Exceptional Holonomy

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KMPB

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algebraic geometry

Bruno Klingler
arithmetic and algebraic geometry

Angela Ortega
algebraic geometry

Chris Wendl
contact and symplectic geometry

Riemannian holonomy, I

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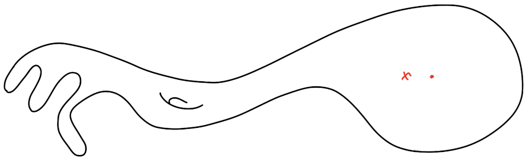

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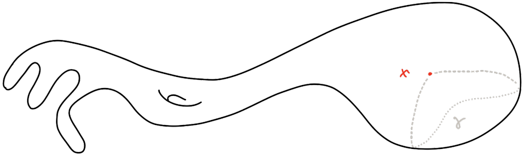

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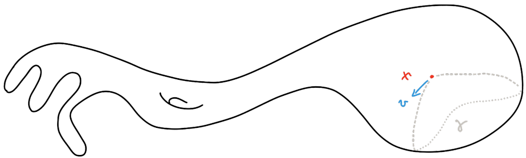

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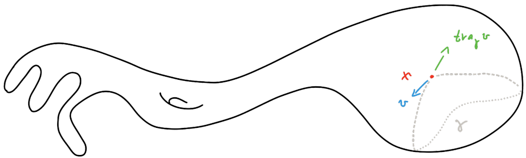

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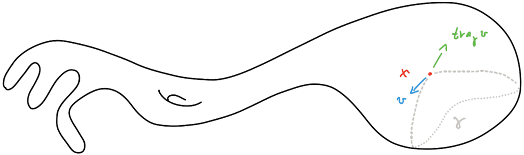

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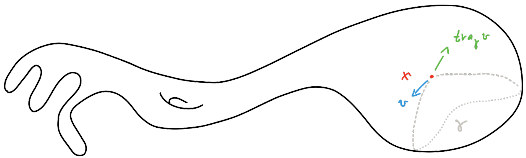

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The conjugacy class of $\mathrm{Hol}(g) \subseteq \mathrm{O}(n)$ is independent of choices.

Riemannian holonomy, II

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If (X, g, J) is a Kähler manifold, e.g., $X \subseteq \mathbf{C}P^N$ is a smooth projective variety, then $\mathrm{Hol}(g) \subseteq \mathrm{U}(n/2)$.

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Chern–Hodge decomposition 1957

The decomposition of $\Lambda^\bullet \mathbf{R}^n$ into irreducible representations G is inherited by the harmonic forms $\mathscr{H}^\bullet(X, g)$ and, therefore, by $\mathrm{H}^\bullet(X; \mathbf{R})$.

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Here Ricci-flatness is always caused by a non-trivial parallel spinor.

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with $\phi_0 := e^{123} - e^{145} - e^{167} - e^{246} - e^{275} - e^{347} - e^{356} \in \Lambda^3(\mathbf{R}^7)^*$ and $\psi_0 := *\phi_0 \in \Lambda^4(\mathbf{R}^7)^*$.

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Holonomy principle $\rightsquigarrow$ **(co)associative calibration** ϕ (ψ) on G_2–manifolds, **Cayley calibration** Φ on Spin(7)–manifolds.

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Kovalev 2003, Kovalev–Lee 2011, Corti–Haskins–Nordström–Pacini 2015. **Twisted connected sum construction** of compact G_2–manifolds. Produces $\geqslant 10^8$ examples.

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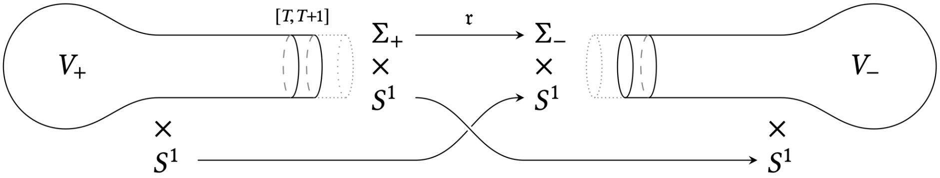

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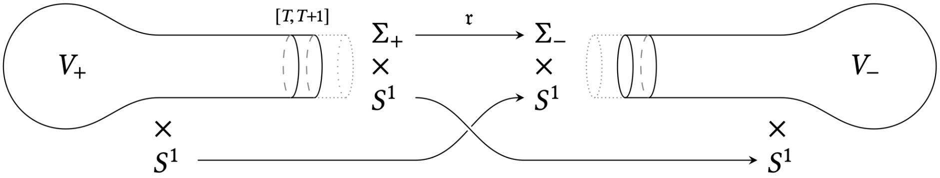

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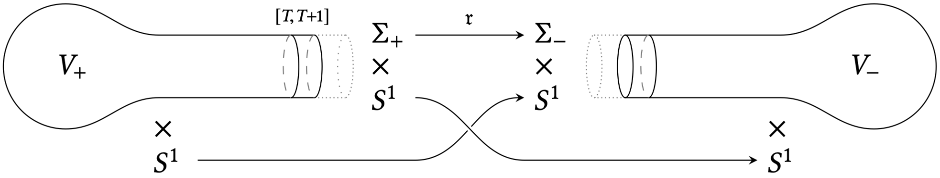

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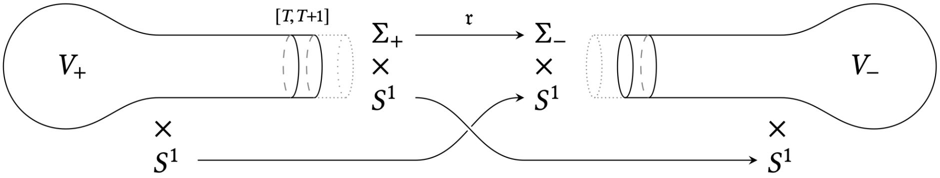

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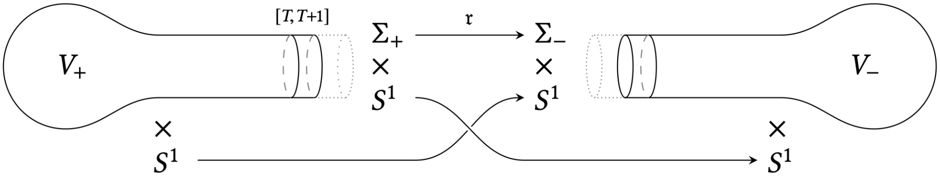

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Calibrated fibrations and adiabatic limits. Analogue of SYZ for Calabi–Yau 3–folds. Most of the above problems have adiabatic counterparts.

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New Construction methods. Physics seems to require G_2–spaces with specific singularities and suggests G_2 geometric transitions (cf. conifold transition).

Gauge theory and calibrated geometry. G_2– and Spin(7) manifolds can admit distinguished solutions of the Yang–Mills equations and minimal submanifolds ("instantons"). Various construction techniques exist, but invariants are so far out of reach. *What is the role of non-removable singularities?*

Topology of G_2– and Spin(7)–manifolds. Which topological restrictions are imposed by the existence of a metric with holonomy G_2 or Spin(7)?

Geometric heat flows. Various heat flows have been considered that should flow towards G_2– and Spin(7)–manifolds, e.g., Bryant's Laplacian flow. *Can the study of singularity models suggest new topological restrictions?*

Calibrated fibrations and adiabatic limits. Analogue of SYZ for Calabi–Yau 3–folds. Most of the above problems have adiabatic counterparts.

Thank you!