Integrability and Quantum Spectral Curves

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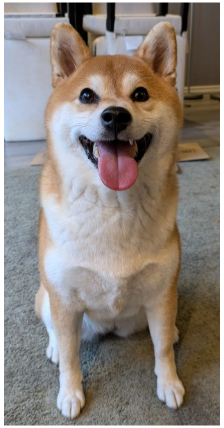
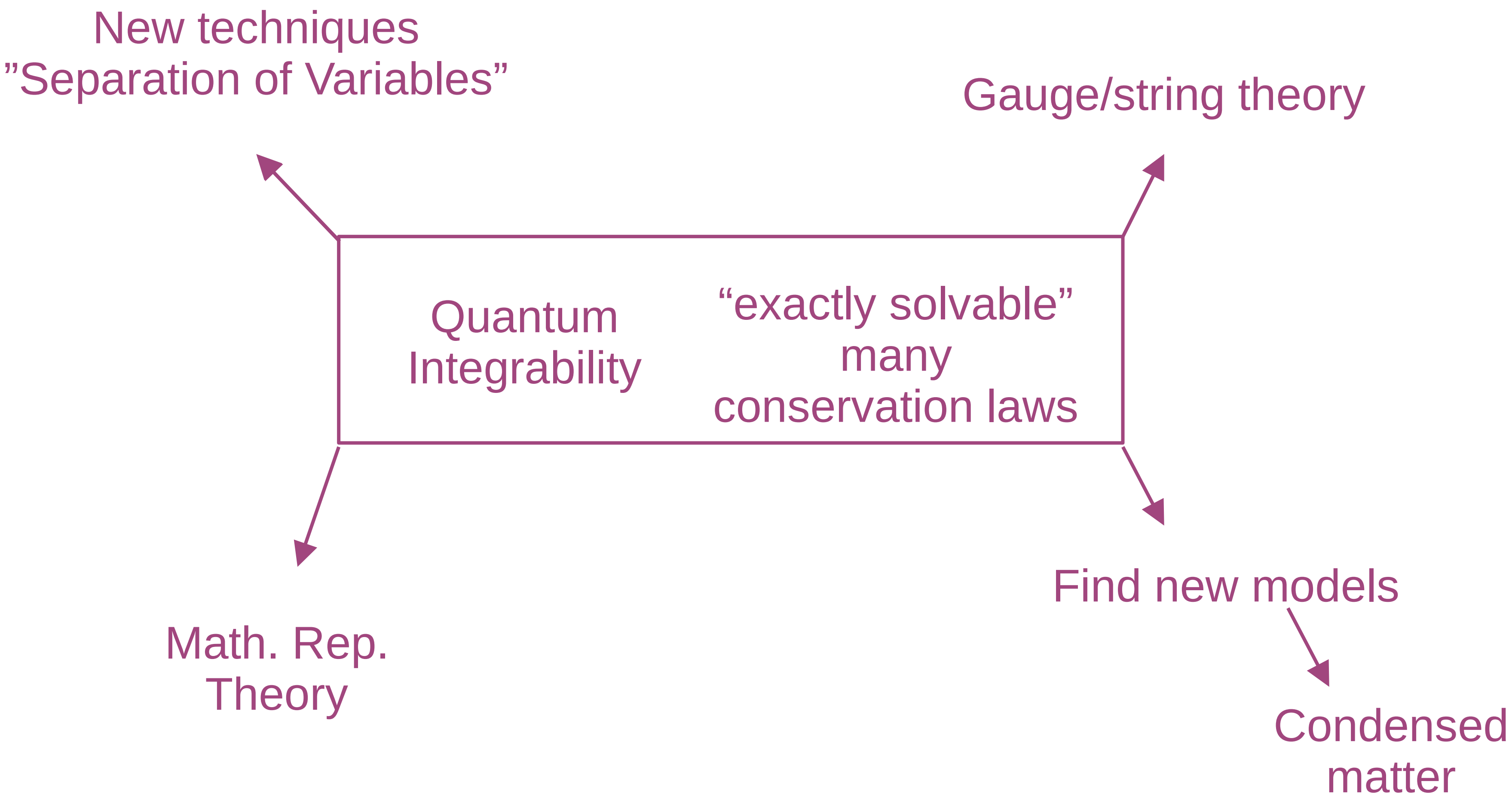
About me

- PhD: Trinity College Dublin, 2021
- King’s College London, 2021 – 2024
- DESY, 2024 -

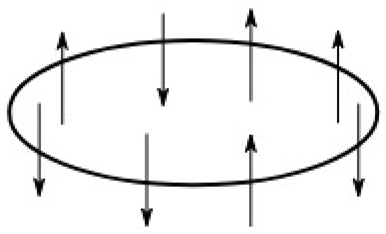


Research


New techniques
”Separation of Variables”
Gauge/string theory
Quantum
Integrability
“exactly solvable”
many
conservation laws
Math. Rep.
Theory
Find new models
Condensed
matter

State of the art: Quantum Spectral Curve

Energy spectrum in integrable systems ↔ Functional equations



$$H = \sum_{n=1}^{L} S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + S_n^z S_{n+1}^z$$

$$\boxed{Q_1(u+\tfrac{i}{2})Q_2(u-\tfrac{i}{2}) - Q_2(u+\tfrac{i}{2})Q_1(u-\tfrac{i}{2}) = u^L}$$

Polynomial solutions ↔ Energy spectrum

$$E = \partial_u \log \frac{Q_j(u+\frac{i}{2})}{Q_j(u-\frac{i}{2})}\bigg|_{u=0}$$

Integrability in gauge / string theory

Planar N=4 Super Yang Mills 4D $\Longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

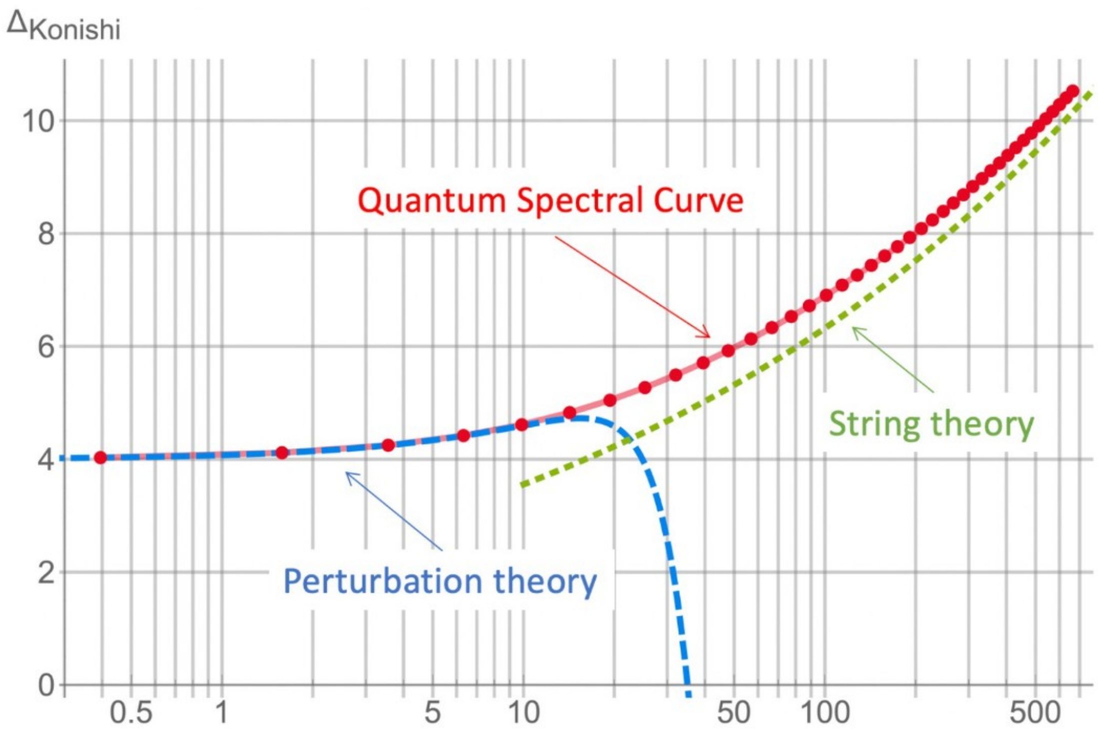
$\boxed{\text{Conformal Field Theory}}$

Theory solved when all 2 and 3 point correlation functions known

Equivalently scaling dimensions and 3 point structure constants

Integrability: scaling dimensions solved via Quantum Spectral Curve

Scaling dimension for Konishi $\mathrm{Tr}(D^2 Z^2)$

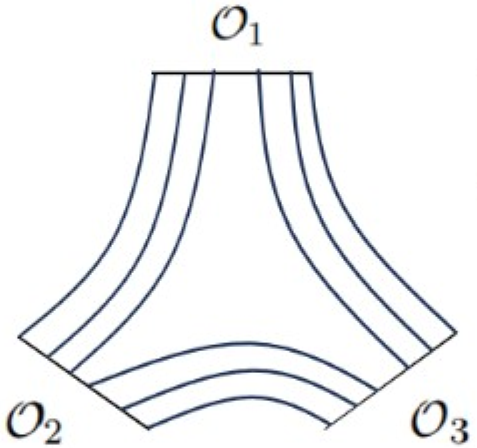


Current direction: Quantum Spectral Curve for 3pt functions

Related to “Separation of Variables” in integrable spin chains

QUANTUM SPECTRAL CURVE

$\mathcal{O} \longrightarrow Q$



SEPARATION OF VARIABLES

$\longrightarrow \mathcal{F}(Q_1, Q_2, Q_3)$

State of the art:

$$C^2_{\bullet\circ\circ} = \frac{(\mathbb{J}!)^2}{(2\mathbb{J})!} \frac{\langle \mathcal{Q}, \mathbf{1}\rangle_\ell^2}{\langle \mathcal{Q}, \mathcal{Q}\rangle_L}$$

Main research goal

$\boxed{\text{Separation of Variables at all loops}}$

- Perturbatively in planar N=4 SYM
- Exactly in “conformal fishnet theory”
- Understanding algebraic integrable structure of gauge theories

Thank you!