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TECHNISCHE UNIVERSITÄT DARMSTADT

IBS Modeling in Reptil

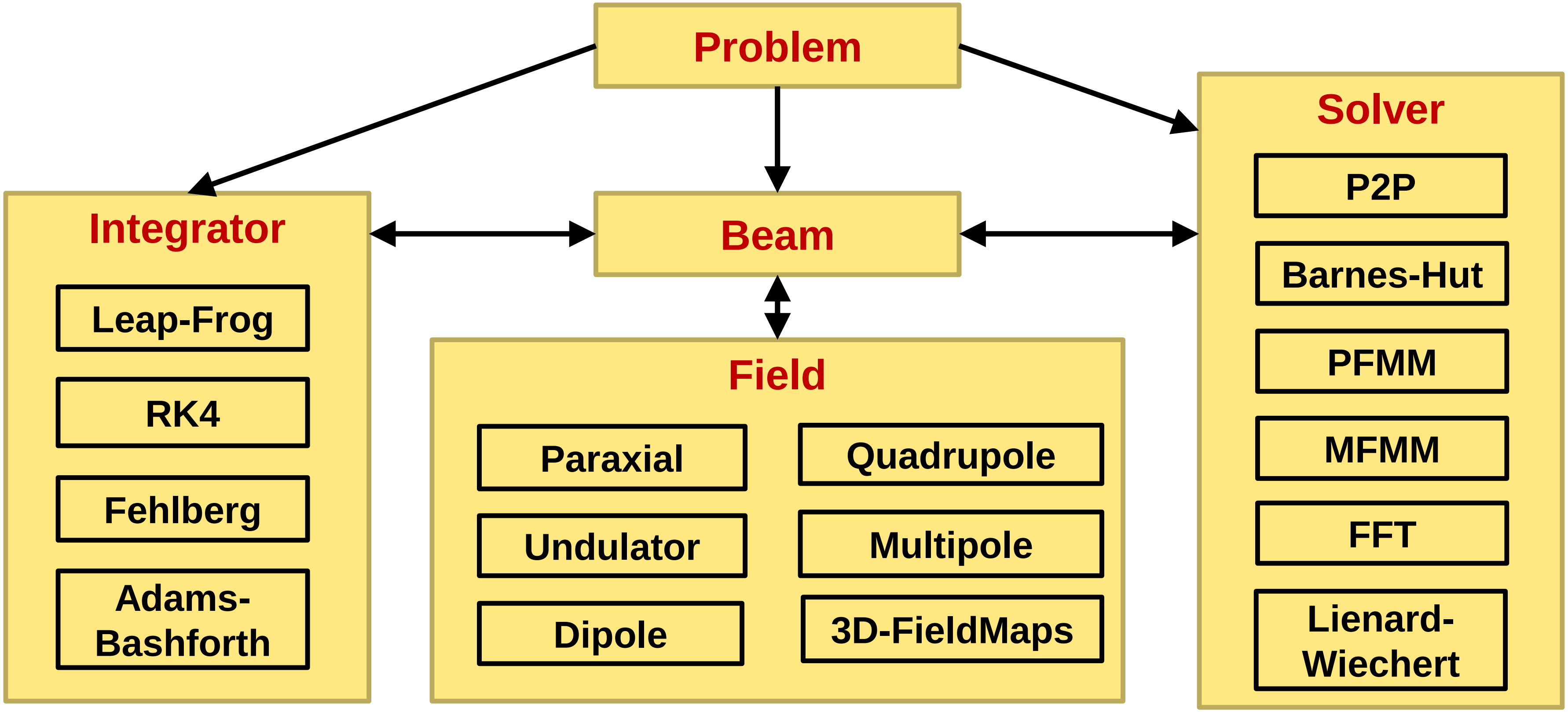
Dec. 5, 2024, DESY

IBS in the XFEL injector

- The Reptil code
- IBS Model
- Validation studies
- Simulations
- Conclusions

The Reptil code

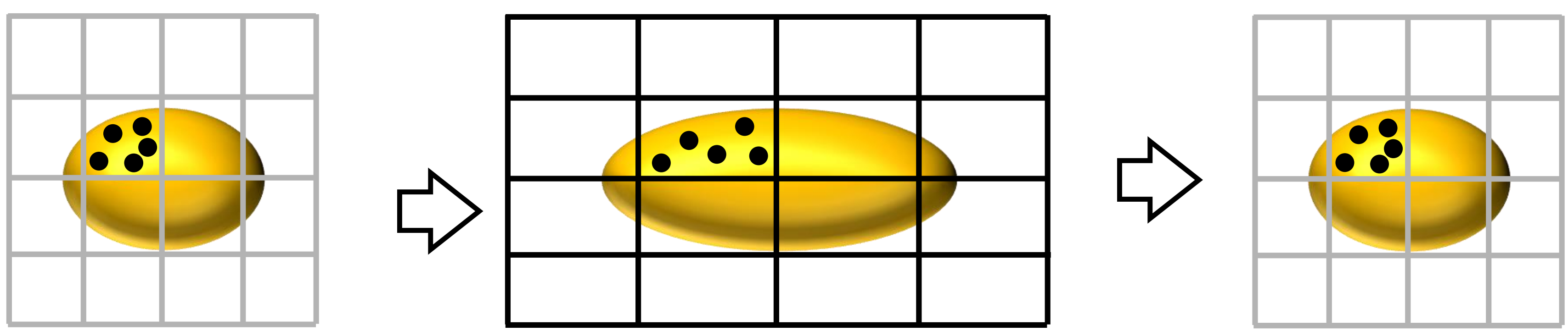
- RElativistc Particle Tracking for Injectors & Linacs



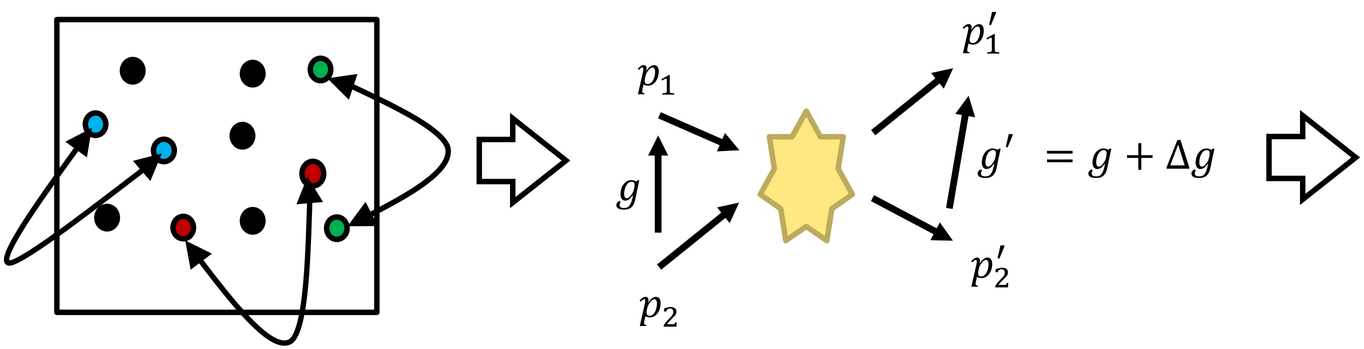
IBS Model in Reptil

- Monte-Carlo collisions (cell wise)

Transform to rest frame Transform to lab frame



Build random particle pairs Effective collision in the CM frame Post-collision momenta

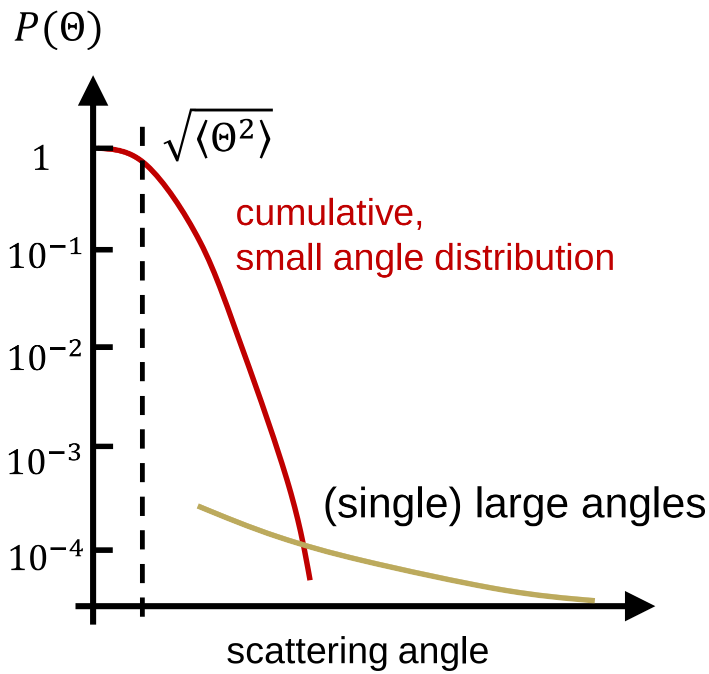


$$p'_1 = p_1 + \frac{1}{2}\Delta g$$

$$p'_1 = p_2 - \frac{1}{2}\Delta g$$

IBS Model in Reptil

- Cumulative scattering angle



For a single (binary) scattering angle event:

$$\langle\theta^2\rangle = \frac{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta)\theta^2 \frac{d\sigma(g,\theta)}{d\Omega}}{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta) \frac{d\sigma(g,\theta)}{d\Omega}} = 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

$$\Lambda_c = \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx \ln\left(\frac{b_{max}}{b_{min}}\right)$$ Coulomb log.

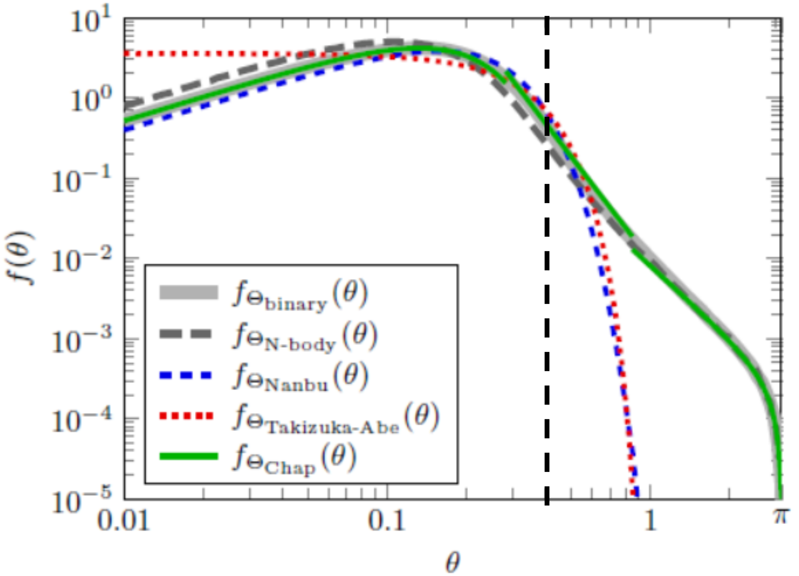
For N (large) scattering events → cumulative angle:

$$P(\Theta) \sim \frac{1}{\sqrt{(2\pi\langle\Theta^2\rangle}} e^{-\frac{\Theta^2}{2\langle\Theta^2\rangle}}$$

$$\langle\Theta^2\rangle = N\langle\theta^2\rangle \quad \text{with:} \quad N = \pi b_{max}^2 n g \Delta t$$

IBS Model in Reptil

- Cumulative scattering angle



Chap & Sedwick, Phys. Rev. E 95, 063209, 2017

For a single (binary) scattering angle event:

$$\langle\theta^2\rangle = \frac{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta)\theta^2 \frac{d\sigma(g,\theta)}{d\Omega}}{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta) \frac{d\sigma(g,\theta)}{d\Omega}} = 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

$$\Lambda_c = \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Coulomb log.

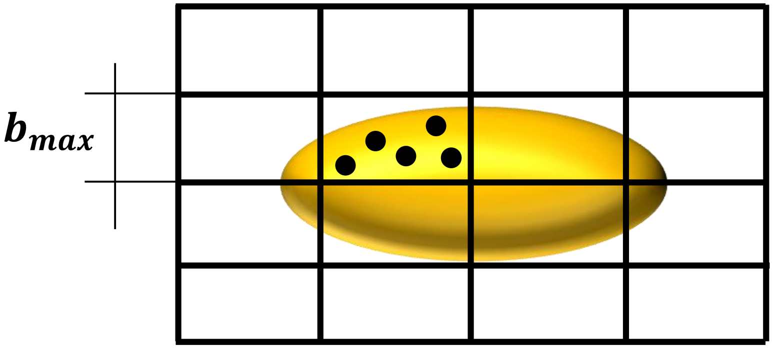
Nanbu's correction for finite N:

- Solve $\coth(A) - A^{-1} = \exp(-2\langle\Theta^2\rangle)$
- Generate cumulative angle as:
  $\theta(u) = \arccos\{\log[\exp(-\mathrm{A}) + 2\,\mathrm{u}\sinh(\mathrm{A})]\}$
  with $\mathrm{u} \in [0,1]$

IBS Model in Reptil

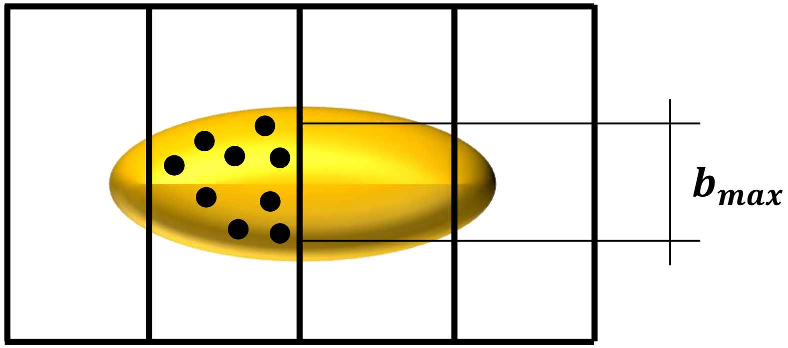
- Choice of Coulomb logarithm

Group particles by cell



Sensitive to mesh refinement!

Group particles by slice



Assumes uniform density in slice!

Shortcomings:

- Does not resolve transverse density variations
- Longitudinal collisions (at very low energies) not included
- Correction for the distribution tail at large angles might be needed → **Coulomb log.**
- **This heuristics cannot be avoided in simulations as well as analytical models**

Validation studies

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- The original Piwinski model:

$$\frac{1}{\tau_\delta} = \frac{1}{\sigma_\delta}\frac{d\sigma_\delta}{dt} = \frac{2 r_e c N_b}{64\pi^2\beta^3\gamma^2\epsilon_x^n\epsilon_y^n\sigma_z\sigma_\delta} f\left(\frac{\sigma_\delta}{\gamma\sigma_{x'}}, \frac{\sigma_\delta}{\gamma\sigma_{y'}}, 2\sigma_\delta\beta\sqrt{\frac{\sigma_y}{r_e}}\right)$$

- The high-energy approximation for round beams (Huang, Bane, 2002):

$$\boxed{\frac{1}{\tau_\delta} = \frac{1}{\sigma_\delta}\frac{d\sigma_\delta}{dt} = \frac{r_e^2 c N_b \Lambda_c}{8\epsilon_x^n\epsilon_x^n\sigma_x\sigma_z\gamma^2\sigma_\delta^2},}$$

$$\Lambda_c = \ln\left(\frac{\Delta\gamma_{max}}{\Delta\gamma_{min}}\right) = \ln\frac{\sigma_x}{r_e/(\gamma^2\sigma_{x'}^2)}$$

($\sigma_x \rightarrow \boldsymbol{b_{max}}$; $r_e/(\gamma^2\sigma_{x'}^2)$: $\boldsymbol{b_{min}}$)

„Cutting“ the tail of scattering angle distribution (Huang, 2002):

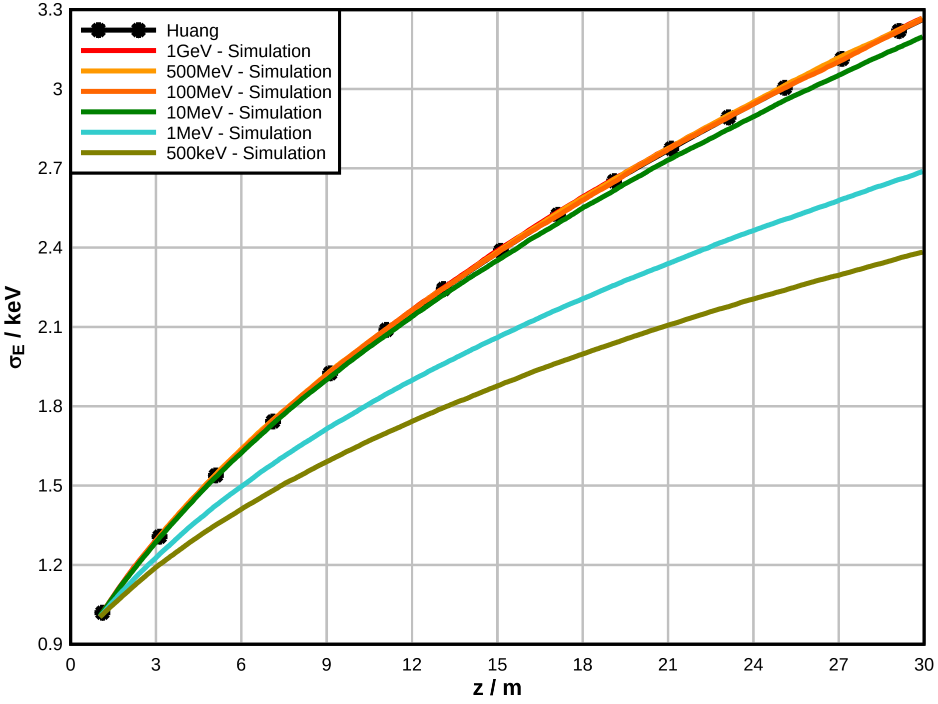
$$\Delta\gamma_{max} \sim \gamma\times 10^{-5} \;\; \rightarrow \Lambda_c = \ln\left(\frac{\epsilon_x^n \times 10^{-5}}{r_e}\right) \approx 8 \qquad \text{for } \epsilon_x^n = 1\mu m$$

This value is wrongly used in several recent papers

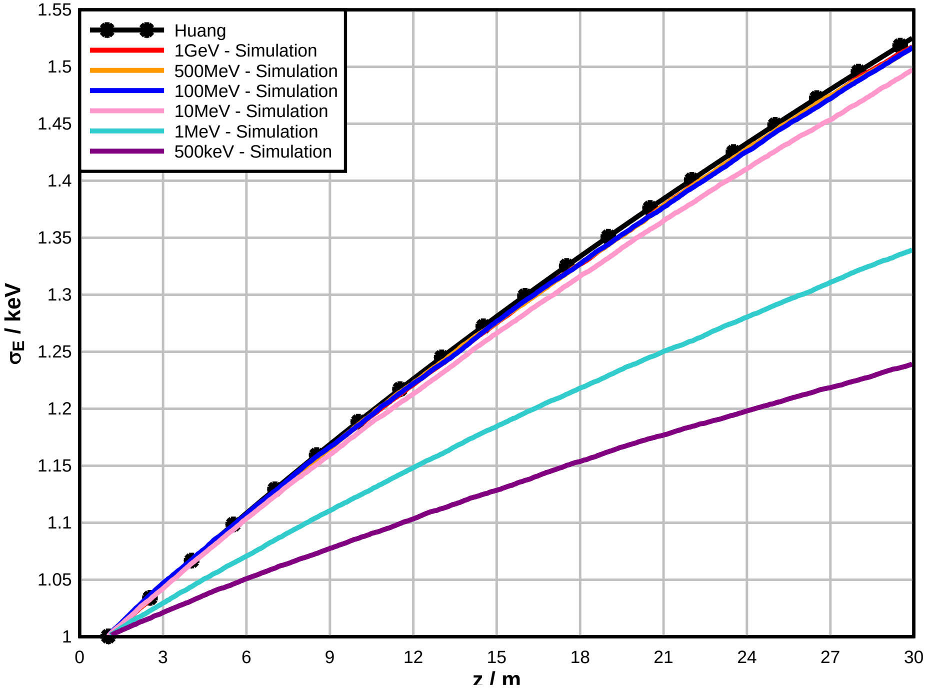
Validation studies

- IBS growth for frozen beam: $\epsilon_x^n = 1\mu m, \sigma_x = 250\mu m, \sigma_z = 1mm$

Pencil beam



Gaussian beam

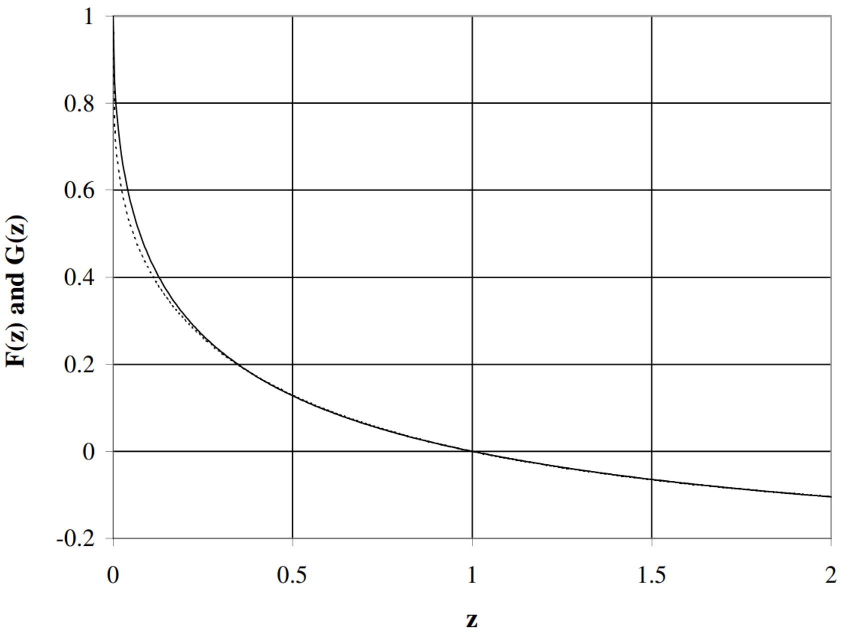


Validation studies

- Low energy IBS fitting formula (Nagaitsev, 2005):

$$\frac{1}{\tau_\delta} = \frac{1}{\sigma_\delta}\frac{d\sigma_\delta}{dt} = \frac{r_e^2 c N_b \Lambda_c}{8\beta^3 \epsilon_x^n \epsilon_x^n \sigma_x \sigma_z \gamma^2 \sigma_\delta^2} \cdot F(\xi)$$

$$\xi = \left(\frac{\sigma_\delta \sigma_x}{\epsilon_x^n}\right)^2 = \left(\frac{\sigma_\gamma \sigma_x}{\gamma \epsilon_x^n}\right)^2$$



$$F(\xi) \approx G(\xi) = (1 - \xi^{\frac{1}{4}}) \frac{\ln(\xi + 1)}{\xi}$$

S. Nagaitsev, PRST-AB, 8, 064403 (2005)

Validation studies

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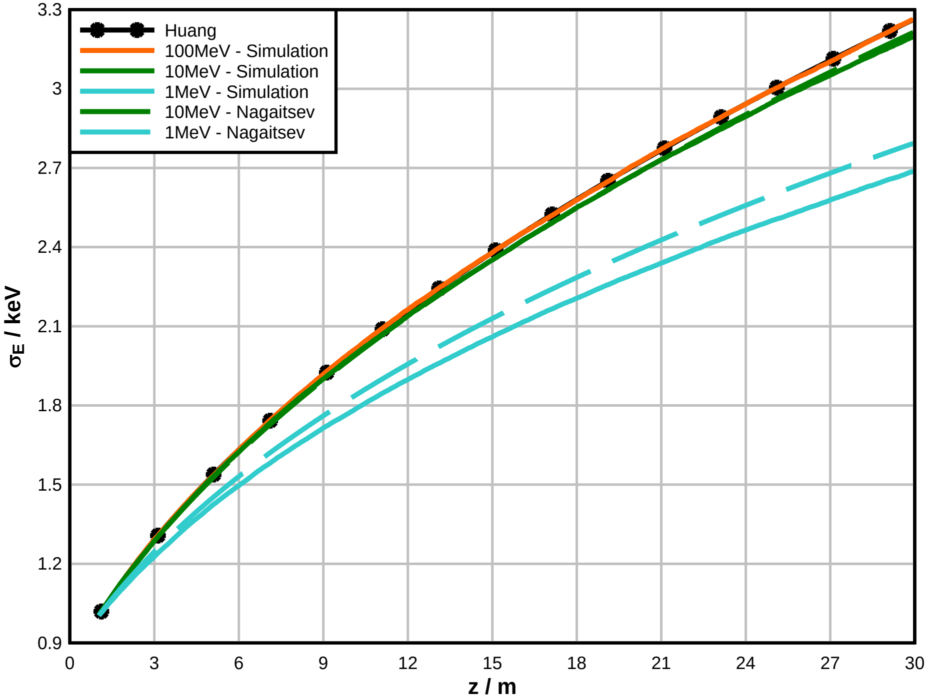
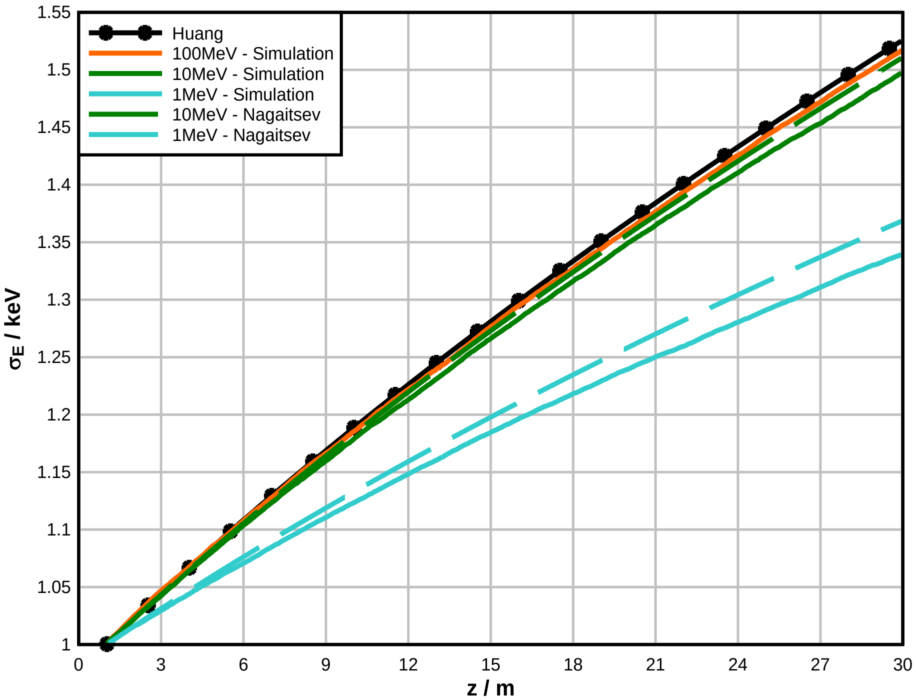
- IBS growth for frozen beam: $\epsilon_x^n = 1\mu m, \sigma_x = 250\mu m, \sigma_z = 1mm$

Pencil beam



Gaussian beam



Simulations for the SwissFEL

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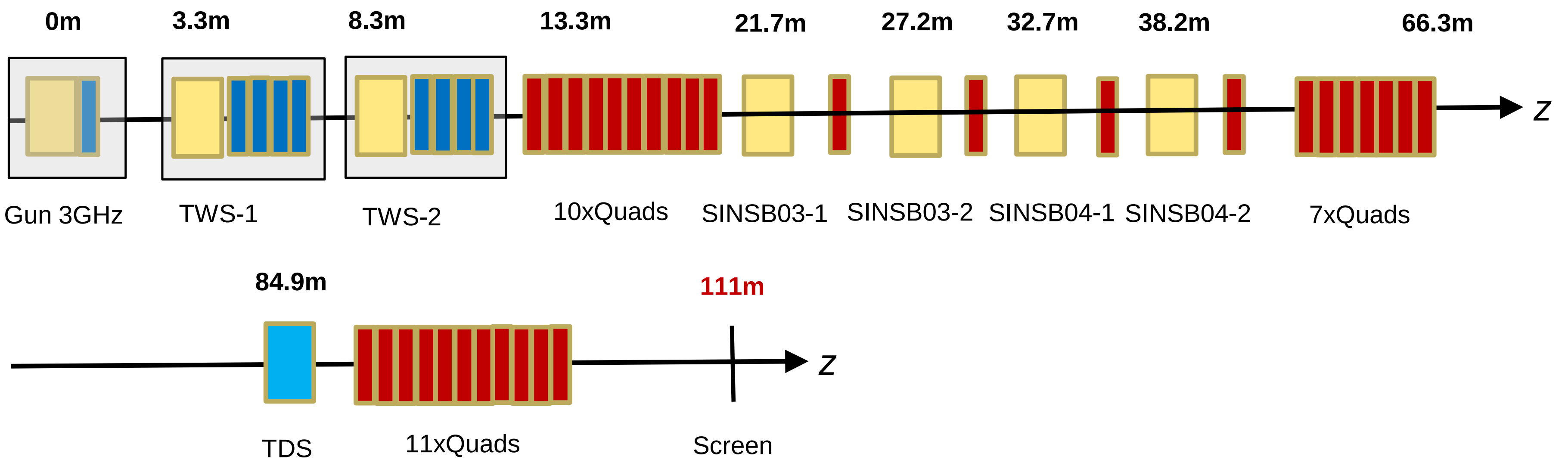
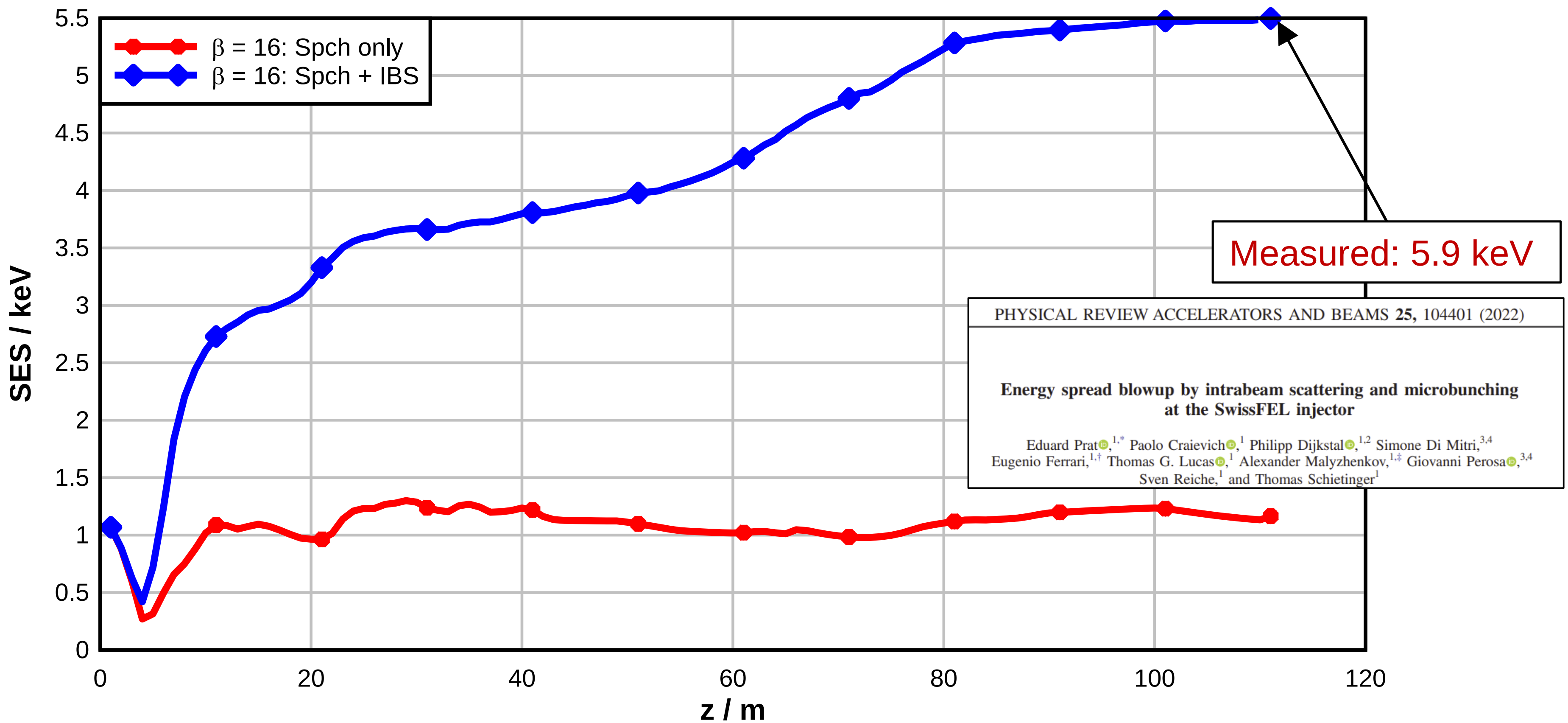
- The SwissFEL injector



- Final energy: 320 MeV
- Bunch charge: 10…200 pC (nominal 200 pC)
- Average β-function: ~16 m (nominal)
- Bunch compressor and laser heater switched off (R_{56}=0)

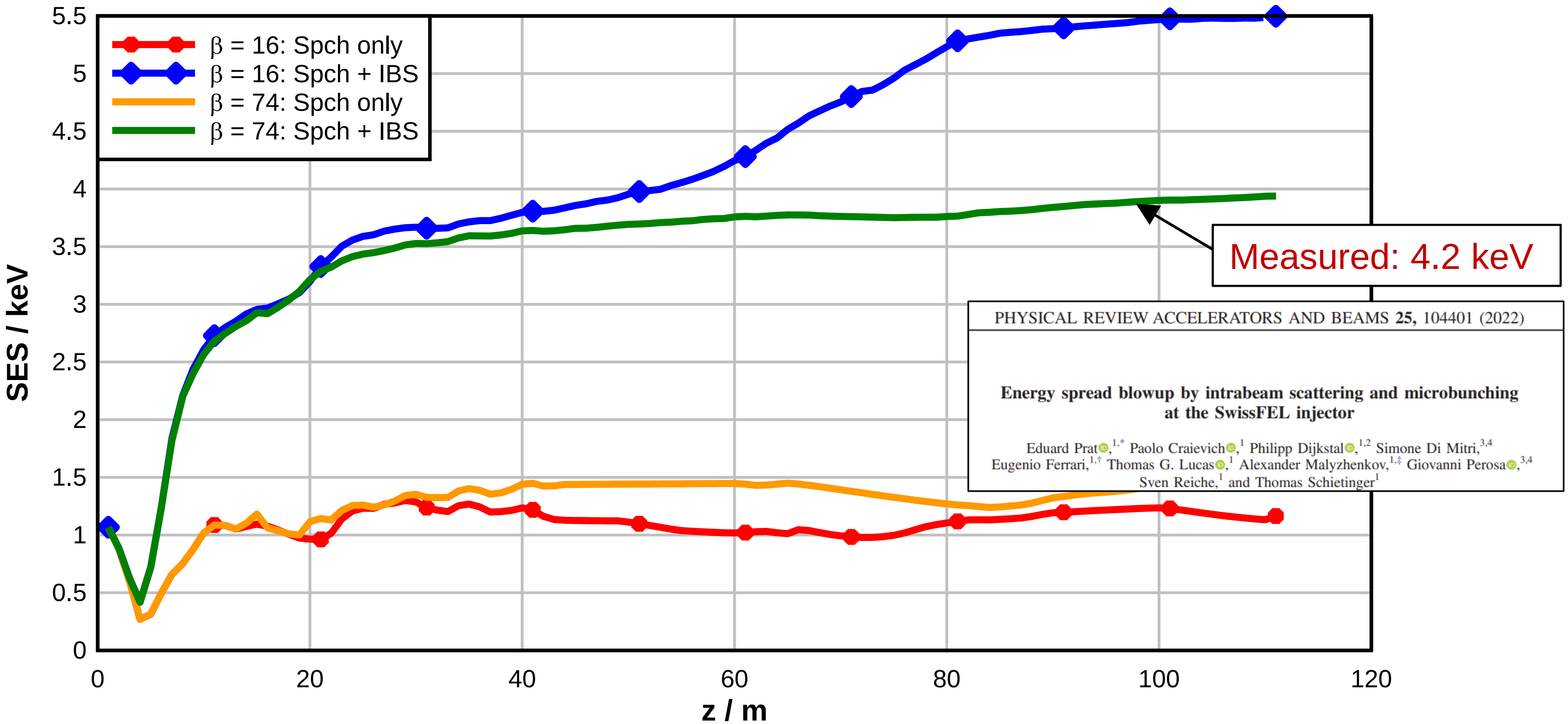
Simulations for the SwissFEL

- Slice energy spread (nominal configuration)



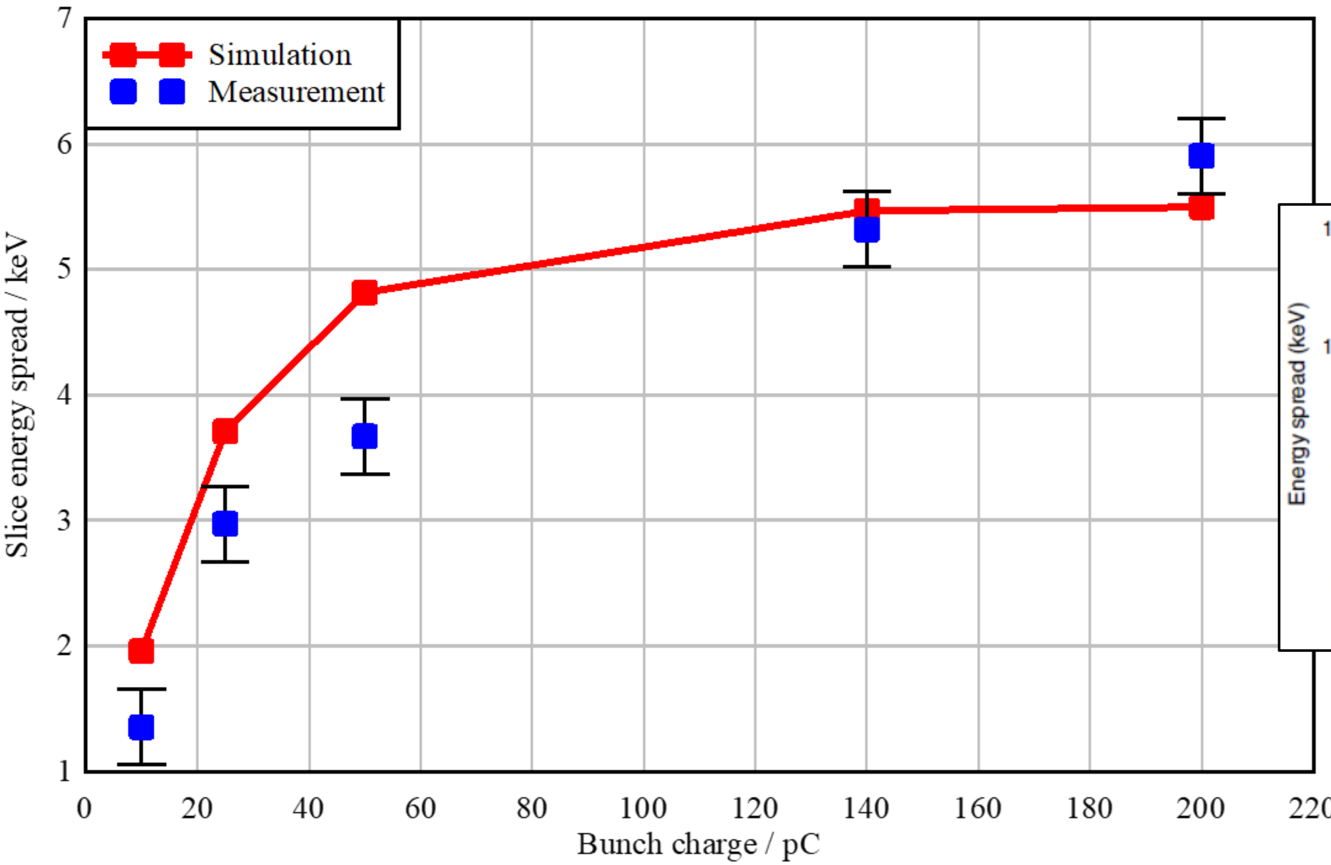
Simulations for the SwissFEL

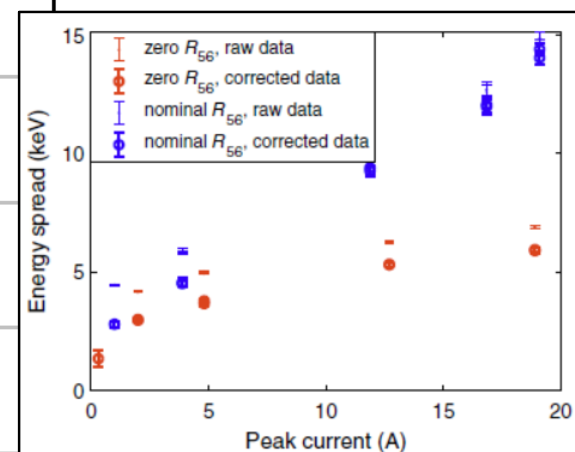
- Slice energy spread ("large" optics)



Simulations for the SwissFEL

- Charge scan

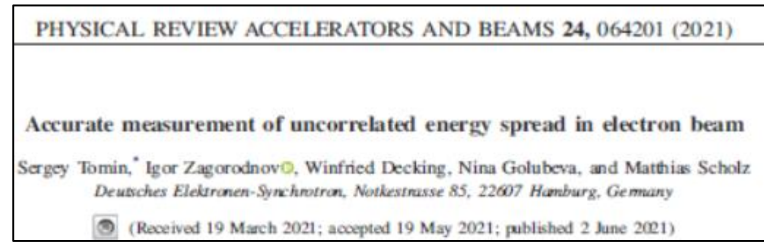




**With permission from Prat et al.*

Simulations for the EuXFEL

- The EuXFEL injector


PHYSICAL REVIEW ACCELERATORS AND BEAMS **24**, 064201 (2021)

Accurate measurement of uncorrelated energy spread in electron beam

Sergey Tomin,* Igor Zagorodnov, Winfried Decking, Nina Golubeva, and Matthias Scholz
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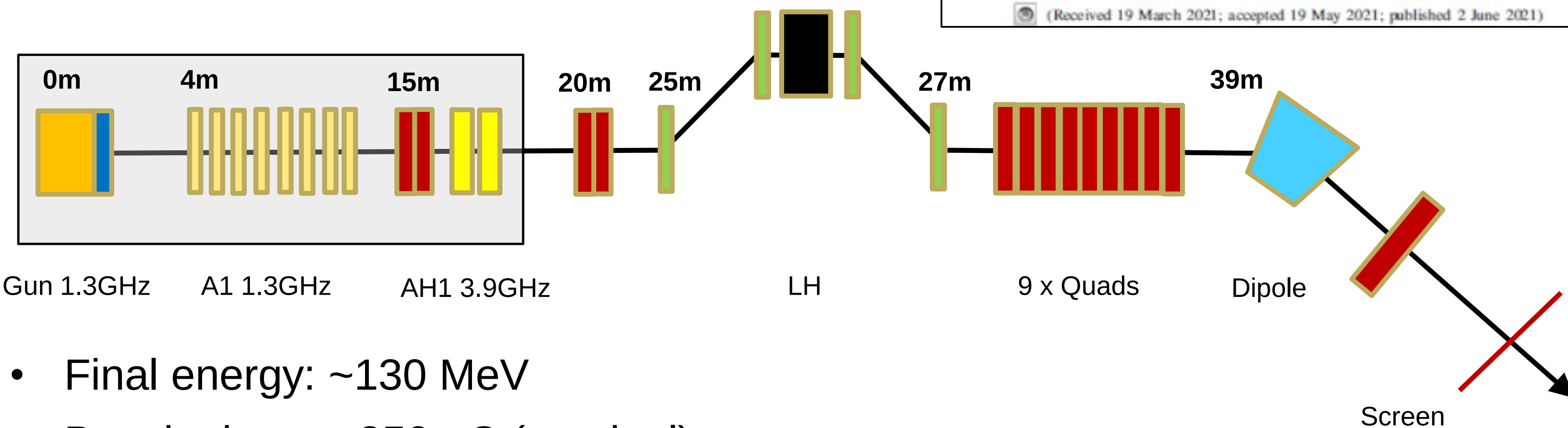
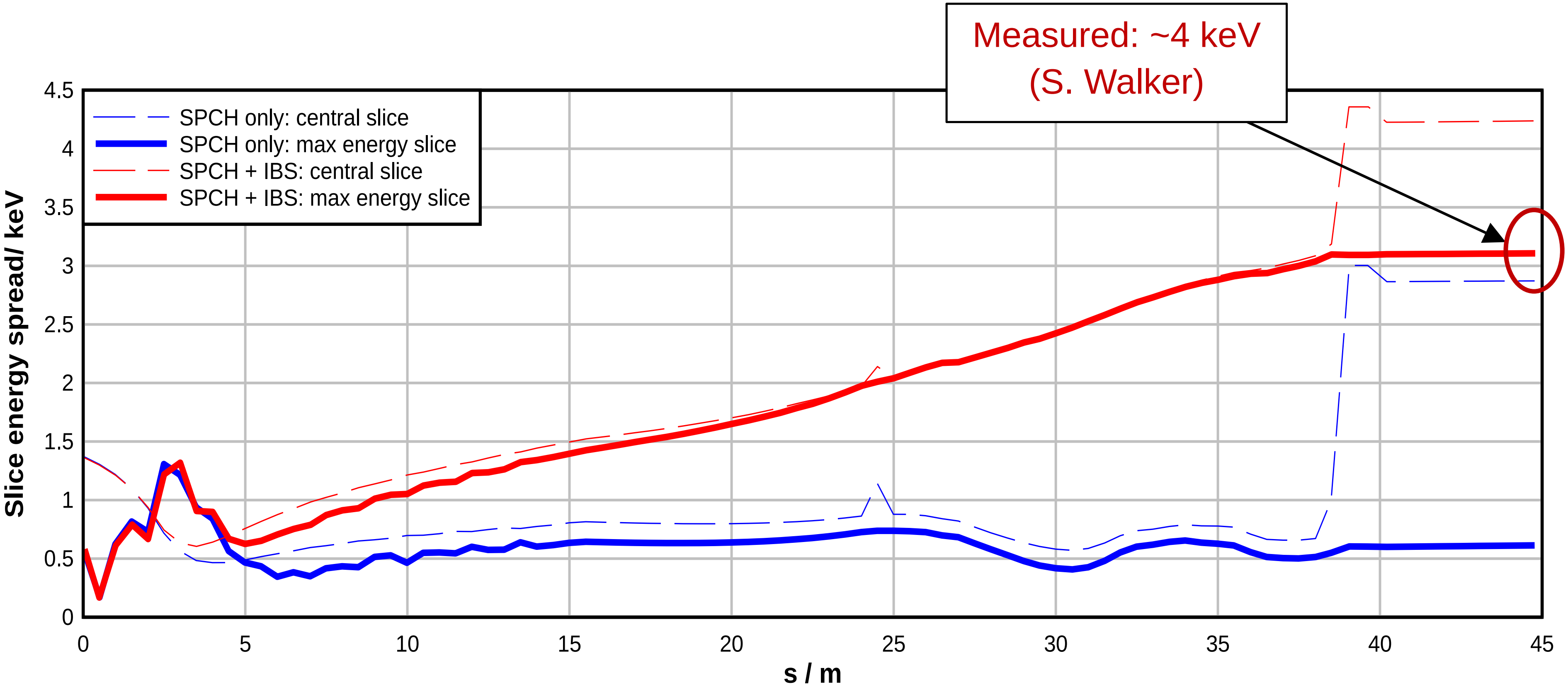


- Final energy: ~130 MeV
- Bunch charge: 250 pC (nominal)
- 3^{rd} harmonic AH1 switched off
- TDS structure (for the moment) ignored
- Energy spread measurement at the maximum energy slice

Simulations for the EuXFEL

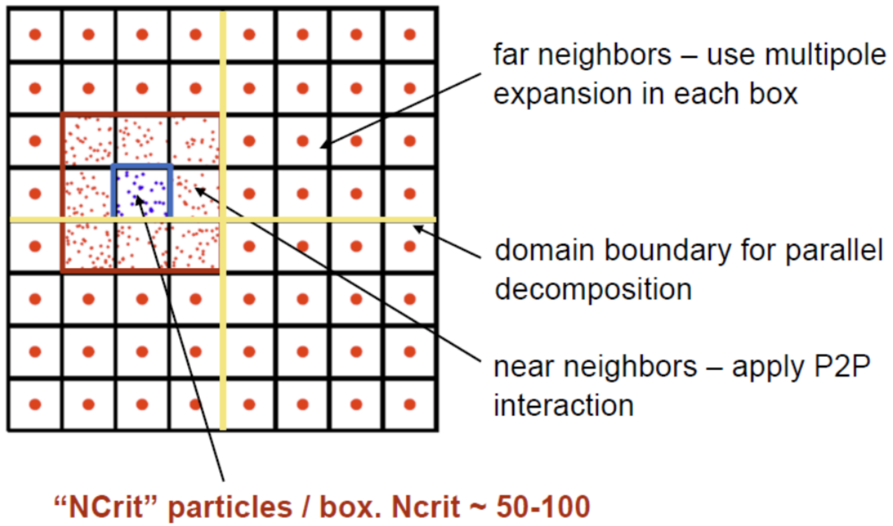
- Slice energy spread calculations

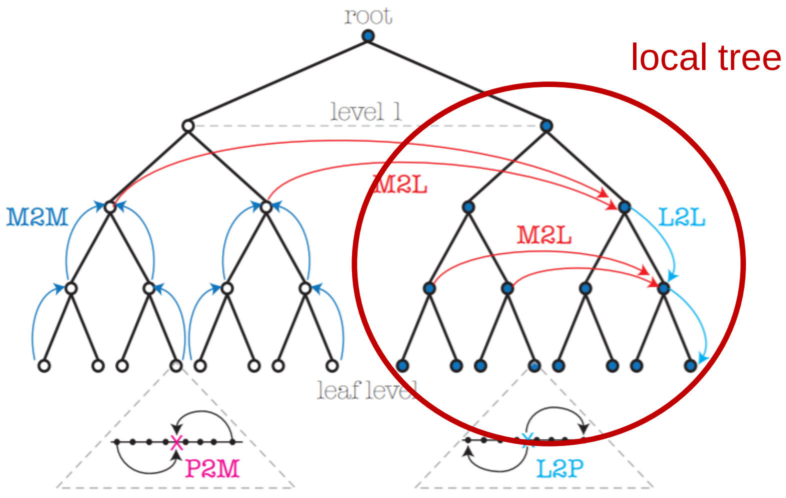


Thank you very much
for your attention

The Reptil code

- The Particle-FMM solver



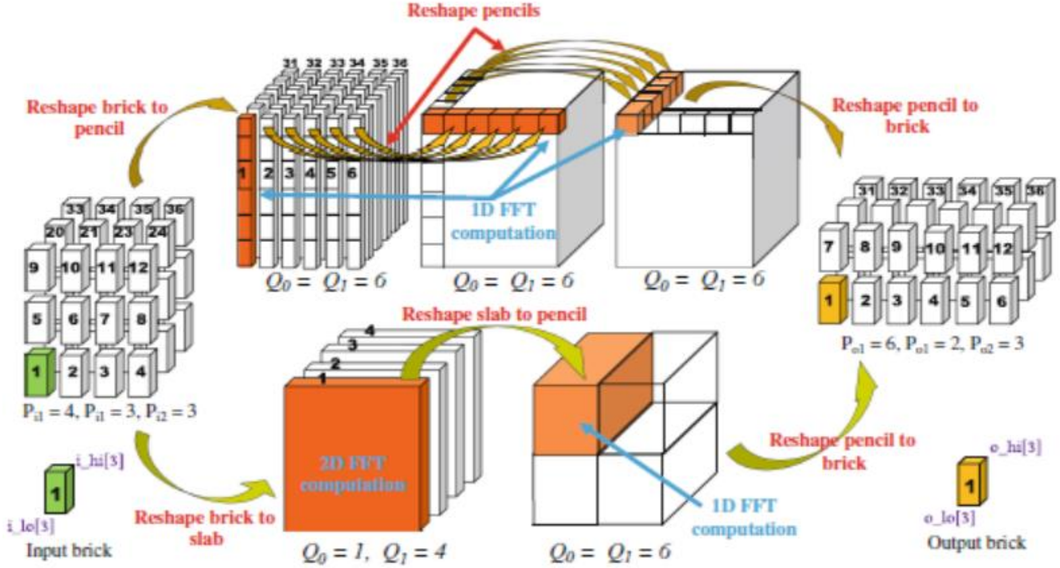


Hierarchic computation of mult. expansion (Rohklin & Greengard, 1987)

- Exponential expansions for M2L transformations -> reduce to O(N)O(P^3)
- MPI-parallelization using local tree decompositions
- Very efficient at high energies – noisy for space-charge dominated beams

The Reptil code

- The FFT-Green function solver
  - Uses Hockney's algorithm on doubled domains using IGF and shifted Green functions for the cathode charge
  - Using parallel 3D-FFT library – HeFFTe[1]
  - Backend FFTW, Intel MKL (tbd. CuFFT)



Decomposition strategies for parallel FFT:

Input data is reshaped such that each processor performs a single 1D-FFT at a time[1]

[1] *A. Ayala, et al., "heFFTe: Highly Efficient FFT for Exascale", (ICCS 2020)*