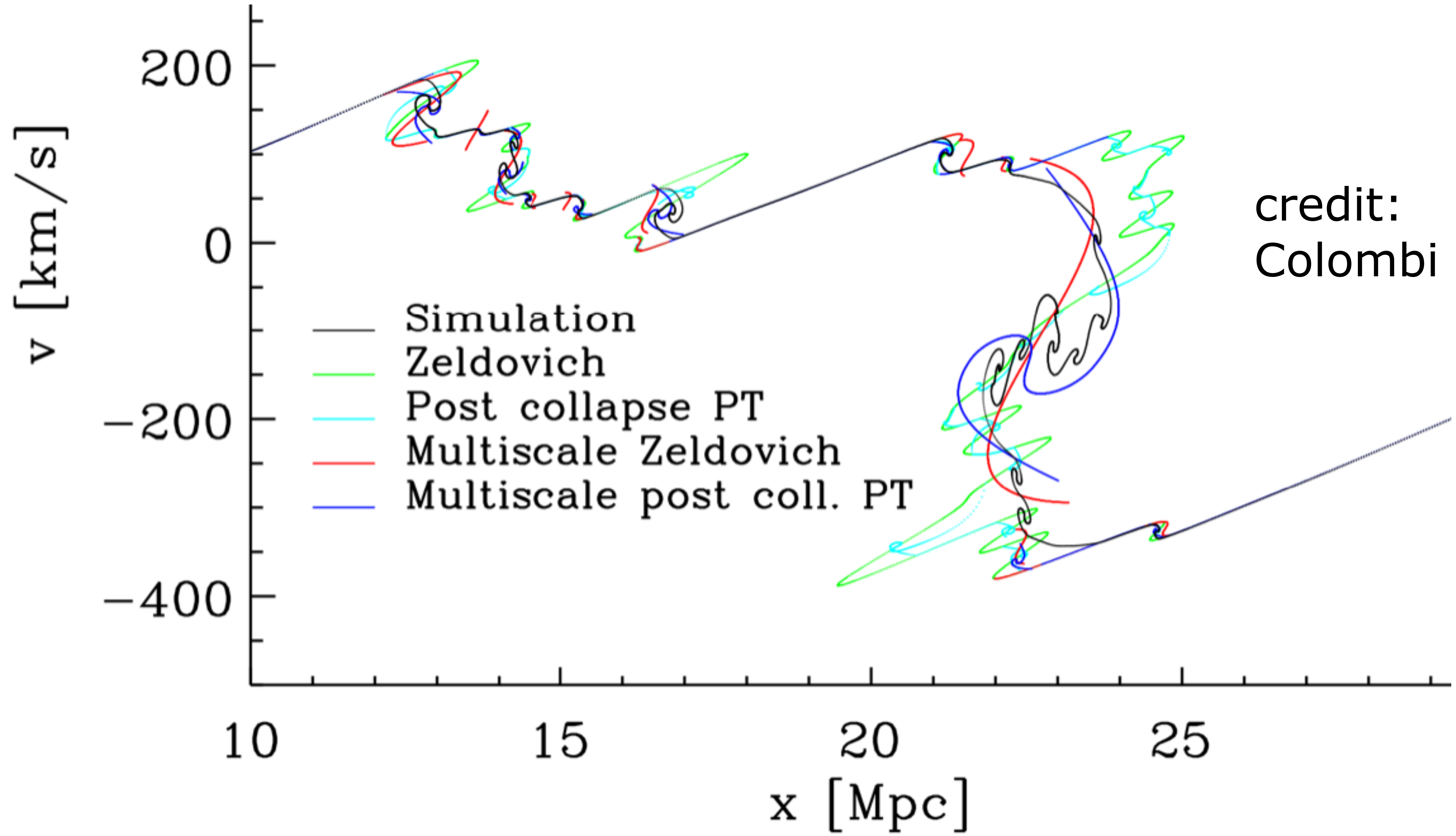
Cold Dark Matter Dynamics

beyond perfect fluid: multi-stream → bound structures



Cold Dark Matter Dynamics

large-scale view



credit:
Colombi & Taruya

Challenges

NUMERICAL	ANALYTICAL
N PARTICLES	2 FIELDS
computational power	perturbative fluid
limited sampling	limited features
large-scale accuracy	small-scale accuracy

One Wavefunction to rule them all?

Numerical

N particles

cold dark matter
particles

Analytical

2 fields

cold dark matter
fluid

1 Complex

Wave Function

wave dark matter

Key Idea

Semiclassical Dynamics

correspondence: classical $\rightleftarrows$ quantum

$$f(\boldsymbol{x},\boldsymbol{p},t) \simeq f_\hbar[\psi(\boldsymbol{x},t)](\boldsymbol{p})$$

3+3 dim　　　3 dim

numerics idea:
Widrow & Kaiser '93

$$\hbar \simeq \frac{\hbar_{\mathrm{phys}}}{m}$$ small parameter

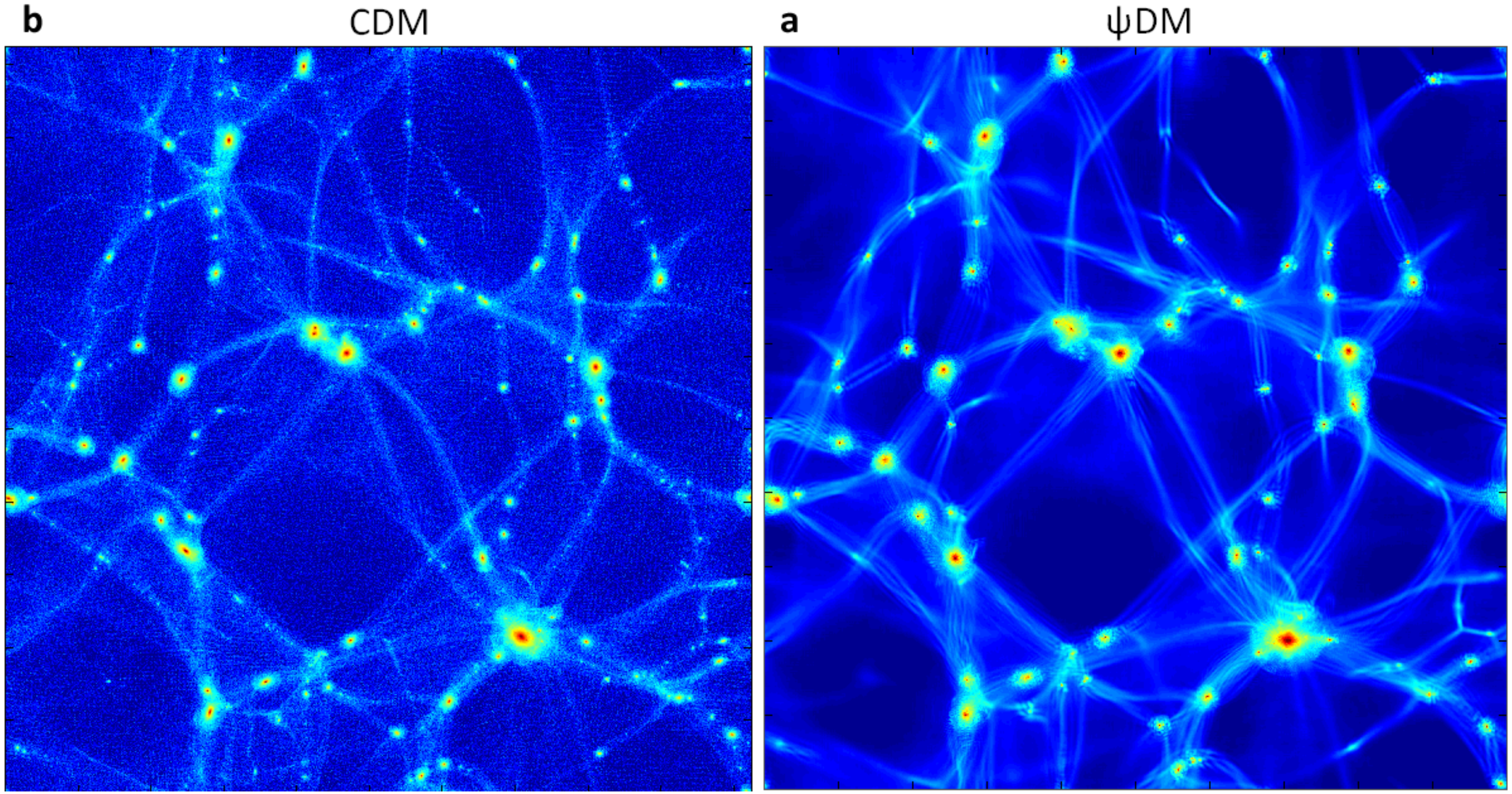
Schrödinger-Poisson equation

$$i\hbar\,\partial_t\psi(\boldsymbol{x},t) = \hat{H}\psi(\boldsymbol{x},t) \qquad \Delta V(\boldsymbol{x},t) \propto |\psi(\boldsymbol{x},t)|^2 - 1$$

fundamental for (ultra-)light scalar fields

Wave Dark Matter

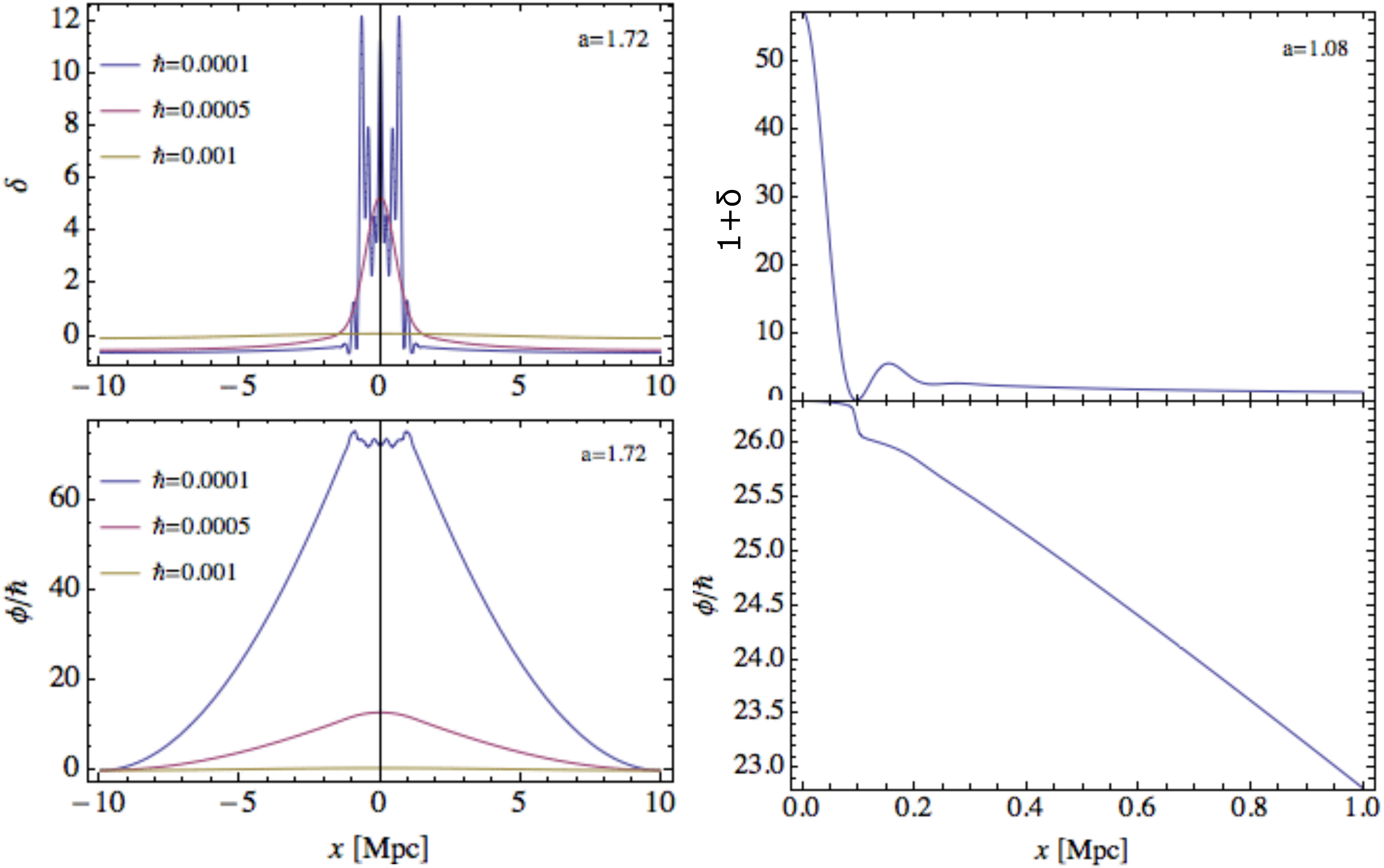
axion-like particles Schive ++ Nature Phys. Lett ‘15



astrophysical imprints: Hui, Ostriker, Tremaine & Witten 17, Hui `21

Wave Dark Matter

shell-crossing: oscillations & phase jumps $\quad \psi \propto \boxed{\sqrt{1+\delta}}\exp[\boxed{i\phi/\hbar}]$



Key Idea

Semiclassical Dynamics

correspondence: classical $\rightleftarrows$ quantum

$$f(\boldsymbol{x},\boldsymbol{p},t) \simeq f_\hbar[\psi(\boldsymbol{x},t)](\boldsymbol{p})$$

3+3 dim 3 dim

$$\partial_t f_W = \left[\frac{\boldsymbol{p}^2}{2a^2m} + mV\right] \frac{2}{\hbar}\sin\left(\frac{\hbar}{2}(\overleftarrow{\boldsymbol{\nabla}}_x\overrightarrow{\boldsymbol{\nabla}}_p - \overleftarrow{\boldsymbol{\nabla}}_p\overrightarrow{\boldsymbol{\nabla}}_x)\right) f_W$$

$$\simeq \left(\overleftarrow{\boldsymbol{\nabla}}_x\overrightarrow{\boldsymbol{\nabla}}_p - \overleftarrow{\boldsymbol{\nabla}}_p\overrightarrow{\boldsymbol{\nabla}}_x\right)$$

CU, Kopp, Haugg PRD '14

Numerical Proof Of Concept

Semiclassical Dynamics

classical $\rightleftarrows$ quantum

$$f(\boldsymbol{x},\boldsymbol{p},t) \simeq f_\hbar[\psi(\boldsymbol{x},t)](\boldsymbol{p})$$

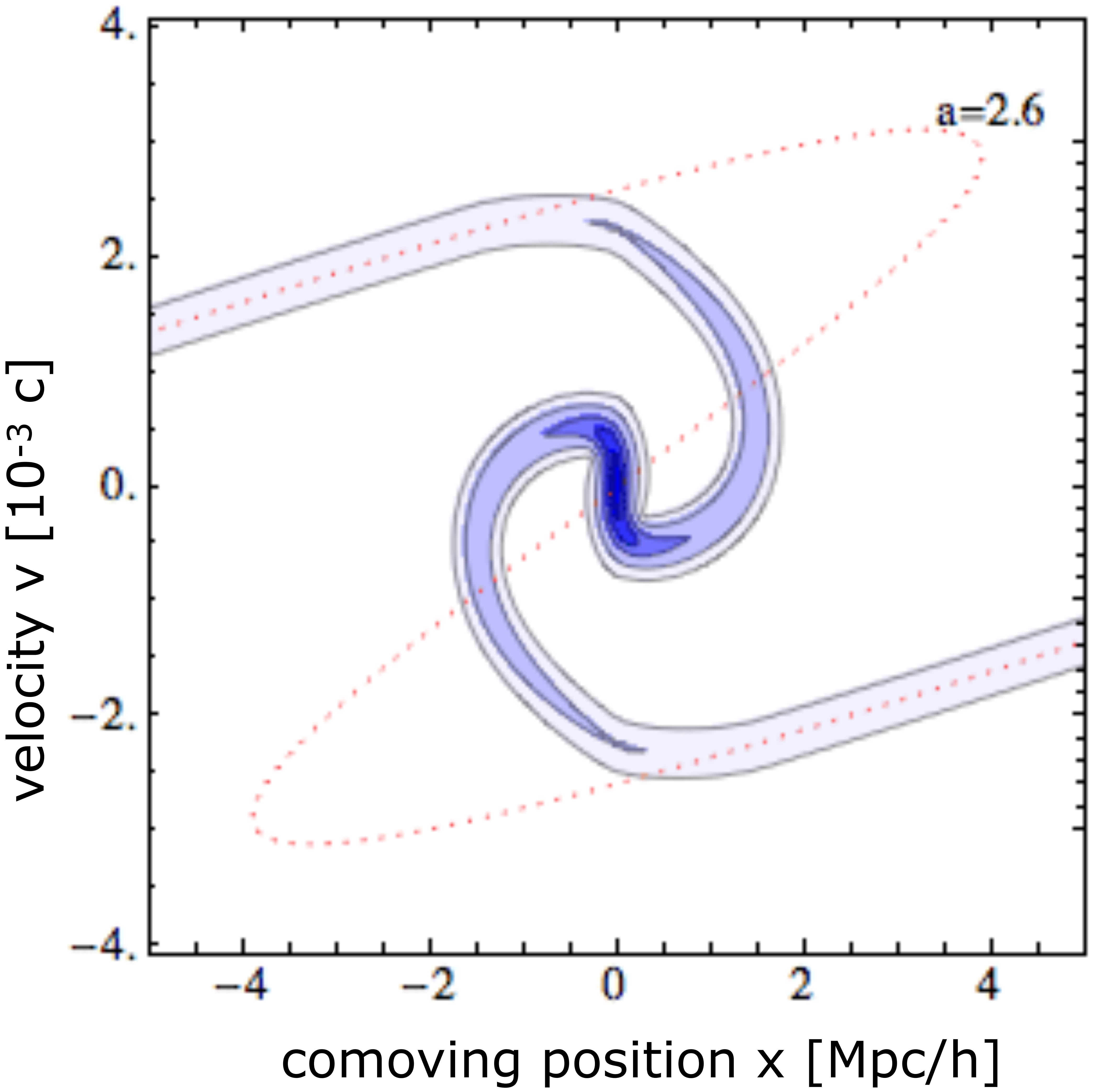
+ coarse-graining $\sigma_x\sigma_p \gtrsim \hbar/2$

multi-stream

→ bound structure

CU, Kopp & Haugg PRD '14

2D: Kopp++ PRD '17



One Wavefunction to rule them all?

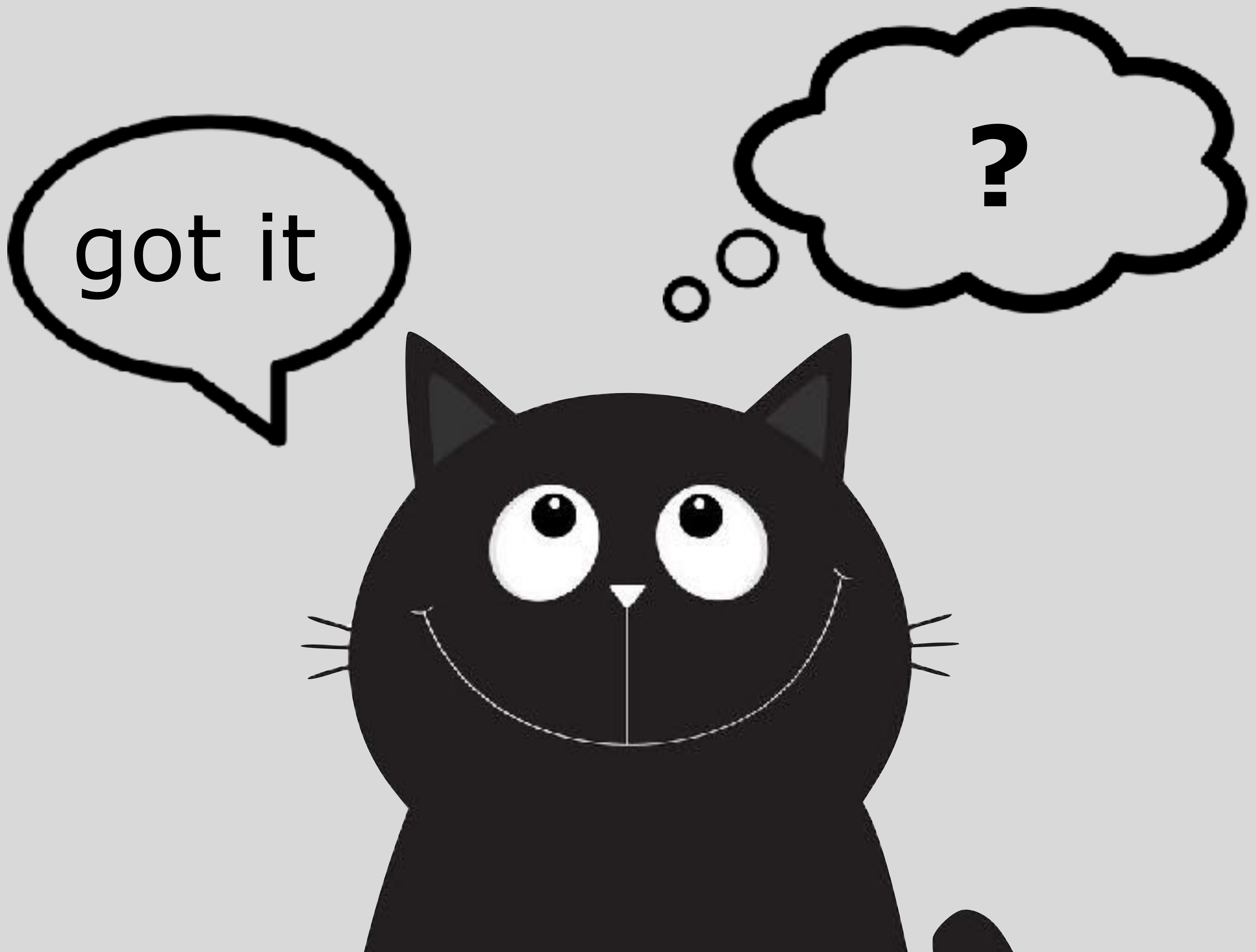
numerical

N particles

analytical

2 fields

1 Wave Function



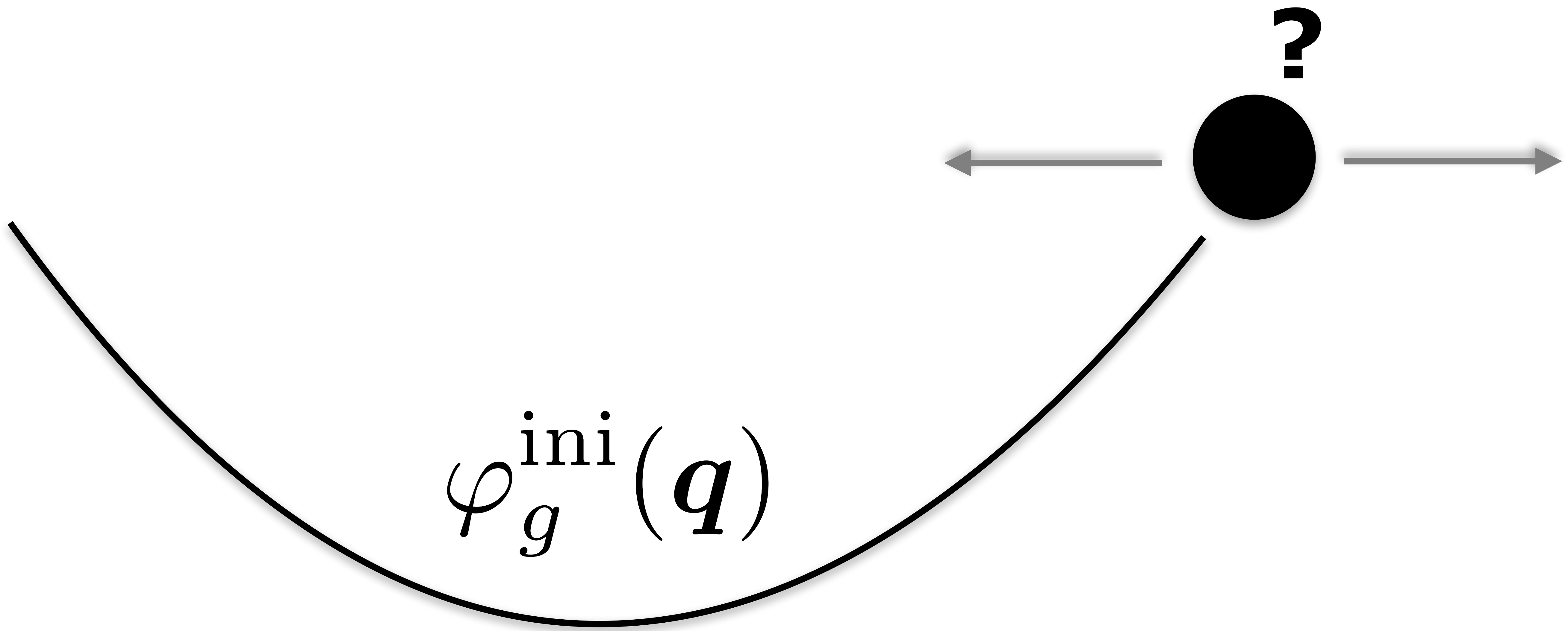
Li, Hui & Bryan 18:
naive wave PT
no good

Classical Dynamics

Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla}\varphi_g^{\text{ini}}(\boldsymbol{q})$$



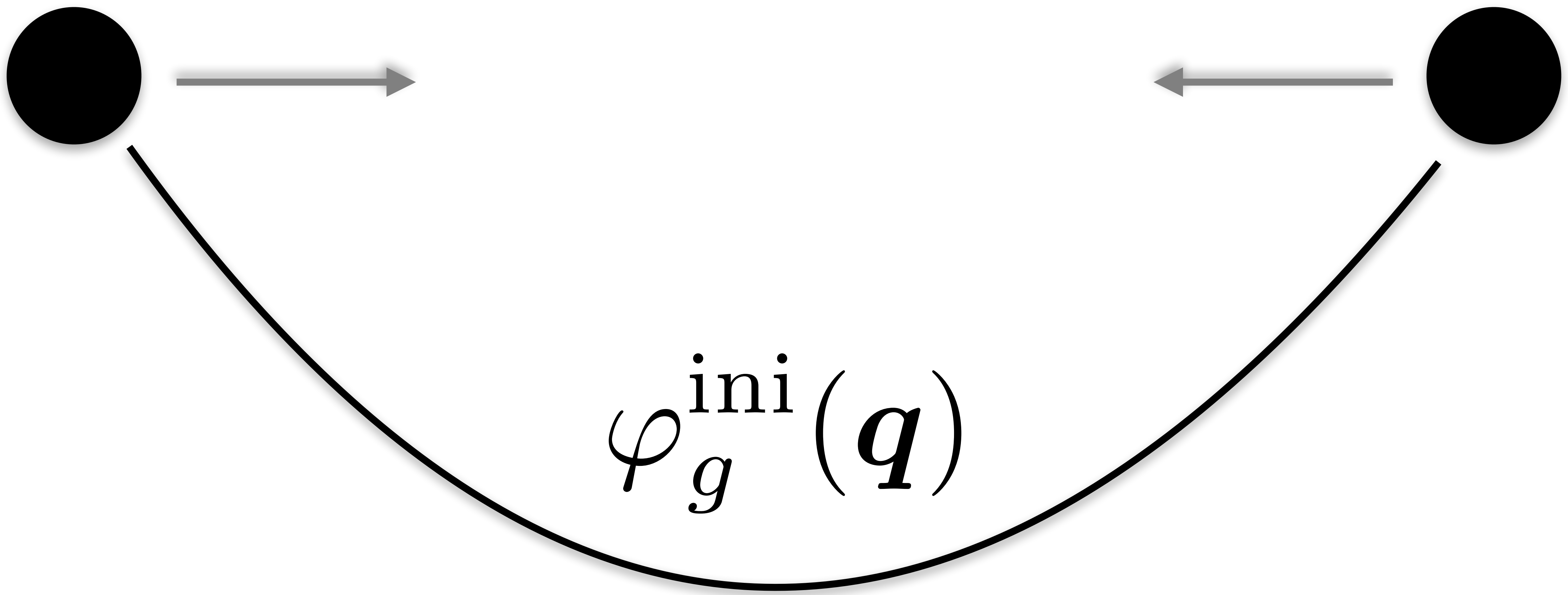
Classical Dynamics

Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q}, a) = -\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$

$$\boldsymbol{x}(\boldsymbol{q}, a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{\mathrm{ini}}(\boldsymbol{q})$$



Classical Dynamics

Approximate: Shoot Particles

follow initial gravitational potential

$$\boldsymbol{v}(\boldsymbol{q},a) = -\boldsymbol{\nabla}\varphi_g^{\text{ini}}(\boldsymbol{q})$$

$$\boldsymbol{x}(\boldsymbol{q},a) = \boldsymbol{q} - a\boldsymbol{\nabla}\varphi_g^{\text{ini}}(\boldsymbol{q})$$

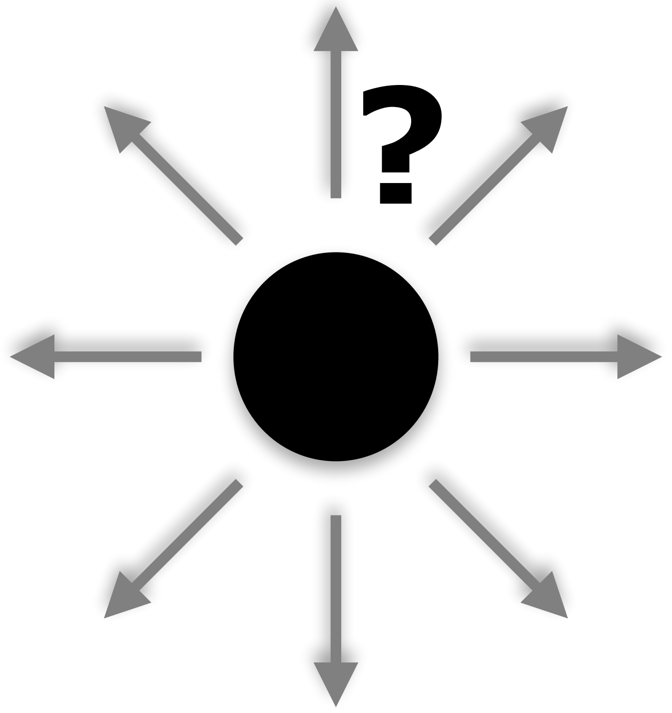
Zel'dovich 1D: exact before shell-crossing

Coordinates & PT

x: 'standard' Eulerian (SPT)

q: Lagrangian (LPT)

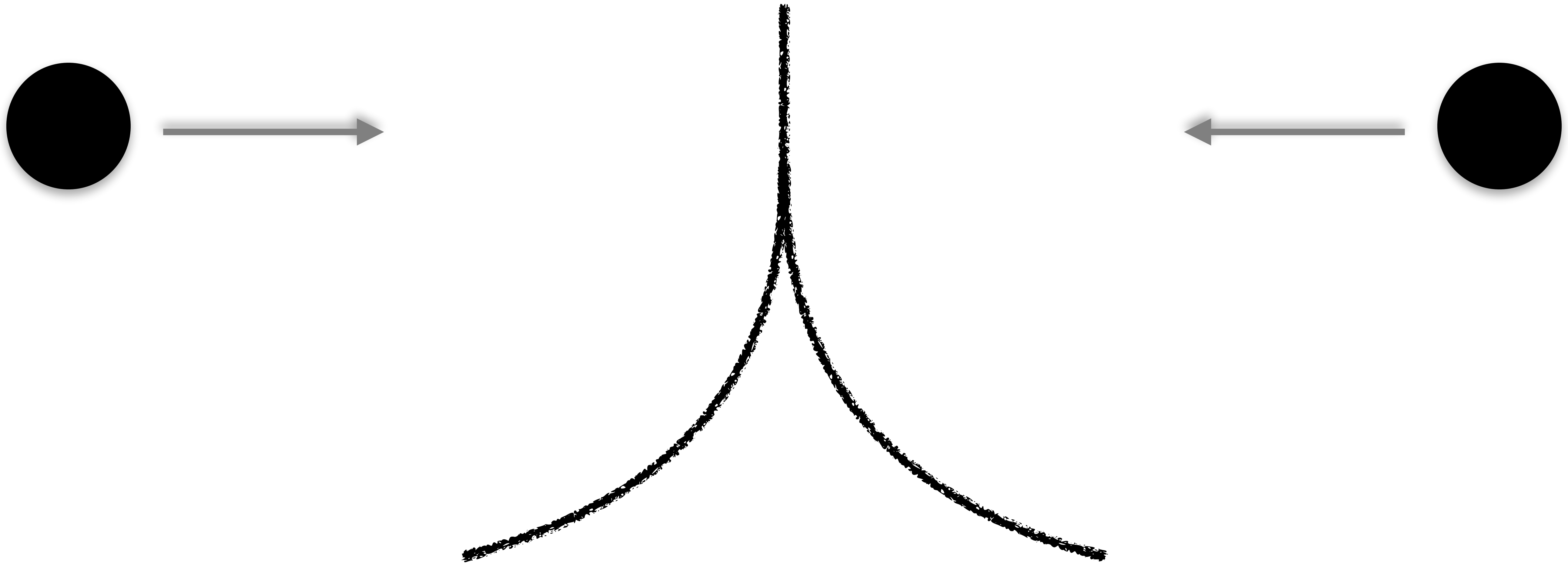
in 2D & 3D:
+ tidal effects



Classical Dynamics

Approximate: Shoot Particles

shell-crossing: singular density



Zeldovich useful until shortly after shell-crossing

no comeback after fly-through

Free Propagation

classical action: displacement × velocity

$$S_0(\boldsymbol{x}, \boldsymbol{q}, a) = \frac{1}{2}(\boldsymbol{x} - \boldsymbol{q}) \cdot \frac{\boldsymbol{x} - \boldsymbol{q}}{a}$$

background expansion

***CU**, Rampf, Gosenca & Hahn 18*

Semiclassical Dynamics

Translate Free Propagation

transition amplitude

$$\psi_0(\boldsymbol{x}, a) = N \int d^3q \exp\left[\frac{i}{\hbar} S_0(\boldsymbol{x}, \boldsymbol{q}, a)\right] \psi_0^{\text{ini}}(\boldsymbol{q})$$

Schrödinger equation

$$i\hbar\partial_a\psi_0 = -\frac{\hbar^2}{2}\nabla^2\psi_0$$

Coles & Spencer 03

***CU**, Rampf, Gosenca & Hahn 18*

in 1D & right coordinates ≅ exact before shell-crossing

Classical Observables

Eulerian Fluid

density & velocity

$$\rho(\boldsymbol{x}) = |\psi(\boldsymbol{x})|^2 \qquad \psi = \sqrt{\rho}\exp[i\phi_v/\hbar]$$

$$\boldsymbol{v}(\boldsymbol{x}) = \frac{i\hbar}{2}\frac{\psi\boldsymbol{\nabla}\bar{\psi} - \bar{\psi}\boldsymbol{\nabla}\psi}{|\psi|^2} = \nabla\phi_v$$

not necessarily potential

+ velocity dispersion, …

Free Wave Evolution

Amplitude: brightness

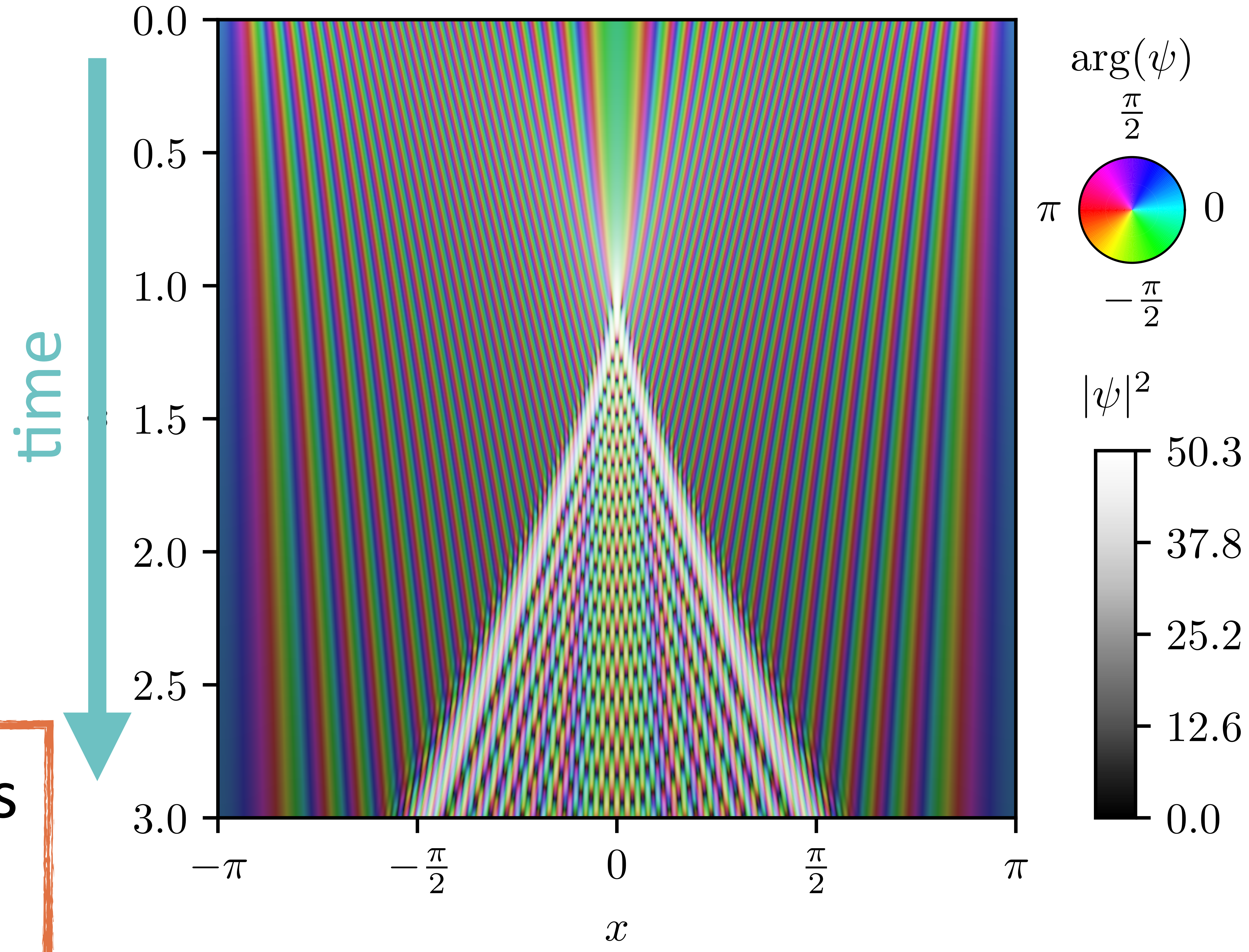
Phase: colour

Features

- Interference
- Regularised caustic

Interfering components

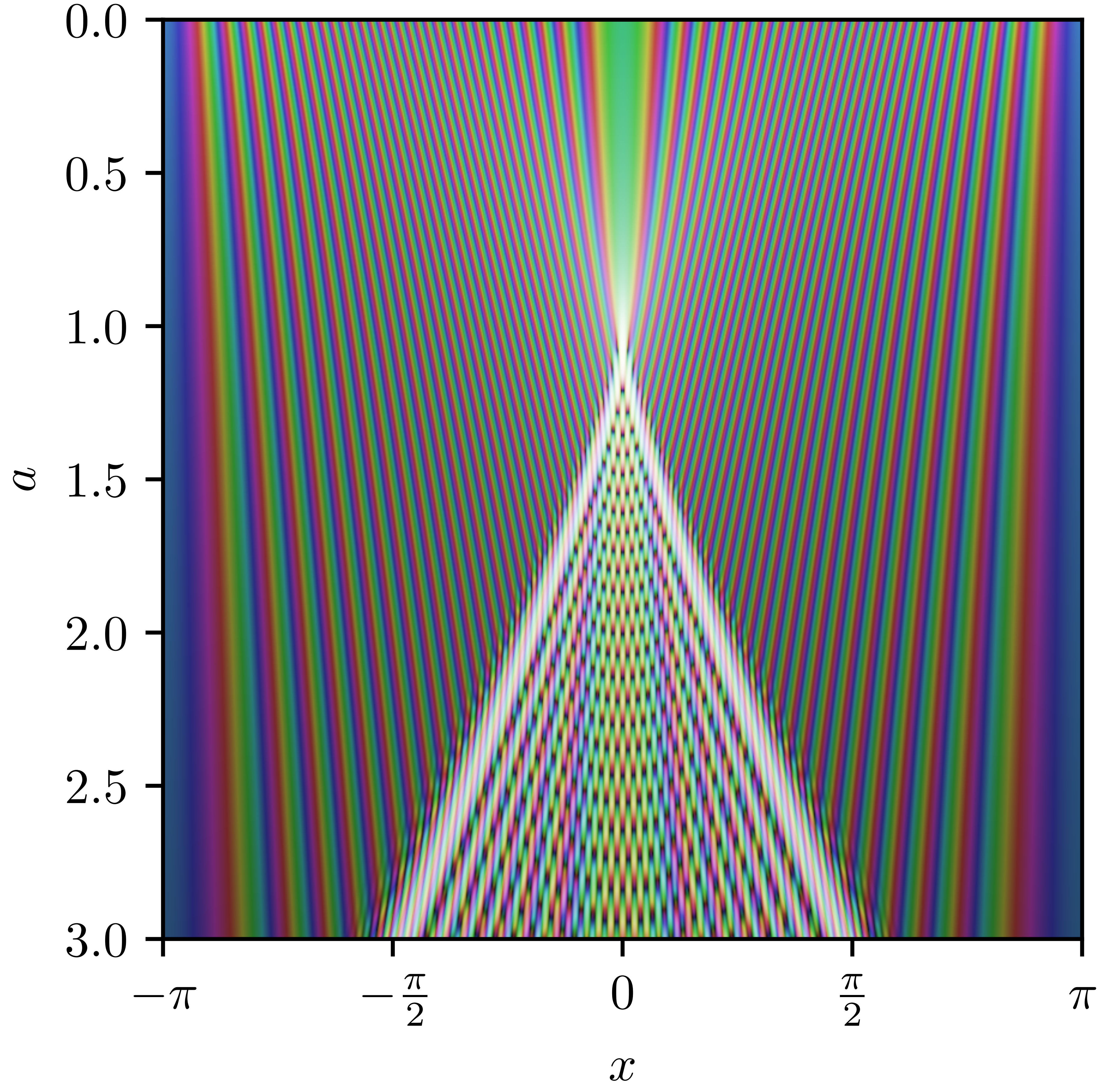
Caustic properties



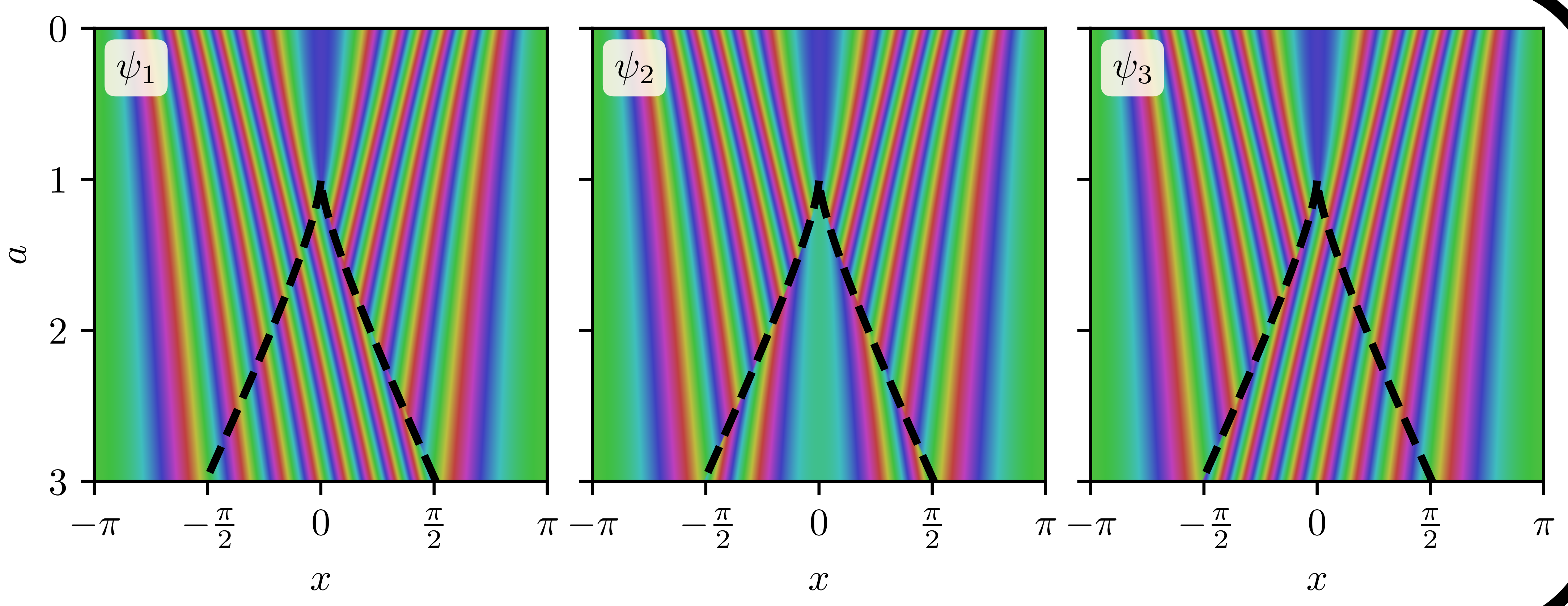
Uhlemann++ PRD 2019

Gough & Uhlemann 2022

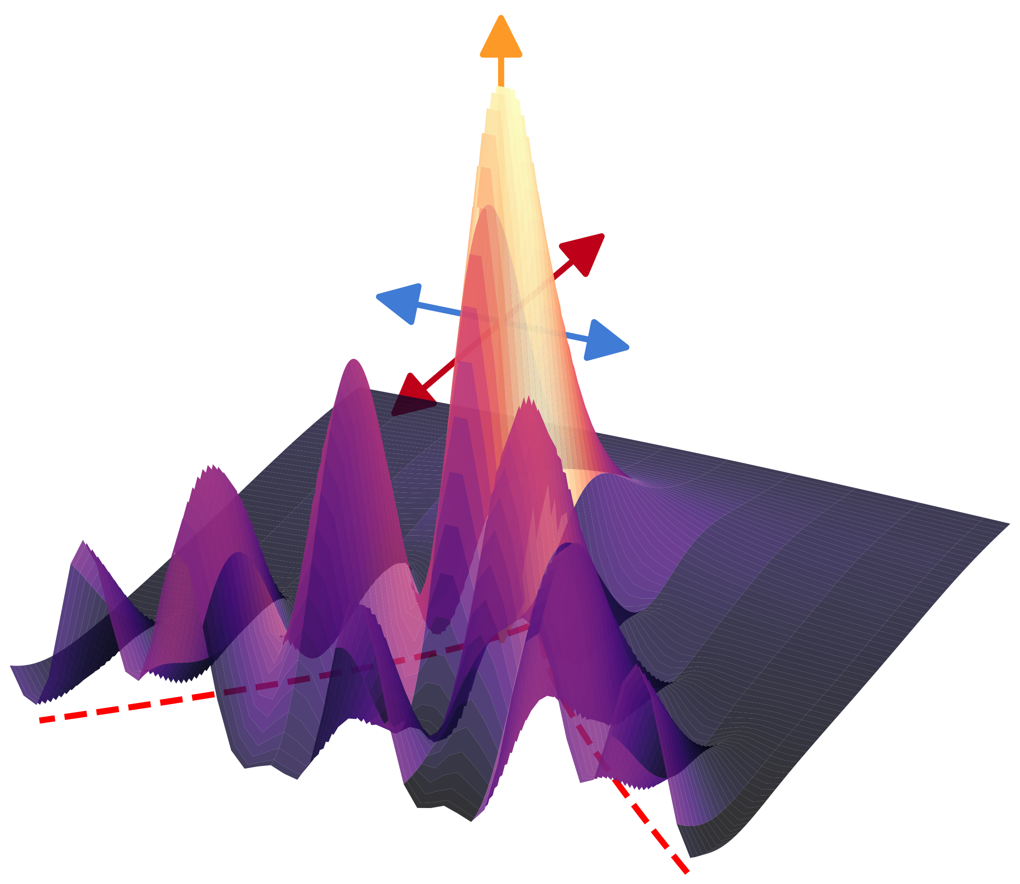
Unweaving the Wavefunction



Stream components



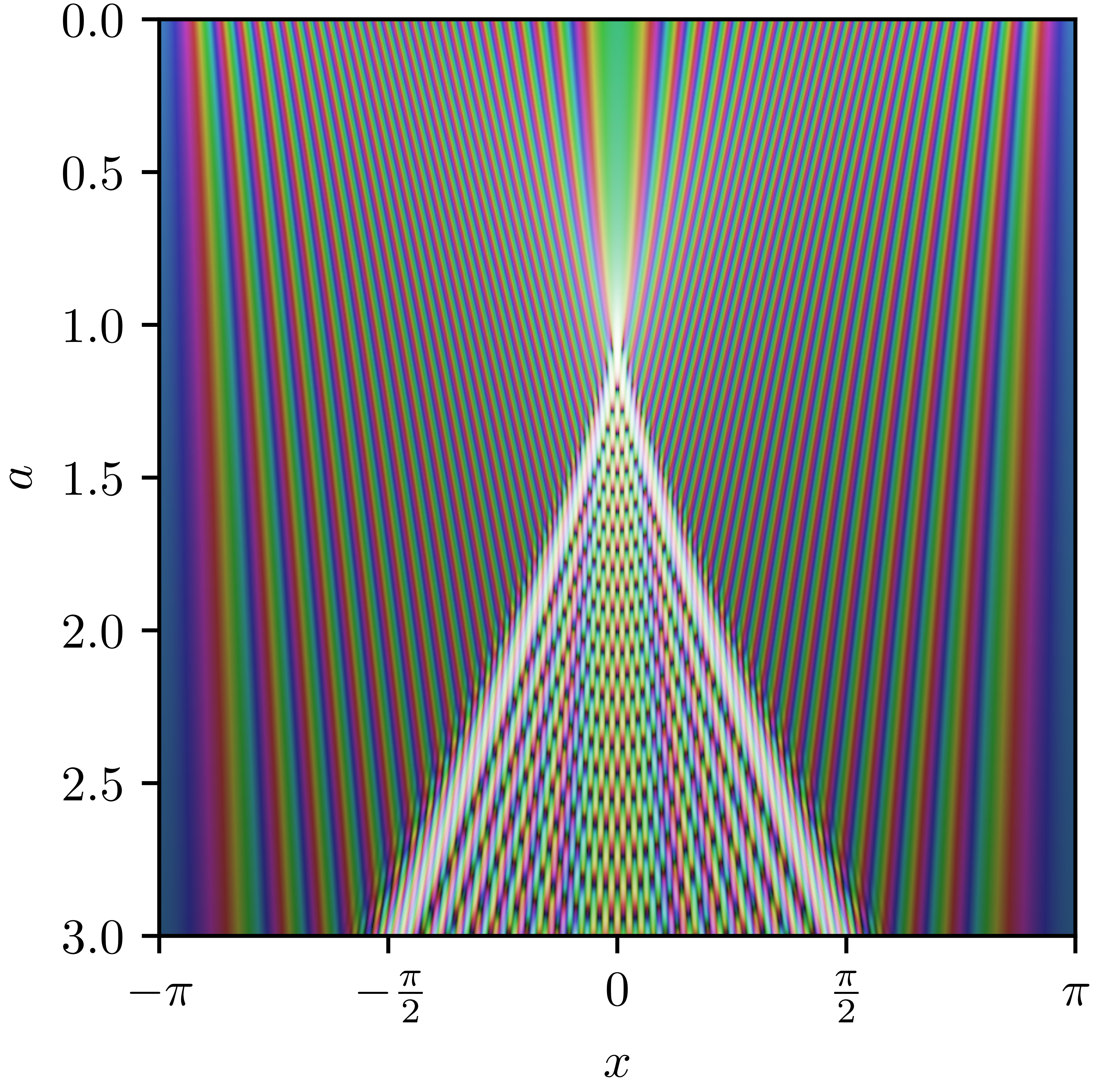
Caustic classification



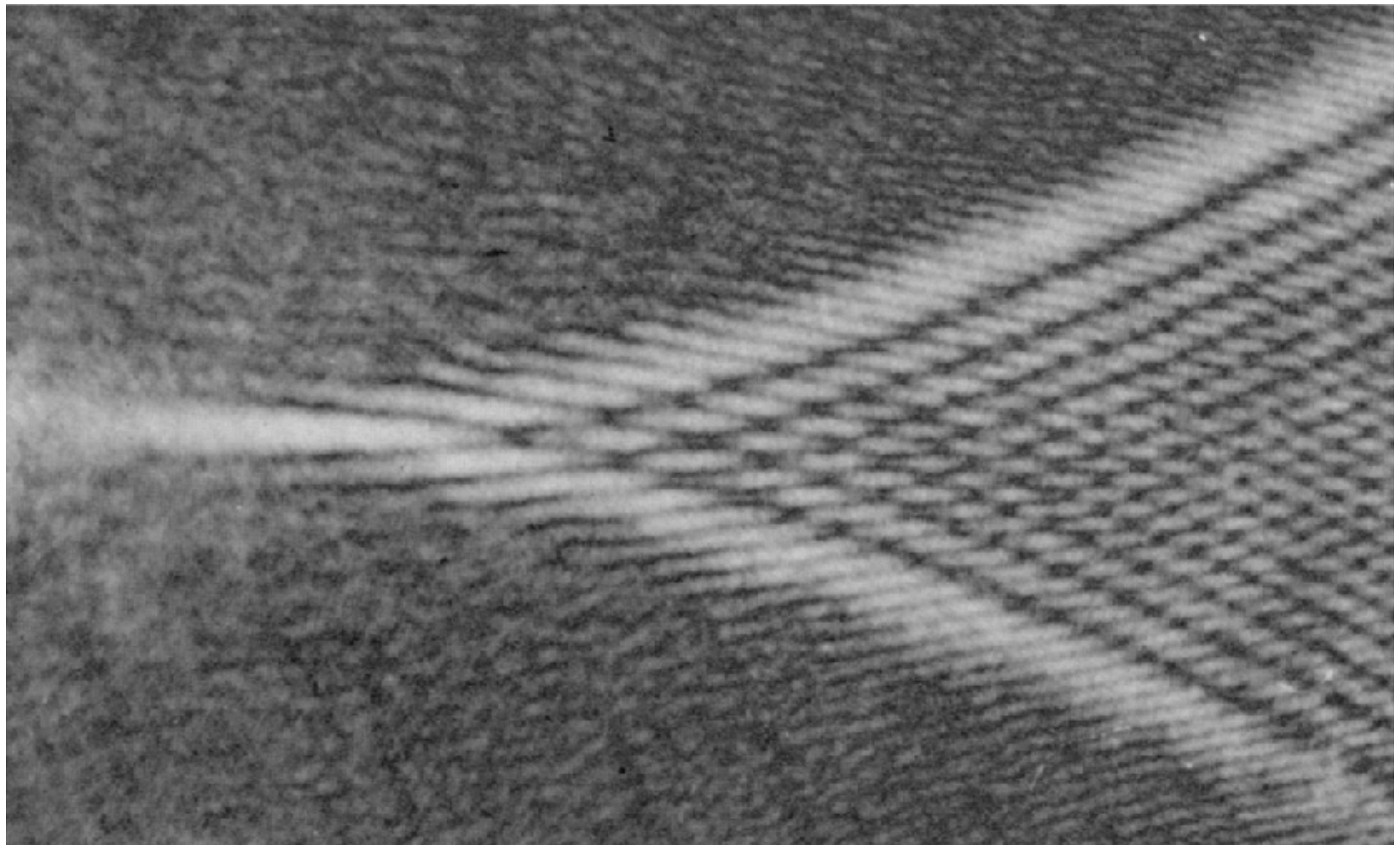
Gough & Uhlemann 2022

Optics Analogy

Dark matter



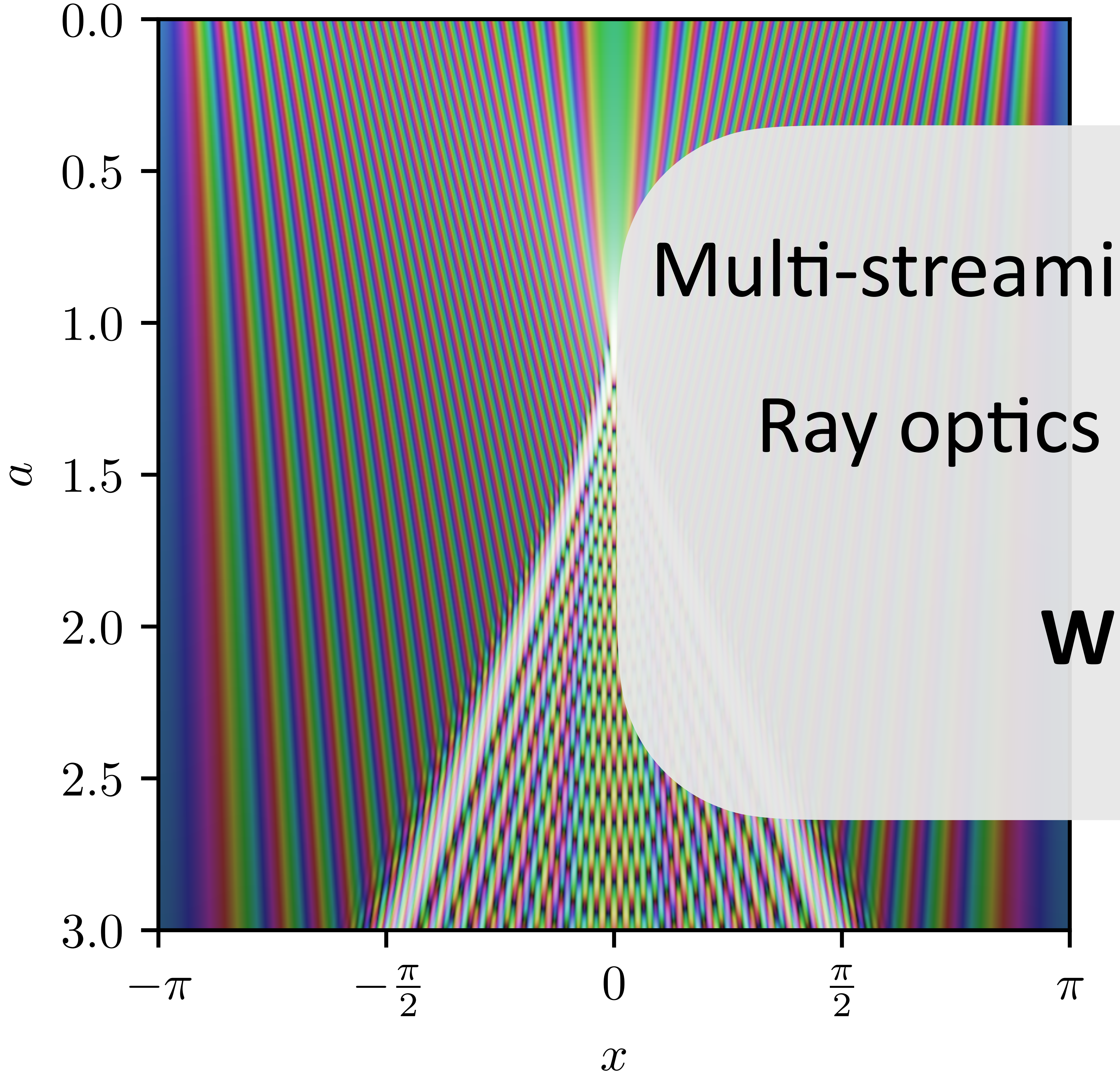
Optics



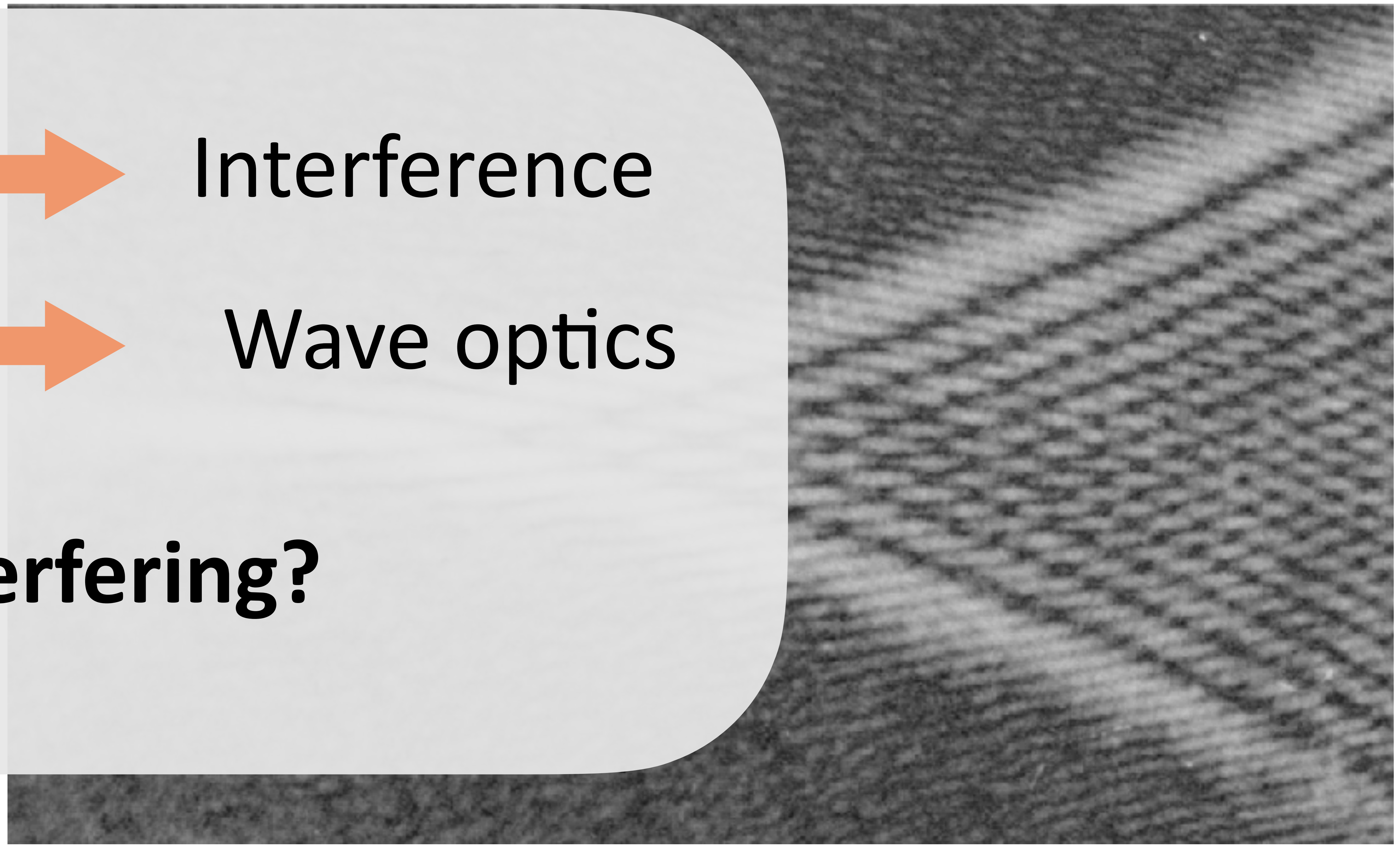
Berry, Nye, Wright `79

Optics Analogy

Dark matter



Optics



Multi-streaming ⟷ Interference

Ray optics ⟷ Wave optics

What is interfering?

Unweaving the Wavefunction

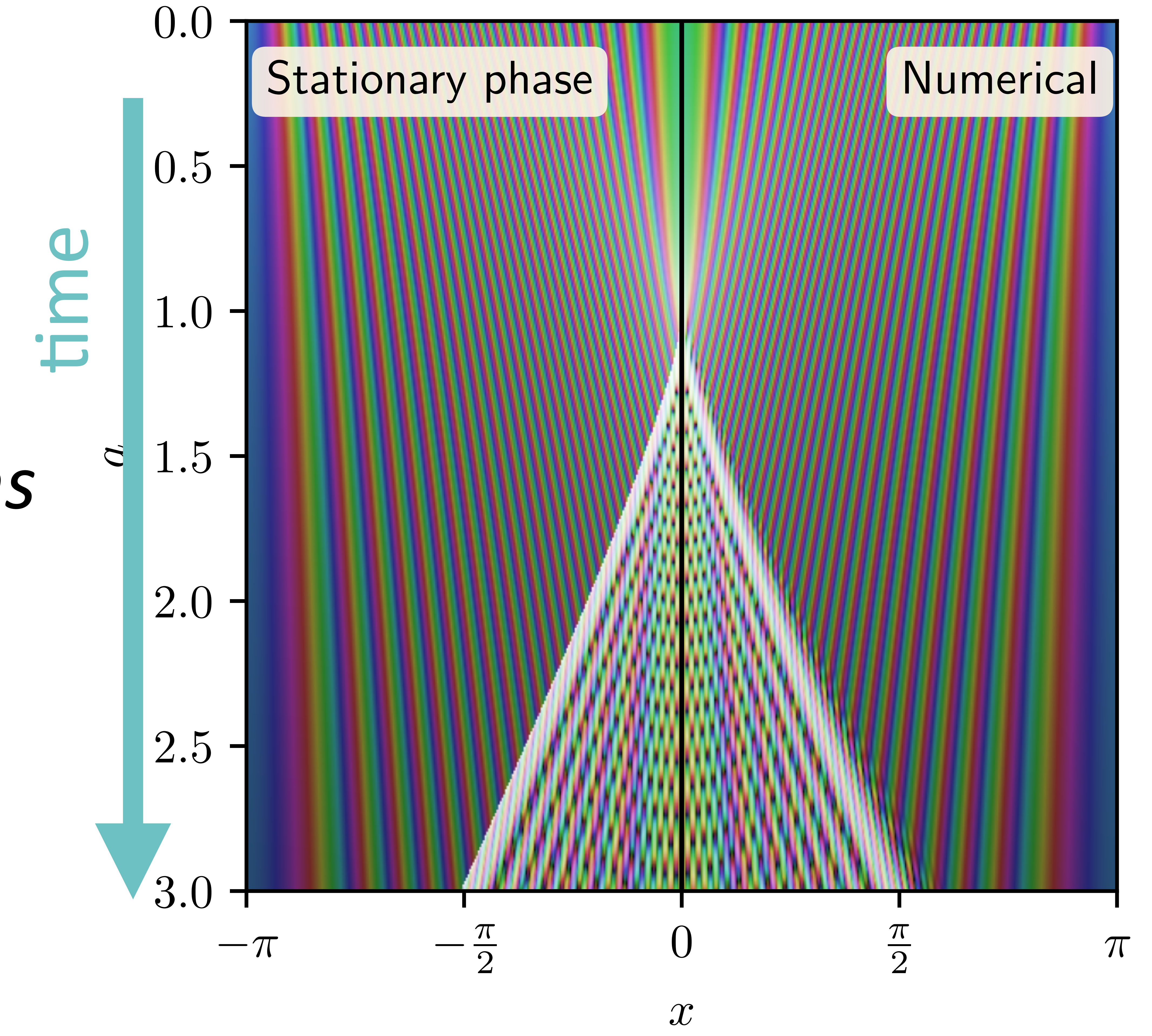
Based on the propagator

$$\psi(x,a) \sim \int \mathrm{d}q \; \underbrace{K_0(q;x,a)\psi^{(\mathrm{ini})}(q)}_{\exp\left[\frac{i}{\hbar}\zeta(q;x,a)\right]}$$

- $\zeta(q;x,a)$ contains *action* & *initial conditions*
- $K(q;x,a)$ transition amplitude
- $\hbar$ small $\rightarrow$ integrand oscillatory

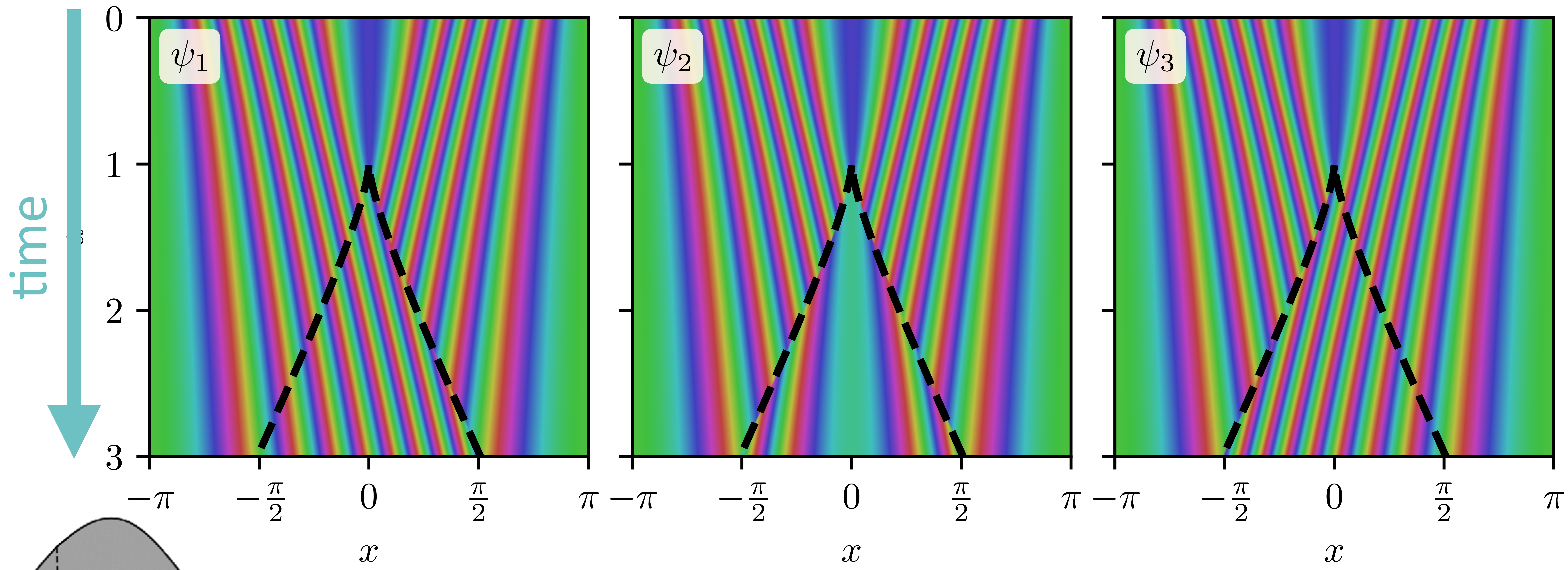
Stationary Phase Approximation

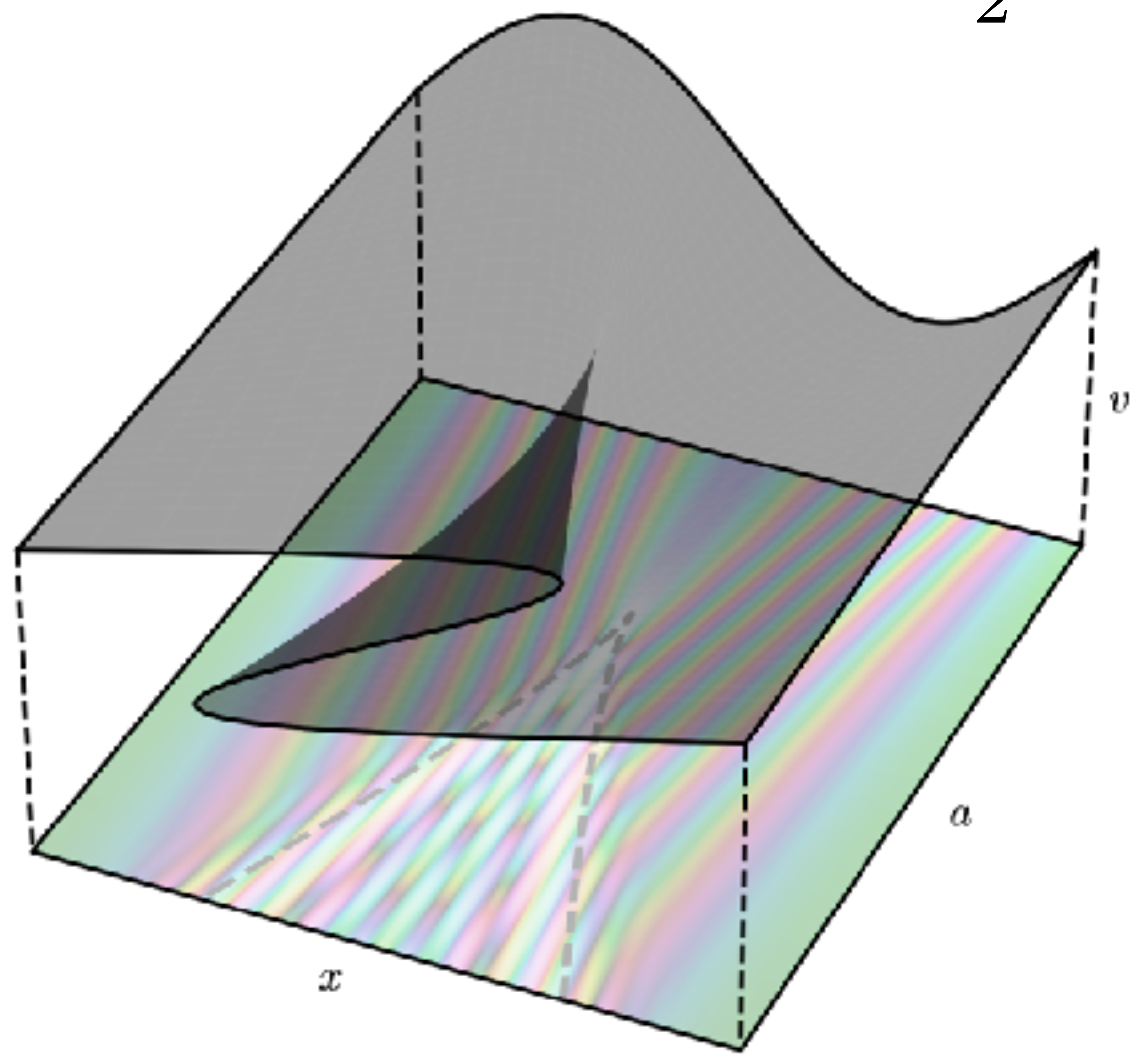
q where $\zeta'(q) = 0$ dominate integral



Gough & Uhlemann 2022

Stream Wavefunctions



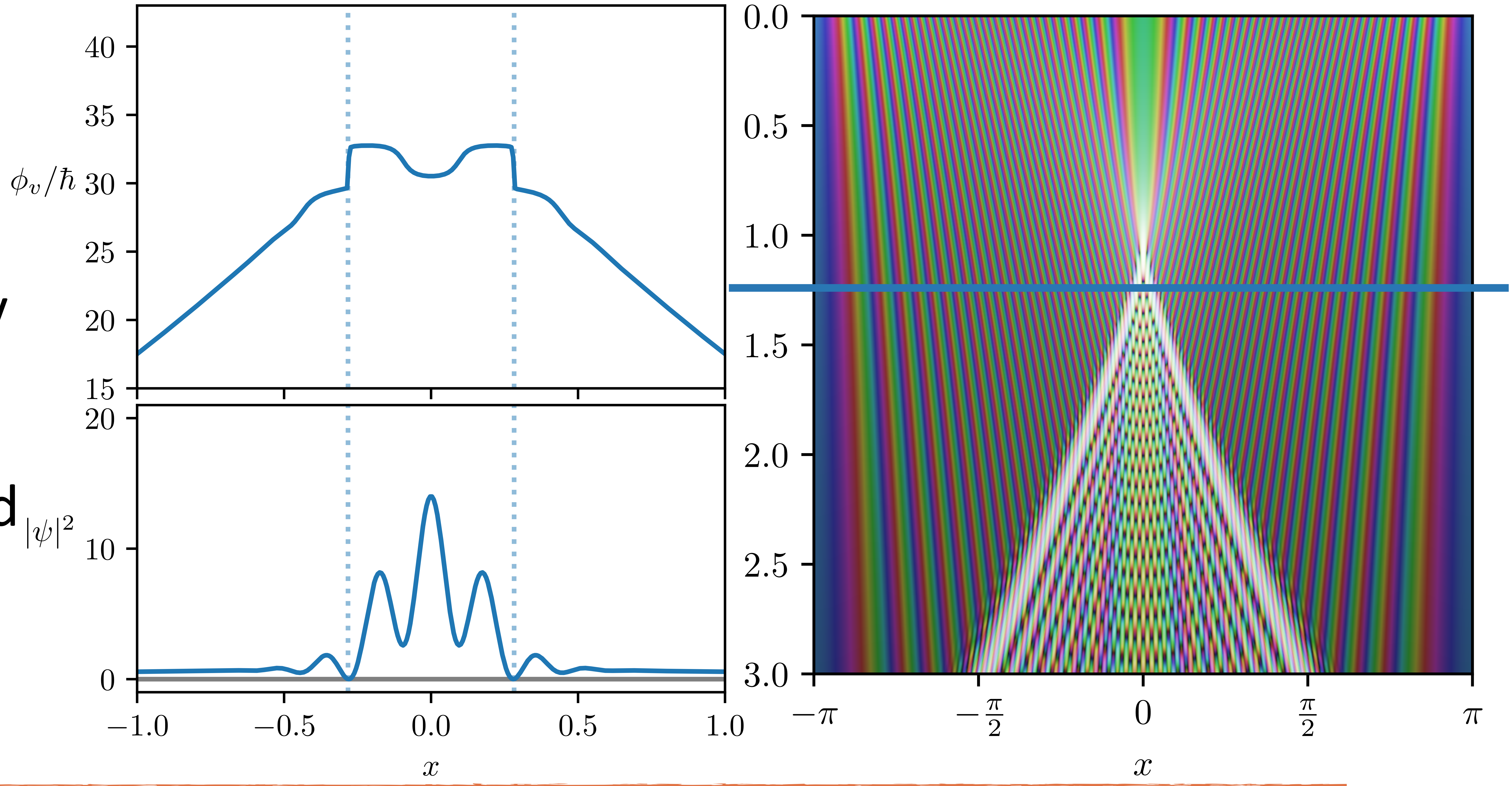


- The classical trajectories interfere!
- Recover classical information from just interference, no need for phase space!

Gough & Uhlemann 2022

Non-Potential Velocity

- Phase jumps correspond to zeros in the density
- ψ encodes information beyond a perfect fluid!



Get effect of stream averaging without explicit dissection of streams!

Gough & Uhlemann 2022

Non-Potential Velocity

- Velocity dispersion from ψ derivatives

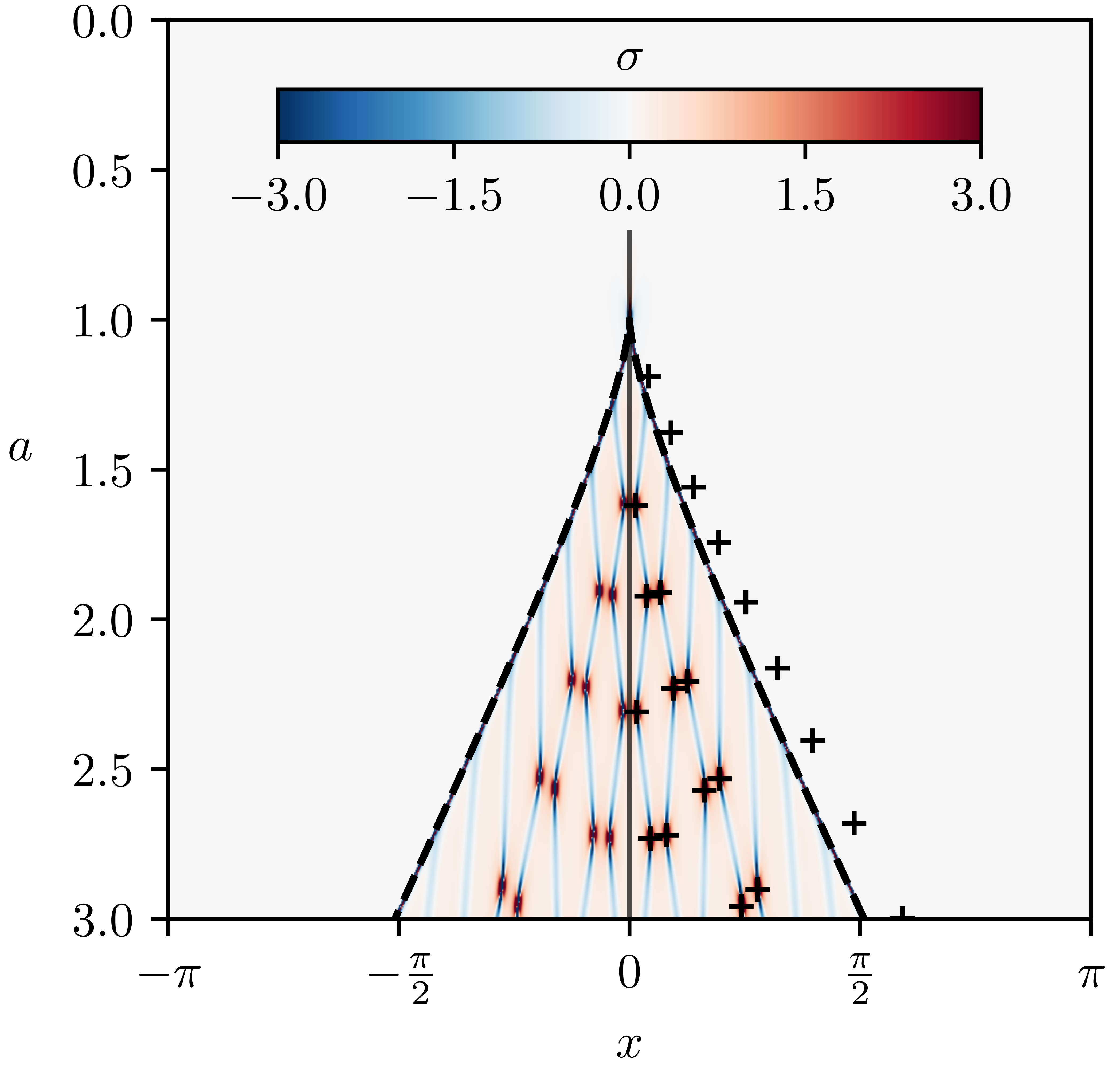
$$\sigma = -\frac{\hbar}{4}\nabla^2 \ln \rho$$

- sourced by phase jumps & density zeros
- Oscillatory part of ψ goes beyond fluid

$$\psi \approx \psi_{\rm avg} \times \psi_{\rm hidden}$$

Fluid part

Oscillatory



Gough & Uhlemann 2022

Interference Features

What phenomena do we see in space-time as observables?

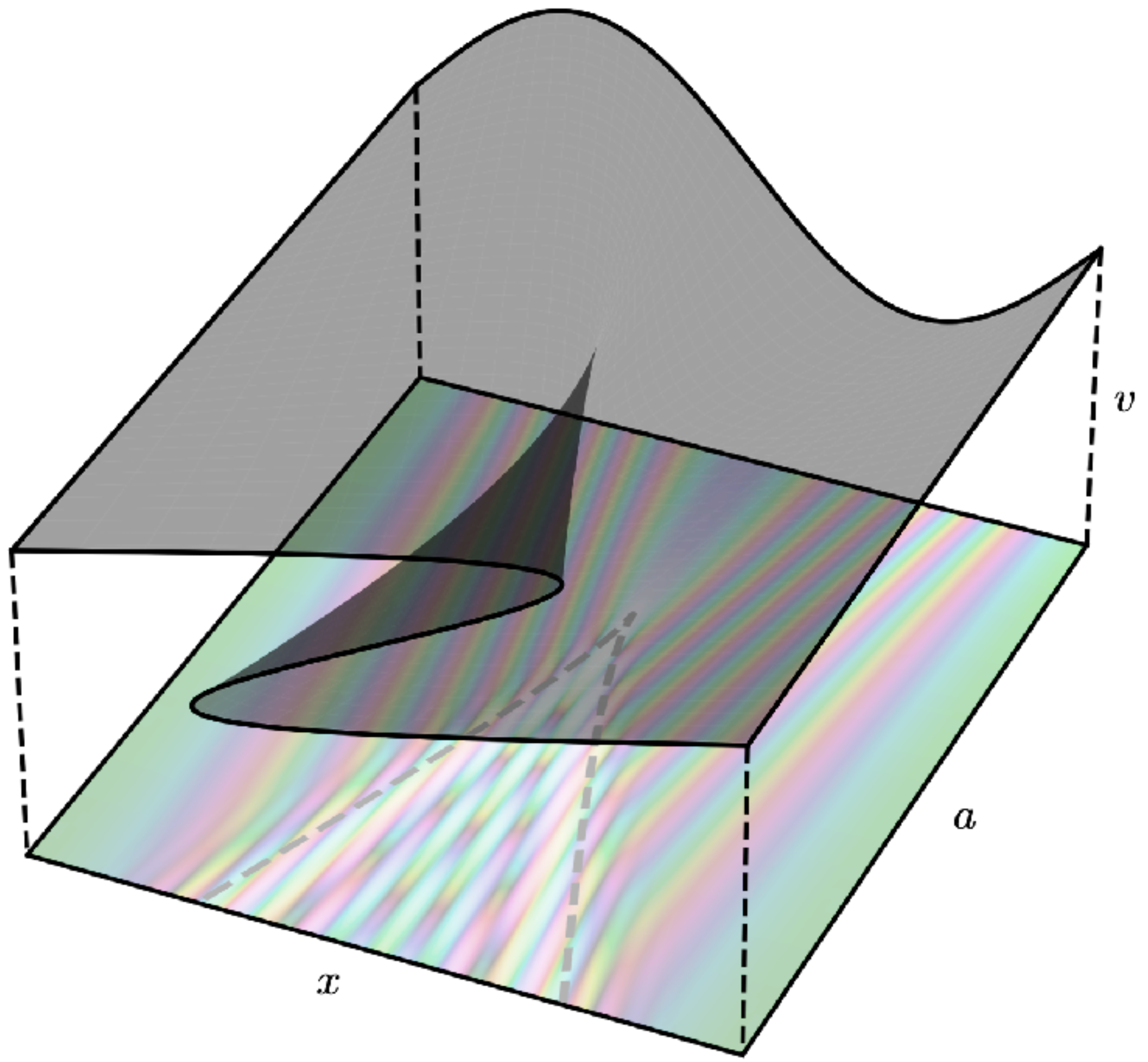
- classically, multiple fluid streams

  → not just fluid density & velocity

  → velocity no longer "potential"

Multi-streaming ↔ Interference

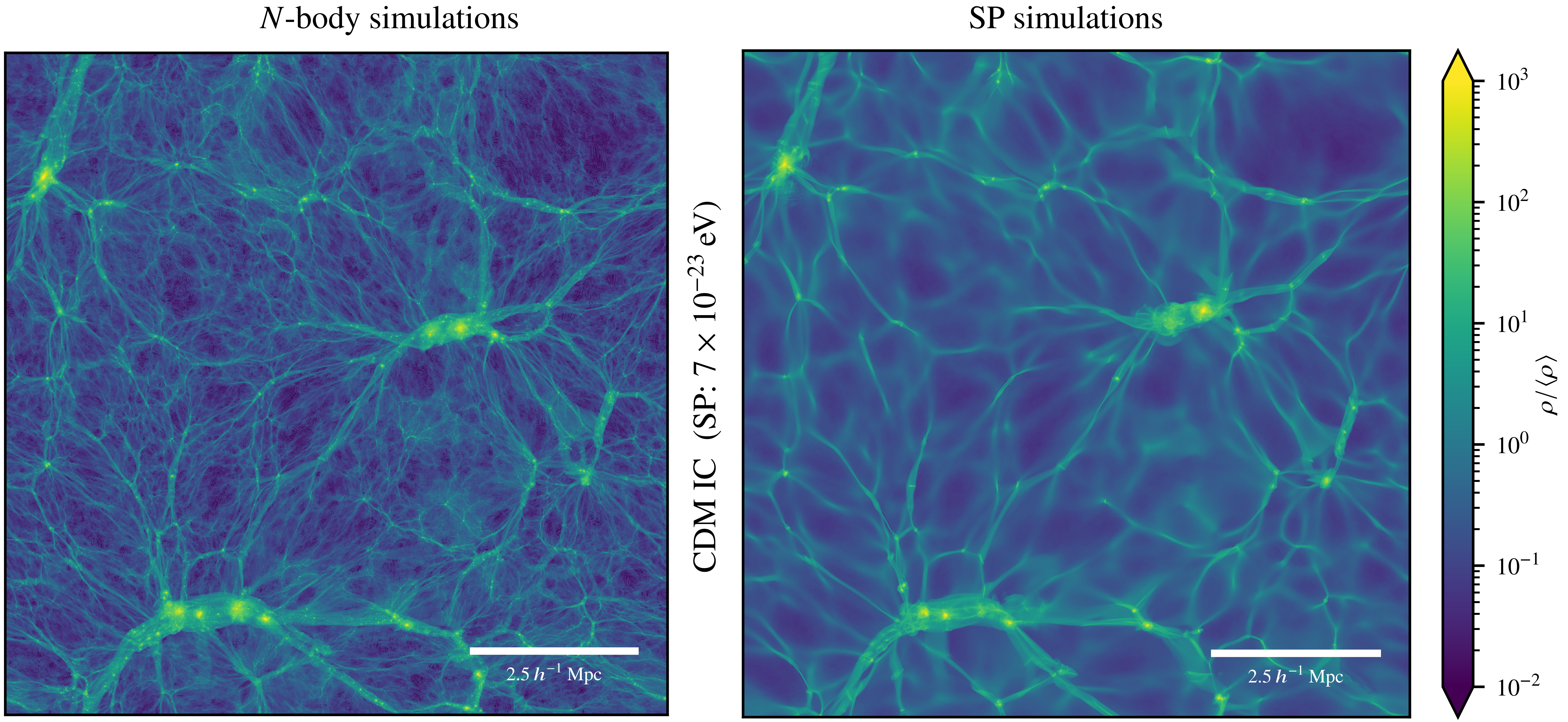
isolate non-potential velocity from interference



Gough & Uhlemann 2022

Cold vs. Wave Dark Matter

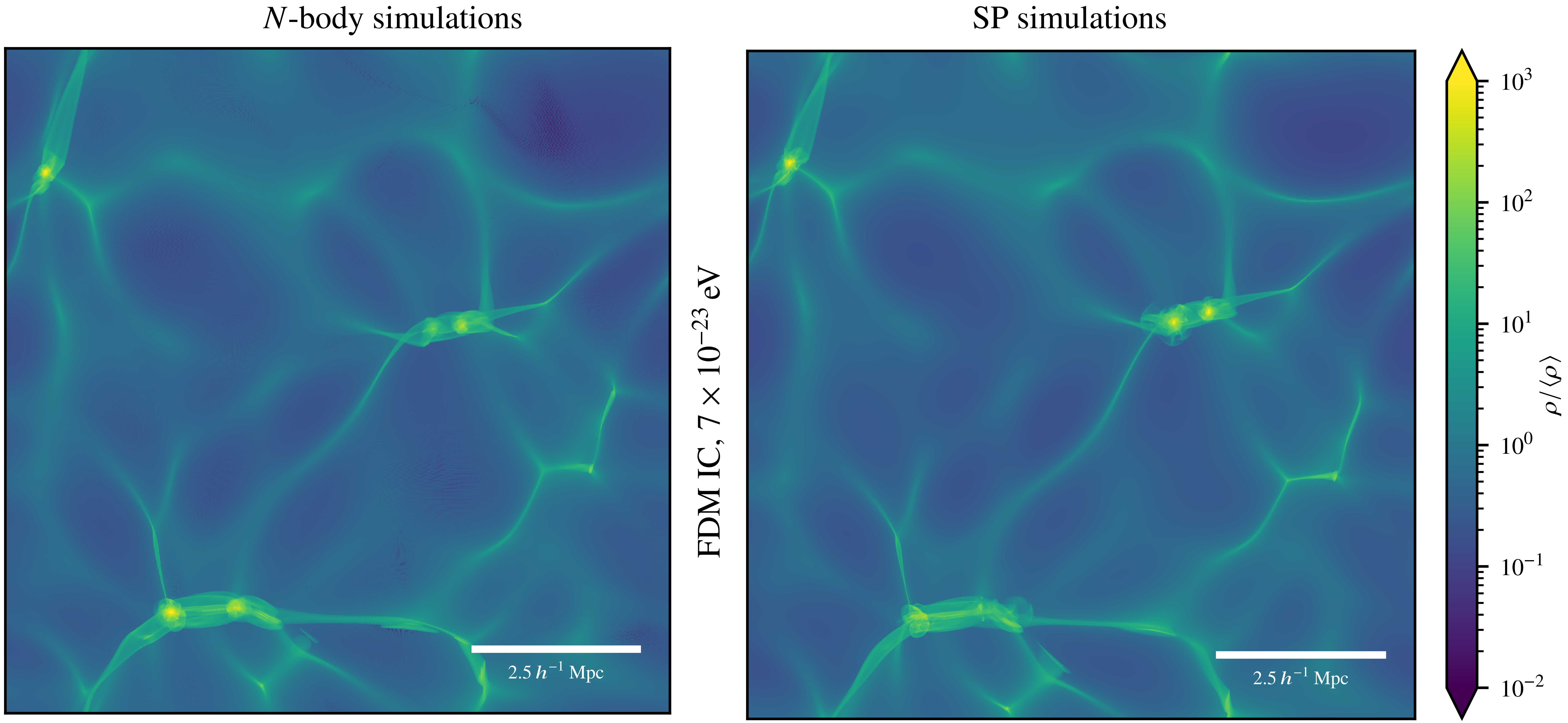
May & Springel `22



astrophysical imprints: Hui, Ostriker, Tremaine & Witten ‘17, Hui ‘21

Cold vs. Wave Dark Matter

May & Springel `22



(hack)
warmish
dark matter

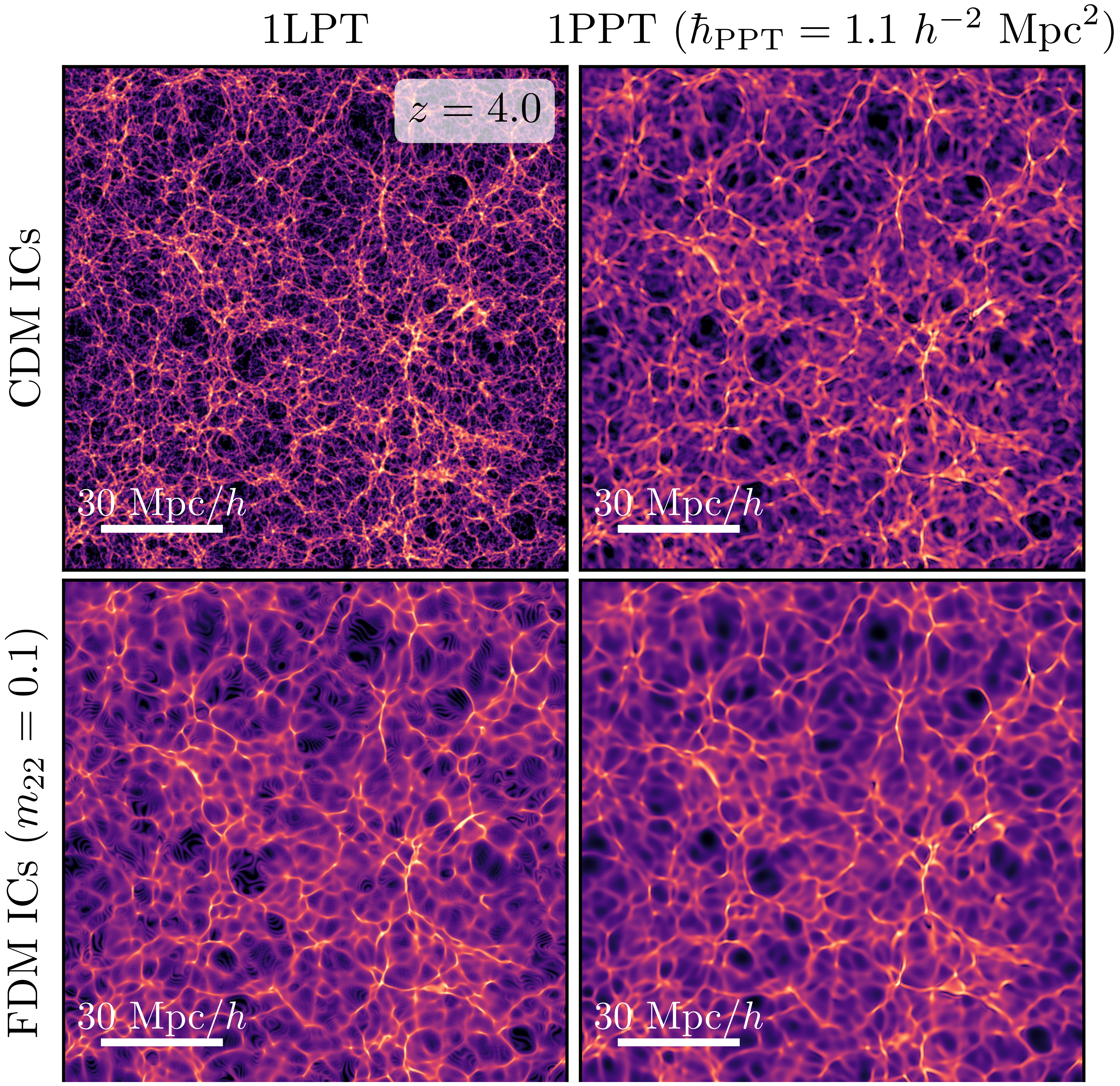
astrophysical imprints: Hui, Ostriker, Tremaine & Witten '17, Hui '21

Cold vs. Wave Dark Matter

cold dark matter

(hack) warmish dark matter

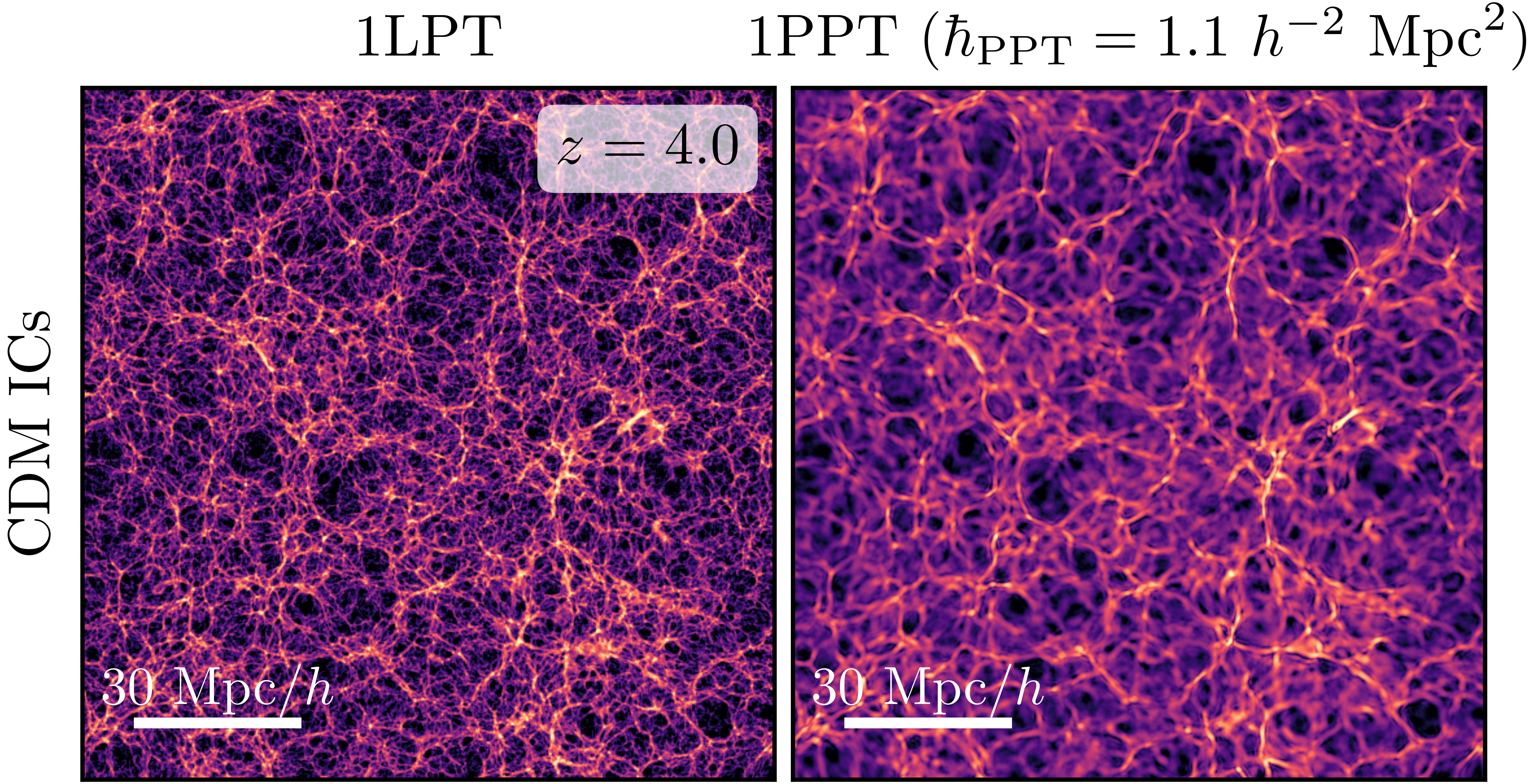
Dome et al. 2022



Gough & CU 2024

wave dark matter

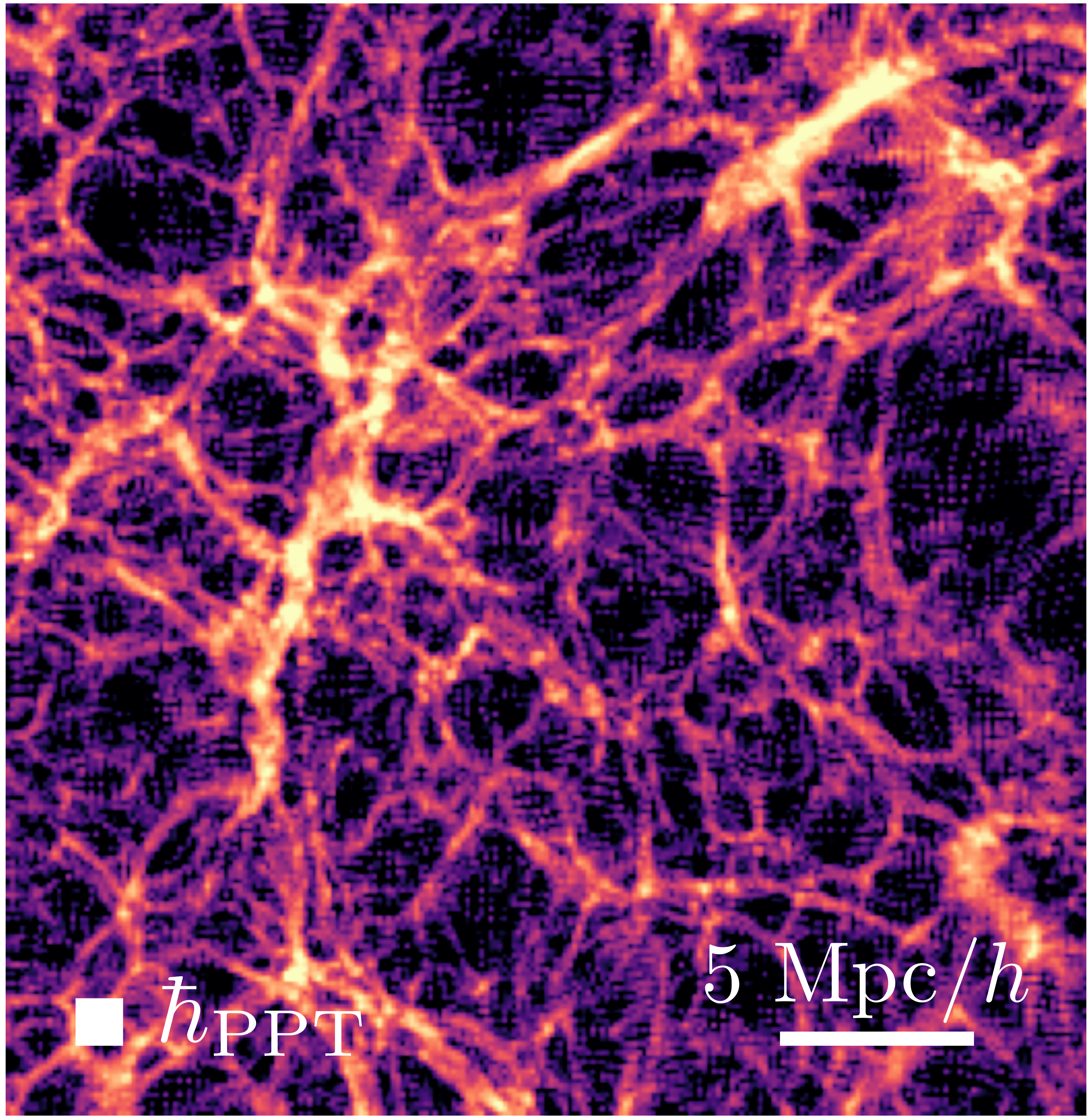
How Classical is Fuzzy DM?



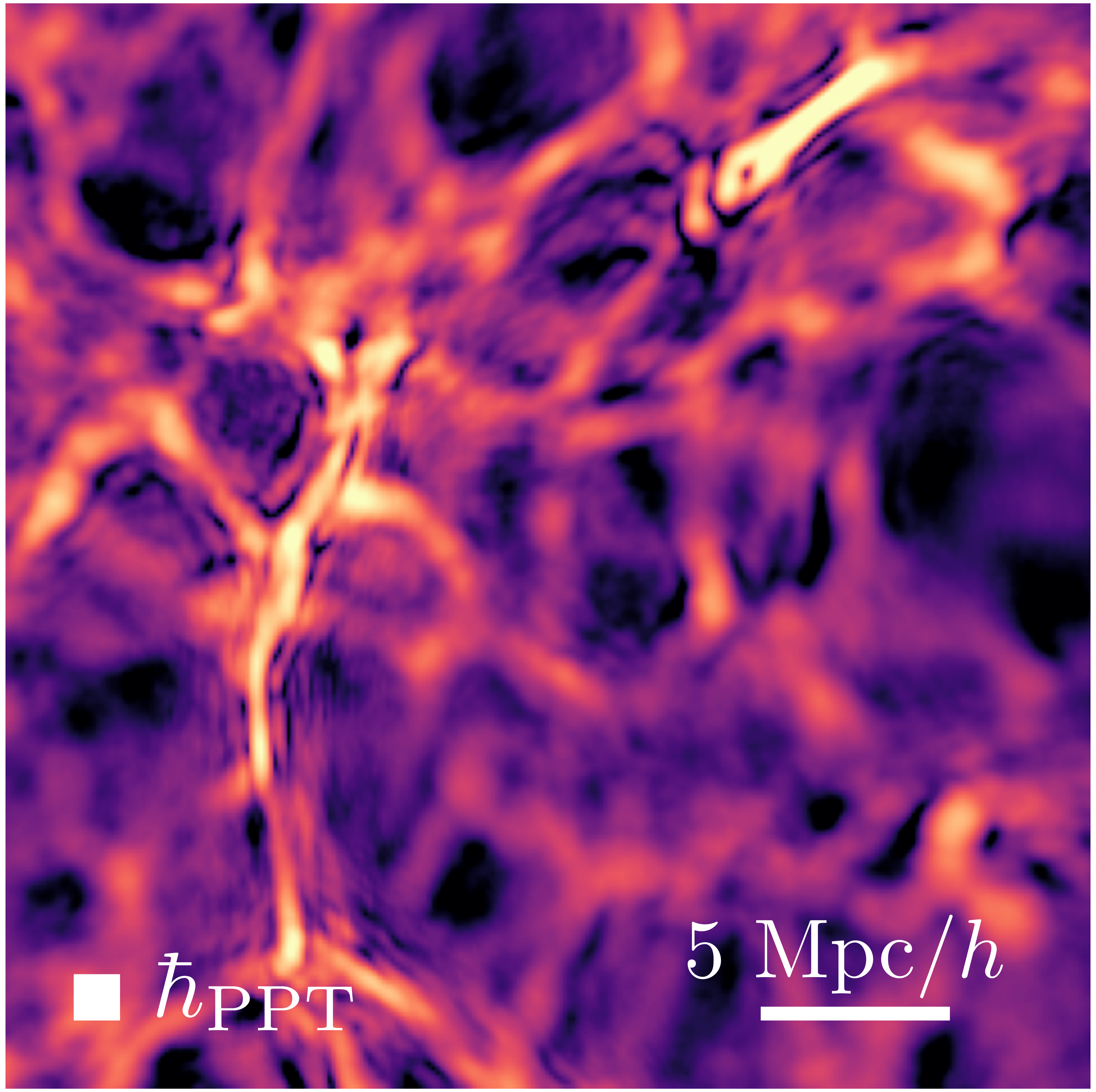
Gough et al. in prep

How Classical is Fuzzy DM?

LPT CDM

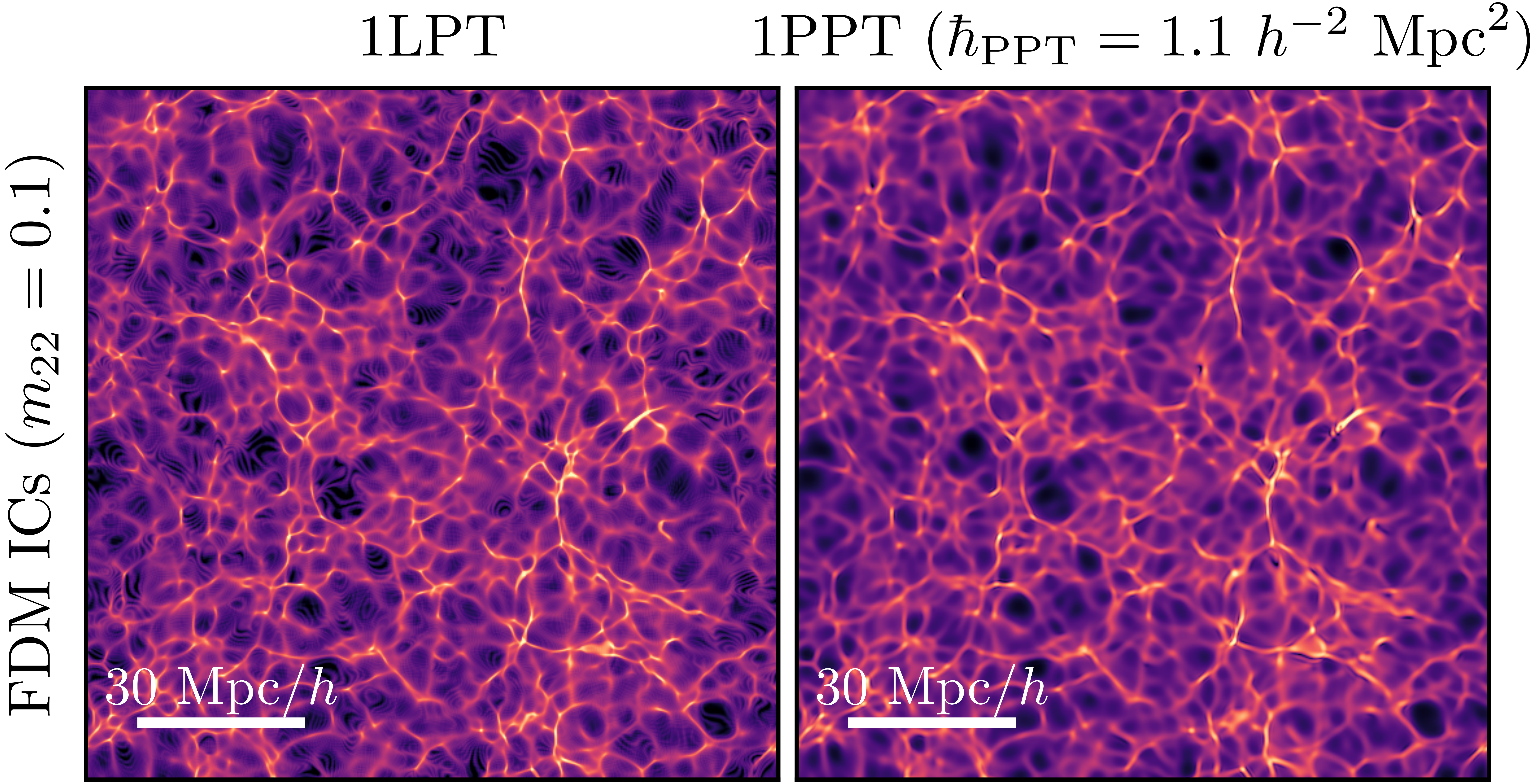


PPT CDM



Gough & Uhlemann, OJA 2024

How Classical is Fuzzy DM?

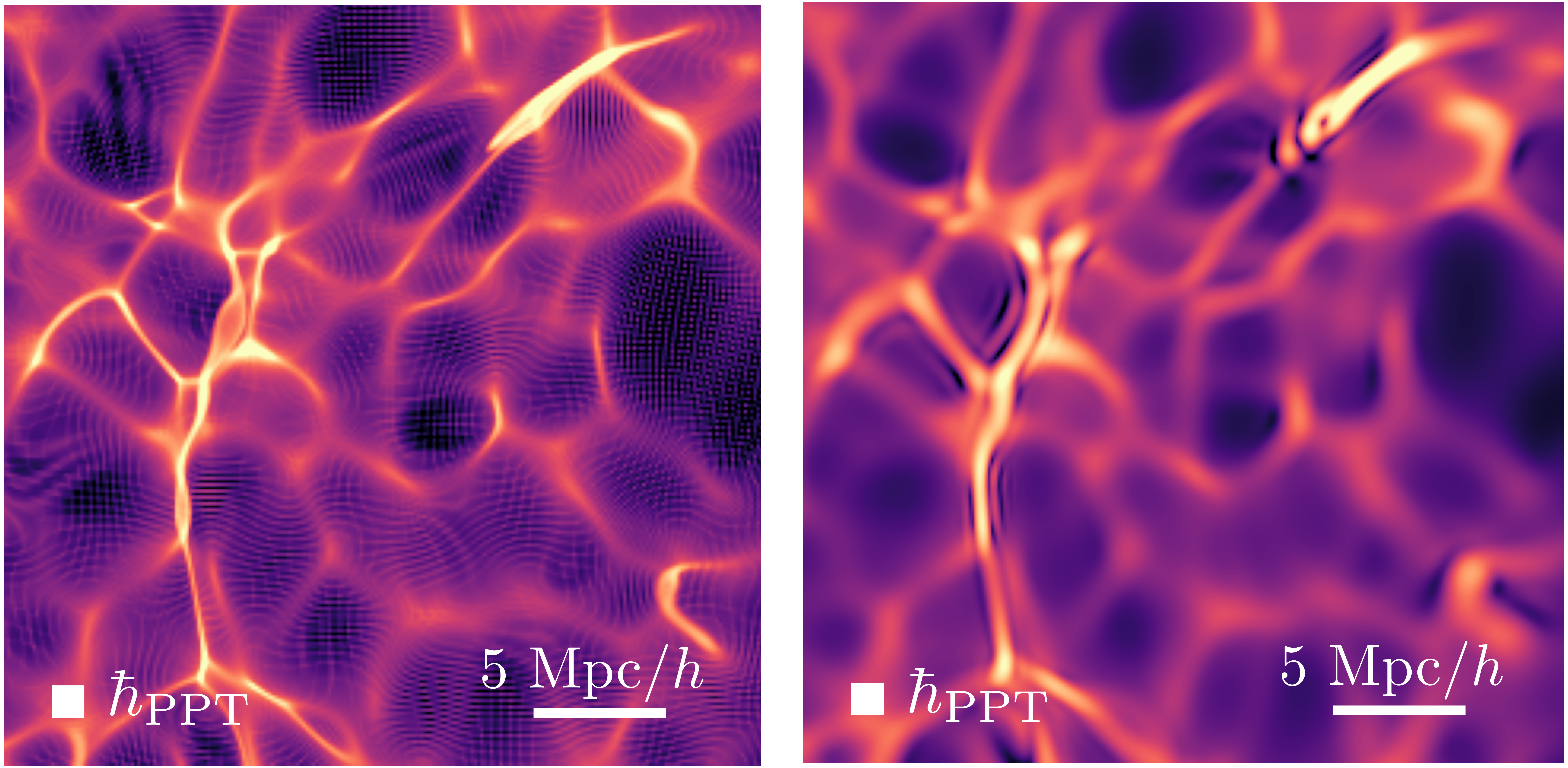


"classical" Fuzzy DM = FDM ICs + CDM dynamics - Dome et al. 2022

Gough & Uhlemann, OJA 2024

How Classical is Fuzzy DM?

LPT FDM PPT FDM



“classical” Fuzzy DM = FDM ICs + CDM dynamics - Dome et al. 2022

Gough & Uhlemann, OJA 2024

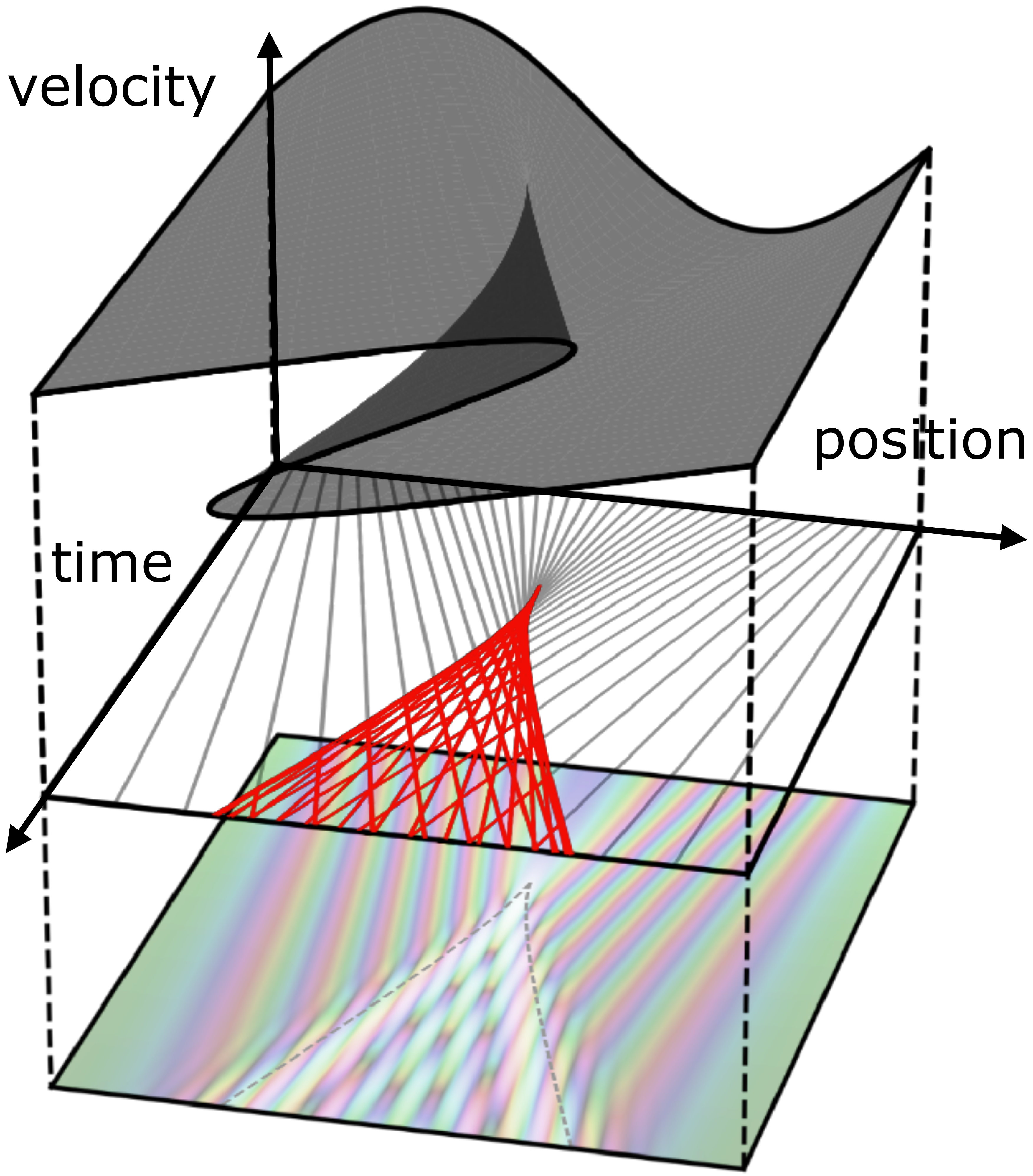
Conclusion: The Sky from Ψ

A new layer of Large-Scale Structure

phase space
high dimensions

particle-based
resolution loss

perturbative fluid
limited physics ✗

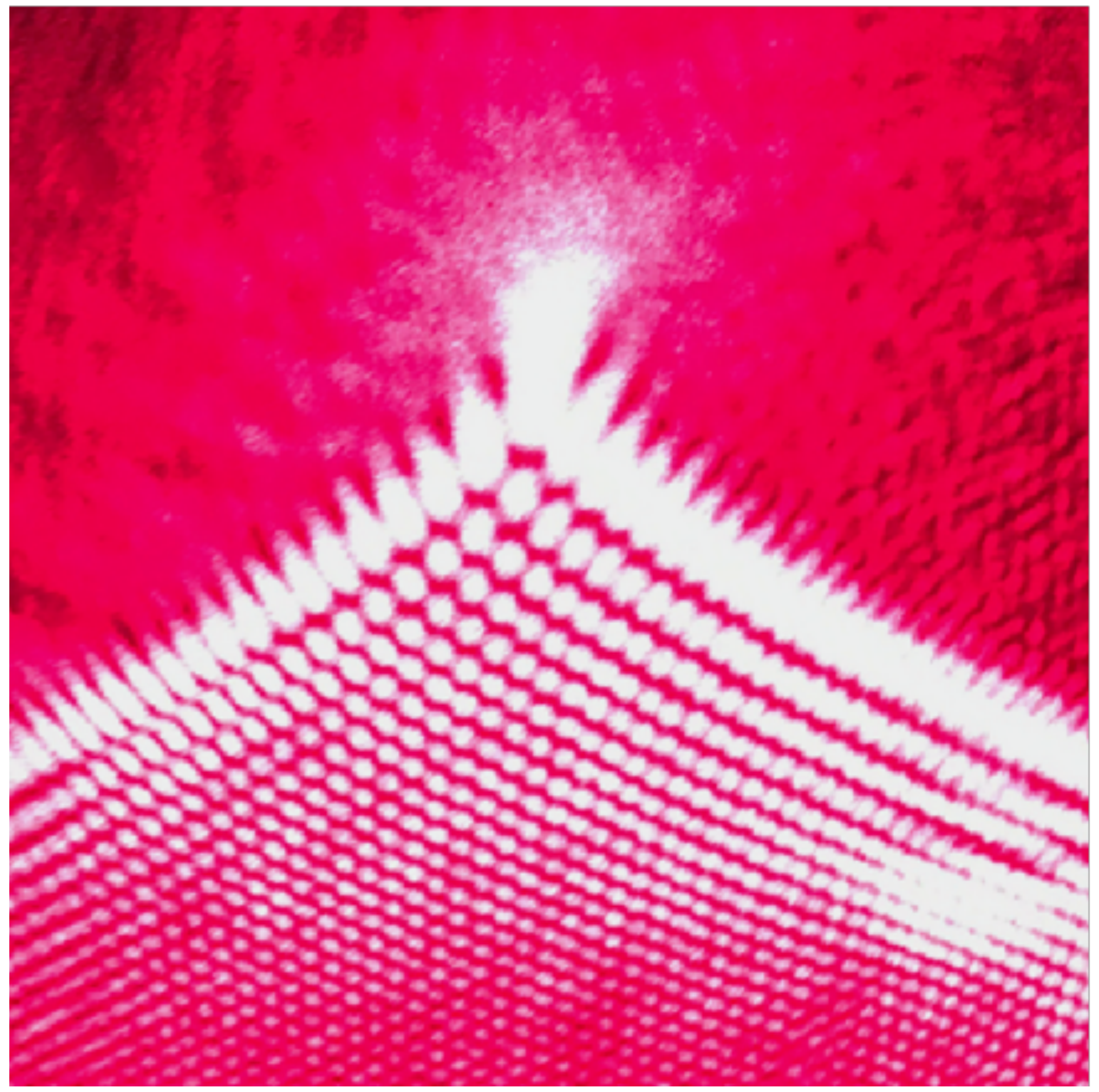


wave space
full physics
half dimensions

map-level predictions
cold dark matter
↑ small ħ/m
wave dark matter

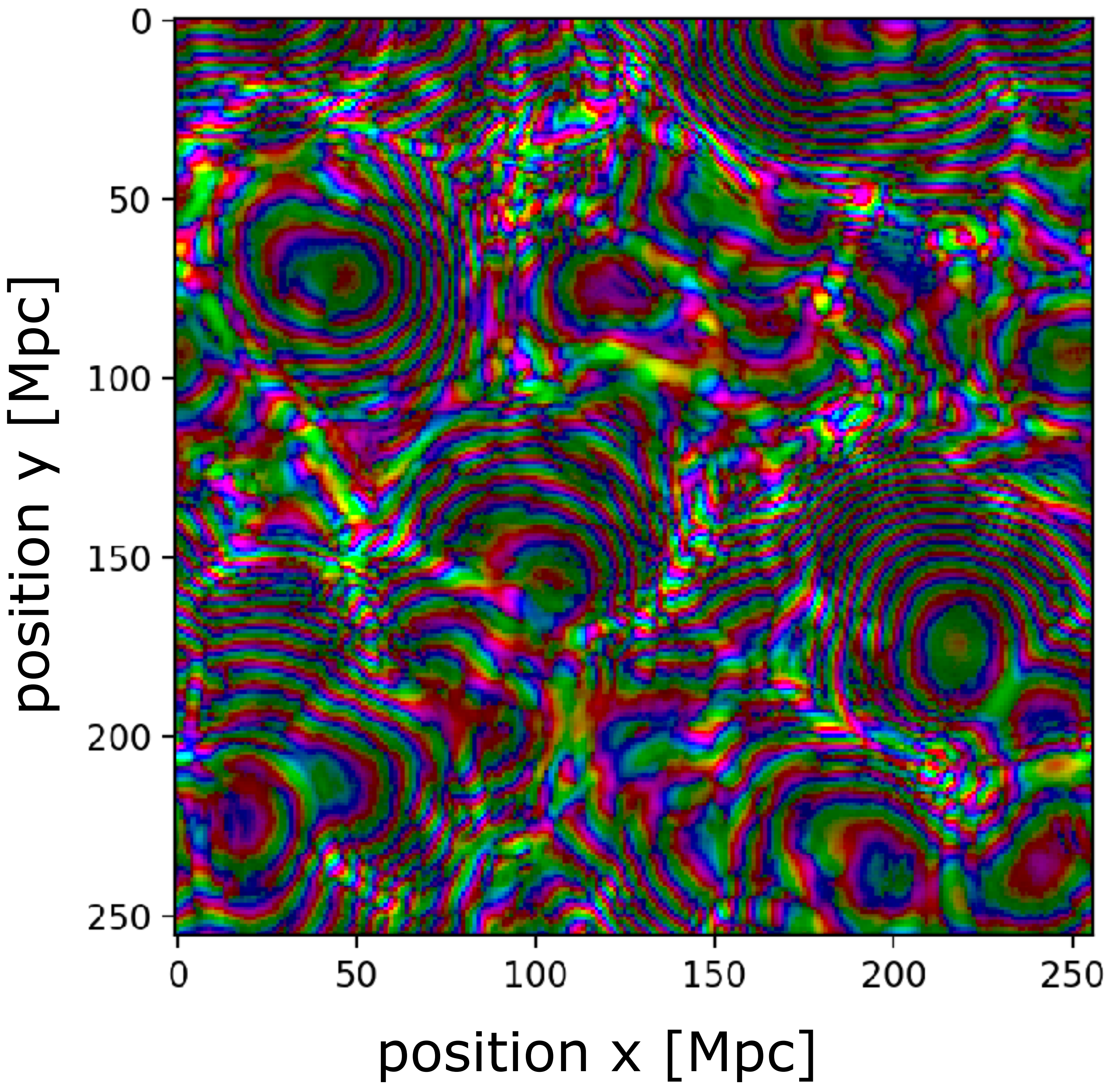
Complexity in a Wavefunction

Diffraction Optics



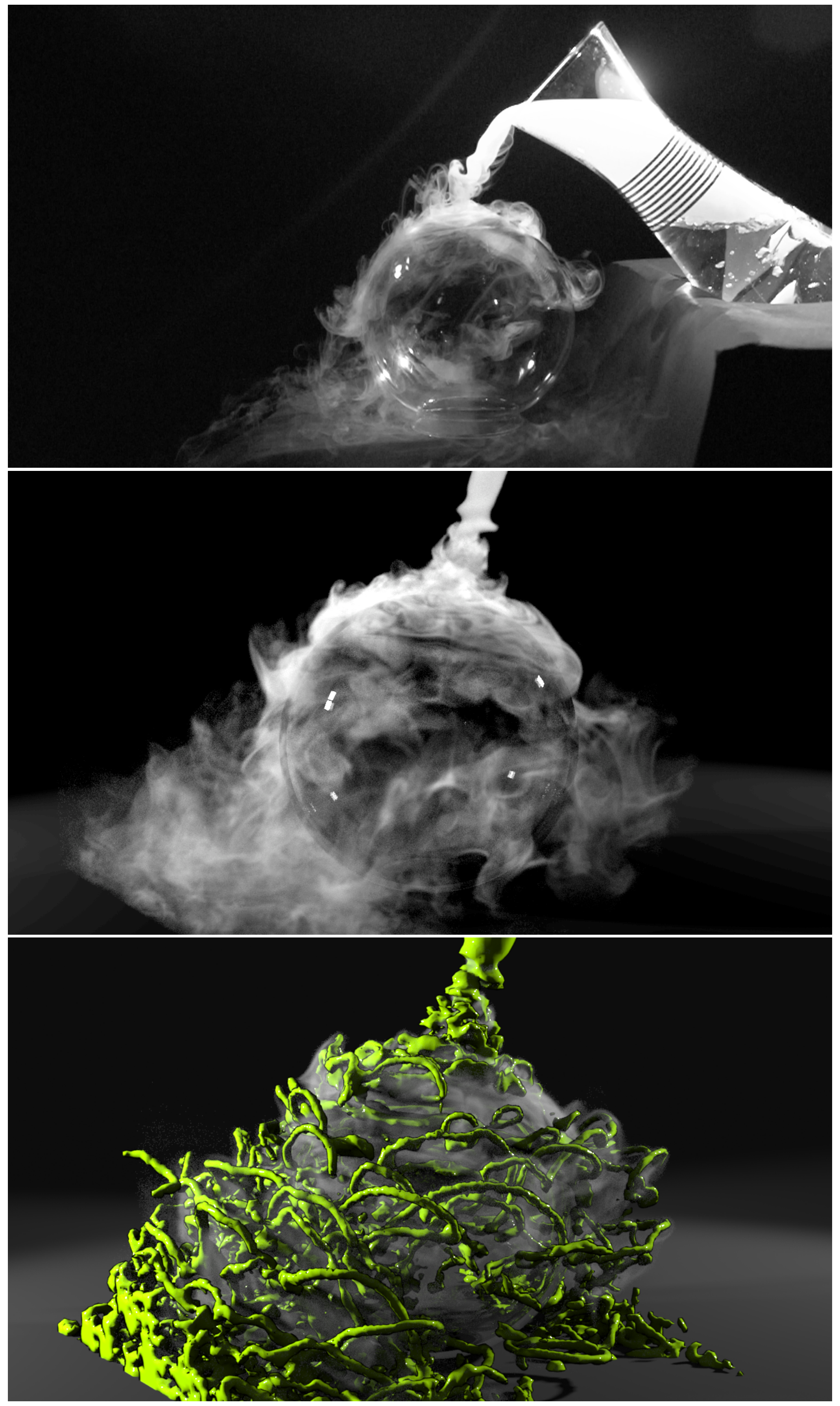
Cusp caustic from
laser droplet diffraction
Wikimedia: Dan Piponi

Cosmic Web



by Oliver Hahn

Vorticity Tracking



Schrödinger's Smoke

Albert Chern
Caltech
Felix Knöppel
TU Berlin
Ulrich Pinkall
TU Berlin
Peter Schröder
Caltech
Steffen Weißmann
Google Inc.

Figure 1: *Comparing experiment (dry ice vapor, top) with ISF simulation (middle), followed by a visualization of the underlying wave function ψ. Vorticity is concentrated within the green region.*

Conclusion

Making Dark Matter waves: cosmic web & wave dark matter

Challenge: wave vs. cold dark matter & large-scale structure

large-scale cosmic web skeleton + bound structures

Wave dark matter as candidate & tool

candidate: (ultra-)light particles described by wavefunction

tool: semiclassical limit

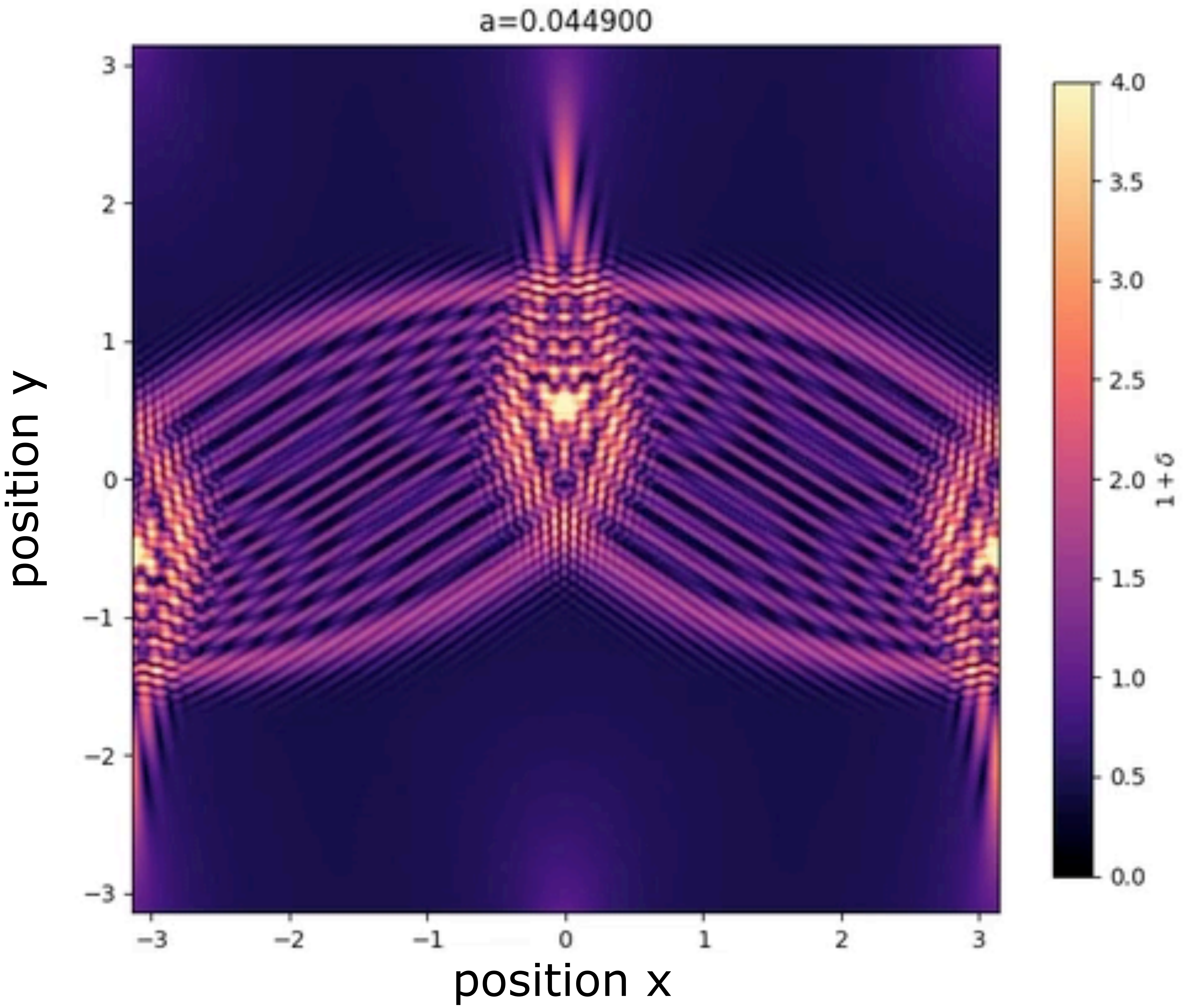
numerical: Schrödinger-Poisson, analytical: Schrödinger + eff. potential

phenomena: classical multi-stream $\rightleftarrows$ wave interference

2D Phased Wave Example

Density

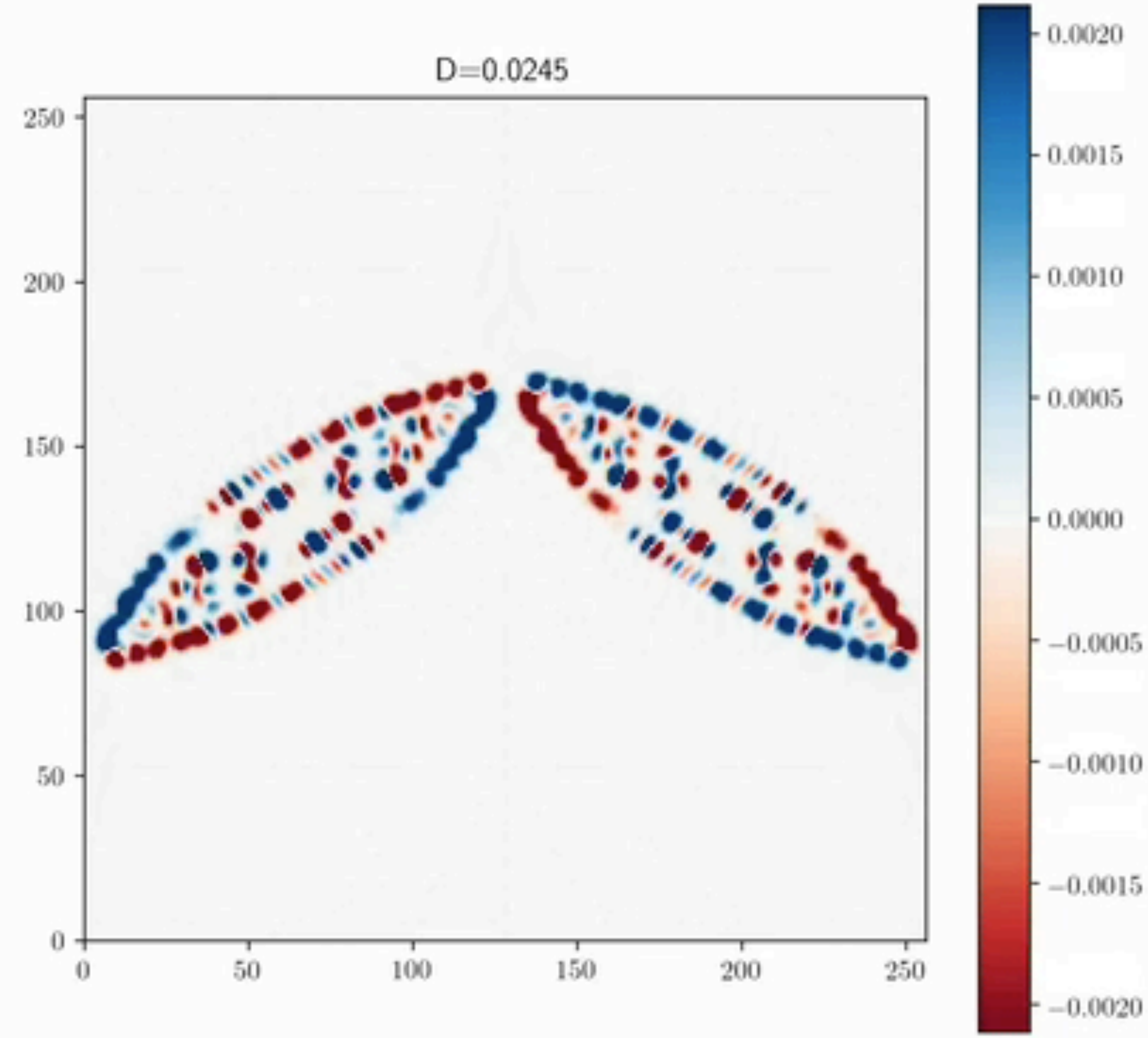
$$1 + \delta(\boldsymbol{x}, a) = |\psi|^2$$



***CU**, Rampf, Gosenca & Hahn 18*

Multi-Stream Regime

Vorticity from phase jumps $v = \nabla\phi_v$ but $\nabla \times v \neq 0$

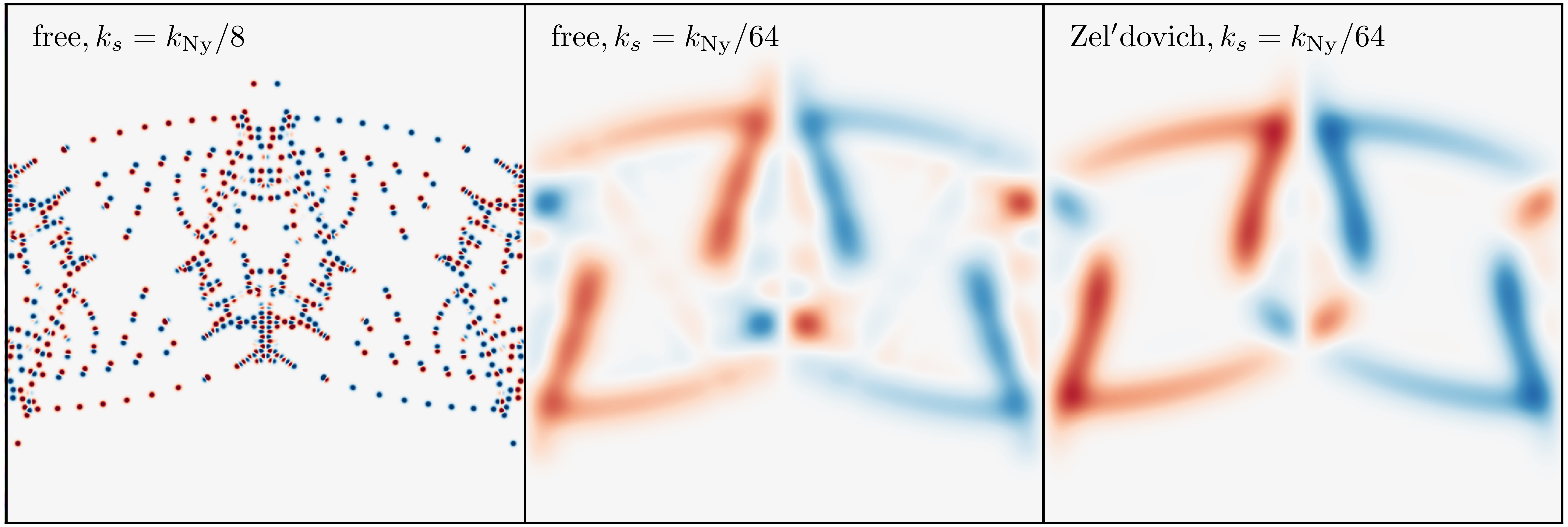


***CU**, Rampf, Gosenca & Hahn 18*

Multi-Stream Regime

Vorticity

small scales　　　　large scales



quantised　　　　classical appearance

analog to Schrödinger-Poisson vortices
2D: Kopp++ ’17, 3D: Hui++ ’20

***CU**, Rampf, Gosenca & Hahn 18*

Key Idea

Semiclassical Dynamics

correspondence: classical $\rightleftarrows$ quantum

$$f(\boldsymbol{x}, \boldsymbol{p}, t) \simeq f_\hbar[\psi(\boldsymbol{x}, t)](\boldsymbol{p})$$

3+3 dim 3 dim

add coarse-graining $\sigma_x \sigma_p \gtrsim \hbar/2$

$$\bar{f}_W(\boldsymbol{x}, \boldsymbol{p}) = \int \frac{d^3\tilde{x} d^3\tilde{p}}{(\pi\sigma_x\sigma_p)^3} \exp\left[-\frac{(\boldsymbol{x}-\tilde{\boldsymbol{x}})^2}{2\sigma_x^2} - \frac{(\boldsymbol{p}-\tilde{\boldsymbol{p}})^2}{2\sigma_p^2}\right] f_W(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{p}})$$

Semiclassical Dynamics

Interactive Propagation

PT or numerics

$$i\hbar\partial_a\psi = -\frac{\hbar^2}{2}\nabla^2\psi + V_{\text{eff}}(\boldsymbol{x},a)\psi$$

propagator

$$\begin{tikzcd}
\psi(\boldsymbol{x},a) \arrow[d, "\text{phase space}"'] & & \psi^{\text{ini}}(\boldsymbol{q}) \arrow[ll, "V_{\text{eff}}(\boldsymbol{x})"'] \arrow[ll, bend left, "V_{\text{eff}}(\boldsymbol{x},a)"] \\
\bar{f}_{\mathrm{W}}(\boldsymbol{x},\boldsymbol{p},a) \arrow[r] & \text{classical observables} &
\end{tikzcd}$$

Classical Observables

Phase-Space Distribution

coarse-grained Wigner $\bar{f}_W[\psi, \hbar \to 0]$

$$f_\mathrm{W}(\boldsymbol{x}, \boldsymbol{p}) = \int \frac{\mathrm{d}^3 x'}{(2\pi)^3} \exp\left[\frac{-\mathrm{i}\boldsymbol{p} \cdot \boldsymbol{x}'}{a^{3/2}}\right] \psi(\boldsymbol{x} + \tfrac{\hbar}{2}\boldsymbol{x}')\, \bar{\psi}(\boldsymbol{x} - \tfrac{\hbar}{2}\boldsymbol{x}')$$

phase-space info in wave function

Classical Observables

Lagrangian Fluid

compare $\bar{f}_W[\psi, \hbar \to 0]$ to

$$f_{\mathrm{fl}}(\boldsymbol{x},\boldsymbol{p}) = \int \mathrm{d}^3q\, \delta_{\mathrm{D}}^{(3)}\left[\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\xi}(\boldsymbol{q})\right] \delta_{\mathrm{D}}^{(3)}\left[\frac{\boldsymbol{p}}{a^{3/2}} - \boldsymbol{v}^{\mathrm{L}}(\boldsymbol{q})\right]$$

displacement velocity

→ usual Lagrangian PT $\boldsymbol{v}^L(\boldsymbol{q}) = \dot{\boldsymbol{\xi}}(\boldsymbol{q})$

Classical Observables

Lagrangian Fluid

velocity beyond $v^L(\boldsymbol{q}) = \dot{\boldsymbol{\xi}}(\boldsymbol{q})$

$$\boldsymbol{v}(\boldsymbol{q}) = -\boldsymbol{\nabla}\varphi_g^{(\mathrm{ini})} - a\boldsymbol{\nabla}V_{\mathrm{eff}}^{(2)}$$

$$+\frac{a^2}{2}\boldsymbol{\nabla}\nabla V_{\mathrm{eff}}^{(2)} \cdot \nabla\varphi_g^{(\mathrm{ini})}$$

vorticity conserver

Classical Observables

Vorticity Conservation

Eulerian: $\boldsymbol{\nabla}_x \times \boldsymbol{v} = 0$

before shell-crossing

Multi-Stream Regime

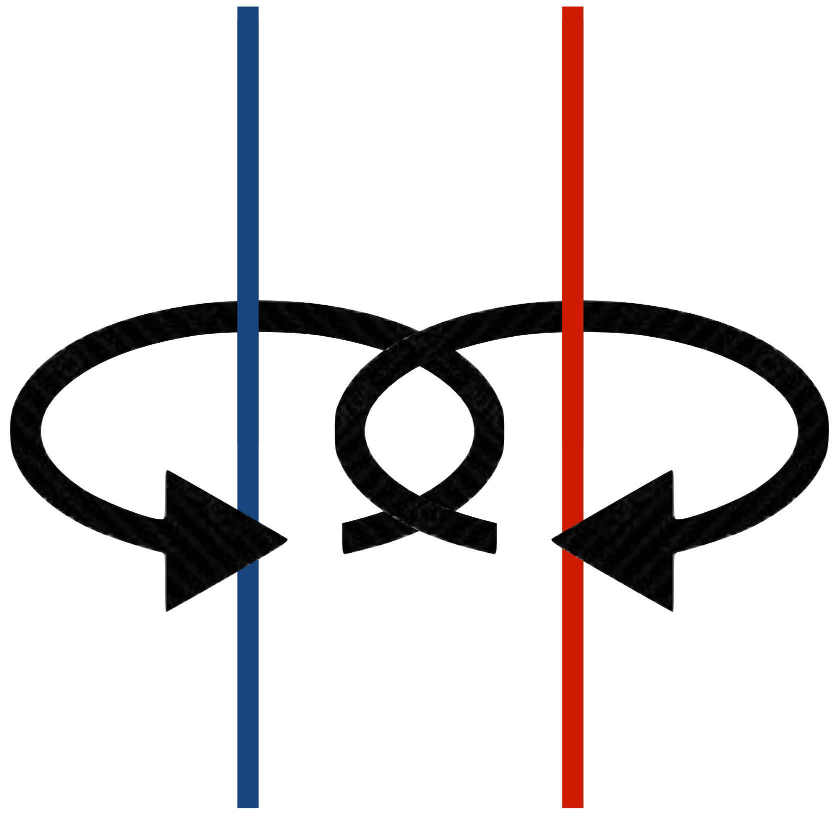
Vorticity

phase jumps → vorticity

$$\psi = \sqrt{\rho}\exp[i\phi_v/\hbar] \qquad \boldsymbol{v} = \frac{i\hbar}{2}\frac{\psi\boldsymbol{\nabla}\bar{\psi} - \bar{\psi}\boldsymbol{\nabla}\psi}{|\psi|^2} = \nabla\phi_v$$

topological defects: rotons

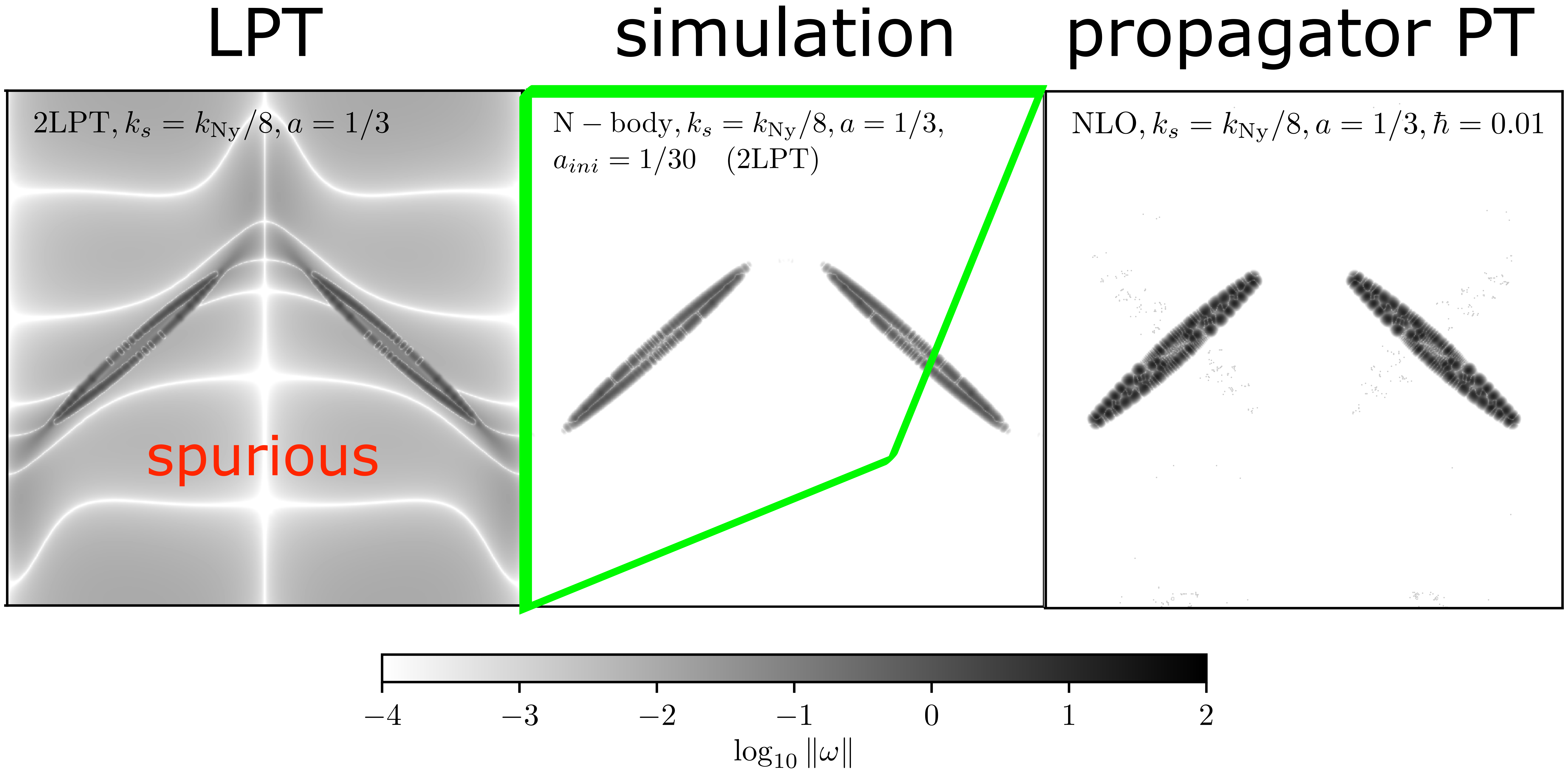
$$\frac{1}{2\pi\hbar}\oint_{C(a)} \boldsymbol{\nabla}\phi_{\rm v}\cdot \mathrm{d}\boldsymbol{x} = n_+ - n_- = 0$$



preserve Kelvin-Helmholtz invariant

Phased Wave Example

Vorticity

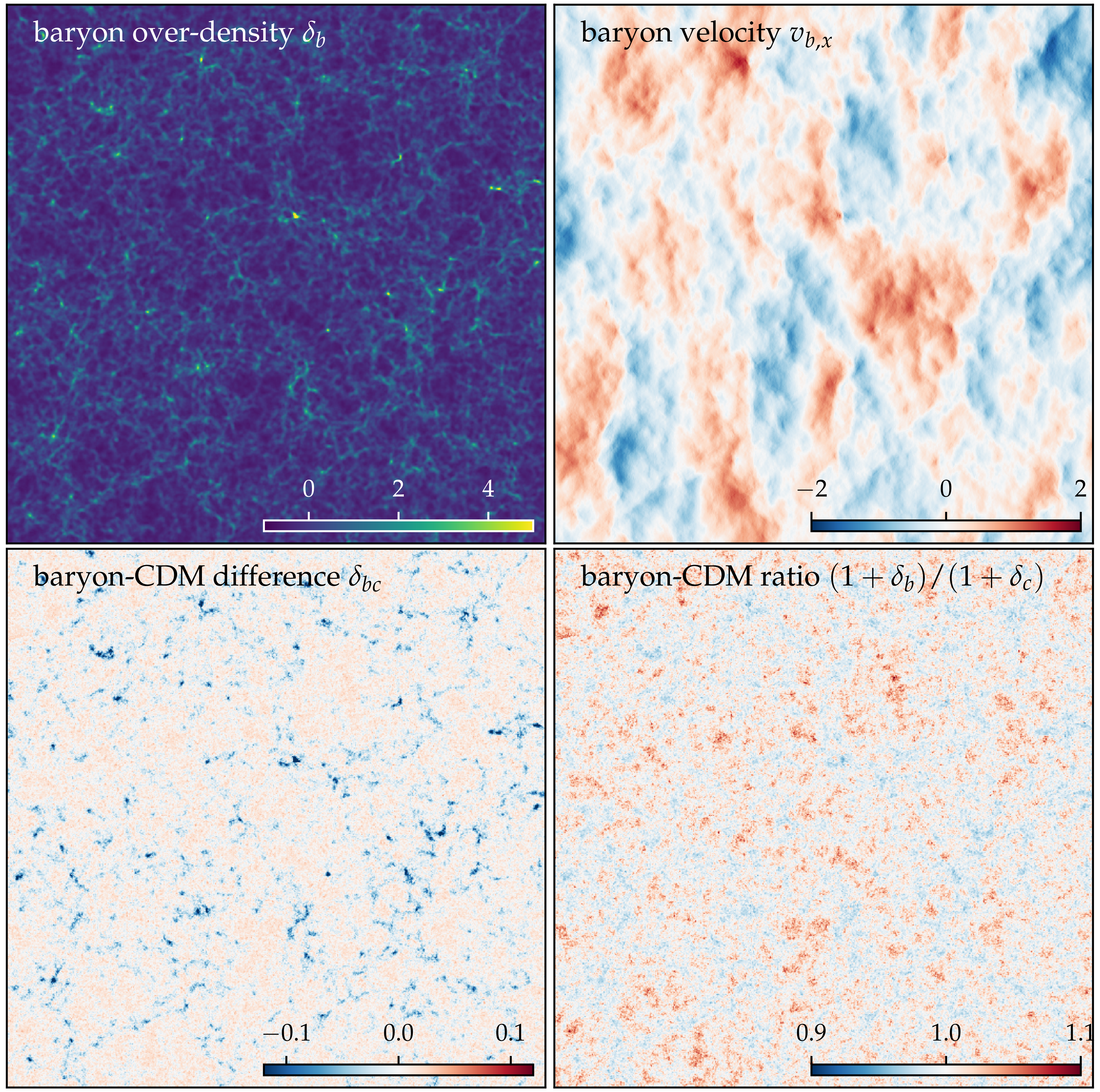


Dark Matter + Baryons: ICs

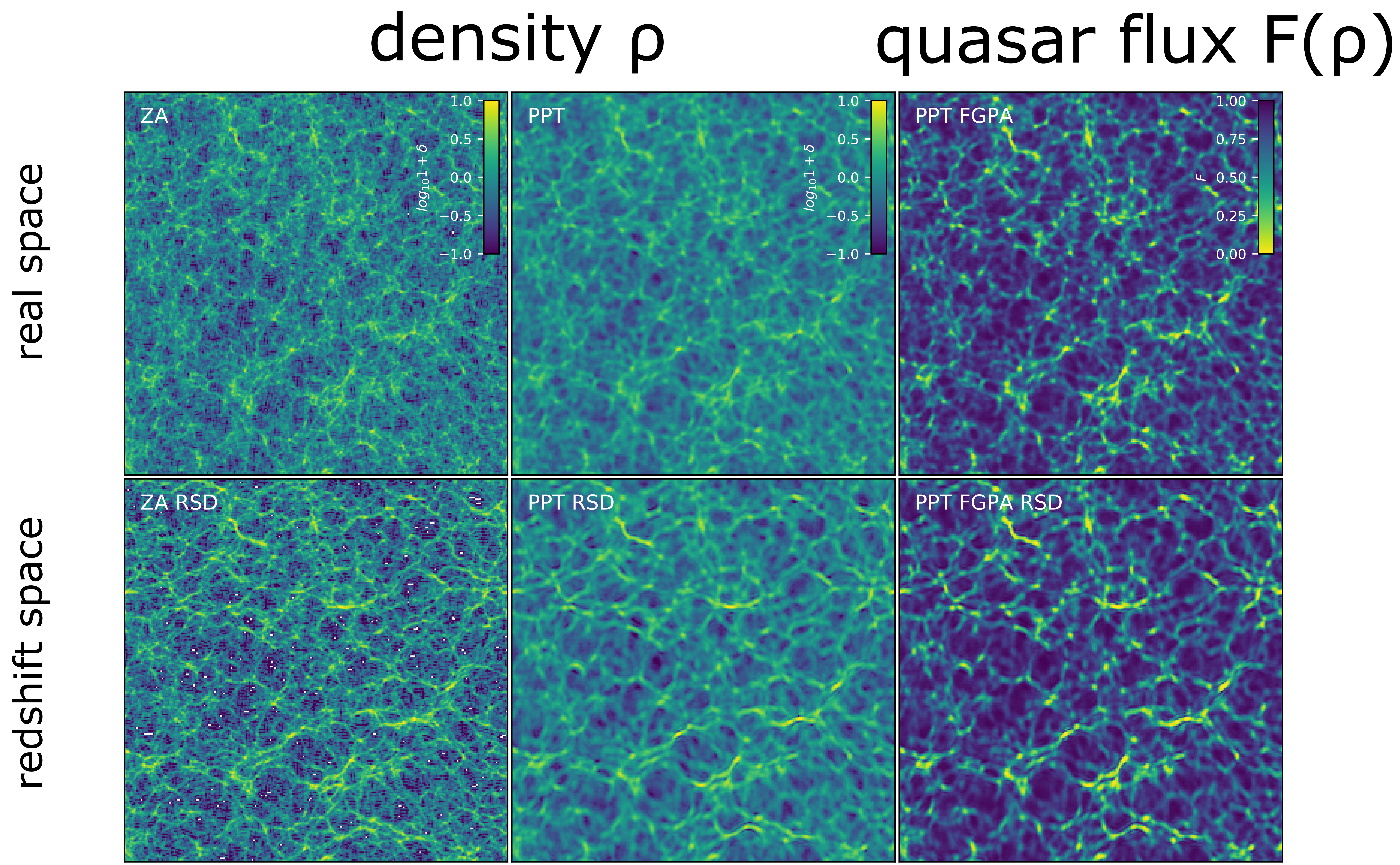
PPT initial conditions for Eulerian codes

evolve one Ψ for each component (valid for non-decaying modes)

Rampf, **CU**, Hahn ’20
Hahn, Rampf, **CU** `20

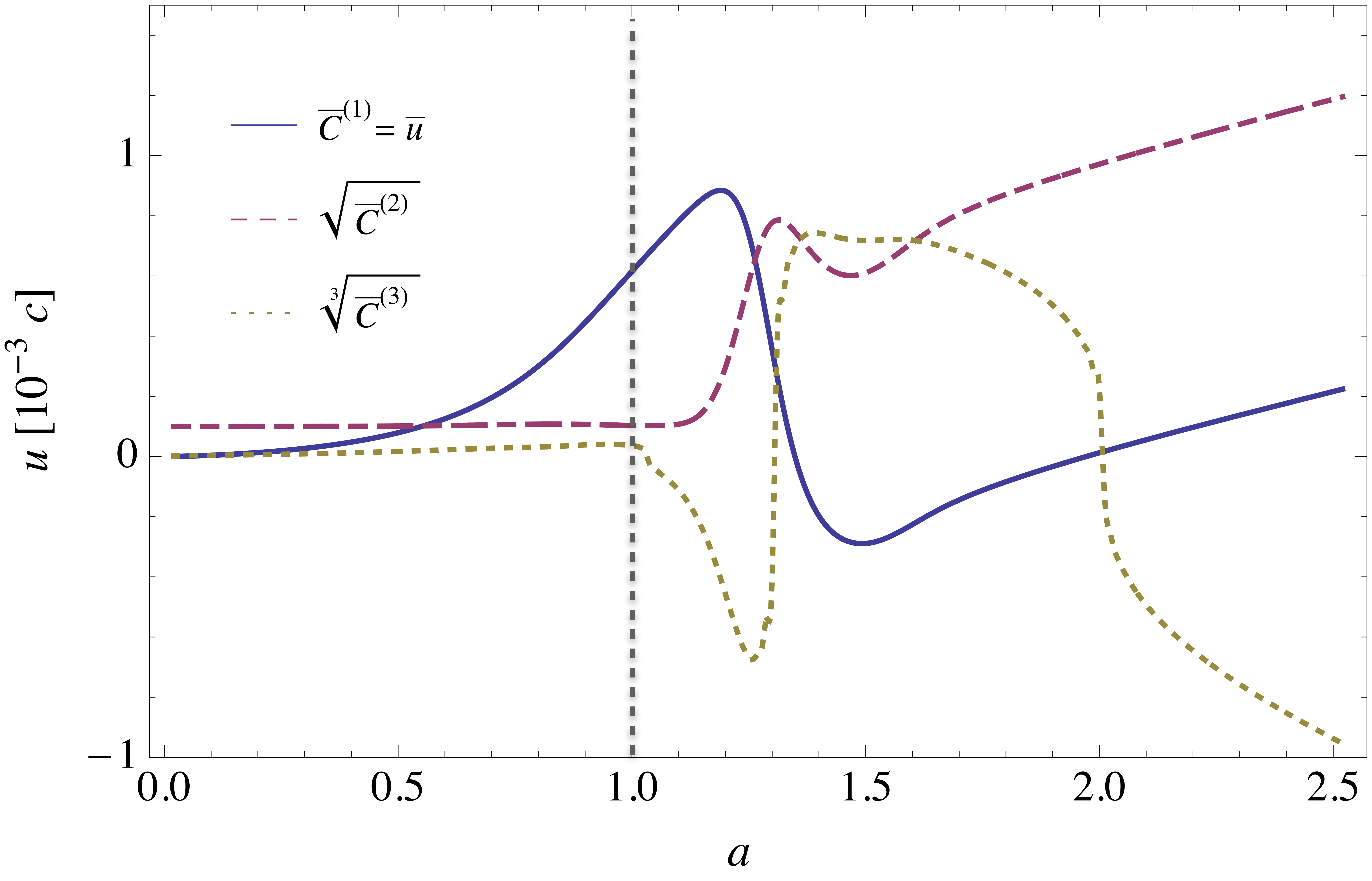


The Sky from Ψ



Porqueres ++ ‘20

Cumulant hierarchy



Multi-Stream Regime

Vorticity

small scales
quantised

large scales
classical

from Kopp++ PRD ’17

Vlasov Schrödinger