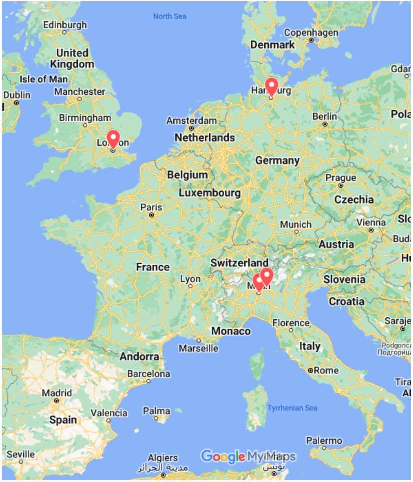
Analytic bootstrap for defect CFTs

Davide Bonomi

DESY

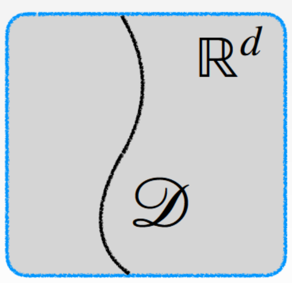
About me

- Name: Davide Bonomi.
- Born in Bergamo, Italy.
- Bachelor's and Master's in Milan.
- PhD in London.
- Hobbies: reading, trekking,...



My research

Main focus: Conformal Field Theories (CFTs) that contain extended objects (defects).

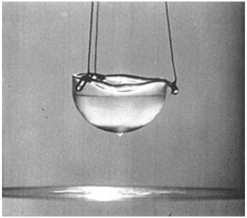


Main strategy: use conformal symmetry + properties of the Operator Product Expansion (OPE) to constrain or solve CFTs (conformal bootstrap).

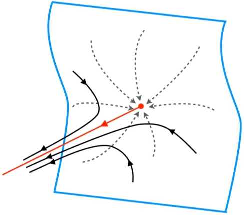
Why CFT?

Conformal invariance $\sim$ Poincarè + scale invariance.

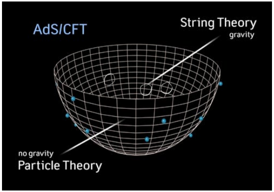
Conformal field theories are ubiquitous in physics:



Phase transitions



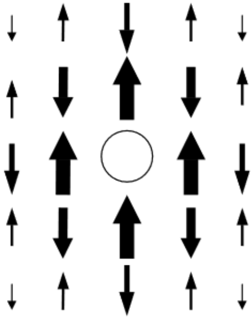
RG flows



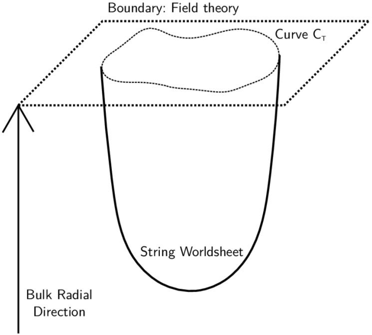
AdS/CFT

Why defects?

Many CFTs admit interesting defects:



Phase transitions with impurities
(e.g. metal with external atom)



Wilson lines in CFTs

$\sim$

Strings and branes in AdS

Conformal Bootstrap

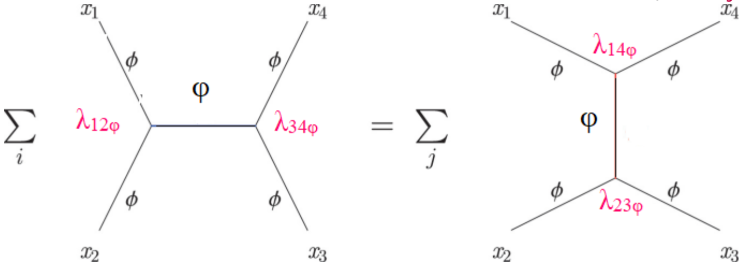
2pt and 3pt fixed up to scaling dimensions and OPE coefficients:

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{x^{2\Delta_{\mathcal{O}}}}, \quad \langle \mathcal{O}_i\mathcal{O}_j\mathcal{O}_k\rangle \sim \lambda_{ijk}$$

All other correlators can be computed using the OPE:

$$\mathcal{O}_i(x)\mathcal{O}_j(0) \overset{\text{OPE}}{=} \sum_k \frac{\lambda_{ijk}\mathcal{O}_k(0)}{|x|^{\Delta_i+\Delta_j-\Delta_k}} + \dots$$

Different OPEs $\rightarrow$ crossing equation $\rightarrow$ constraints on $\{\Delta_i, \lambda_{ijk}\}$



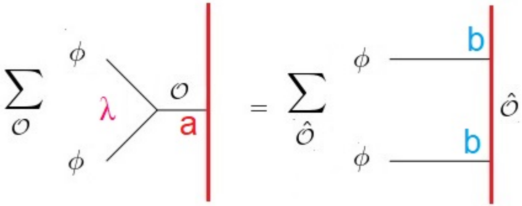
The crossing eq is positive definite and can be studied numerically.

Defect bootstrap

A p-dimensional defect breaks the bulk conformal group as

$$\underbrace{SO(d+1,1)}_{\mathcal{O}_{\Delta,\ell}} \to \underbrace{SO(p+1,1)\times SO(d-p)}_{\hat{\mathcal{O}}_{\hat{\Delta},\hat{\ell},s}}$$

Bulk and defect OPE $\to$ crossing equation $\to \left\{\hat{\Delta}_i, \hat{\lambda}_{ijk}, a_i, b_{ij}\right\}$



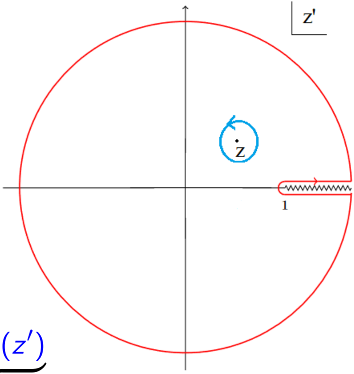
Problem: not positive definite $\to$ no numerical bootstrap.

Analytic bootstrap

The OPE controls the singularities of correlators. We can use this input to reconstruct correlators and extract CFT data. No need of a positive definite expansion.

E.g.: consider a function $f(z)$ such that

- $f(z)$ has a branch cut for $z > 1$.
- $|f(z)/z| \to 0$ as $|z| \to \infty$.



Cauchy's theorem implies:

$$f(z) = \frac{1}{2\pi i} \oint \frac{dz'}{z' - z} f(z')$$

By deforming the contour:

$$f(z) = \frac{1}{2\pi i} \int_1^\infty \frac{dz'}{z' - z} \underbrace{\mathrm{Disc} f(z')}_{f(z' + i\epsilon) - f(z' - i\epsilon)}$$

Example: defects in the $O(N)$ model

We studied the critical $O(N)$ model

$$S = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi_i)^2 + \frac{\lambda}{4!} (\phi_i \phi_i)^2 \right]$$

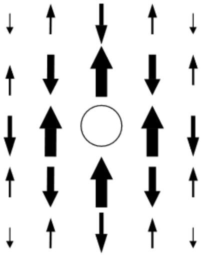
in presence of defects in $d = 4 - \varepsilon$ with $\varepsilon \ll 1$.

- Localized magnetic field

$$D = e^{-h_0 \int d\tau \, \phi_1(\tau)}$$

- Spin impurity

$$D_j = Tr_{2j+1} \left(P e^{\frac{\zeta_0}{\sqrt{\kappa}} \int d\tau \phi^i T^i} \right), \quad T^i \in \mathfrak{su}(2)$$



Main result: $\langle \phi \phi \rangle$ and ∞ new defect CFT data from few bulk data.

Ongoing work and future directions

- Combine analytic bootstrap with numerical bulk data.
- Correlators of defects (e.g. correlator of two impurities).
- Holographic defects and interplay with integrability, localization ect.

Thank you for your attention!