Achieved Information Gain

as a Sustainability Measure

Torsten Enßlin
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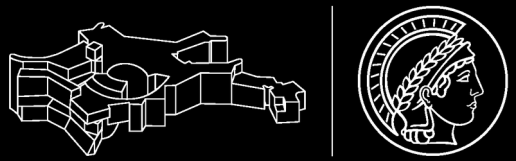
$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{A}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$



MAX PLANCK INSTITUTE
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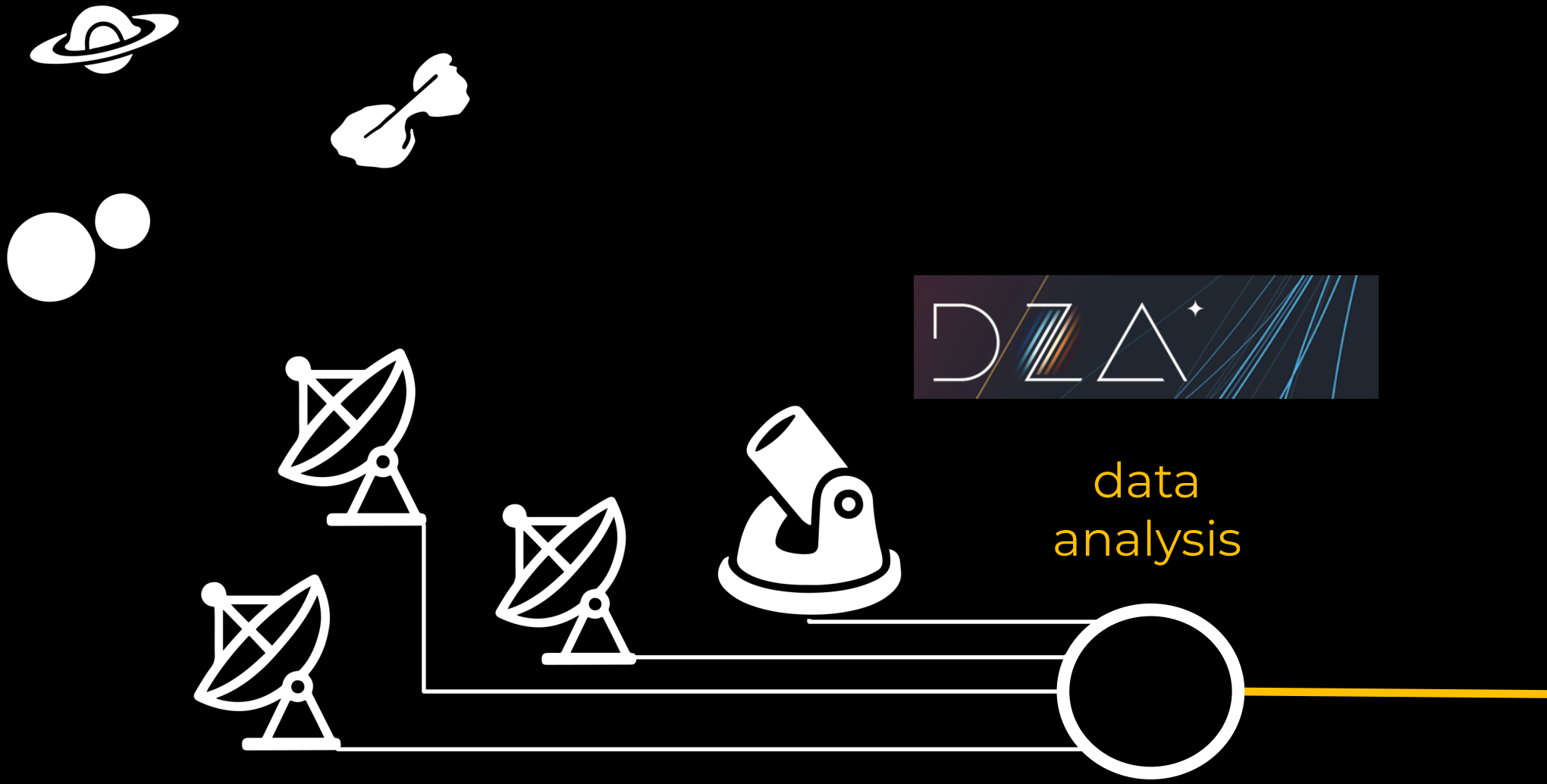
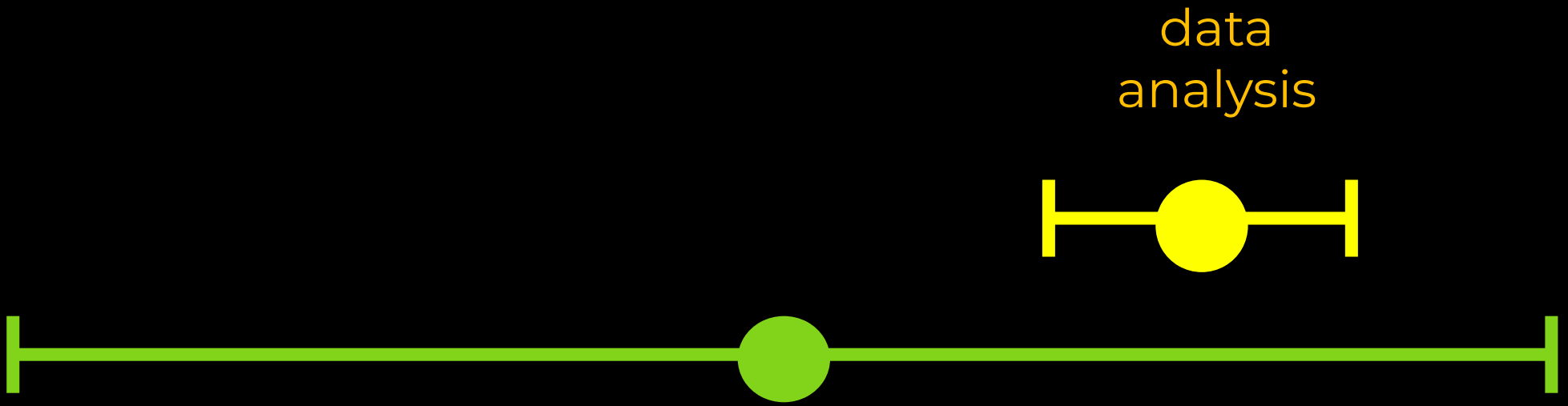
$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{A}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$

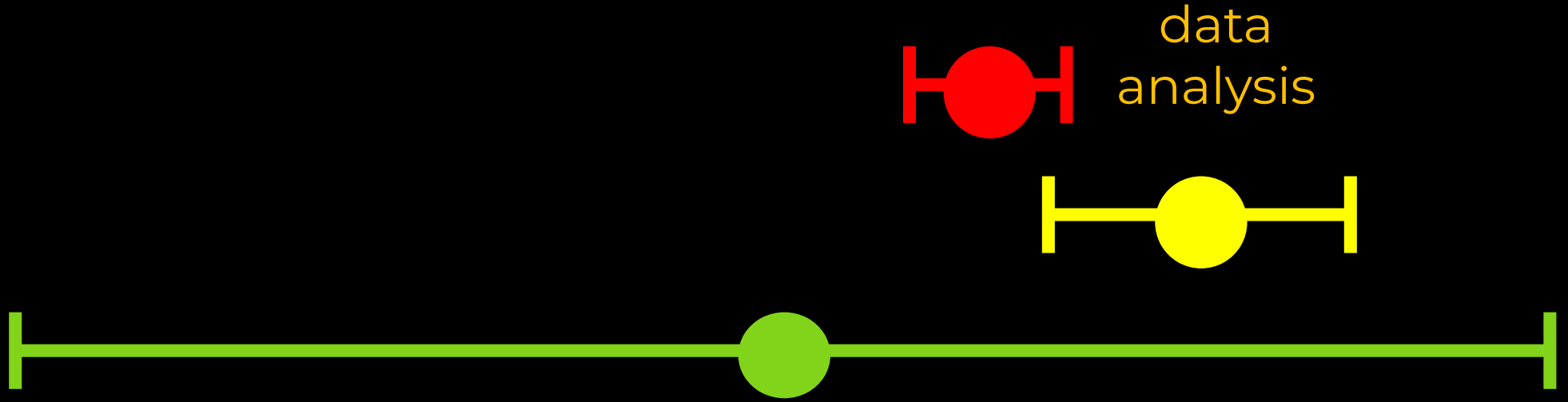

DZA
data
analysis

$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{B}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$



$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{A}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$

$$= \mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_0) - \mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}})$$



$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{A}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$

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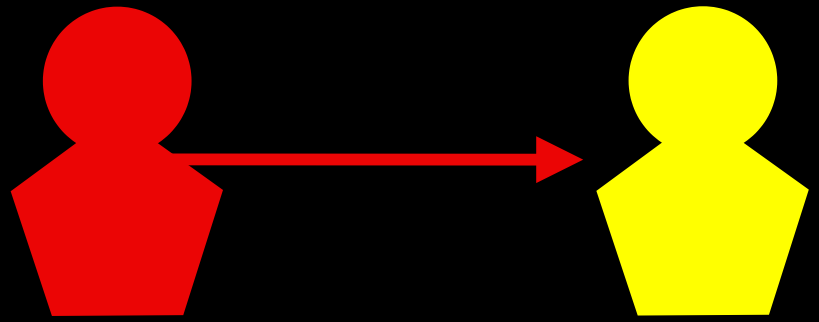
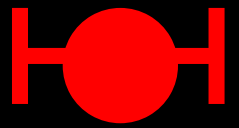
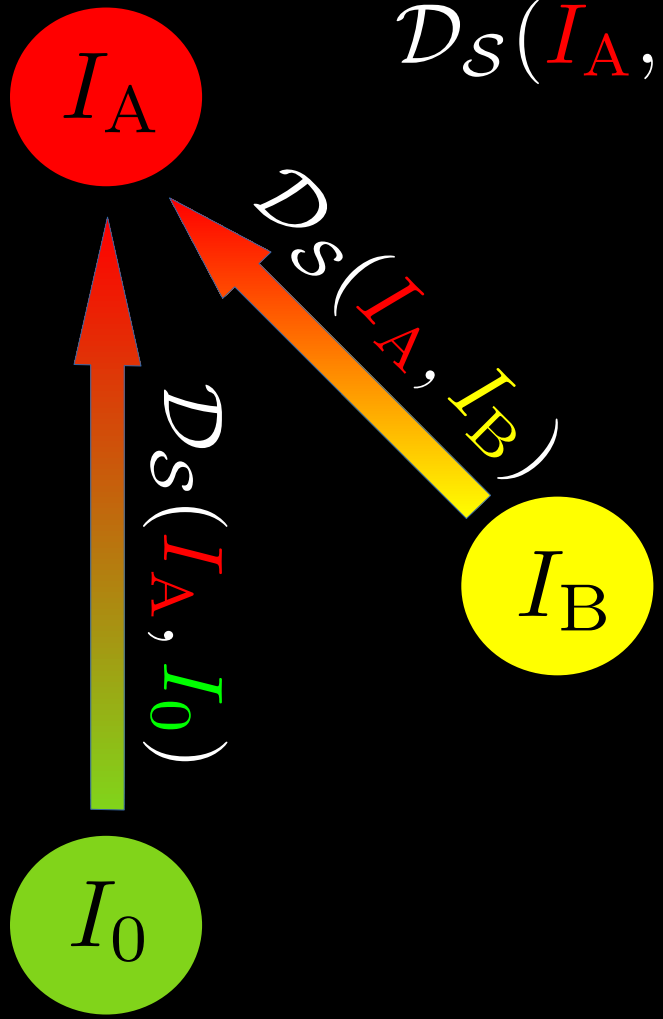




data
analysis



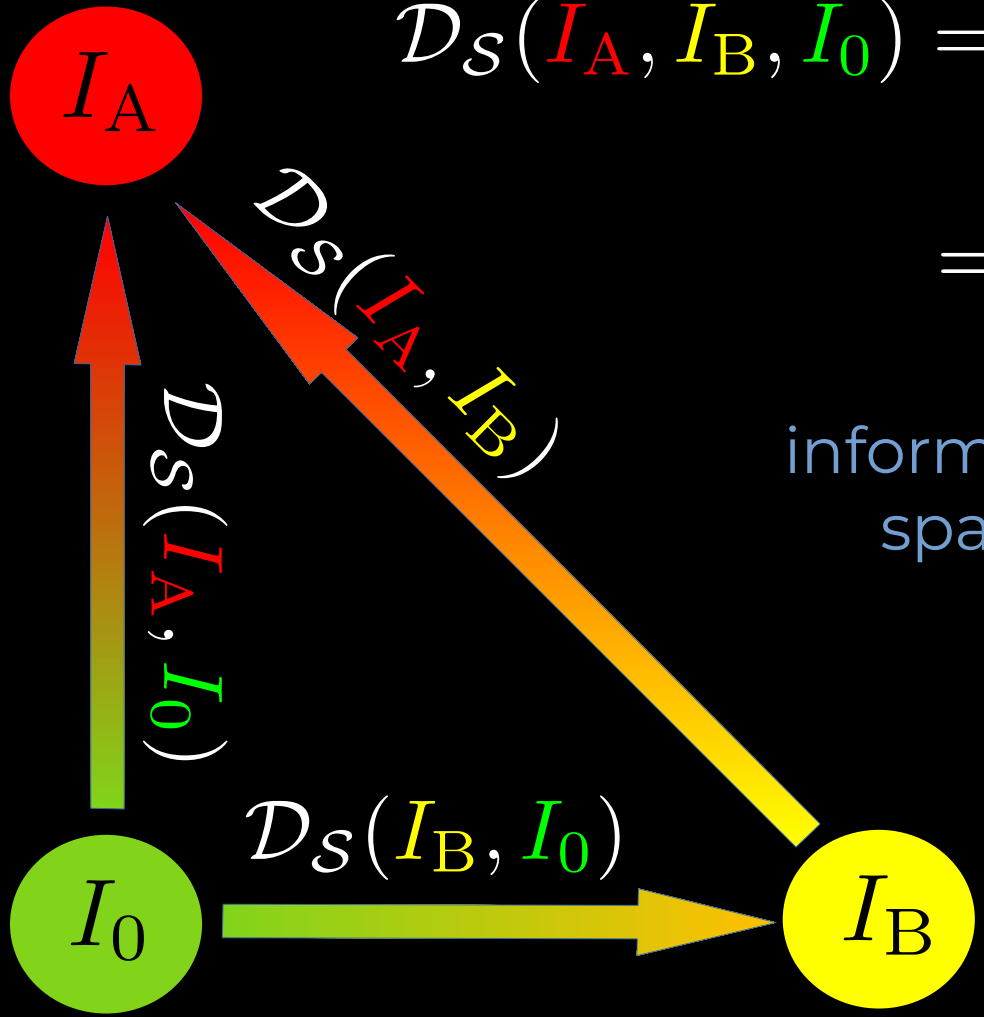
$$\mathcal{D}s(I_\mathrm{A}, I_\mathrm{B}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_\mathrm{A}) \ln \frac{\mathcal{P}(s|I_\mathrm{B})}{\mathcal{P}(s|I_0)}$$

$$= \mathcal{D}s(I_\mathrm{A}, I_0) - \mathcal{D}s(I_\mathrm{A}, I_\mathrm{B})$$

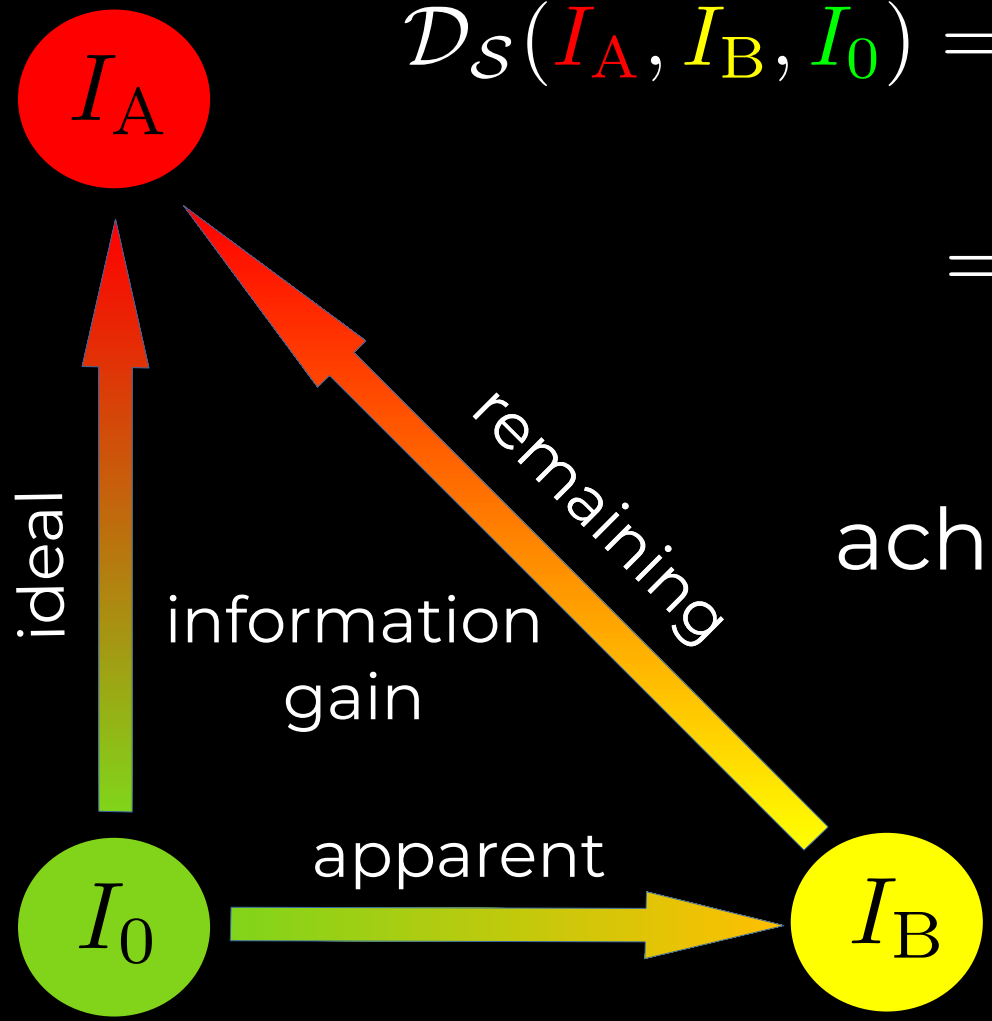
information space

$$\mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}}, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_{\mathrm{A}}) \ln \frac{\mathcal{P}(s|I_{\mathrm{B}})}{\mathcal{P}(s|I_0)}$$

$$= \mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_0) - \mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}, I_{\mathrm{B}})$$



information space

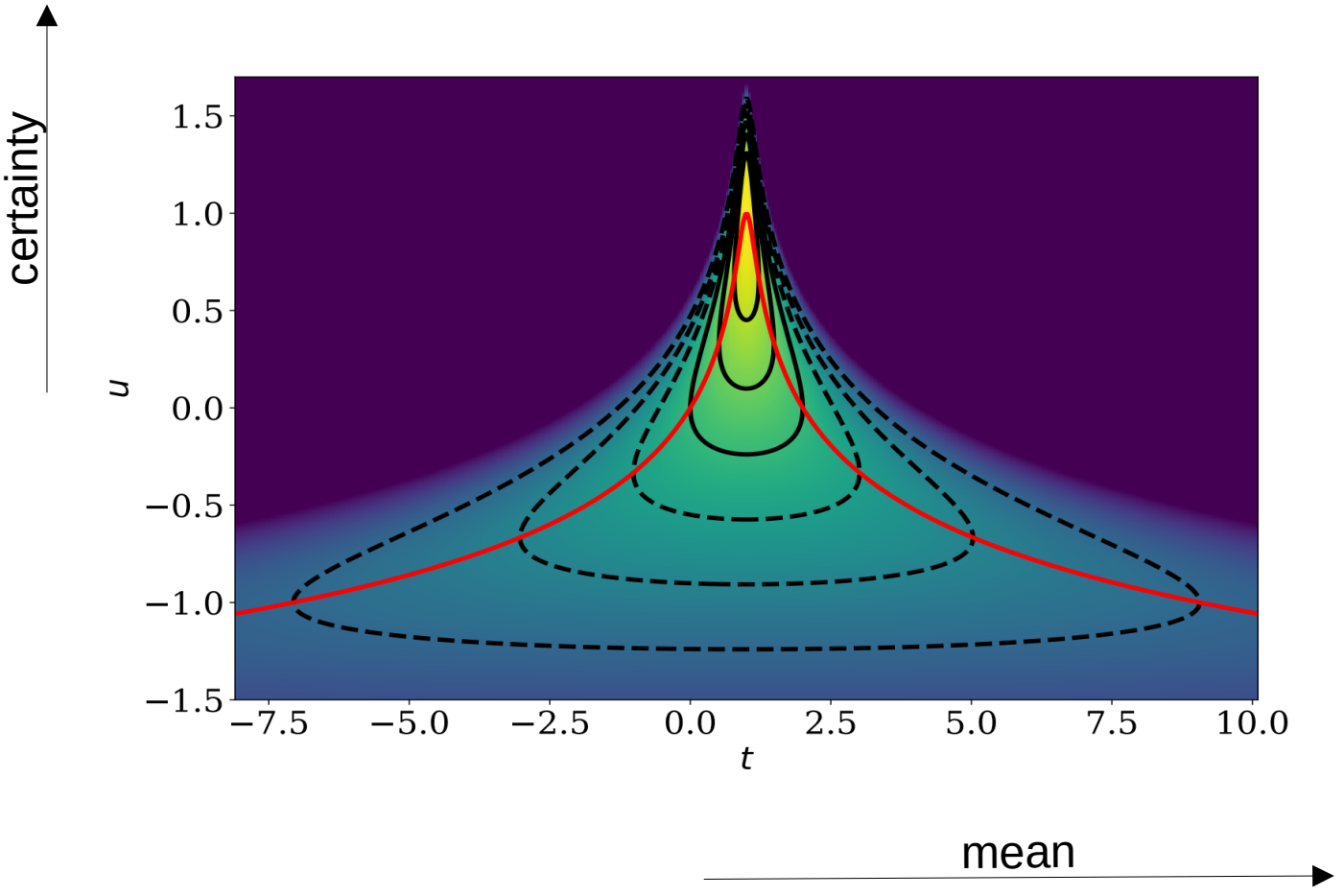


$$\mathcal{D}_{\mathcal{S}}(I_A, I_B, I_0) = \int_{\mathcal{S}} \mathrm{d}s\, \mathcal{P}(s|I_A) \ln \frac{\mathcal{P}(s|I_B)}{\mathcal{P}(s|I_0)}$$

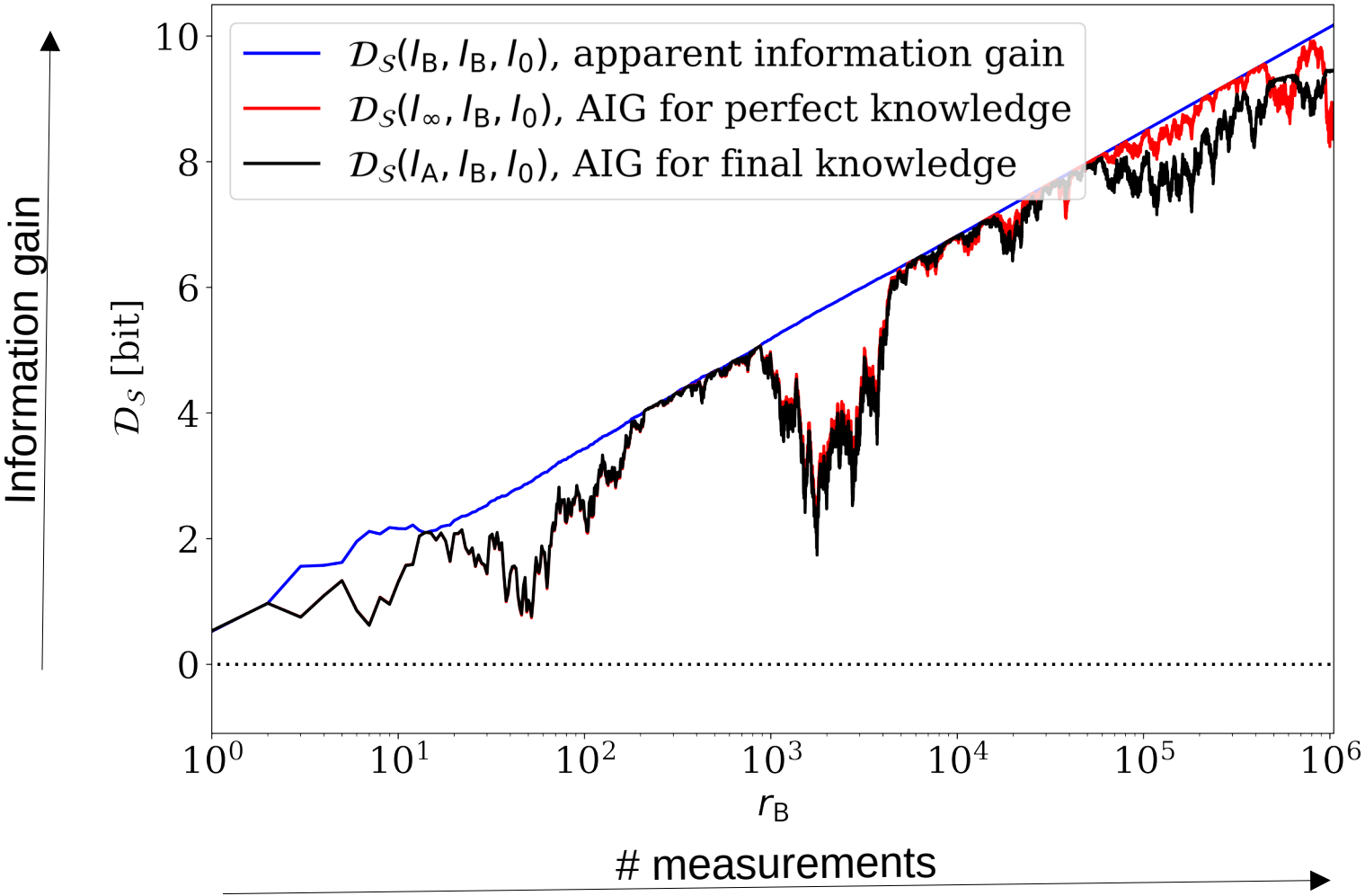
$$= \mathcal{D}_{\mathcal{S}}(I_A, I_0) - \mathcal{D}_{\mathcal{S}}(I_A, I_B)$$

information gain
achieved = ideal- remaining

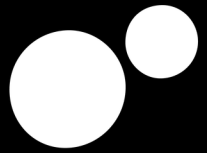
Gaussian Probabilities



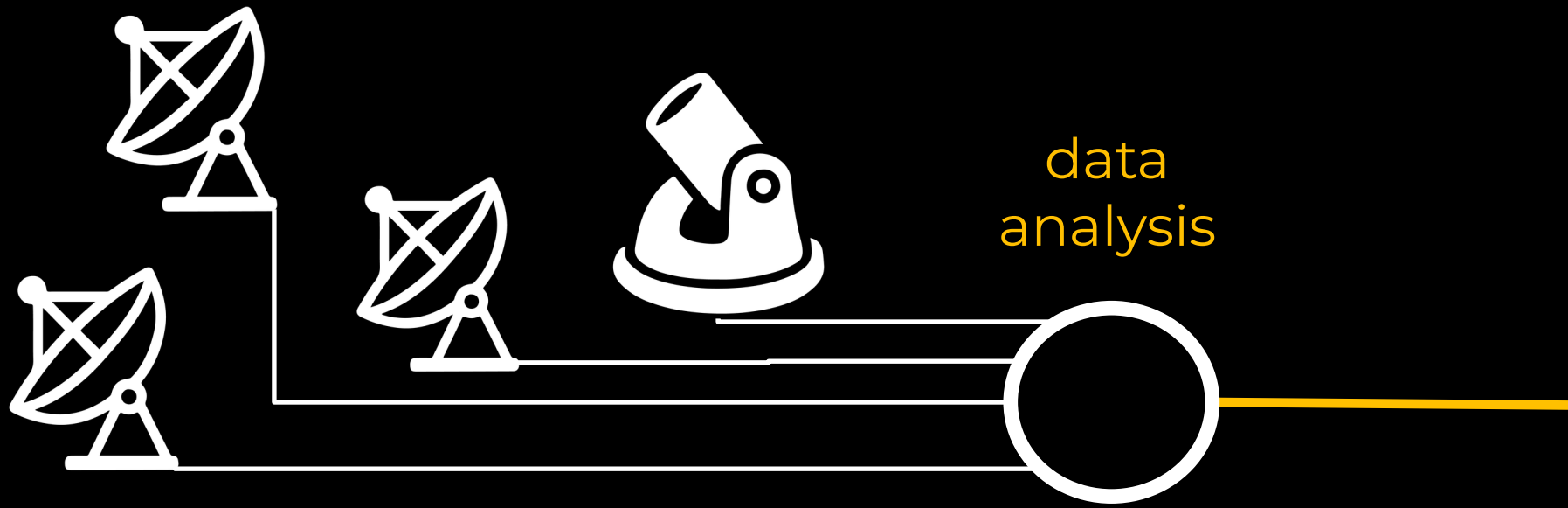
Repeated Gaussian Measurements



$$\mathcal{D}_\mathcal{S}(I_\mathrm{A}, I_\mathrm{B}, I_0) \propto \ln(\#\ \text{meas.})$$



large facility price tag = 0.1 G€ / a
two analysis methods
difference in information efficiency: 20%
same information gain: 1 day ↔ 1.5 day
break even point: 135 k€ extra costs



Achieved Information Gain

How to calculate it?

$I_{\mathrm{B}}(d)$ approximative

$I_{\mathrm{A}}(d)$ perfect $\qquad \mathcal{P}(s|I_{\mathrm{A}}(d)) = \dfrac{\mathcal{P}(d|s)\mathcal{P}(s|I_0)}{\mathcal{P}(d|I_0)}$

synthetic signal & data: $s_i \hookleftarrow \mathcal{P}(s_i|I_0) \;\; d_i \hookleftarrow \mathcal{P}(d_i|s_i)$

$$\langle \mathcal{D}_{\mathcal{S}}(I_{\mathrm{A}}(d), I_{\mathrm{B}}(d), I_0) \rangle_{(d|I_0)} \approx \left\langle \ln \frac{\mathcal{P}(s_i|I_{\mathrm{B}}(d_i))}{\mathcal{P}(s_i|I_0)} \right\rangle_i$$

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RESEARCH ARTICLE

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Quantifying Imperfect Cognition Via Achieved Information Gain

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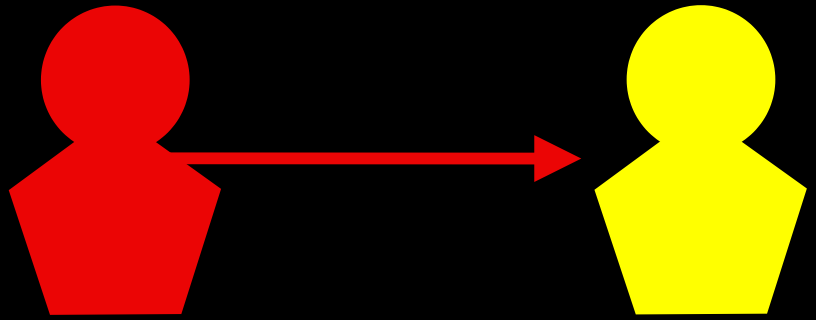

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