A soliton-halo bound? Looking at the Ultralight Dark Matter soliton-halo relation

DESY

Teodori Luca

December 2024

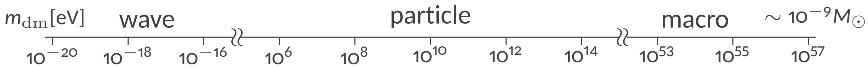
UNDARK

Funded by
the European Union

WEIZMANN INSTITUTE OF SCIENCE

Based on:
K. Blum, M. Gorghetto, E. Hardy, S. Sibiryakov, LT
[250X.XXXXX]

Ultralight Dark Matter with gravity alone



- Investigate very light fields, possibly arising from misalignment

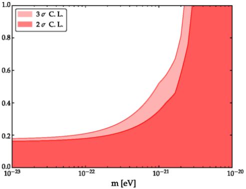
$$\Omega_{\rm a} \sim 0.1 \left(\frac{f}{1 \times 10^{17}\ \text{GeV}} \right)^2 \left(\frac{m}{1 \times 10^{-22}\ \text{GeV}} \right)^{1/2}$$

- Generically, couplings suppressed by large scale $f \implies$ investigate via gravity alone, m and the fraction $\Omega_{\rm a}/\Omega_{\rm dm}$ being the only parameters
- Cosmology constraints from small-scale suppression, $m \gtrsim 1 \times 10^{-21}$ eV

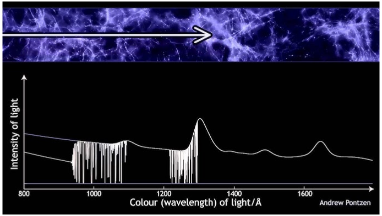
Ultralight Dark Matter with gravity alone

$m_{\rm dm}$[eV] wave particle macro $\sim 10^{-9} M_\odot$

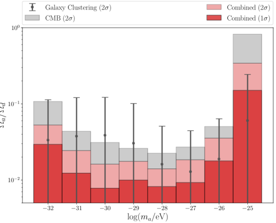
10^{-20} 10^{-18} 10^{-16} ≀≀ 10^{6} 10^{8} 10^{10} 10^{12} 10^{14} ≀≀ 10^{53} 10^{55} 10^{57}



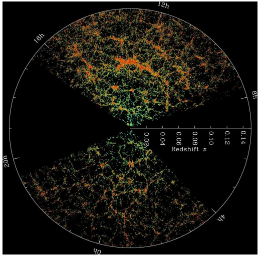
Kobayashi et al 2017 [1708.00015]



UCL Mathematical and Physical Sciences



. Laguë et al 2021 [2104.07802]



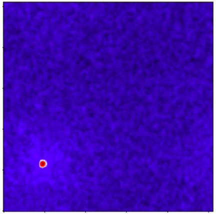
- Investigate very light fields, possibly arising from misalignment

$$\Omega_{\rm a} \sim 0.1 \left(\frac{f}{1\times 10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{1\times 10^{-22}\,\mathrm{GeV}}\right)^{1/2}$$

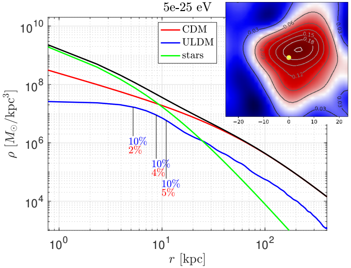
- Generically, couplings suppressed by large scale f $\Longrightarrow$ investigate via gravity alone, m and the fraction $\Omega_{\rm a}/\Omega_{\rm dm}$ being the only parameters
- Cosmology constraints from small-scale suppression, $m \gtrsim 1 \times 10^{-21}$ eV

Astrophysics for ULDM

- Main features: wave-interference phenomena and inner cored profile, called solitons
- Gravitational lensing anomalies; time delays (subdominant ULDM)
- Galaxy rotation curves and stars dynamical heating: dependence on how big the soliton forms, given an halo, i.e. on the soliton-halo relation
- All consistent with $m \gtrsim 1 \times 10^{-21}$ eV
- Can we understand the soliton-halo relation? Can we use such understanding to say something meaningful about phenomenological consequences of ULDM?

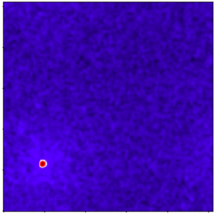


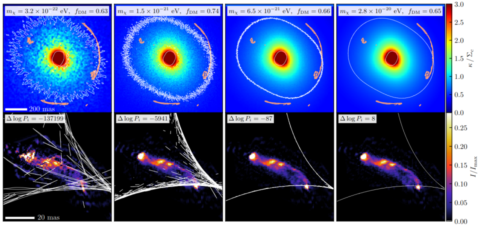
Astrophysics for ULDM



K. Blum and LT [2409.04134]

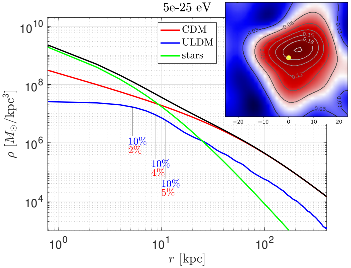
- Main features: wave-interference phenomena and inner cored profile, called solitons
- Gravitational lensing anomalies; time delays (subdominant ULDM)
- Galaxy rotation curves and stars dynamical heating: dependence on how big the soliton forms, given an halo, i.e. on the soliton-halo relation
- All consistent with $m \gtrsim 1 \times 10^{-21}$ eV
- Can we understand the soliton-halo relation? Can we use such understanding to say something meaningful about phenomenological consequences of ULDM?





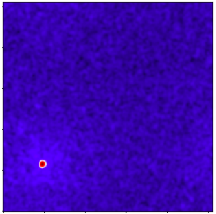
D.M. Powell et al 2023 [2302.10941]

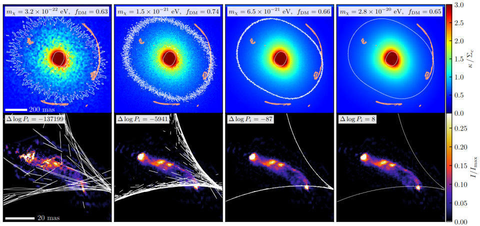
Astrophysics for ULDM



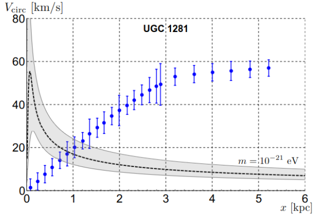
K. Blum and LT [2409.04134]

- Main features: wave-interference phenomena and inner cored profile, called solitons
- Gravitational lensing anomalies; time delays (subdominant ULDM)
- Galaxy rotation curves and stars dynamical heating: dependence on how big the soliton forms, given an halo, i.e. on the soliton-halo relation
- All consistent with $m \gtrsim 1 \times 10^{-21}$ eV
- Can we understand the soliton-halo relation? Can we use such understanding to say something meaningful about phenomenological consequences of ULDM?



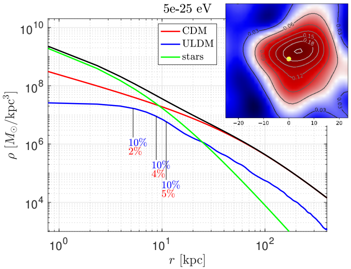


D.M. Powell et al 2023 [2302.10941]



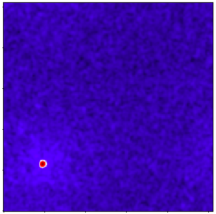
N. Bar et al 2018 [1805.00122]

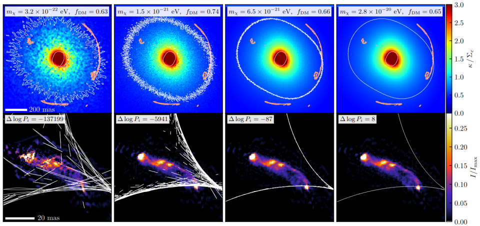
Astrophysics for ULDM



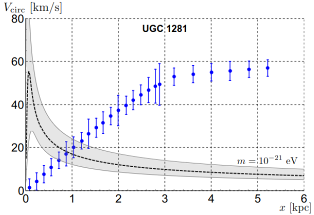
K. Blum and LT [2409.04134]

- Main features: wave-interference phenomena and inner cored profile, called solitons
- Gravitational lensing anomalies; time delays (subdominant ULDM)
- Galaxy rotation curves and stars dynamical heating: dependence on how big the soliton forms, given an halo, i.e. on the soliton-halo relation
- All consistent with $m \gtrsim 1 \times 10^{-21}$ eV
- Can we understand the soliton-halo relation? Can we use such understanding to say something meaningful about phenomenological consequences of ULDM?



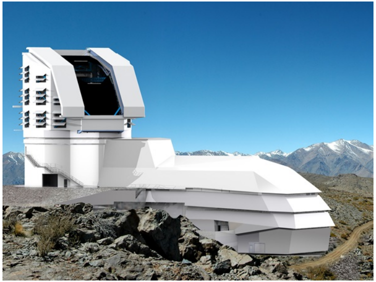


D.M. Powell et al 2023 [2302.10941]

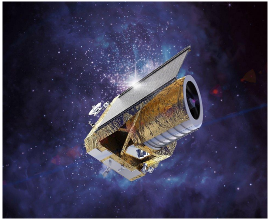


N. Bar et al 2018 [1805.00122]

Data incoming!

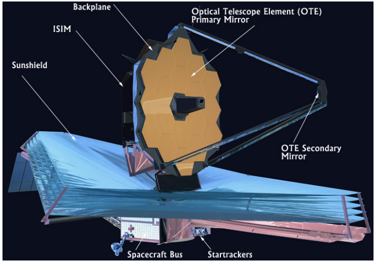


LSST artistic



Euclid

- Euclid: Cosmology, observing $\sim 10^9$ galaxies, across more than a third of the sky, up to $z \sim 2$
- JWST: High redshift universe, precise stellar kinematics
- LSST: wide field of view allows it to observe large areas of the sky at once, look for transients. Expected orders of magnitude more of gravitational lens systems
- ... and many more!

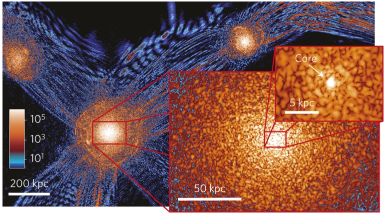


JWST

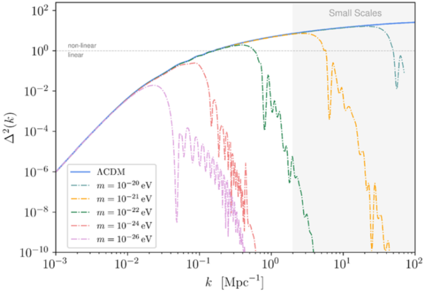
Outline

- ULDM essentials
- The many tales of the soliton-halo relation
- A soliton-halo lower bound? Investigate through simulations

Ultralight Dark Matter essentials



H.-Y. Schive et al 2014 [1406.6586]



E.G.M. Ferreira 2021 [2005.03254]

- De-Broglie relevant in astrophysical scales

$$\lambda_{\rm dB} = \frac{2\pi}{k} = \frac{2\pi}{mv} \simeq 3.8\,\text{pc}\left(\frac{10^{-20}\,\text{eV}}{m}\right)\left(\frac{10^2\,\text{km s}^{-1}}{v}\right)$$

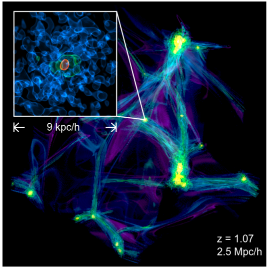
- Cannot squeeze too much mass in little volume (from uncertainty principle): small scales power spectrum suppression, plus formation of cored profiles
- Huge occupation number $\Longrightarrow$ classical field

$$\mathcal{N} \simeq \frac{\rho_{\rm dm}}{m(mv)^3} \simeq 10^{84}\left(\frac{\rho_{\rm dm}}{0.4\,\text{GeV cm}^{-3}}\right)\left(\frac{10^{-20}\,\text{eV}}{m}\right)^4$$

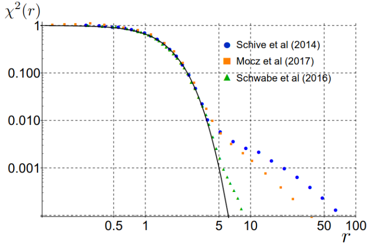
- In NR limit, we have Schrödinger-Poisson equations

$$i\partial_t\psi = -\frac{\nabla^2\psi}{2m} + m\Phi\psi\,,$$
$$\nabla^2\Phi = 4\pi G|\psi|^2\,.$$

Solitons



J. Veltmaat et al 2018 [1804.09647]



N. Bar et al 2018 [1805.00122]

- Ground state solution of Schrödinger-Poisson

$$\psi(\vec{x},t) = \frac{mM_{\rm pl}}{\sqrt{4\pi}} e^{-i\gamma mt}\chi(\vec{x})\,,\; x = rm$$

$$\partial_x^2\chi + \frac{2}{x}\partial_x\chi = 2(\Phi + \Phi_{\rm ext} - \gamma)\chi$$

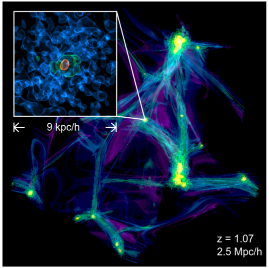
$$\partial_x^2\Phi + \frac{2}{x}\partial_x\Phi = \chi^2$$

- Schrödinger-Poisson invariant under

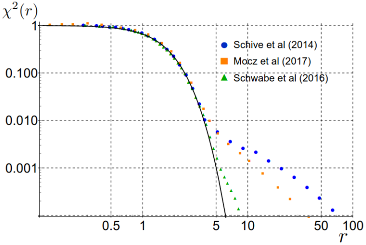
$$\tilde{\psi} = \frac{1}{\lambda^2}\frac{\sqrt{4\pi G}}{m}\psi\,,\; \tilde{\vec{x}} = \lambda m\vec{x}\,,\; \tilde{t} = \lambda^2 mt\,,\; \tilde{\Phi} = \Phi/\lambda^2$$

which implies

$$\frac{E}{M^3}\frac{1}{G^2m^2} = (4\pi)^2\frac{\tilde{E}}{\tilde{M}^3} \implies \text{Invariant}$$



J. Veltmaat et al 2018 [1804.09647]



N. Bar et al 2018 [1805.00122]

- Ground state solution of Schrödinger-Poisson

$$\psi(\vec{x},t) = \frac{mM_{\rm pl}}{\sqrt{4\pi}} e^{-i\gamma mt}\chi(\vec{x}) \,,\; x = rm$$

$$\partial_x^2\chi + \frac{2}{x}\partial_x\chi = 2(\Phi + \Phi_{\rm ext} - \gamma)\chi$$

$$\partial_x^2\Phi + \frac{2}{x}\partial_x\Phi = \chi^2$$

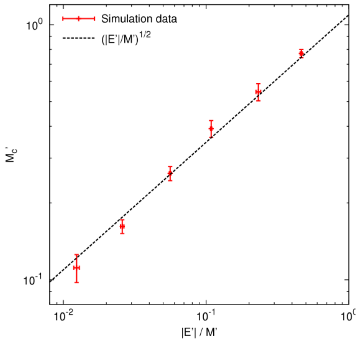
- Schrödinger-Poisson invariant under

$$\tilde{\psi} = \frac{1}{\lambda^2}\frac{\sqrt{4\pi G}}{m}\psi \,,\; \tilde{\vec{x}} = \lambda m\vec{x} \,,\; \tilde{t} = \lambda^2 mt \,,\; \tilde{\Phi} = \Phi/\lambda^2$$

which implies

$$\frac{E}{M^3}\frac{1}{G^2m^2} = (4\pi)^2\frac{\tilde{E}}{\tilde{M}^3} \implies \text{Invariant}$$

Is there a soliton-halo relation?



- Parametrize the relation as

$$\frac{M_{\rm sol}}{M_{\rm halo}} = \alpha \left(\frac{|E_{\rm halo}|}{M_{\rm halo}^3} \frac{1}{G^2 m^2} \right)^\beta$$

- Schive et al, $\alpha \simeq 4.1$, $\beta = 1/2$. Intuitive understanding: kinetic equilibration (N. Bar et al 2018 [1805.00122])

$$\frac{E_{\rm sol}}{M_{\rm sol}} = \frac{E_{\rm halo}}{M_{\rm halo}}$$

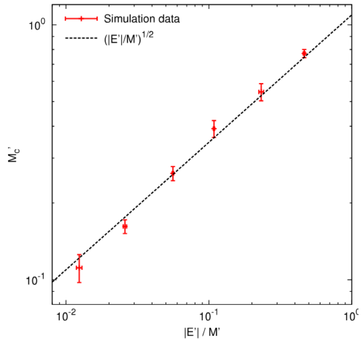
- On cosmological simulations, the relation is recast with

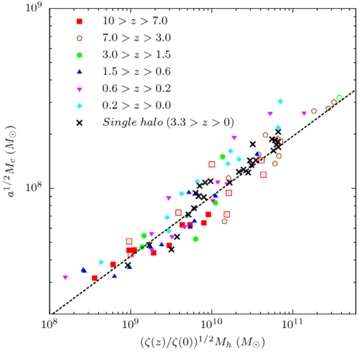
$$E_{\rm halo} \sim \frac{GM_{\rm halo}^2}{r_{\rm vir}}, \; M_{\rm halo} = \frac{4\pi \rho_{m,0} r_{\rm vir}^3}{3} \zeta(z)$$

$$\implies M_c = \left(\frac{E_{\rm halo}}{M_{\rm halo}} \right)^{1/2} \sim M_h^{1/3}$$

- Seen in cosmological and toy initial conditions (multiple identical solitons)
- Is this relation an attractor?

Is there a soliton-halo relation?





H.-Y. Schive et al 2014 [1406.6586]

- On cosmological simulations, the relation is recast with

$$E_{\rm halo} \sim \frac{GM_{\rm halo}^2}{r_{\rm vir}}\,,\quad M_{\rm halo} = \frac{4\pi\rho_{m,0} r_{\rm vir}^3}{3}\zeta(z)$$

$$\implies M_c = \left(\frac{E_{\rm halo}}{M_{\rm halo}}\right)^{1/2} \sim M_h^{1/3}$$

- Parametrize the relation as

$$\frac{M_{\rm sol}}{M_{\rm halo}} = \alpha\left(\frac{|E_{\rm halo}|}{M_{\rm halo}^3}\frac{1}{G^2m^2}\right)^{\beta}$$

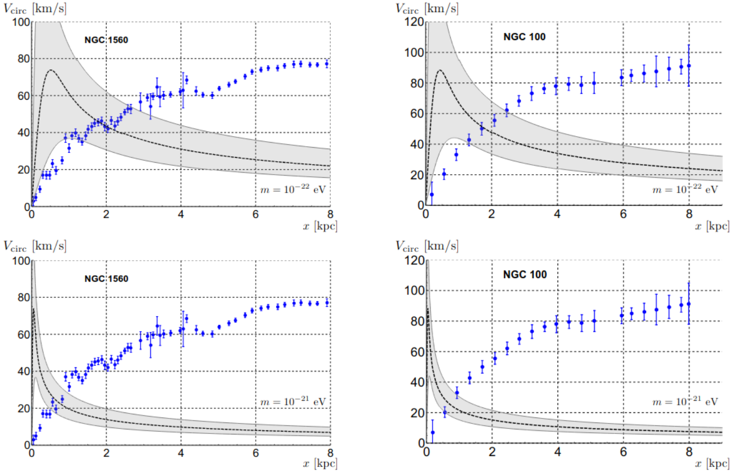
- Schive et al, $\alpha \simeq 4.1$, $\beta = 1/2$. Intuitive understanding: kinetic equilibration (N. Bar et al 2018 [1805.00122])

$$\frac{E_{\rm sol}}{M_{\rm sol}} = \frac{E_{\rm halo}}{M_{\rm halo}}$$

- Seen in cosmological and toy initial conditions (multiple identical solitons)
- Is this relation an attractor?

Soliton-halo for stellar kinematics

Disk galaxies, N. Bar et al 2018 [1805.00122]



N. Bar et al 2018 [1805.00122]

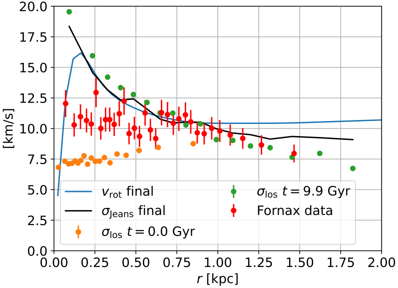
- Given an halo, one can predict the soliton size
- Rotation curves

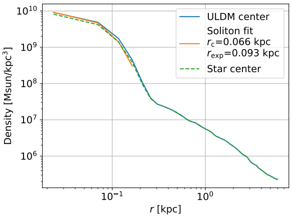
$$v_{\rm rot}(r) = \sqrt{\frac{GM(r)}{r}}$$

- Soliton-halo relation: velocity curve has two peaks with the same velocity
- $m \lesssim 1 \times 10^{-21}$ eV incompatible with data, if soliton-halo relation works

Soliton-halo for stellar kinematics

Dwarf galaxies, K. Blum, A. Caputo, LT (in preparation)



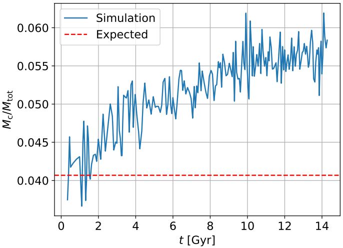


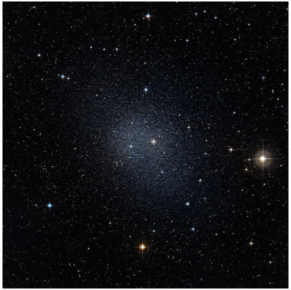
- Different mass profile affects velocity dispersion expected

$$\sigma_r^2(r) = \frac{G}{\rho_*(r)} \int_r^\infty \mathrm{d}s \, \frac{\rho_*(s) M(s)}{s^2}$$

$$\sigma_{\rm los}^2(r) = \frac{2}{\Sigma_*(r)} \int_r^\infty \mathrm{d}s \, \frac{\rho_* \sigma_r^2}{\sqrt{s^2 - r^2}}$$

- Soliton accretes mass, becomes heavier: soliton-halo relation a lower bound?

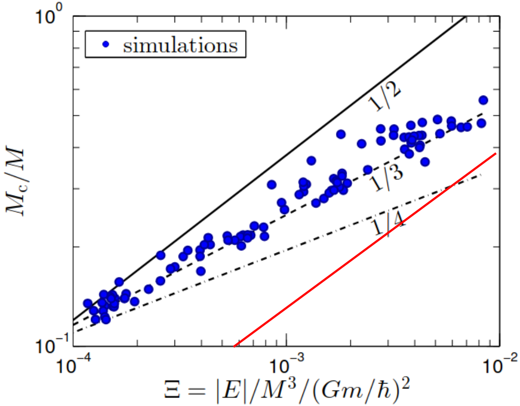




ESO/Digitized Sky Survey 2

The many tales of the soliton-halo relation

P. Mocz et al 2017 [1705.05845]



P. Mocz et al 2017 [1705.05845]

- Toy initial condition: multiple solitons with varying mass
- A different relation

$$M_c = 2.6\left(\frac{|E_{\rm halo}|}{G^2m^2}\right)^{1/3}$$

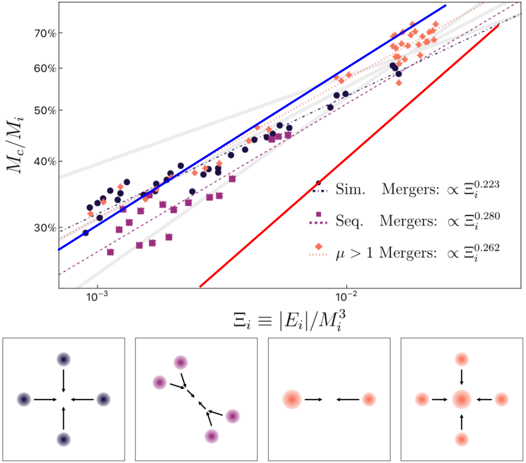
- Note: both Schive and Mocz relations hold for an “halo” with a single soliton in it

$$\frac{M_c}{M} = 1 = \alpha_{\rm Schive}\left(\frac{|E_{\rm sol}|}{M_{\rm sol}^3}\frac{1}{G^2m^2}\right)^{\beta_{\rm Schive}}$$

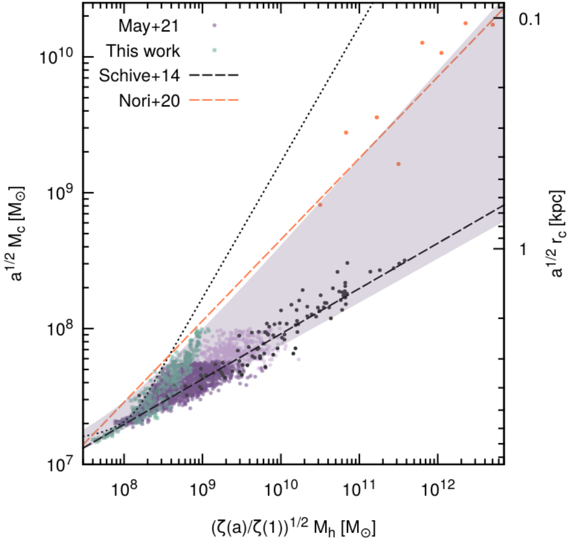
$$= \alpha_{\rm Mocz}\left(\frac{|E_{\rm halo}|}{M_{\rm halo}^3}\frac{1}{G^2m^2}\right)^{\beta_{\rm Mocz}}$$

The many tales of the soliton-halo relation

J. L. Zagorac et al 2023 [2212.09349]



- Symmetric configuration soliton merger: simultaneous mergers, sequential mergers, different mass solitons merger; sponge boundary conditions
- Claim no universal soliton-halo relation
- Notice: all above Schive, and mostly between Schive and Mocz relation; soliton is sizeable percentage of the halo for all the runs considered



- Recall

$$E_{\rm halo} \sim \frac{GM_{\rm halo}^2}{r_{\rm vir}}\,,\; M_{\rm halo} = \frac{4\pi\rho_{\rm m,0} r_{\rm vir}^3}{3}\zeta(z)$$

$$\implies M_{\rm c} = \left(\frac{E_{\rm halo}}{M_{\rm halo}}\right)^{1/2} \sim M_{\rm h}^{1/3}$$

- Analysis of condensation and evaporation of a soliton in a homogeneous background with maxwellian velocity distribution ("gas"), scattering problem

$$\frac{T_{\rm sol}}{T_{\rm gas}} \lesssim 0.08 \implies \text{soliton evaporates}$$

$$T_{\rm sol} := \frac{2mE^{\rm kin}_{\rm sol}}{3M_{\rm sol}} \simeq 0.15\frac{k^2_{\rm sol}}{m}\ , \ k_{\rm sol} \sim \frac{1}{\lambda_{\rm dB}}\ ; \quad T_{\rm gas} := \frac{k^2_{\rm gas}}{2m}$$

- Another way of seeing this: if $\lambda^{\rm gas}_{\rm dB} < \lambda^{\rm sol}_{\rm dB}$, gas disrupts the soliton
- Extension to a non-homogeneous halo? Soliton that forms should have $\mathcal{O}(1)$ the temperature of the halo

$$T_{\rm halo} \sim m\frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}} \implies \frac{E^{\rm kin}_{\rm sol}}{M_{\rm sol}} \sim \frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}}$$

- Schive relation might be close to the soliton evaporation condition, which would be the actual lower bound
- Try to test it via pseudo-spectral Schrödinger-Poisson solver (no cosmology!)

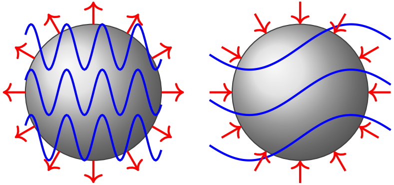
A soliton-halo bound?

J.H. Chan et al 2023 [2207.04057]

- Analysis of condensation and evaporation of a soliton in a homogeneous background with maxwellian velocity distribution ("gas"), scattering problem

$$\frac{T_{\rm sol}}{T_{\rm gas}} \lesssim 0.08 \implies \text{soliton evaporates}$$

$$T_{\rm sol} := \frac{2mE^{\rm kin}_{\rm sol}}{3M_{\rm sol}} \simeq 0.15\frac{k^2_{\rm sol}}{m}\ ,\ k_{\rm sol} \sim \frac{1}{\lambda_{\rm dB}}\ ;\quad T_{\rm gas} := \frac{k^2_{\rm gas}}{2m}$$



- Another way of seeing this: if $\lambda^{\rm gas}_{\rm dB} < \lambda^{\rm sol}_{\rm dB}$, gas disrupts the soliton
- Extension to a non-homogeneous halo? Soliton that forms should have $\mathcal{O}(1)$ the temperature of the halo

$$T_{\rm halo} \sim m\frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}} \implies \frac{E^{\rm kin}_{\rm sol}}{M_{\rm sol}} \sim \frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}}$$

- Schive relation might be close to the soliton evaporation condition, which would be the actual lower bound
- Try to test it via pseudo-spectral Schrödinger-Poisson solver (no cosmology!)

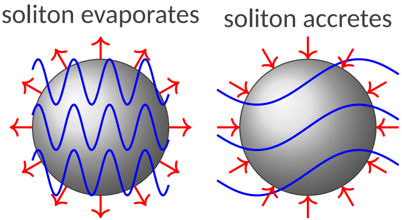
A soliton-halo bound?

J.H. Chan et al 2023 [2207.04057]

- Analysis of condensation and evaporation of a soliton in a homogeneous background with maxwellian velocity distribution ("gas"), scattering problem

$$\frac{T_{\rm sol}}{T_{\rm gas}} \lesssim 0.08 \implies \text{soliton evaporates}$$

$$T_{\rm sol} := \frac{2mE^{\rm kin}_{\rm sol}}{3M_{\rm sol}} \simeq 0.15\frac{k^2_{\rm sol}}{m}\,,\ k_{\rm sol} \sim \frac{1}{\lambda_{\rm dB}}\ ;\quad T_{\rm gas} := \frac{k^2_{\rm gas}}{2m}$$



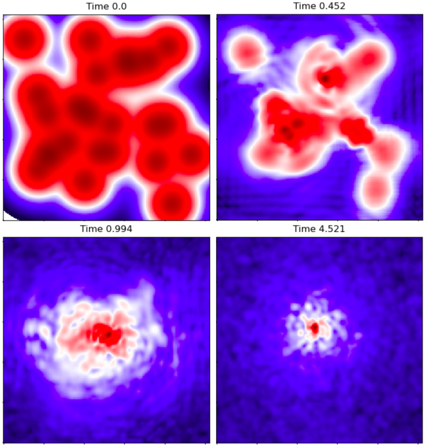
- Another way of seeing this: if $\lambda^{\rm gas}_{\rm dB} < \lambda^{\rm sol}_{\rm dB}$, gas disrupts the soliton
- Extension to a non-homogeneous halo? Soliton that forms should have $\mathcal{O}(1)$ the temperature of the halo

$$T_{\rm halo} \sim m\frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}} \implies \frac{E^{\rm kin}_{\rm sol}}{M_{\rm sol}} \sim \frac{E^{\rm kin}_{\rm halo}}{M_{\rm halo}}$$

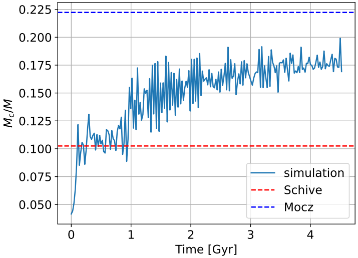
- Schive relation might be close to the soliton evaporation condition, which would be the actual lower bound
- Try to test it via pseudo-spectral Schrödinger-Poisson solver (no cosmology!)

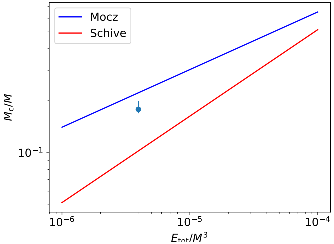
Soliton-Halo relation from many initial conditions

Example: multi-soliton

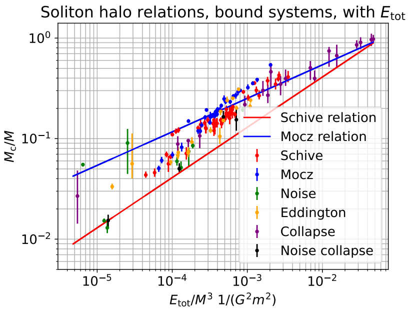


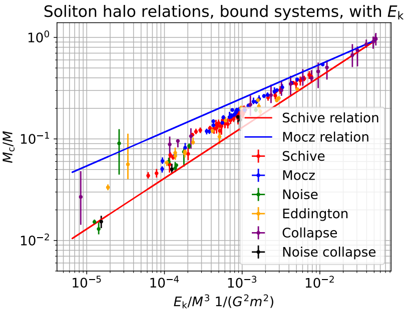
- Multiple soliton toy initial condition
- Density fluctuations plus secular growth, when to compute the soliton-halo relation?





Is the soliton-halo relation an attractor?





- Multiple solitons with same radius, Schive
- Multiple solitons with different radii, Mocz
- Homogeneous background, Noise
- NFW halo initialized with Eddington
- Elliptical overdensity, Collapse
- Elliptical overdensity plus noise, Noise collapse

Summary

WEIZMANN INSTITUTE OF SCIENCE

- ULDM paradigm can be investigated using astrophysical probes, since soliton formation changes the expected dark matter density profile
- Estimates of expected soliton size within a given halo are important for such constraints: soliton-halo relation
- Many claims about existence or otherwise of a soliton-halo relation
- Is the soliton-halo relation a lower bound? Soliton evaporation in a gas background suggests the existence of a possible lower bound also in haloes (non homogeneous)
- We performed many numerical simulations, with different initial conditions, hinting at the lower bound possibility, and possibly reconciling the different claims in the literature
- Astrophysical bounds on ULDM which use the Schive soliton-halo relation are conservative, if such a relation is a lower bound on the mass of the soliton.

Relaxation and secular growth

- Relaxation time

$$t_{\rm rel} \simeq \frac{0.7\sqrt{2}\sigma^6 m^3}{12\pi^3 G^2 \rho^2 \ln\Lambda}$$

- Soliton expected two growth regimes (B. Eggemeier and J.C. Niemeyer 2019 [1906.01348])

$$M_{\rm sol}(t) \sim t^{1/2} \text{ relaxation regime },\ M_{\rm sol} \sim t^{1/8} \text{ saturation regime}$$

Eddington initialized halos

Initialize halos as

$$\psi(x) = (\Delta v)^{3/2} \sum_{\vec{v}} \sqrt{f(\mathcal{E}(x, v))} \mathrm{e}^{\mathrm{i} m_i \vec{x} \cdot \vec{v} + \mathrm{i} \varphi_{\vec{v}}} ,$$

where $\varphi_{\vec{v}}$ is a random phase dependent on $\vec{v}$, Δv is the velocity spacing allowed by resolution in the simulation, and

$$f(\mathcal{E}) = \frac{2}{\sqrt{8}\pi^2} \int_0^{\sqrt{\mathcal{E}}} \mathrm{d}Q \, \frac{\mathrm{d}^2 \rho}{\mathrm{d}\Psi^2}(Q) ,$$

$$\mathcal{E} = \Psi(r) - \frac{v^2}{2} , \; \Psi = -\Phi + \Phi(r_{\max}) ,$$

where $Q = \sqrt{\mathcal{E} - \Psi}$ and Φ is the overall gravitational potential. As target density, we use a Navarro-Frenk-White profile,

$$\rho = \frac{\rho_0 r_{\mathrm{s}}^3}{r(r + r_{\mathrm{s}})^2} ,$$

with varying ρ_0 and r_{s}.

Other initial conditions

- Elliptical collapse

$$\psi(\vec{x}) = \sqrt{N} \mathrm{e}^{-x_1^2/2R_1^2 - x_2^2/2R_2^2 - x_3^2/2R_3^2}$$

- Homogeneous background (noise) initial conditions

$$\langle \psi^*(x)\psi(y) \rangle = C(x-y)\,, C(x) = A\mathrm{e}^{-x^2/l^2}$$

which can be realized with

$$\psi(x) = \frac{(\Delta k)^3}{L^3} \sum_{\vec{k}} \sqrt{\frac{\hat{C}_{\vec{k}} L^3}{2}} (A_{\vec{k}} + \mathrm{i} B_{\vec{k}}) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\,, A_{\vec{k}}, B_{\vec{k}} \sim \mathcal{N}(0,1)$$

$E_{\rm tot}$ or $E_{\rm kin}$?

- Notice that

$$\frac{E_{\rm sol}}{M_{\rm sol}} \simeq 0.23\lambda^2 \;, \; M_{\rm sol} = \lambda\lambda M_1$$

- The code computes $E_{\rm pot}$ up to a constant (Φ in the code is determined such that $\langle\Phi\rangle = 0$). To recover the constant which makes $\Phi(r \to \infty) \to 0$, use

$$\Phi(R) = \Phi_{\rm code}(R) + c = -\frac{GM}{R} \implies c = -\frac{GM}{R} - \Phi_{\rm code}(R)$$

- How reliable? If virial theorem holds, we can think of substituting $|E_{\rm tot}| \to E_{\rm kin}$

Energy spectrum

- Energy spectrum

$$F(t,\omega) = 2\,\mathrm{Re}\int_0^\infty \frac{\mathrm{d}t'}{2\pi}\,\mathrm{d}^3x\,\psi^*(\vec{x},t)\psi(\vec{x},t+t')\mathrm{e}^{\mathrm{i}\omega t'-t'^2/\tau^2}$$

- Soliton arising at energies

$$\omega = \omega_{\mathrm{sol}} + \omega_{\mathrm{halo}}\;,\omega_{\mathrm{halo}} \sim -4\pi G\int_{r_t}^\infty \mathrm{d}y\,y\rho_{\mathrm{halo}}(r)$$