SM 2L Renormalization Equations

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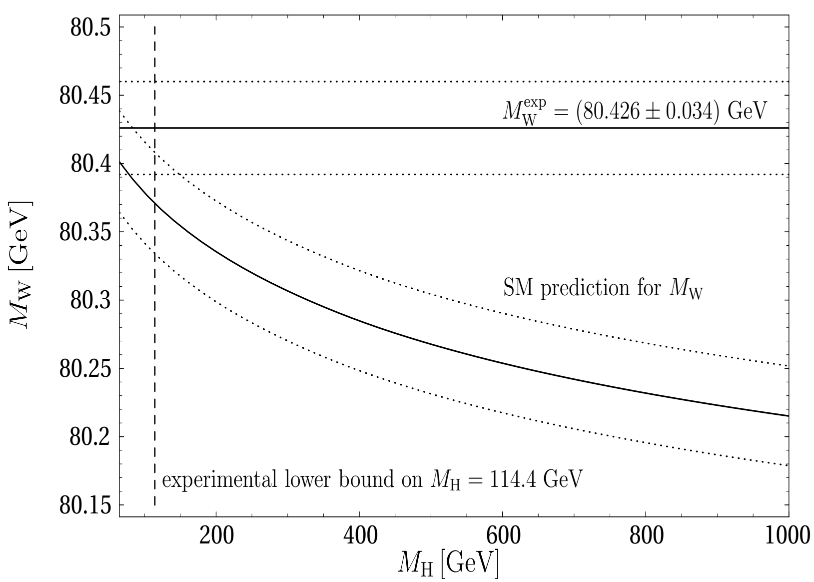
M_W at Two Loops

$$M_W^2 = M_Z^2 \left\{ \frac{1}{2} + \left[\frac{1}{4} - \frac{\pi\,\alpha}{\sqrt{2}\,G_F\,M_Z^2} \left(1 + \underbrace{\Delta r}_{\text{muon decay}} \right) \right]^{1/2} \right\}$$

$$\Delta r = \underbrace{\Delta r^{(\alpha)}}_{1EW} + \underbrace{\Delta r^{(\alpha\alpha_S)}}_{1EW\times 1QCD} + \underbrace{\Delta r^{(\alpha\alpha_S^2)}}_{1EW\times 2QCD} + \underbrace{\Delta r^{(\alpha^2)}_{fer}}_{2EW} + \underbrace{\Delta r^{(\alpha^2)}_{bos}}_{2EW}$$

$$\Delta r = \Delta r(\,\underline{M_W}\,, M_Z, M_H, m_t, \ldots)$$



$m_t^{exp} = 174.3 \pm 5.1$ GeV $\quad \delta M_W^{exp} \sim \delta M_W^{th}$

- Marciano-Sirlin (1980)
- Chetyrkin-Kühn-Steinhauser (1996)
- Freitas-Hollik-Walter-Weiglein, Awramik-Czakon-Onishchenko-Veretin (2002)

different scales

$I.$ LHC

$\delta M_W^{exp} \sim 15$ MeV

$\delta m_t^{exp} \sim 1$ GeV $\Rightarrow \delta M_W^{exp} \sim \delta M_W^{th}$

$II.$ $M_W \;\; sin^2\theta_{eff}^{lep} \Leftrightarrow M_H$

Awramik-Czakon-Freitas-Weiglein [hep-ph:0311148]

Renormalization Equations

* QED/QCD
  - ⇒ IBP [Chetyrkin-Tkachov (1981)] + LI [Gehrmann-Remiddi (1999)] ⟶ Reduction
  - ⇒ MI ⟶ Mellin-Barnes technique [Smirnov-Tausk (1999)]/differential equations [Gehrmann-Remiddi (2001)]/...
* EW
  numerical results 2 loops, 2/3 legs [Ferroglia-Passarino-Passera-Uccirati] (2001-2003) ⟶ Tested ? [St.A.-Passarino]

- Input data

$$\left[\begin{array}{rl} \alpha & = 1/137.03599911(46) \\ G_F & = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2} \\ M_Z & = 91.1876 \pm 0.0021\,\text{GeV} \end{array}\right.$$

$\alpha = 1/137.03599911(46)$	e anomalous magnetic moment
$G_F = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$	muon decay
$M_Z = 91.1876 \pm 0.0021\,\text{GeV}$	Z lineshape

- Renormalization eqs. $\alpha = \alpha(\{\text{bareSM}\}) \qquad G_F = G_F(\{\text{bareSM}\}) \qquad M_Z = M_Z(\{\text{bareSM}\})$
- Renormalization eqs. ⇒ α running + M_W

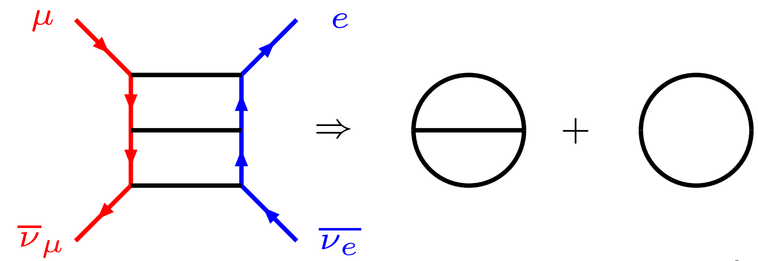
M_Z, M_W ⇒ unstable particles ⇒ definition of mass

* Re$P^{-1}(m^2_{OS}) = 0$ — gauge-parameter dependent
* $P^{-1}(\mu^2) = 0 \qquad \mu^2 = m^2_{CP} - i\,\gamma\,m_{CP}$ — gauge-parameter independent

$m_{CP} \neq M_{exp} \qquad m_{CP} \Rightarrow M_{exp}$

Renormalization Equation For G_F

- GraphShot [FORM] $\Rightarrow$ Amplitudes 2Loops 4Legs
  [St.A-Ferroglia-Passarino-Passera]
- Projection [amplitude SM] $\Rightarrow$ [amplitude FT]
- Subtraction [amplitude FT $\times$ QED]
- 2L diags $\sim$ Tadpoles $\longrightarrow$ IBP $\longrightarrow$ Reduction



soft scales nullified

$\Rightarrow$ $\underbrace{\frac{G_F}{\sqrt{2}}}_{\text{exp}} = \underbrace{\frac{g^2}{8 M_W^2}\left(1 + \Delta g^{(1)} + \Delta g^{(2)}\right)}_{\text{bareSM}}$ (α, M_Z) $\Rightarrow$ inversion $\longrightarrow$ [bareSM] $-$ (input data)

$$\alpha \Rightarrow D_{AA}(0) \quad \Rightarrow D_{AA}(p^2) \quad \Rightarrow \underline{\alpha \quad \text{running}}$$

$$M_Z \Rightarrow D_{ZZ}(\mu_Z^2) \Rightarrow D_{WW}(\mu_W^2) \Rightarrow \underline{M_W}$$

$\ast$ resummations ?

$\Rightarrow$ higher-order leading effects

$\Rightarrow$ gauge-parameter invariance beyond one loop

Counterterms

bare-renormalized parameters $M_b = Z_M^{1/2} M_r$

counterterms (UV divergencies, γ, $\ln \pi$) $Z_M \to \delta Z_M$ (two loops)

$$\delta Z_M = \frac{149}{12} + \frac{379}{12}\frac{1}{c_\theta^6 x_H} + \frac{19}{2}\frac{x_t}{c_\theta^4 x_H} - \frac{289}{6}\frac{1}{c_\theta^4 x_H} - \frac{701}{96}\frac{1}{c_\theta^4} - 40\frac{x_t}{c_\theta^2 x_H}$$
$$+ \frac{8}{3}\frac{x_t^2}{c_\theta^2 x_H} + \frac{77}{3}\frac{1}{c_\theta^2 x_H} - \frac{3}{4}\frac{x_H}{c_\theta^2} - \frac{85}{48}\frac{x_t}{c_\theta^2} + \frac{73}{6}\frac{1}{c_\theta^2} + 35\frac{x_t}{x_H} - \frac{8}{3}\frac{x_t^2}{x_H}$$
$$- 15\frac{x_t^3}{x_H} - \frac{256}{3}\frac{1}{x_H} + \frac{9}{4}x_H x_t - \frac{3}{2}x_H + \frac{63}{32}x_H^2 - \frac{43}{24}x_t - \frac{21}{16}x_t^2$$
$$\ldots$$

Numerical Methods

- STWI $\Rightarrow$ $\underbrace{\text{standard}}_{\text{Passarino-Veltman}(1979)}$ $+$ $\underbrace{\text{sub-loop}}_{\text{Bohm-Scharf-Weiglein}(1993)}$ tensor reduction $\Rightarrow$ Gram determinants

- Algebraic check $\neq$ Realistic computation

> $*$ One Loop $\Rightarrow$ Stuart (1988), Campbell-Glover-Miller (1996), Denner-Dittmaier (2005) $\Rightarrow$ Avoid Gram-det. problems
>
> $*$ Two Loops $\Rightarrow$? $\Rightarrow$ Avoid tensor reduction (tensors $\sim$ scalars) [St.A-Ferroglia-Passarino-Passera-Uccirati] (2004)

$I.$ Observable $\longrightarrow$ gauge-parameter independent blocks $\mathcal{B}_i$

$II.$ $\mathcal{B} \longrightarrow \underbrace{\frac{1}{S}}_{\text{sing.}} \times \int dx\, \underbrace{G(x)}_{\text{smooth}}$ smooth — N (large) continuous derivatives

> Bernstein-Tkachov algorithm (1996) $+$ Sector decomposition [Binoth-Heinrich] (2000)
>
> $\Rightarrow$ numerical results 2 loops, 2/3 legs [Ferroglia-Passarino-Passera-Uccirati] (2001-2003)

$\star$ N increases $\longrightarrow$ number of terms in G increases $\longrightarrow$ numerical instabilities

$\star$ around zeros of S $\longrightarrow$ singular behaviour overestimated

Conclusions

- Renormalization equations G_F, α, M_Z
- Numerical results 2 loops, 2/3 legs completed

∗ M_W $\sim \Delta r$ [Freitas-Hollik-Walter-Weiglein] [Awramik-Czakon-Onishchenko-Veretin] (2002)

∗ α running $\sim$ BFM [Degrassi-Vicini] (2003) almost completed