Study and Measurement of the Non-Linear Response of Silicon Photomultipliers

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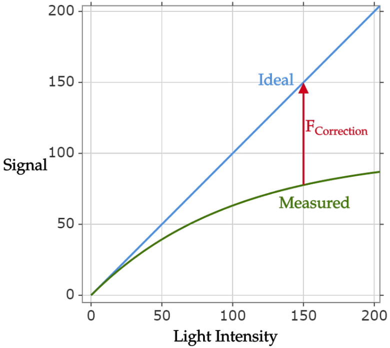
Motivation and Goal

Motivation

- SiPMs used for single photon detection in linear regime
- Array of single-photon avalanche diodes (binary devices)
- Pixel-like design introduces non-linearity at high photon numbers



Goal

- Measure and determine the applicability of a correction function

Understanding SiPM Non-Linear Response

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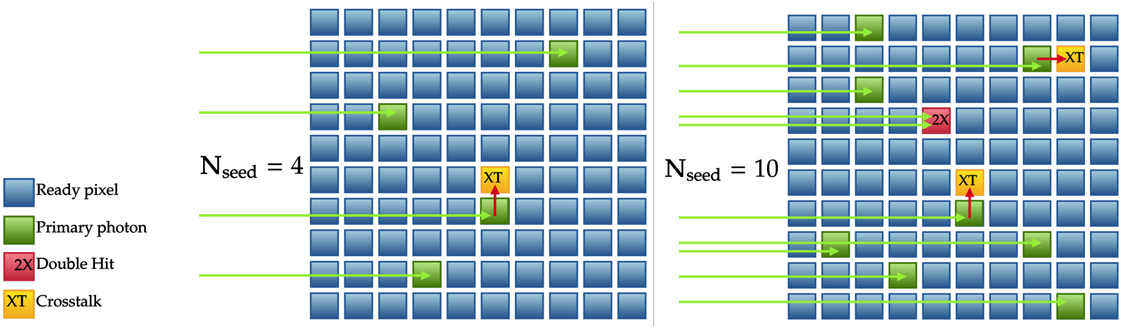
- Charge generated by pixel avalanche q_{pixel}
- SiPM signal is charge generated by all fired pixels N_{fired}

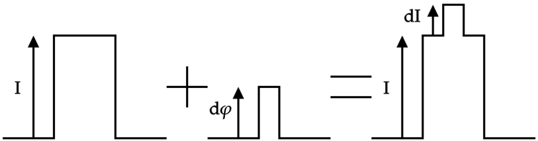
$$Q = q_{\text{pixel}} \cdot N_{\text{fired}}$$

$\longrightarrow$ SiPM response is linear when each incoming photon triggers a different pixel

$\longrightarrow$ Challenge: Photon time distribution (late arriving photons on partially recovered pixels)



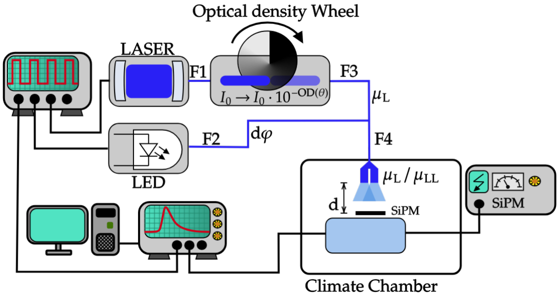
- **Method:** Determine non-linearity by measuring the change in amplitude when a fixed, small light pulse is added to a variable intensity base pulse.



I : Laser $\quad$ $\mathrm{d}\varphi$: LED $\quad$ $\mathrm{d}I$: effective LED light

- Add fixed, small amplitude $\mathrm{d}\varphi$ to the existing amplitude I, resulting in $I + \mathrm{d}I$
- Measure non-linearity with $(I + \mathrm{d}I) - I \leq \mathrm{d}\varphi$

> $\mathrm{d}\varphi$: Fixed calibrated light source
> I : No calibration necessary



1. Set Temperature and bias voltage
2. Start of one saturation curve
3. Rotate wheel to first position and take laser+led **measurement** and laser **measurement**
4. Repeat for every wheel position
5. Led only **measurement**
6. End of one saturation curve
7. Change Temperature and bias voltage for the next scan

Measurement Space and SiPM Parameters

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- On each SiPM three measurement types are performed
  1. Gain measurement (not presented today)
  2. "Corner" measurement
  3. "Matrix" measurement

$$\begin{array}{c|ccc} 20.0\,^\circ\mathrm{C} & 2.0\,\mathrm{V} & \dots & 4.0\,\mathrm{V} \\ \vdots & \vdots & & \vdots \\ 0.0\,^\circ\mathrm{C} & 2.0\,\mathrm{V} & \dots & 4.0\,\mathrm{V} \\ \vdots & \vdots & & \vdots \\ -20.0\,^\circ\mathrm{C} & 2.0\,\mathrm{V} & \dots & 4.0\,\mathrm{V} \end{array}$$

Parameter	Symbol	Value
Name	Ketek PM1150T SB	
Effective photosensitive area	-	$1.3 \times 1.3\,\mathrm{mm}$
Pixel pitch	-	$50.0\,\mathrm{\mu m}$
Number of pixels	N_{pixel}	576
Breakdown voltage	V_{br}	$26.0\,\mathrm{V}$
Integration gate	t_{gate}	$120.0\,\mathrm{ns}$

"Corner" Measurements

V_{over}	T	I_{LED}	N
2.0 V, 4.0 V	20.0 °C, 0.0 °C, −20.0 °C	≈ 25	50000

- High statistics runs

$\rightarrow$ **Determine Correction Function**

"Matrix" Measurements

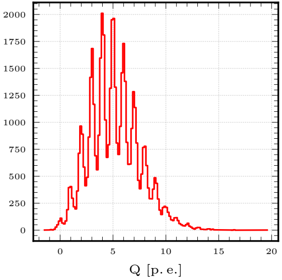
V_{over}	T	I_{LED}	N
2.0 V to 4.0 V	−20.0 °C to 20.0 °C	0	10000

- Low statistics laser only runs

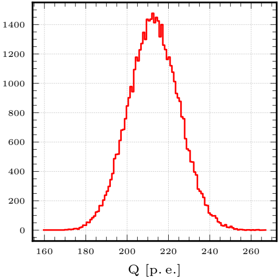
$\rightarrow$ **Apply Correction Function**

Integrated Charge for different Intenisties

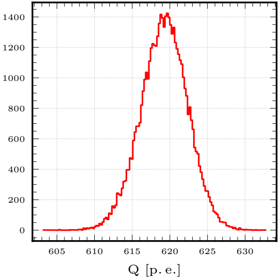
Low-light intensity



Medium-light intensity

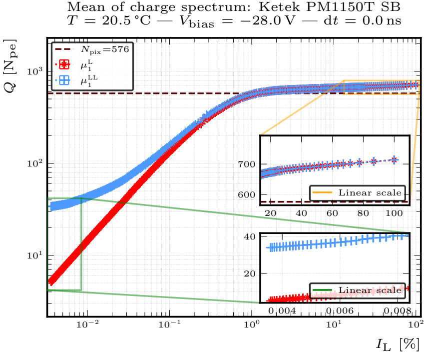


High-light intenisty



- For each measurement the Mean, RMS and Skewness are calculated
- Spectra range from low-intensity single photo electron spectra to high-intensity normal distributions

Mean of the Integrated Charge



μ_{L}: Laser

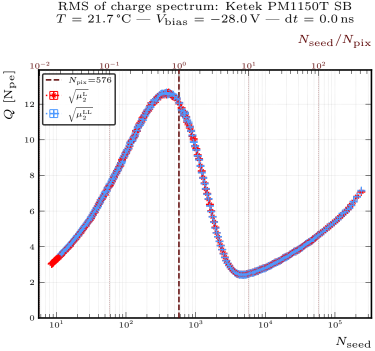
μ_{LL}: Laser+LED

- Mean values of μ_L and μ_{LL} versus linear laser intensity
- Y-axis converted to unitless number of photo electrons (npe):

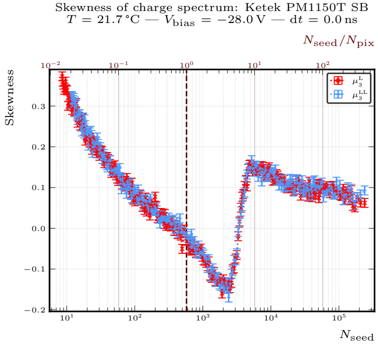
$$\mu[npe] = \frac{\mu}{R_{\mathrm{L}} \cdot e \cdot G}$$

- Laser+LED and Laser signal differ at low-intensity
- Difference decreases with increasing laser intensity
- SiPM response exceeds 576 physical pixels
- Full saturation does not occur for this SiPM

The RMS $\sqrt{\mu_2}$ and Skewness μ_3



- Increase of RMS with light intensity
- Reduction of RMS due to total number of physical pixels
- Increase at high light intensity not yet understood



- Positive skewness for low-intensity
- Negative skewness due to the total number of physical pixels
- No explanation behaviour for high light intensity

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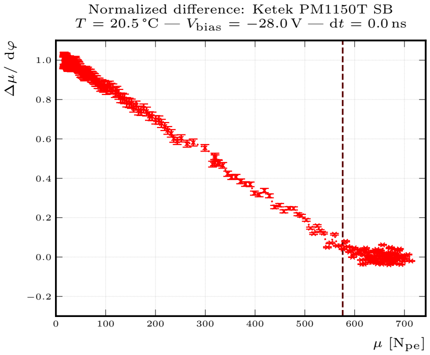
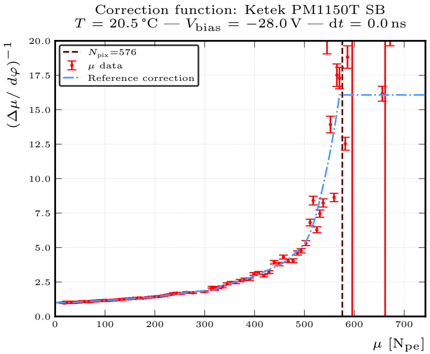
Step 1
Calculate the difference between laser and laser + led measurement normalized to led $\frac{(\mu_{LL}-\mu_{L})}{d\varphi}$



Step 2
The correction function is 1 divided by the normalized difference



Application of the Correction Function

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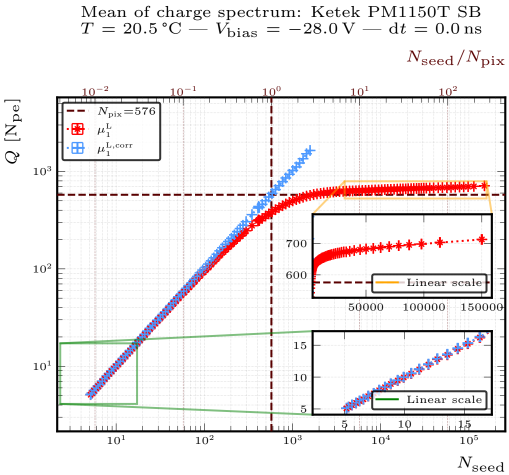
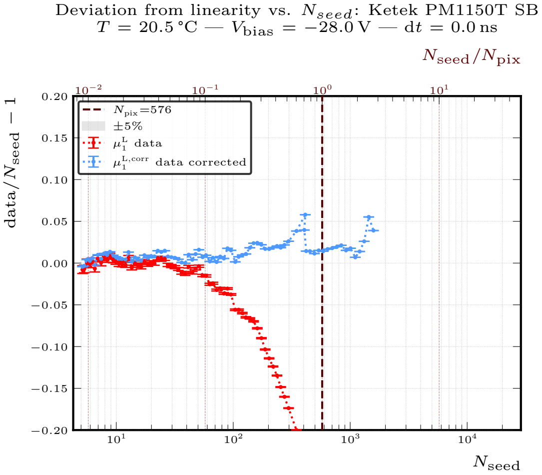


- Apply the correction function on event by event basis to obtain corrected charge φ:

$$\varphi = \int_0^{\mu_{\text{meas}}} f(\mu) d\mu$$

- Correction function only applied to data with $\mu \leq N_{\text{pix}}$
- Non-Linear response successfully corrected

Deviation from Linearity Score



- Plot the deviation from the number of seed photons
- Data begins to diverge from ±5% linearity after $N_{\mathrm{seed}} = 100$, equivalent to 0.17 N_{pix}
- Corrected data remains within ±5% linearity across the entire correction range, except for 2 outliers
- Linearity range is extended by a factor of 10 with the applied correction
- Evaluate the correction performance by the average absolute deviation:

$$\sum \left| \frac{\text{data}}{N_{\text{seed}}} - 1 \right| / N$$

How Universal is the Correction?

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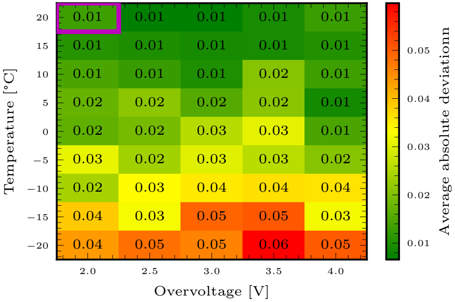
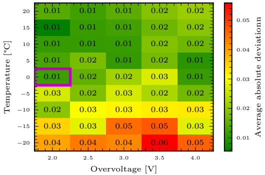
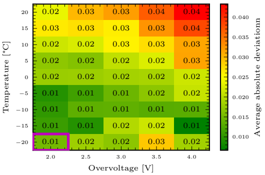
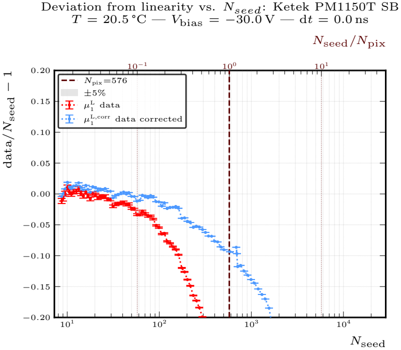


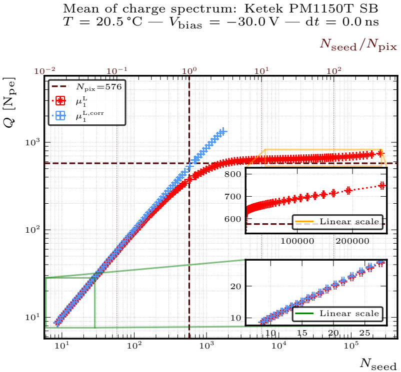




- The correction, determined at $T = 20.0\,^\circ\mathrm{C}$ and $V_{\mathrm{over}} = 2.0\,\mathrm{V}$ (magenta box), is applied to every measurement in the matrix
- No overvoltage but temperature dependence
- The correction is valid within a range of $T \pm 2.5\,\mathrm{K}$ and $V_{\mathrm{over}} \pm 0.5\,\mathrm{V}$ around the original parameters

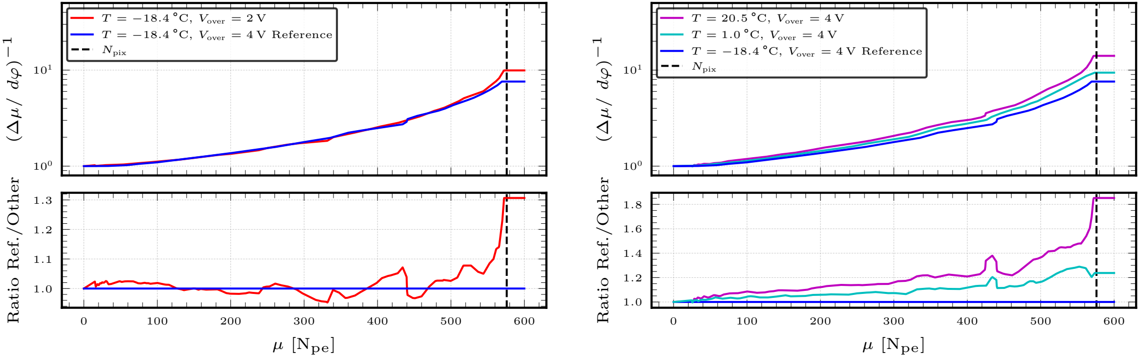
Worst Case Scenario





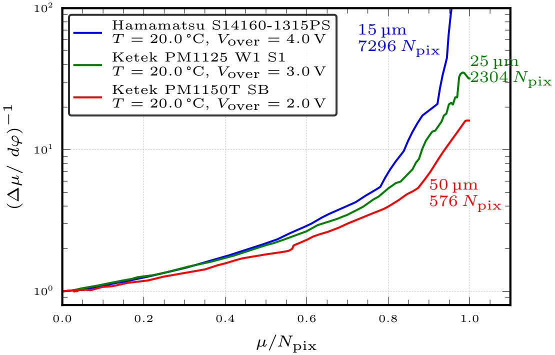
- Applying the correction measured at $T = -20.0$ $V_{\mathrm{over}} = 2.0\,\mathrm{V}$ to data at $T = 20.0$ $V_{\mathrm{over}} = 4.0\,\mathrm{V}$
- **DIFFERENCE OF 40 K !**

Voltage and Temperature Dependence



- No significant voltage dependence of the correction function in the measured range
- Temperature dependence significant for differences $\geq 10.0\,^\circ\mathrm{C}$

Pixel Capacitance Dependence



- Comparison of three different SiPMs with different pixel capacitance

	Hamamatsu S14160-1315PS	Ketek PM1125 W1 S1	Ketek PM1150T SB
Area	1.3×1.3 mm	1.2×1.2 mm	1.3×1.3 mm
Pixel pitch	15.0 µm	25.0 µm	50.0 µm
N_{pixel}	7296	2304	576

- Correction functions are scaled to N_{pix} for comparison
- Larger correction function for smaller pixel sizes

Conclusion and Outlook

Conclusion

- Automated setup to perform the measurements required to extract the correction function
- Unified code to analyse the data SUMLab (Massimiliano Antonello's next talk)
- Correction function shows temperature but no voltage dependence
- Shown effects of pixel pitch on the correction function
- Recovery linearity within 1% in range $\pm 2.0\,\mathrm{V}$ and $\pm 5.0\,\mathrm{K}$
- Even "possible" for temperature differences of $40.0\,\mathrm{K}$

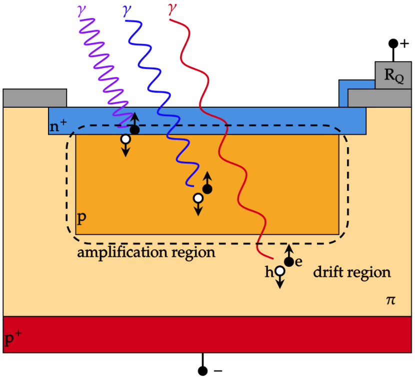
Outlook

- Finish the analysis of the $25.0\,\mu\mathrm{m}$ SiPM
- Perform more systematic studies on further SiPMs

Backup

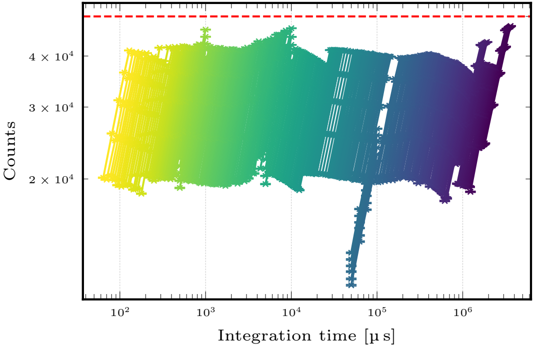
What is a SiPM?

- A Silicon Photomultiplier (SiPM) is a photo-detector operating in the red-to-near UV range
- Some useful properties:
  - High photon-detection efficiency ($> 50\%$)
  - Good time resolution ($< 100.0\,\mathrm{ps}$)
  - Low noise
  - Single-photon counting capability
  - Insensitivity to magnetic fields
- Used for Particle Physics Experiments, Medical Imaging, LiDAR (Light Detection and Ranging), ...

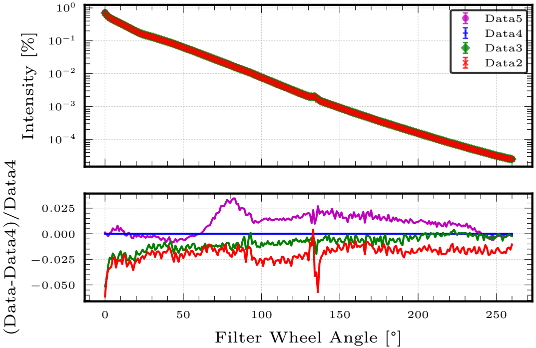


Single SiPM cell

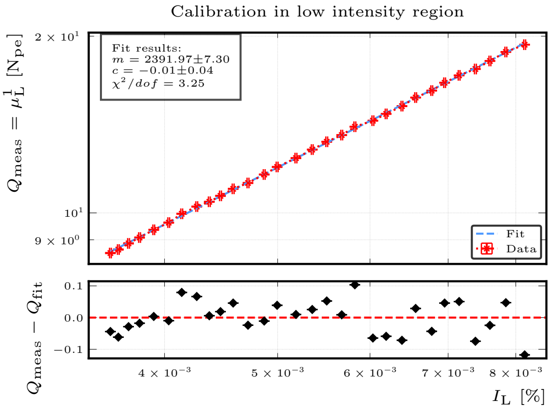
The count rate (Data4) shows linear behaviour for all angles.



Calculate the intensity as the ratio of the reference and angle slope $I_i = \frac{a_{\mathrm{ref}}}{a_i}$



Assumptions in Seed Photon Calculations

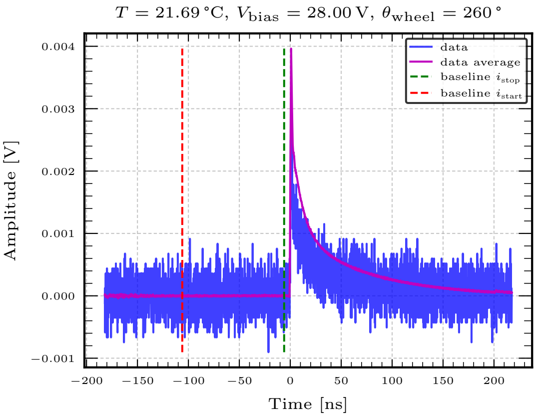


- **Assumptions:**
  - Correct determination of the Gain
  - Correct determination of intensity from wheel calibration
  - Correlated noise ignored/small effect
  - $N_{\mathrm{seed}} = Q_{\mathrm{meas}}$ for low intensity
- Perform linear fit in range $3 < Q_{\mathrm{meas}} < 20$
- Check: $c = 0$, no light no Q_{meas}
- Assign number of seed photons for each intensity

$$N_{\mathrm{seed}} = m \times I + c$$

- Correction does not depend on this, only used for plotting and validation!

Performing Baseline Subtraction



Calculate the baseline in specified range from i_{start} to i_{stop} for each waveform

$$\text{baseline} = \frac{1}{N} \sum_{i_{\mathrm{start}}}^{i_{\mathrm{stop}}} y_i$$

Subtract the baseline (b) from the waveform y data resulting in waveform corrected $\hat{y}$

$$\hat{y} = y - b$$

Note: Waveform in figure already baseline corrected

Waveform Integration in Uncorrelated Bins

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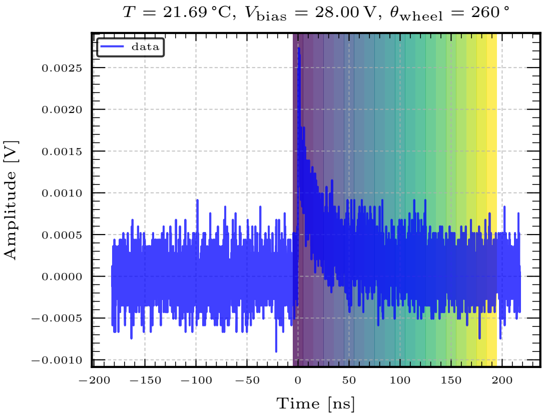


Integration of the waveform in specified uncorrelated bins as the sum of the $\hat{y}$ data times the resolution dt (bin height$\times$bin width)

$$Q = \left(\sum_{i_{\mathrm{start}}}^{i_{\mathrm{stop}}} \hat{y} \right) \times dt$$

$$[Q] = [\mathrm{V\,s}]$$

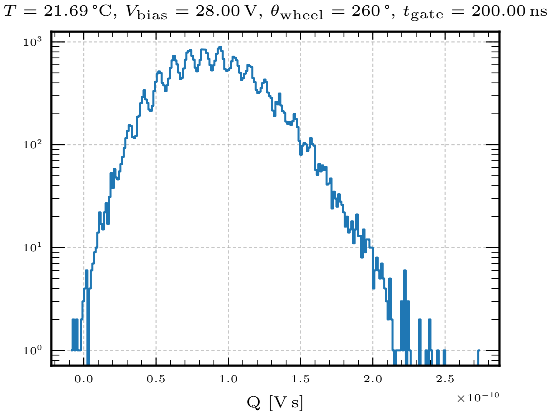
$$Q = Q/G$$

$$[Q] = [\mathrm{p.\,e.}]$$

Bin	0	1	2	3	...
Time [ns]	-5 - 5	5 - 15	15 - 25	25 - 35	...
Edge index	3551 - 3751	3751 - 3951	3951-4151	4151 - 4351	...

Note: Stop index is not included in array indexing, there is no overlap.

Calculating the First Three Moments of a Histogram



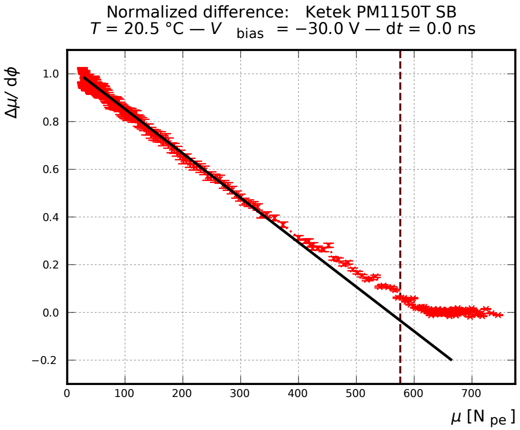
Calculate the mean μ_1, standard deviation $\sqrt{\mu_2}$ and skewness μ_3 of the Q_i array for each measurement

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{N} Q_i = \bar{Q}$$

$$\sqrt{\mu_2} = \sqrt{\frac{\sum_i \left|Q_i - \bar{Q}\right|^2}{N}}$$

$$\mu_3 = \frac{\sqrt{N(N-1)}}{N-2}\,\frac{\frac{1}{N}\sum_{i=1}^{N}(Q_i - \bar{Q})^3}{\left(\frac{1}{N}\sum_{i=1}^{N}(Q_i - \bar{Q})^2\right)^{3/2}}$$

Normalized Difference non linear



The normalized difference is generally not a linear function of μ which prohibits us from performing a linear fit.

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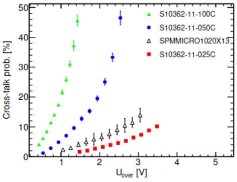
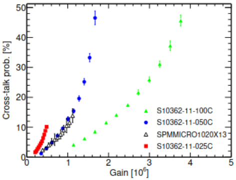
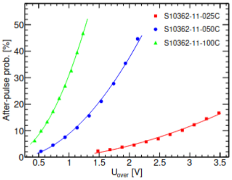
K. Tadday, "Scintillation Light Detection and Application of Silicon Photomultipliers in Imaging Calorimetry and Positron Emission Tomography," Ph.D. dissertation, Heidelberg U., 2011.



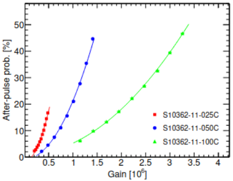
(a) Cross-talk probability vs. over-voltage.



(b) Cross-talk probability vs. gain.



(a) After-pulse probability vs. over-voltage.



(b) After-pulse probability vs. gain.

Response function

$$Q[N_{\text{pe}}] = \frac{Q(T,V)}{G(T,V)} = e\,N_{\text{seed}}\,CT(T,V)\,(1 + AP(T,V))$$

$$CT = \text{Crosstalk probability}$$
$$AP = \text{Afterpulse probability}$$
$$e = \text{Electric charge}$$
$$Q = \text{Integrated charge}$$
$$G = \text{Gain}$$
$$N_{\text{pe}} = \text{Number of photo electrons}$$

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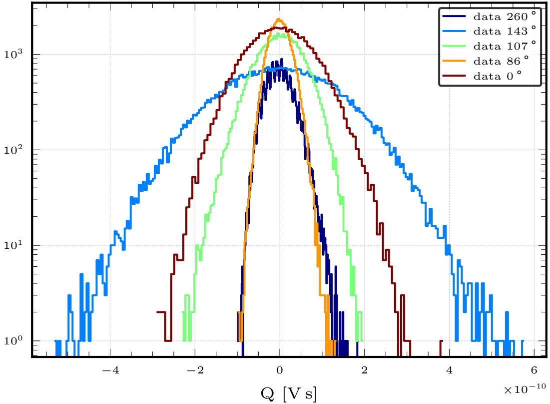
Comparison of all histograms centred around their mean.



Illumination study

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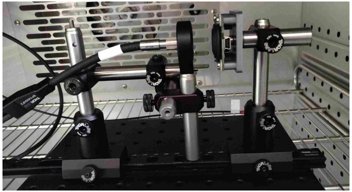
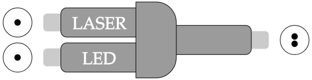
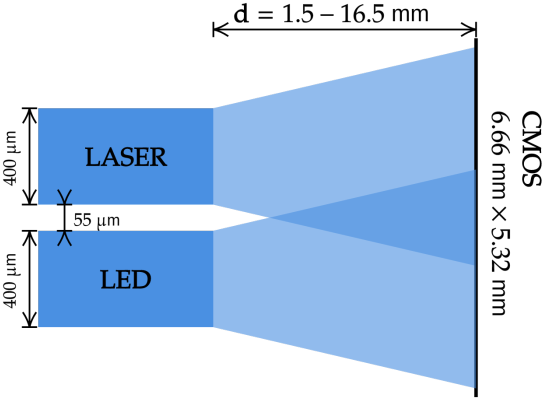
- Motivation: Check spatial uniformity of LASER/LED light
- Method: Measure spatial distribution with CMOS camera at various distances
- Goal: Find minimal distance at which overlap of one standard deviation occurs

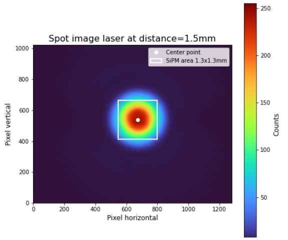


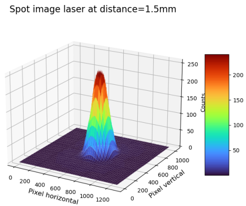


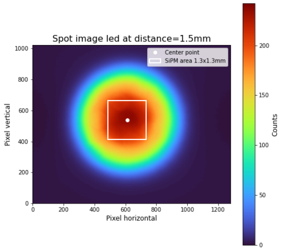


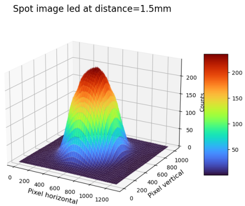
Images of LASER and LED spots

- Both light sources exhibit Gaussian intensity profiles
- LED produces a spatial larger profile
- SiPM represented as a white box
- Calculate mean position and standard deviation for d ranging from 1.5 mm to 16.5 mm

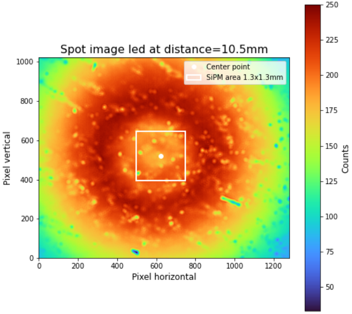


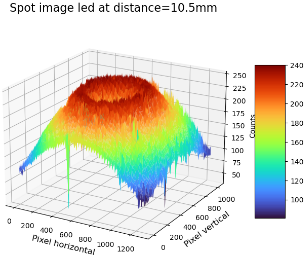






50/50 Splitter wrong end





Non-Gaussian intensity profile from tap port

Illumination Study Results

- SiPM centered around the LASER spot, using the larger LED spot
- Distance d of 7.5 mm is sufficient for achieving coverage within one standard deviation
- However, this distance results in a reduced overall light intensity