Electrostatic Time Dilation and Redshift

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As I have argued in my work M. Özer, Electrostatic Time Dilation and Redshift, Phys. Lett. B, **802** (2020) 135212, I believe the gravitational time dilation effect has an Electrostatic analog, namely The Electrostatic Time Dilation Effect.

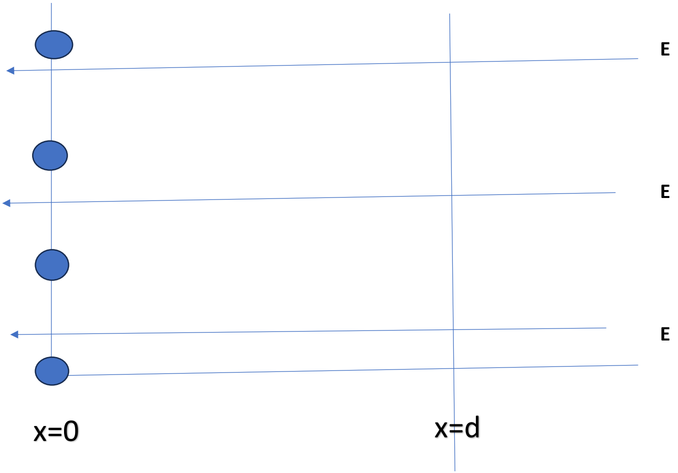
I will, first, obtain the Electrostatic Tine Dilation expression imitating the derivation of the Gravitational Time Dilation Expression

To this end, let us consider an electric field $\mathbf{E}$ in a region of the xy plane directed from right to left in the $-x$ axis.

Assume there are four identical positively charged atoms (cations) in their ground states at a level on the x axis where the electrical potential will be taken to be zero.

The atoms are prevented from interacting with each other through some mechanism.

WARNING: This should not be confused with the Stark Effect which takes place in an electric field as a result of the interaction of the field with the electric dipole moment of the atom. The effect considered here takes place for electrically charged systems with zero elecric dipole moment too, such as a proton, for example. Furthermore, two charged atoms with the same electric dipole moment in a uniform electric field but at different potentials would have the same Stark effect energy level splittings whereas their aging would be different due to the different electric potentials they are exposed to.


E
E
E
x=0
x=d

The energies of the atoms at this zero level of potential are given by

$$E(0) = m_i c^2 \tag{1}$$

Then we move one of the atoms to the right to a level d on the x axis. The energy of this atom is

$$E(d) = m_i c^2 + q \mid \mathbf{E} \mid d, \tag{2}$$

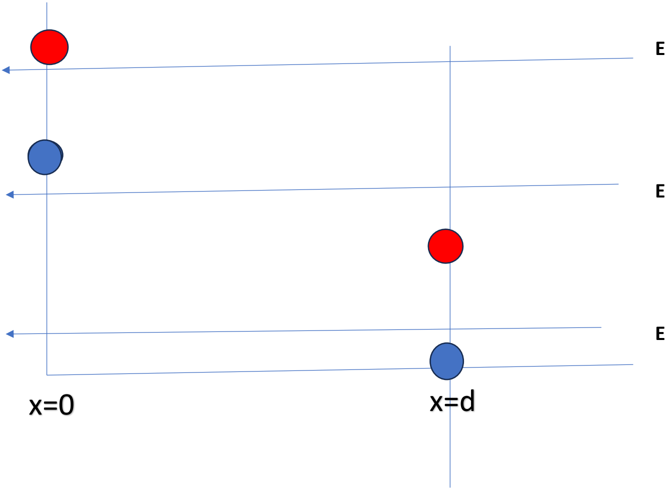
where q is the positive charge of the atom.

Then, we excite two of the atoms by letting them absorb a photon of energy $\mathcal{E}$ increasing their energy to

$$E^*(0) = m_i c^2 + \mathcal{E}. \tag{3}$$

Afterwards, we move one of these excited atoms to the right to the same level d as before. The energy of this atom will be

$$E^*(d) = m_i c^2 + \mathcal{E} + q \mid \mathbf{E} \mid d. \tag{4}$$


E
E
E
x=0
x=d

The fractional changes in the energies and vibrational frequencies are

$$\frac{E(d) - E(0)}{E(0)} = \frac{\delta E}{E(0)} = \frac{\delta f}{f(0)} = \frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2} = \frac{q}{m_i}\frac{\Delta\phi_e}{c^2}, \qquad (5)$$

in the ground state, and

$$\frac{\delta E^*}{E^*(0)} = \frac{\delta f^*}{f^*(0)} = \frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2\left(1 + \frac{\mathcal{E}}{m_i c^2}\right)} \approx \frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2} = \frac{q}{m_i}\frac{\Delta\phi_e}{c^2}, \quad (6)$$

in the excited state where $\mathcal{E}/m_i c^2 \ll 1$ has been implemented.

Here $f = E/h$, with h being the Planck constant, is the oscillation frequency of the atom and $\Delta\phi_e =\mid \mathbf{E} \mid d > 0$ is the electrical potential difference between the positions of the two atoms.

As a result of the moving of the atoms, the oscillation frequency of the atom changes from $f(0)$ to $f(d)$ in the ground state, and from $f^*(0)$ to $f^*(d)$ in the excited state. These frequencies at different positions are related to each other in the ground and excited states, respectively as

$$\begin{aligned} f(d) &= f(0)\left(1+\frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2}\right) = f(0)\left(1+\frac{q}{m_i}\frac{\Delta\phi_e}{c^2}\right), \\ f^*(d) &= f^*(0)\left(1+\frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2}\right) = f^*(0)\left(1+\frac{q}{m_i}\frac{\Delta\phi_e}{c^2}\right), . (7) \end{aligned}$$

Taken as a frequency reference, such an atom can be considered as a clock whose rate, its oscillation frequency, is faster the farther the position d is. Thus, the time intervals measured by such a clock are proportional to its oscillation frequencies and are given by

$$\begin{aligned} \Delta T(d) &= \Delta T(0)\left(1+\frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2}\right) = \Delta T(0)\left(1+\frac{q}{m_i}\frac{\Delta\phi_e}{c^2}\right), \\ \Delta T^*(d) &= \Delta T^*(0)\left(1+\frac{q}{m_i}\frac{\mid \mathbf{E} \mid d}{c^2}\right) = \Delta T^*(0)\left(1+\frac{q}{m_i}\frac{\Delta\phi_e}{c^2}\right), \end{aligned} \quad (8)$$

which indicate how such atoms/clocks age.

Thus we find out that atoms at farther positions, or higher potentials, age faster than those at lower positions in the electric field.

Note that this Electrical Time Dilation Effect has been obtained using simple classical physics namely electrostatics.

Therefore the results obtained must be a fact of Nature.

As I will show later, the same result can be obtained from the spacetime metric I proposed for a metric unification of Gravity and Electromagnetism outside a spherical object of mass M and electric charge Q, which is

$$
\begin{aligned}
ds^2 = & -\left(1 - 2\frac{m_p}{m_i}\frac{GM}{c^2 r} + 2\frac{q}{m_i}\frac{k_e Q}{c^2 r}\right) c^2 dt^2 \\
& + \left(1 - 2\frac{m_p}{m_i}\frac{GM}{c^2 r} + 2\frac{q}{m_i}\frac{k_e Q}{c^2 r}\right)^{-1} dr^2 \\
& + r^2 \left(d\theta^2 + sin^2\theta d\phi^2\right)
\end{aligned}
\tag{9}
$$

I have provided five different arguments along with a formal derivation of the above metric on my web page www.muratozer.org

It should be noted that these electrical time dilation effects become contraction effects for anions, atoms with a total negative electric charge, for $\Delta\phi_e > 0$.

This is an effect with no counterpart in the gravitational case due to the absence of negative inertial mass (in our part of the Universe).

The attentive listener may feel disturbed by the presence of the term charge-to-mass ratio, q/m_i, in the above equations.

The common understanding, so far, has been that the gravitational aging expression at a height H from the surface of the earth is given by

$$\Delta T(H) = \Delta T(0)\left(1 + \frac{gH}{c^2}\right),$$

and the gravitational time dilation is a property of the point under consideration in space, or the gravitational potential at that point.

However, as is clear in Eq. (9), the g_{00} component of the metric does contain the term m_p/m_i as a result of which the gravitational time dilation is a property of the test particle present at the given height H. The aging of the test particle is actually given by

$$\Delta T(H) = \Delta T(0)\left(1 + \frac{m_p}{m_i}\frac{gH}{c^2}\right), \tag{10}$$

which shows very clearly that the time dilation effect is a property of the test particle and the gravitational potential. The fractional time dilation is equal to the gravitational potential energy of the test particle divided by its rest energy.

For example, considered over the total time-span of Earth (4.6 billion years), a clock set in a geostationary position at an altitude of 9,000 meters above sea level, such as perhaps at the top of Mount Everest (prominence 8,848 m), would be about 39 hours ahead of a clock set at sea level,

The Equivalence Principle for Electricity

I believe that there is an Equivalence Principle for Electric and Magnetic fields, too.

Let us introduce the Equivalence Principle for Electricity:

Let us consider a collection of charged particles with different q/m_i ratios moving in an external electric field $\mathbf{E}$. The equation of motion of the *nth* particle will be

$$m_i^{(n)} \frac{d^2\mathbf{r}^{(n)}}{dt^2} = q^{(n)}\mathbf{E} + \sum_M \mathbf{F}(\mathbf{r}^{(n)} - \mathbf{r}^{(M)}), \qquad n = 1, 2, ..., N, \tag{11}$$

where $\mathbf{F}$ denotes the interparticle interactions. The spacetime transformations

$$
\begin{aligned}
t' &= t, \qquad (12)\\
\mathbf{r}^{\,\prime(n)} &= \mathbf{r}^{\,(n)} - \frac{1}{2}\left(q^{(n)}/m_i^{(n)}\right)\mathbf{E}\,t^{\,\prime 2}, \qquad n = 1, 2, ..., N,
\end{aligned}
$$

cast Eq. (11) to

$$
\begin{aligned}
m_i^{(n)}\frac{d^2\mathbf{r}^{\,\prime(n)}}{dt'^2} + m_i^{(n)}\left(q^{(n)}/m_i^{(n)}\right)\mathbf{E} &= q^{(n)}\mathbf{E} + \sum_M \mathbf{F}(\mathbf{r}^{\,\prime(n)} - \mathbf{r}^{\,\prime(M)})\\
m_i^{(n)}\frac{d^2\mathbf{r}^{\,\prime(n)}}{dt'^2} &= \sum_M \mathbf{F}(\mathbf{r}^{\,\prime(n)} - \mathbf{r}^{\,\prime(M)}). \qquad (13)
\end{aligned}
$$

The electric force on the *nth* particle

$$
\mathbf{F}_E^{(n)} = q^{(n)}\mathbf{E} \qquad (14)
$$

has been canceled by the following fictitious force

$$
\mathbf{F}_{fict}^{(n)} = -m_i^{(n)}\left(q^{(n)}/m_i^{(n)}\right)\mathbf{E} = -q^{(n)}\mathbf{E}. \qquad (15)
$$

In other words,

$$\mathbf{F}_{E}^{(n)} + \mathbf{F}_{fict}^{(n)} = 0. \tag{16}$$

As is seen clearly from Eq. (15), it is not required for the cancellation of the external electric force on the particle locally that the ratio $(q^{(n)}/m_i^{(n)})$ be 1.

This ratio can be equal to any value, as is the case in Nature. Furthermore, the cancellation of the electric field in the vicinity of

the *nth* particle occurs independently of the cancellation of the electric field for the other particles.

So, the ratio $(q^{(n)}/m_i^{(n)})$ is not required to be the same for all particles.

Thus we can state the electrical equivalence principle, adapting the statement of the gravitational equivalence principle in [R. C. Tolman, Relativity Thermodynamics And Cosmology, (Dover Publications, Inc., 1987] to electricity, as

"It is always possible at any space-time point of interest to transform to coordinates such that the effects of electricity will disappear over a differential region in the neighborhood of that point, which is taken small enough so that the spatial and temporal variation of electricity within the region may be neglected."

or as

"It is impossible to distinguish the fictitious inertial force from the real electrical force in a local region containing a single particle."

So, this discussion shows that just as the equivalence of an accelerating frame in deep space and a frame at rest in a local uniform gravitational field, a similar equivalence exists for electricity too.

A frame containing a charged particle of mass m_i and charge q and accelerating in deep space at an acceleration $\mathbf{a} = -(q/m_i)\mathbf{E}$ is equivalent to a laboratory frame at rest where there is an identical particle in it and a uniform electric field $\mathbf{E}$.

The electricity experiments involving the charged particle in these two frames would give identical results.

When a charged particle or a collection of them move in a region where there are superimposed gravitational and electric fields we add the term $m_g^{(n)} g$ to the right hand side of Eq. (10) and the term $-(1/2) m_g^{(n)} g t'^2$ to the right hand side of Eq. (11) combining the gravitational transformations and the electrical ones. Applying them simultaneously on the *n*th charged particle we see that in the immediate neighborhood of a charged particle moving in a combined gravitational and electric field both fields are canceled and the particle "falls" freely. In the absence of interparticle interactions, each particle, then, follows its own geodesic according to

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0, \tag{17}$$

where the Christoffel symbols (connection coefficients) $\Gamma^\mu_{\alpha\beta}$ depend on the m_p/m_i ratio and the q/m_i ratio of the test particle and λ is an affine parameter, such as the proper time τ or the proper length s, of the geodesic.

It is often said that had the ratio q/m_i been the same for all charged particles a general relativistic theory of electromagnetism could have been formulated B. Mashhoon, F.Gronwald, H.I.M. Lichtenegger, Lect. Notes Phys. **562**, 83 (2001).

The resulting theory would have been a single-metric theory like Gravity.

However, that the ratio q/m_i is different for differently charged particles is no obstacle to the formulation of a general relativistic theory of electromagnetism.

Such a theory would be a multi-metric theory.

In other words, there would be a different metric for each charged particle; the theories for electrons and protons, for example, would be different due to their q/m_i ratios.

EXPERIMENT should be the ultimate arbiter.

Obtaining the Electrostatic Time Dilation from the Equivalence Principle for Electricity

The electrostatic time dilation effect can also be obtained from the electrical equivalence principle introduced above.

Let us then consider a cabin in a rocket in deep space where there are no fields of any kind.

Let there be two observers at the bottom and the top of the cabin of height d.

Let us establish a Cartesian coordinate frame and choose the direction of motion of the rocket as the upward z-axis.

Let the bottom of the cabin and the origin of the coordinate frame overlap at time $t = 0$.

Let the rocket accelerate upward at $a_{Rocket} = \frac{q}{m_i}|\mathbf{E}|$, where q is the electric charge of some chosen atom, m_i is its inertial mass, and $\mathbf{E}$ is some known electric field directed in the negative z direction as the gravitational field for convenience.

We will assume q to be positive for simplicity. The positions of the bottom and top observers will be given by

$$\begin{aligned} z_B(t) &= \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|t^2, \\ z_T(t) &= d + \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|t^2. \end{aligned} \tag{18}$$

Let the observer at the bottom send up two light pulses at $t = t_1$ and at $t = t_2 = t_1 + \Delta\tau_B$ separated by a time interval of $\Delta\tau_B$.

Let these pulses be received by the observer at the top at times $t = t_1^{'}$ and $t = t_2^{'} = t_1{'} + \Delta\tau_T$ separated by a time interval of $\Delta\tau_T$.

The distance traveled by the first pulse is

$$z_T(t_1^{'}) - z_B(t_1) = c(t_1{'} - t_1), \tag{19}$$

which gives

$$d + \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|t_1^{'\,2} - \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|t_1^2 = c(t_1^{'} - t_1). \tag{20}$$

Similarly, the distance traveled by the second pulse is

$$z_T(t_1^{'} + \Delta\tau_T) - z_B(t_1 + \Delta\tau_B) = c(t_1{'} + \Delta\tau_T - t_1 - \Delta\tau_B), \tag{21}$$

which gives

$$d + \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|(t_1^{'} + \Delta\tau_T)^2 - \frac{1}{2}\frac{q}{m_i}|\mathbf{E}|(t_1 + \Delta\tau_B)^2 = c(t_1{'} + \Delta\tau_T - t_1 - \Delta\tau_B), \tag{22}$$

Using Eq. (20) in Eq. (22) and setting $t_1^{'} = t_1 + d/c$ and neglecting all terms second order in time, we get

$$\Delta\tau_T(c - \frac{q}{m_i}|\mathbf{E}|\frac{d}{c}) \approx c\Delta\tau_B \tag{23}$$

or,

$$\Delta\tau_T \approx \Delta\tau_B(1 + \frac{q}{m_i}|\mathbf{E}|\frac{d}{c^2}) = \Delta\tau_B\left(1 + \frac{q\Delta\phi_e}{m_i c^2}\right). \tag{24}$$

This is the same as in Eq. (8) above.

Now, the electrical equivalence principle tells us that this accelerated frame in deep space where there exist no fields of any kind is equivalent to a stationary frame (a lab) on Earth and the elapsed times between two events separated by a distance d in an electric field is given by Eq. (24).

Note that we can recover the gravitational time dilation effect at once by replacing $q/m_i \longrightarrow m_p/m_i = 1$, $|\mathbf{E}| \longrightarrow g$, and $d \longrightarrow H$.

The result is

$$\Delta\tau_T \approx \Delta\tau_B\left(1 + \frac{m_p}{m_i} g \frac{H}{c^2}\right) = \Delta\tau_B\left(1 + g\frac{H}{c^2}\right), \tag{25}$$

which is the correct expression for gravitational time dilation.

Isn't the similarity between the gravitational and electrical effects striking enough?

Obtaining the Electrostatic Time Dilation from the Metric of Our Unified Theory

We will now show that the electrostatic time dilation effect can also be obtained from the line element of our unified theory.

$$
\begin{aligned}
ds^2 = &-\left(1 - 2\frac{m_p}{m_i}\frac{GM}{c^2 r} + 2\frac{q}{m_i}\frac{k_e Q}{c^2 r}\right) c^2 dt^2 \\
&+ \left(1 - 2\frac{m_p}{m_i}\frac{GM}{c^2 r} + 2\frac{q}{m_i}\frac{k_e Q}{c^2 r}\right)^{-1} dr^2 \\
&+ r^2\left(d\theta^2 + sin^2\theta d\phi^2\right), \qquad (26)
\end{aligned}
$$

We contemplate a laboratory experiment with a charged metal sphere of radius of half to one meter. The proper time interval $d\tau$ for a test particle located at the coordinate r is related to its coordinate time interval dt by the relation

$$d\tau = \left(1 + 2\frac{q}{m_i}\frac{k_e Q}{c^2 r}\right)^{1/2} dt, \tag{27}$$

where the gravitational term due to the mass of the sphere is much smaller than the electrical term and can be neglected.

We assume that the sphere is negatively charged ($Q = -|Q|$) so that its electric field is directed from right to left along the horizontal towards the sphere.

The ratio of the proper time intervals for two clocks (test particles) at $r + d$ and r for $r > R$ (the radius of the sphere) is thus

$$
\begin{aligned}
\frac{d\tau(r+d)}{d\tau(r)} &= \frac{\Delta T(d)}{\Delta T(0)} \\
&= \left(1 - 2\frac{q}{m_i c^2}\frac{k_e|Q|}{c^2(r+d)}\right)^{1/2} \Big/ \left(1 - 2\frac{q}{m_i c^2}\frac{k_e|Q|}{c^2 r}\right)^{1/2} \\
&= \left(1 + 2\frac{q}{m_i c^2}\phi_e(r+d)\right)^{1/2} \Big/ \left(1 + 2\frac{q}{m_i c^2}\phi_e(r)\right)^{1/2}
\end{aligned}
$$

because the two coordinate time intervals are equal and $\phi_e(r) = -k_e|Q|/r$ is the electric potential.

Since the rest energy $m_i c^2$ of the test particle is much larger than its potential energy $q\phi_e$ the above ratio is approximated by

$$\frac{\Delta T(d)}{\Delta T(0)} \approx \left(1 + \frac{q\Delta\phi_e}{m_i c^2}\right), \tag{28}$$

where $\Delta\phi_e = \phi_e(r+d) - \phi_e(r) > 0$ is the electric potential difference.

This is the same expression as those in Eq. (8).

It should be noted that even though the electric potential difference in Eq. (28) is due to a charged metal sphere, the result is general, as is the case in gravity.

The potential difference may be that in an electric field created by other means, as in Eq. (8).

Proposed Experiment to Measure the Electrostatic Time Dilation

This predicted electrostatic time dilation effect can be measured by high precision clocks similar to the atomic ones.

What is required for such a measurement is *ionic clocks* whose oscillators are positively (or negatively) charged ionic atoms whose energy levels will split in an electric field depending on their positions in the field.

T. Rosenband et al., Frequency Ratio of Al^+ and Hg^+ Single-Ion Optical Clocks; Metrology at the 17th Decimal Place, Science **319** (2008) 1808.

C. W. Chou. D. B. Hume, J. C. J. Koelemeij, D. J. Wineland, T. Rosenband, Frequency Comparison of Two High-Accuracy Al^+ Optical Clocks, Phys. Rev. Lett., **104** (2010) 070802.

M. G. Kozlov, M. S. Safronova, J. R. Crespo Lopez-Urritia, P. O. Schmidt, Highly charged ions: Optical clocks and applications in fundamental physics, Rev. Mod. Phys., **90** (2018) 045005.

For example, the fractional change in the oscillation frequencies of two $^{27}Al^{+}$ optical ion clocks in a static electric field for a potential difference of $\Delta\phi_e(inV)$ between them would be

$$\frac{\delta f}{f(0)} = \frac{q}{m}\frac{\Delta\phi_e}{c^2} = 3.979 \times 10^{-11}\Delta\phi_e/V, \tag{29}$$

where $q = e = 1.602 \times 10^{-16}C$ is the charge of $^{27}Al^{+}$ and $m = 4.480 \times 10^{-26}kg$ its mass.

This would be much larger than the special relativistic and gravitational time dilation effects reported in the second reference above ($\frac{\delta f}{f(0)} \approx 10^{-16} - 10^{-18}$) depending on the value of $\Delta\phi_e$.

The verification of this fractional frequency change would be a clear indication that the unification metric in Eq. (26) is correct.

Bound Muon Decay Rate Experiments

Experiments performed around the 60s showed that the decay lifetimes of muons bound to atomic nuclei were larger than the lifetimes of free muons.

As shown by Apsel the experimental data were better explained when the alleged electrical time dilation effect was included along with the special relativistic time dilation .

D. Apsel, Time Dilations in Bound Muon Decay, Gen. Rel. Grav., **13** (1981) 605 and references therein

First, let us obtain the expression for the proper time of a particle moving in a gravitational, electric or magnetic field.

Assuming that ds^2 is diagonalized, it can be cast into

$$
\begin{aligned}
ds^2 &= \left[1 - \frac{-g_{11}(dx^1/dt)^2 - g_{22}(dx^2/dt)^2 - g_{33}(dx^3/dt)^2}{g_{00}c^2}\right] c^2 g_{00} dt^2 \\
&= \left(1 - \frac{v^2}{c^2}\right) c^2 g_{00} dt^2, \qquad (30
\end{aligned}
$$

where the velocity of the muon is defined through

$$
v^2 = \frac{-g_{11}(dx^1/dt)^2 - g_{22}(dx^2/dt)^2 - g_{33}(dx^3/dt)^2}{g_{00}}. \qquad (31)
$$

Thus the proper time of the muon is given by

$$
\begin{aligned}
d\tau &= \left(1-\frac{v^2}{c^2}\right)^{1/2}(-g_{00})^{1/2}dt \\
&= \left(1-(\alpha Z)^2\right)^{1/2}\left(1-2(\alpha Z)^2\right)^{1/2}dt, \qquad (32)
\end{aligned}
$$

where in the Bohr model of the atom $v^2 = k_e Z e^2/m_\mu r$, $r = r_1 = \hbar^2/(Z k_e m_\mu e^2)$ in the ground state, Z is the atomic number, $\alpha = k_e e^2/\hbar c \approx 1/137$ is the fine structure constant, and the g_{00} in Eq. (26) has been used. *

Similar predictions are also presented by T. Yarman, A. L. Kholmetskii, O. V. Missevitch, Int. J. Theor. Phys. 50 (2011) 1407.

*In Apsel's work $d\tau/dt = \left(1-(\alpha Z)^2\right)^{1/2} - (\alpha Z)^2$.

Z	R_{UGE}	R_{Apsel}	$R_{Exp.}$
6	0.997	0.997	1.00 ± 0.02
13	0.99	0.99	0.99 ± 0.04
20	0.97	0.97	1.00 ± 0.03
26	0.95	0.95	0.97 ± 0.04
30	0.93	0.93	0.95 ± 0.03
42	0.86	0.86	0.93 ± 0.05
48	0.81	0.81	0.84 ± 0.07
53	0.77	0.77	0.66 ± 0.07
74	0.54	0.55	0.53 ± 0.05
82	0.43	0.44	0.40 ± 0.10

Tablo: Comparison of theory and experiment as a function of Z.

In this Table the ratios $R = d\tau/dt$ of the muon's proper lifetime to its (laboratory) lifetime in the present theory and that of Apsel's as a function of the atomic number Z are compared with the experimental values provided in Apsel's paper. As is seen clearly, the existence of the electrical time dilation effect is indispensable.

*T*HERE IS MUCH BRAND NEW and GRAND NEW PHYSICS

AWAITING DISCOVERY

THAT CAN BE TESTED BY SIMPLE EXPERIMENTS

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Thanks for listening...