# Using soft emissions for high-precision calculations

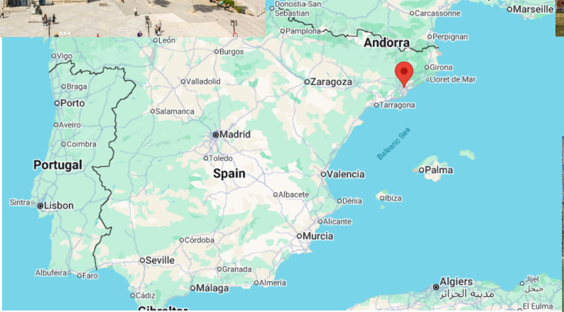
Roger Balsach

Theory Fellows' meeting 2024

November 18, 2024

# About myself

## Sabadell ∼30min from Barcelona




# Hobbies

Food: Mexican & Indian

Listening to Music (all styles)

Reading (specially Science-fiction and Fantasy)

Chess & Table Tennis

# Physics background

**Bachelor of Physics (2015-2019)**

Bachelor thesis: *"Effective Lagrangians of the Standard Model"*

**Postgraduate Program in High Energy Physics, Astrophysics & Cosmology (2019-2020)**

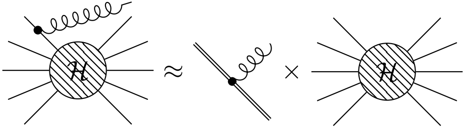
Master's thesis: *"What can the anomalous magnetic moment of the leptons teach us about new physics"*

**PhD in Physics at the University of Münster (2021-2024)**

Dissertation: *"Soft-Emission Effects in High-Energy Processes"*

# Soft emissions

- The emission of a particle with energy much lower than any other scale relevant to the process.
- Relevant for defining infrared-safe observables and cancelling infrared divergences higher-order calculations.
- Soft-emission approximation allows for more complex calculations that would be unfeasible otherwise.




**Theorem (Eikonal approximation)**

$$\mathcal{A}_n \approx \eta Q \frac{\epsilon^*(k) \cdot p}{k \cdot p} \hat{\mathbf{T}}_n \mathbf{H}$$

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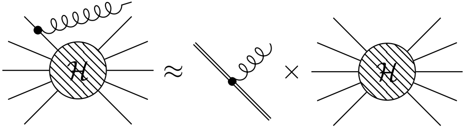


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**Factorisation:**
The radiative amplitude $\mathcal{A}$ is obtained by multiplying the hard amplitude $\mathcal{H}$ with a **process-independent** factor.

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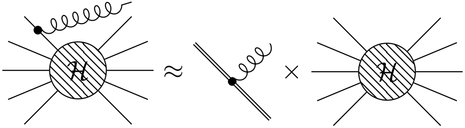


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**Colour space:**

Amplitude $\mathcal{H}$ can be decomposed as a vector in an abstract vector space: $\mathbf{H}$
The emission/absorption of a gluon can be represented as a map between colour spaces, $\hat{\mathbf{T}}_n$.

# Bremsstrahlung vs Resummation

**Photon bremsstrahlung:**

- Study the emission of a soft photon from a given process, e.g., $pp \to \mu^+\mu^-\gamma$
- Only one photon is radiated and observed in the detector.
- We can measure the energy of the photon, thus we can study how good the approximation is at different energies.

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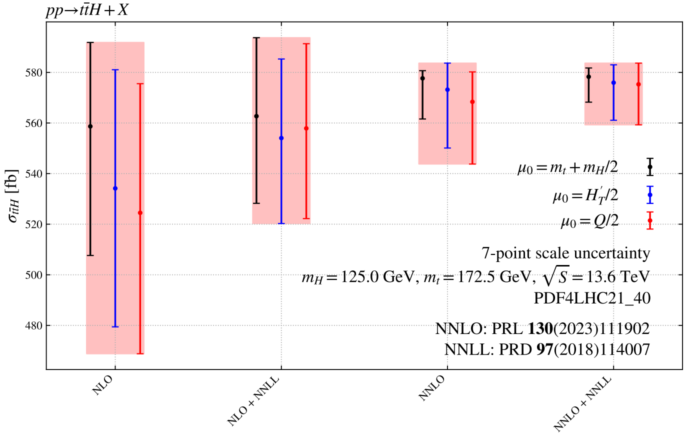
**Soft-gluon resummation:**

- Studied process does not involve soft radiation: $pp \to t\bar{t}H$, $pp \to t\bar{t}\gamma$, $pp \to t\bar{t}Z$
- Soft radiation is used to compute higher-order corrections.
- The number of soft gluons and their momenta are not known, we need to sum over all possible states.

# Resummation for $t\bar{t}H$ production




$$\sigma_{t\bar{t}H}^{\mathrm{NNLO+NNLL}} = 515^{+1.3(4)\,\%}_{-3.2(2)\,\%}\ \mathrm{fb} \qquad \sigma_{t\bar{t}H}^{\mathrm{exp}} = (557 \pm 18\,\%)\ \mathrm{fb} \qquad @13\ \mathrm{TeV}$$

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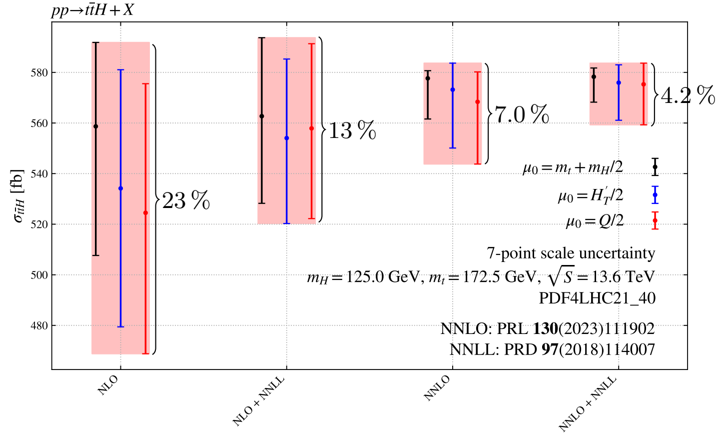


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# Soft photon emission beyond LP

Soft theorem has subleading corrections

**Theorem (Low-Burnet-Kroll)**

$$\overline{|\mathcal{A}|}^2_{\mathrm{LP+NLP}} = -\sum_{i,j} \frac{(Q_i p_i)\cdot(Q_j p_j)}{(p_i\cdot k)(p_j\cdot k)} \left[1 + \frac{(p_j\cdot k) p_{i\mu}}{p_i\cdot p_j} G_j^{\mu\nu} \frac{\partial}{\partial p_j^\nu}\right] \overline{|\mathcal{H}|}^2(p)$$
$$= -\left(\sum_{i,j} \frac{(Q_i p_i)\cdot(Q_j p_j)}{(p_i\cdot k)(p_j\cdot k)}\right) \overline{|\mathcal{H}(p+\delta p)|}^2$$

Recently, some people claimed that the LBK theorem was inconsistent and ambiguous.

- I proved that ambiguities in the LBK theorem are beyond NLP and thus not in the range of validity of the theorem.
- I found an alternative definitions of the shifts $\delta p$ that keep masses of external particles unchanged to all orders in the soft expansion. Simplifying the numerical implementation and allowing the use of on-shell amplitude generators.

# Thank You!

*Questions?*

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