Superconductivity: Past, Present and Future

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Collaborators:

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'Super-C' collaboration (P. Torma)

PRO-QM EFRC (BES DE-SC0019443): Columbia MRSEC (DMR-2011738)
The Simons foundation; KEELE FOUNDATION

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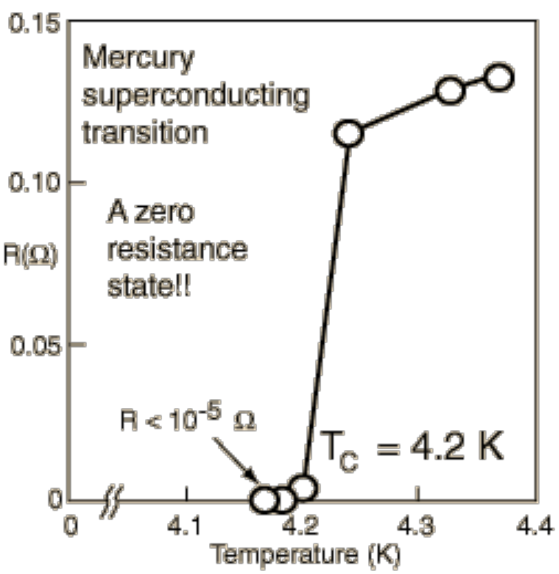
Superconductivity:

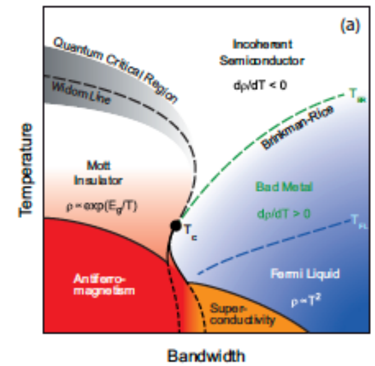
Serendipitous discovery

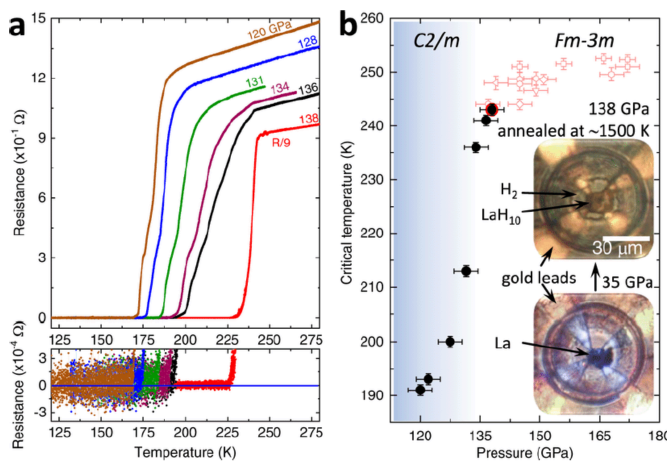
Heike Kamerlingh-Onnes
1911-12

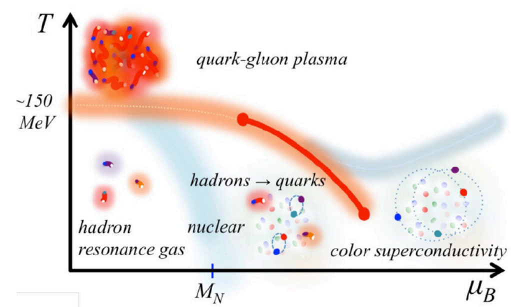


Many instances

- ‘Conventional’: Hg, Pb, Nb3Sn, MgB2…
- Unconventional -conventional: A_3C_{60}; LaH_{10}
- Heavy fermion
- Organic: BEDT…
- Copper-Oxide
- Iron-pnictide
- ^{3}He
- Condensed atomic gasses
- Neutron Stars
- High density nuclear matter







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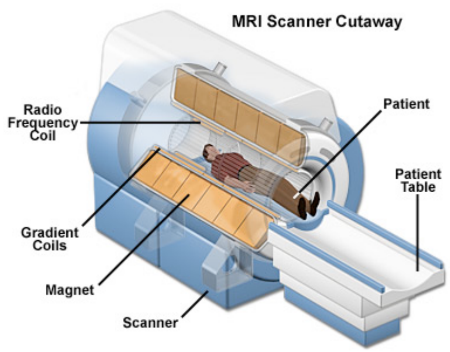
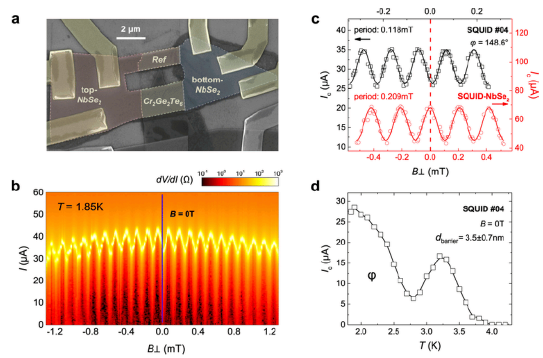
Superconductivity in practice

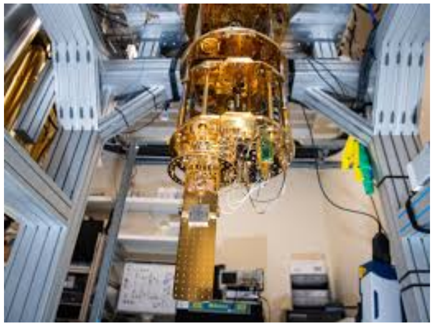
High field magnets



Precision measurement



Superconducting Qubits



Dreams: loss-free power transmission

Powering cities with minimal thermal load

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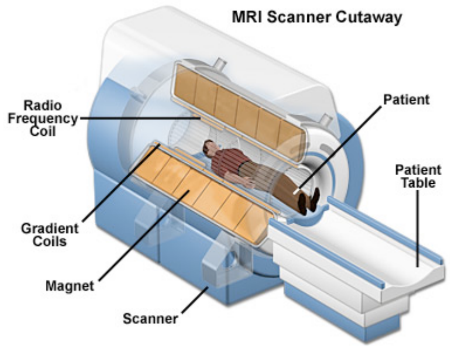
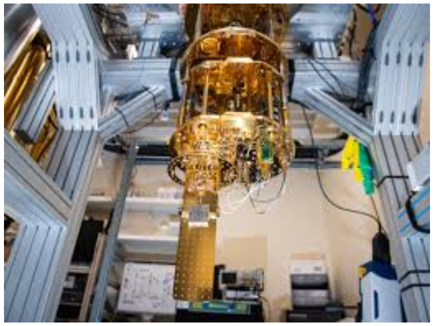
Three needs

High Transition temperature



High Critical Current

‘Protected’ Excitations



?What materials support these behaviors?

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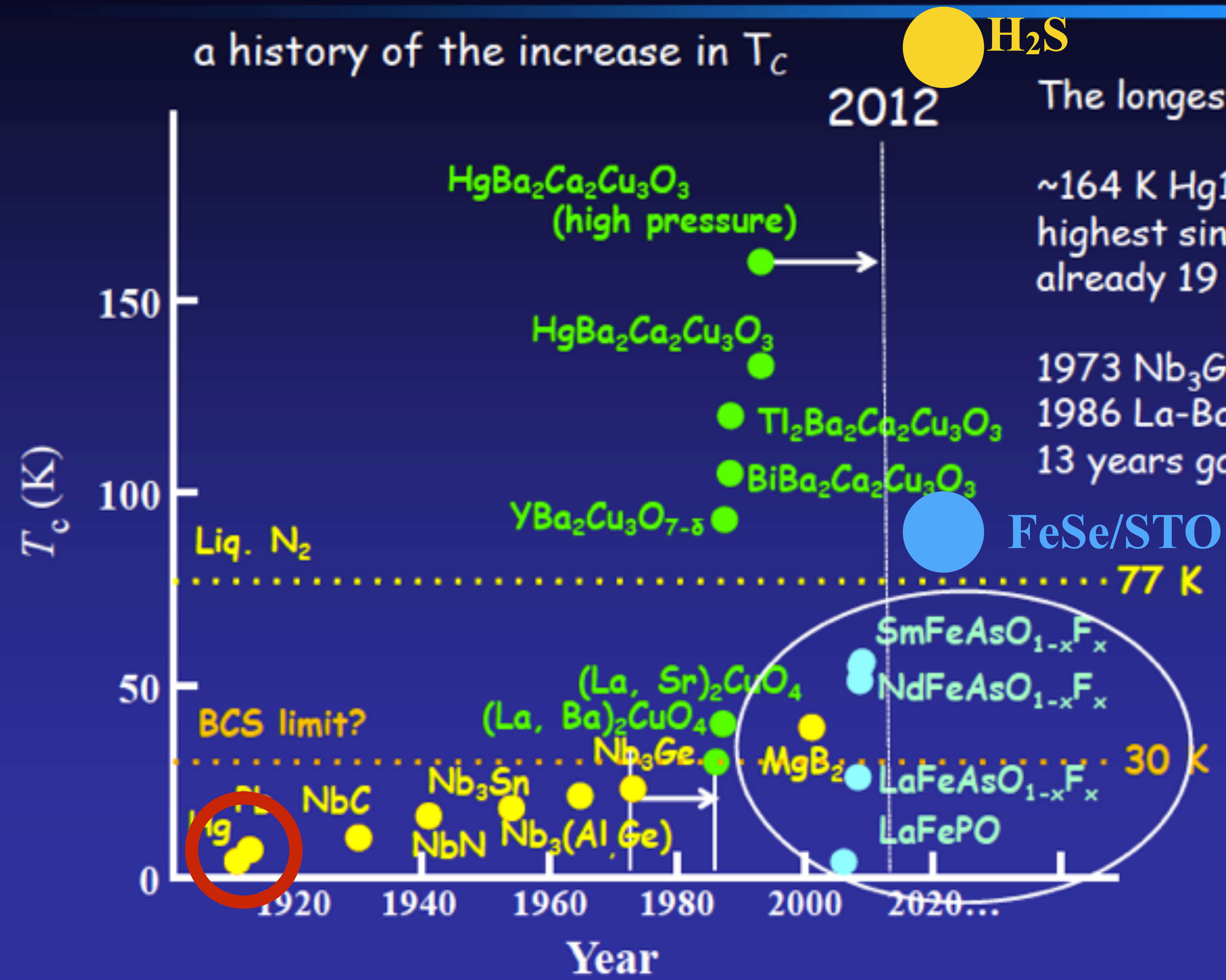
Where are we?

a history of the increase in T_c



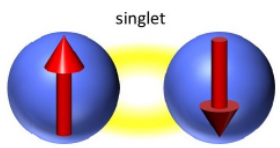
The longest silence

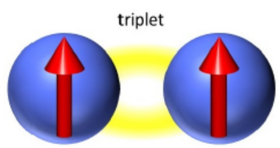
~164 K Hg1223
highest since 1993
already 19 years gap

1973 Nb_3Ge (23 K)
1986 La-Ba-Cu-O (30 K)
13 years gap

Theory

Superconductivity is based on pairing





Pairing => order parameter

$$\mathbf{\Delta(R)} = \int \mathbf{dr}\ \mathbf{D}_{\alpha\beta}(\mathbf{r}) \left\langle \mathbf{c}_\alpha\left(\mathbf{R}+\frac{\mathbf{r}}{2}\right) \mathbf{c}_\beta\left(\mathbf{R}-\frac{\mathbf{r}}{2}\right)\right\rangle$$

- **"Pair wave function" D encodes symmetry of pairing state**
- **$\Delta(\mathbf{R}) = e^{i\theta(\mathbf{R})}|\Delta(\mathbf{R})|$ has magnitude, phase and may depend on internal spin/orbital indices**
- **Order parameter non-classical: sc state is coherent superposition of states of different particle number. $[\theta, N] \neq 0$**

- **Long ranged superconducting order implies non-zero 'phase stiffness' ρ_S: term in free energy $F = \frac{1}{2}\rho_S(\nabla\theta)^2$**

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Theory: Status

- **Qualitative properties understood**
  - **Symmetries classified**
  - **Elementary excitations determined**

- **Open questions**
  - **Mechanism: “what causes that”**
  - **Transition temperature: “how big can it be”**
  - **Phase stiffness: “how tough is the superconductor?”**

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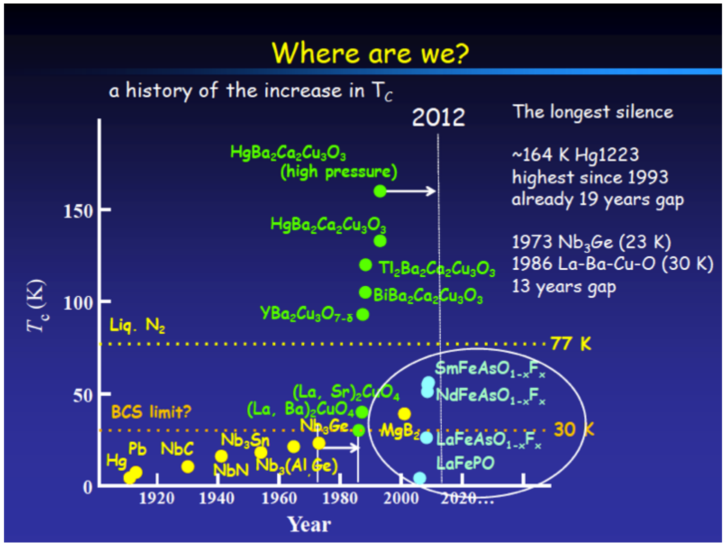
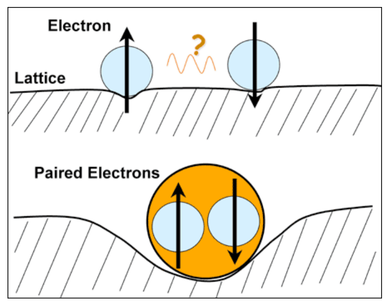
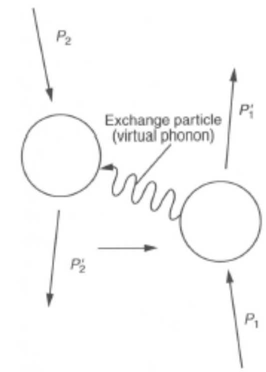
‘Conventional’ superconductors: BCS/Migdal-Eliashberg Theory



Pairing of electrons via exchange of lattice vibrations (“phonons”)





Key fact: in conventional metals, phonons are much slower than electrons:

Typical phonon frequency $\omega_{\mathrm{Debye}} \ll$ typical electron energy E_F

=>

Controlled ‘Migdal-Eliashberg’ theory

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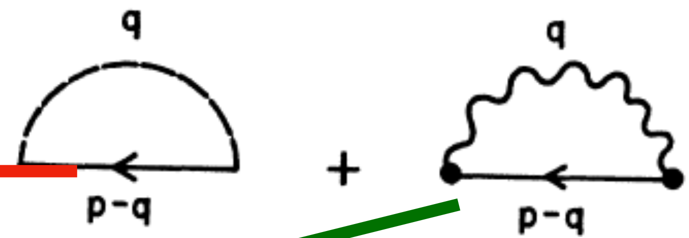
BCS/Migdal-Eliashberg Theory

Scalapino, Schreiffer, Wilkins PR148 263 (1966)



Electron-electron interaction
Treated as an instantaneous repulsion

electron -phonon interaction.
Treated dynamically.

$$\Sigma(\mathbf{p},i\omega_n)=-\frac{1}{\beta}\sum_{\mathbf{p}'n'}\tau_3 G(\mathbf{p}',i\omega_{n'})\tau_3$$
$$\times\Big\{\sum_{\lambda}|\bar{g}_{\mathbf{pp}'\lambda}|^2 D_{\lambda}(\mathbf{p}-\mathbf{p}',\, i\omega_n-i\omega_{n'})$$
$$+V(\mathbf{p}-\mathbf{p}')\Big\}$$

Defines electron-phonon coupling λ

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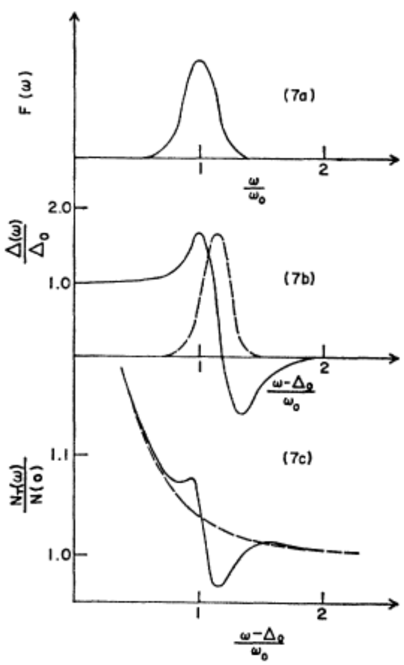
How do we know this is right?

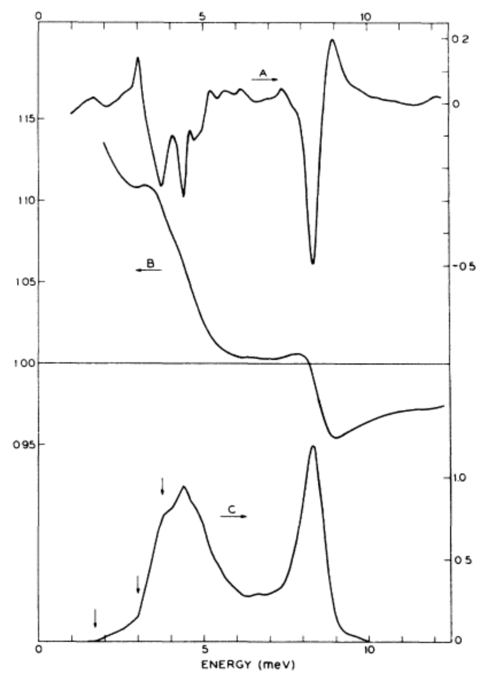
McMillan and Rowell, PRL 14 108 1965

gap function $\Delta(\omega)$ has real and imaginary parts with structure related to phonon frequencies

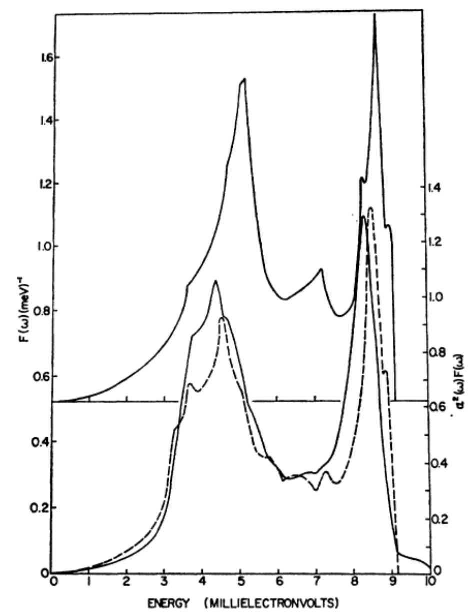
Scalapino, Schreiffer, Wilkins
PR148 263 (1966)



McMillan and Rowell
PRL 14 108 1965



Rowell, McMillan and Feldman
PR 178 897 1969



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Implications

- ’S-wave’: Δ has point symmetry of material
- Transition temperature:

Tc increase ‘self limited’: increasing λ increases electron mass: cuts off Tc increase

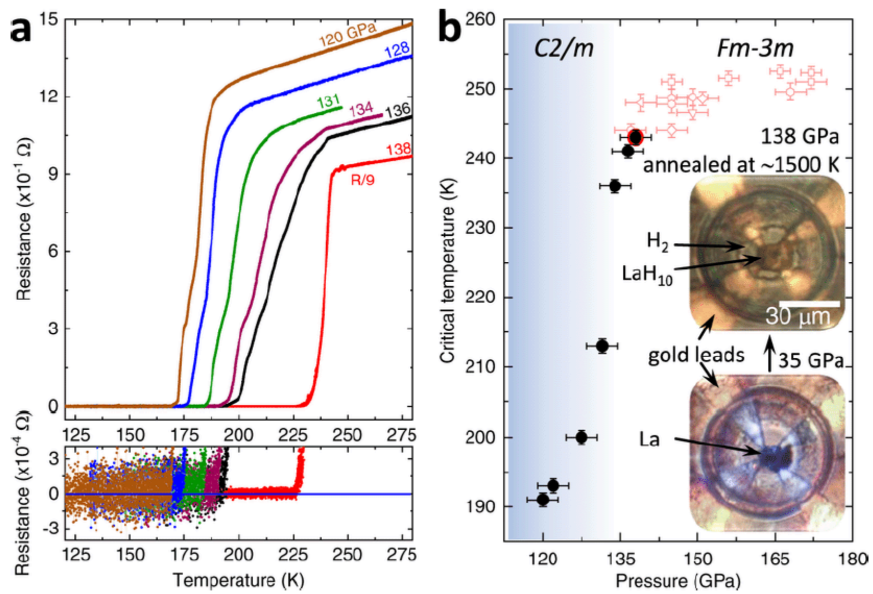
$$T_c = \omega_{\mathrm{Debye}}\, e^{-\frac{1+\lambda}{\lambda-\mu^\star}}$$

Electron-electron interaction bad

Increasing coupling increases Tc

Fundamental limit: Phonon frequency

200K superconductivity in H3S, LaH10,… Small mass of H means high frequency



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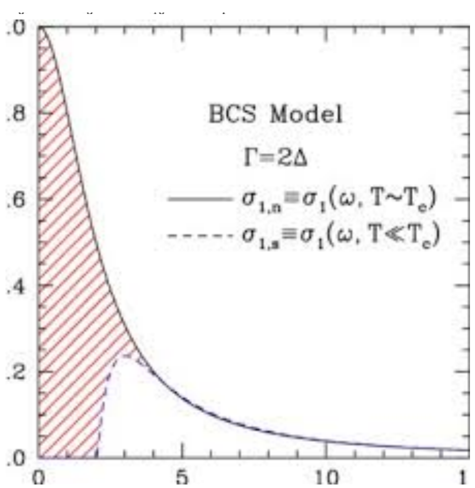
Implications

- **Electron mass <=> density of states**
- **Big mass => large coupling (good)**
- **But: big mass =>low ρ_s (bad)**



Normal state conductivity: $\sigma(\omega) = \frac{\mathbf{ne^2}}{\mathbf{m}} \frac{\Gamma}{\omega^2 + \Gamma^2}$

Stiffness: $\rho_S = \frac{\mathbf{ne^2}}{\mathbf{m}} \mathbf{f}\left(\frac{\Gamma}{\Delta}\right)$

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Assumptions

- **Electrons well described by (effectively non-interacting) density functional (DFT) theory: $-\nabla^2\psi + V_{KS}\left[\{n(r), R_n\}\right]\psi = E\psi$**

- **Static approximation for electron-electron interaction. Exact (up to corrections $\mathcal{O}\left(\frac{\omega_{\text{Debye}}}{E_F}\right)$) and neglect of anharmonicity for phonon**

From DFT eigenstates ψ

From dependence of DFT energy on atomic positions

$$\Sigma(\mathbf{p},i\omega_n) = -\frac{1}{\beta}\sum_{\mathbf{p}'n'}\tau_3 G(\mathbf{p}',i\omega_{n'})\tau_3 \times\left\{\sum_\lambda |\bar{g}_{\mathbf{p}\mathbf{p}'\lambda}|^2 D_\lambda(\mathbf{p}-\mathbf{p}', i\omega_n - i\omega_{n'}) + V(\mathbf{p}-\mathbf{p}')\right\}$$

From scattering amplitude implied by $\delta V_{KS}/\delta R_n$

Semiphenomenological

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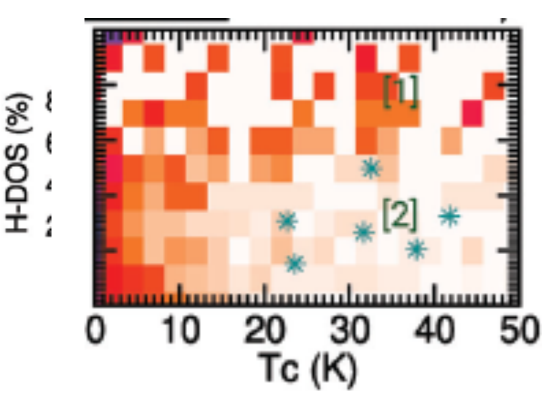
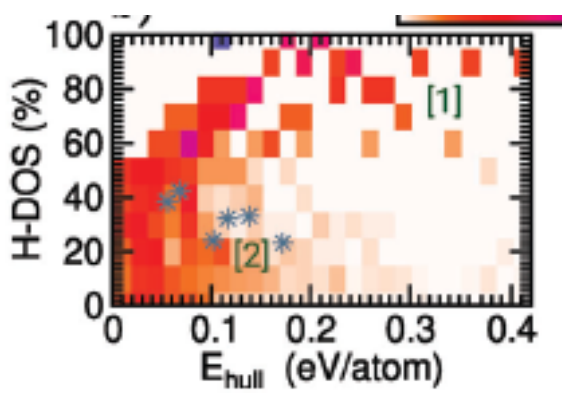
Extension: Materials Searches
High-T_c hydrides at ambient pressure
(M. Marques, Super-C)

- **Assumption: standard DFT+Migdal-Eliashberg works**
- **Database:**
  - **~10^6 band structures of compounds**
  - **~10^4 detailed electron-phonon calculations**
- **=>train neural network to "predict" (identify) high Tc candidates**
- **Detailed study of candidate materials: verification of transition temperature, material stability**





High hydrogen DOS at fermi level

- **Good for high Tc**
- **Bad for thermodynamic stability**

Adv Mat. 36, e2307085 2024
Adv. Funct. Mat. 342404043 (2024)

No room temperature-stable high Tc compounds found

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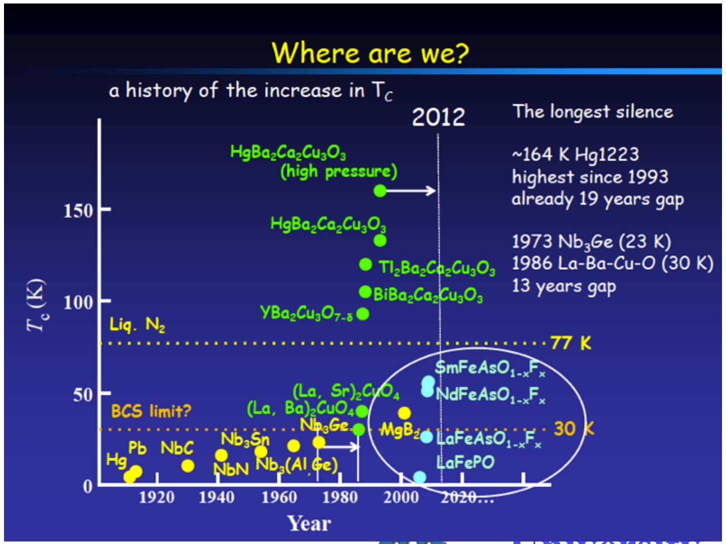
Relaxing the assumptions

Extending the conventional theory:

=>beyond the static Coulomb approximation; non-adiabatic phonons

Electrons not well described by (effectively non-interacting) density functional (DFT) theory:

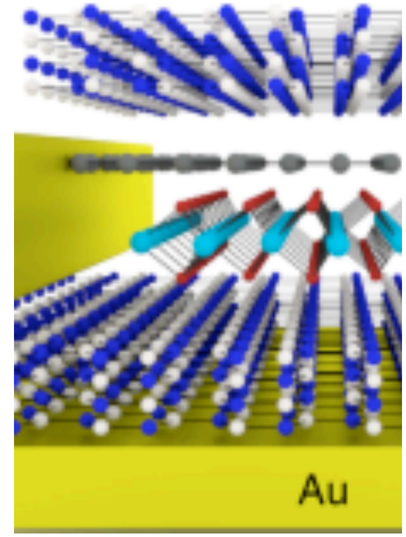
=>Hubbard model; quantum critical pairing; doped spin liquids



Dynamical Coulomb interaction in 2d materials

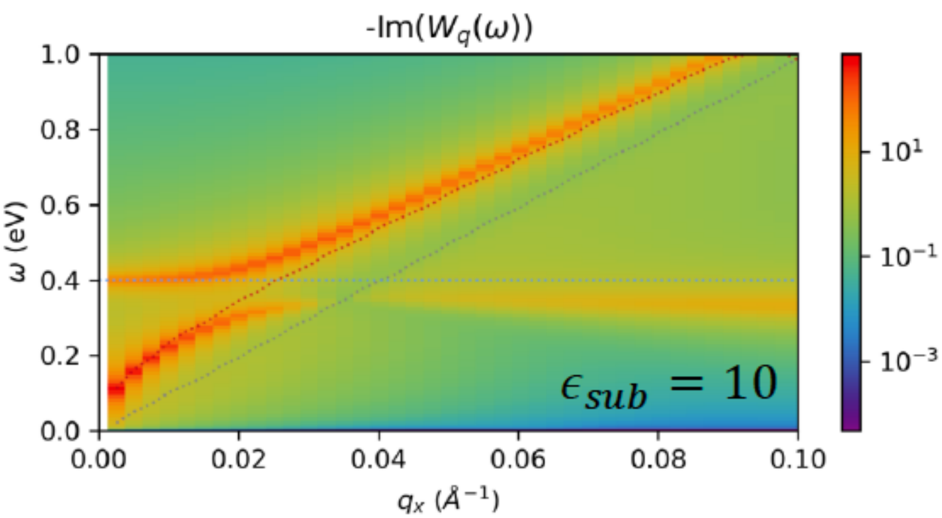
Yann in ’t Veldt; M. Roesner, M. Katsnelson, AJM.

2D Materials 10, 045031 (2023) and to appear



- Low carrier density superconductivity (NbSe2, MoTe2..). Can be fabricated in monolayer form with controllable surroundings

- Soft tunable plasmon (variable ϵ and layer number) mixes with longitudinal phonons



Model system square lattice tight binding k_F=0.2, E_F=2

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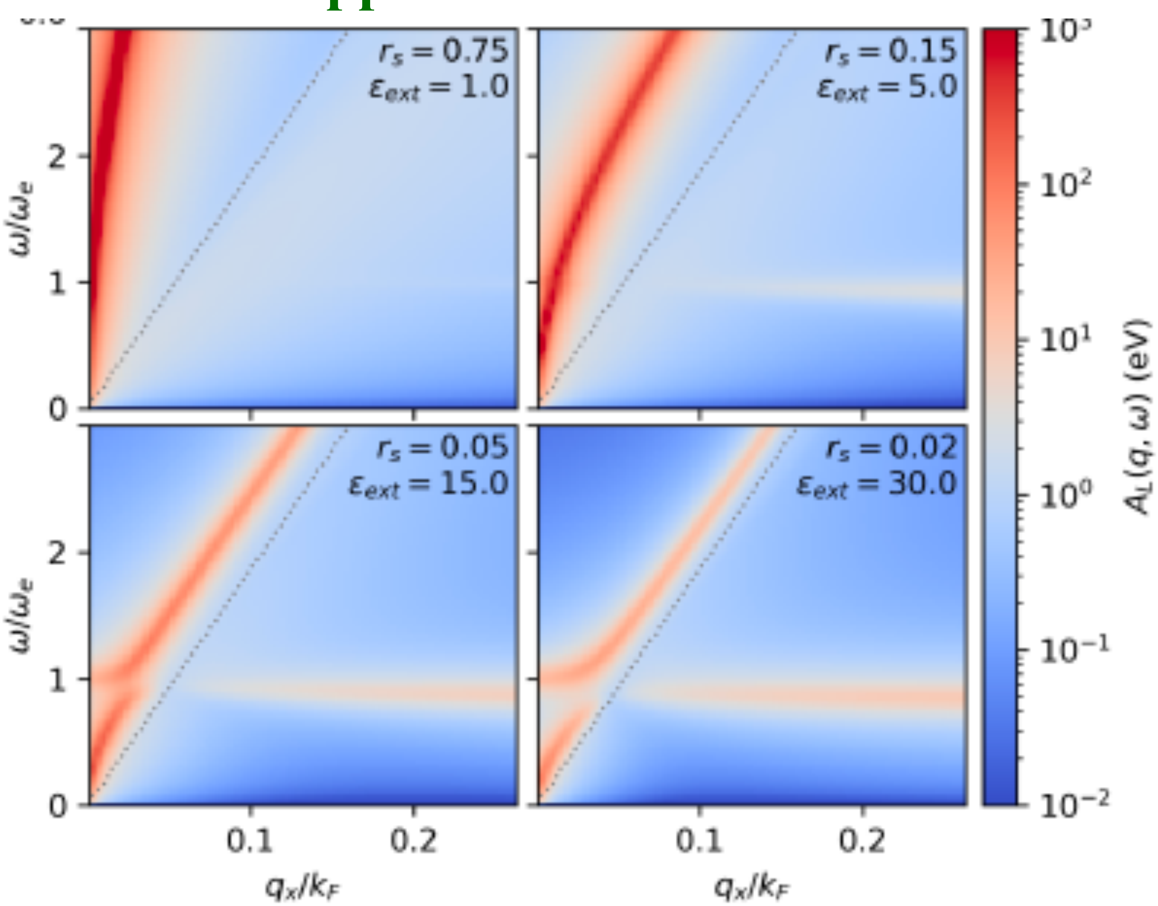
One loop theory
k-dependent self energy

2D electrons, tight binding model, approx 1/3 electron/site



The physical (dynamically screened) interaction.

$$(q, i\nu_n) = \frac{v_C(q) + v_{ph}^{LO}(i\nu_n)}{1 - [v_C(q) + v_{ph}^{LO}(i\nu_n)]\Pi_0(q, i\nu_n)},$$

Static screening

$$I_L^{stat}(q, i\nu_n) = \frac{v_C(q) + v_{ph}^{LO}(i\nu_n)}{1 - [v_C(q) + v_{ph}^{LO}(i\nu_n)]\Pi_0(q, i\nu_n = 0)}$$

Separate

$$I_L^{C+ph}(q, i\nu_n) = \frac{v_C(q)}{1 - v_C(q)\Pi_0(q, i\nu_n)} + \frac{v_{ph}^{LO}(i\nu_n)}{1 - v_{ph}^{LO}(i\nu_n)\Pi_0(q, i\nu_n)}$$

ty so to plasmon
W0

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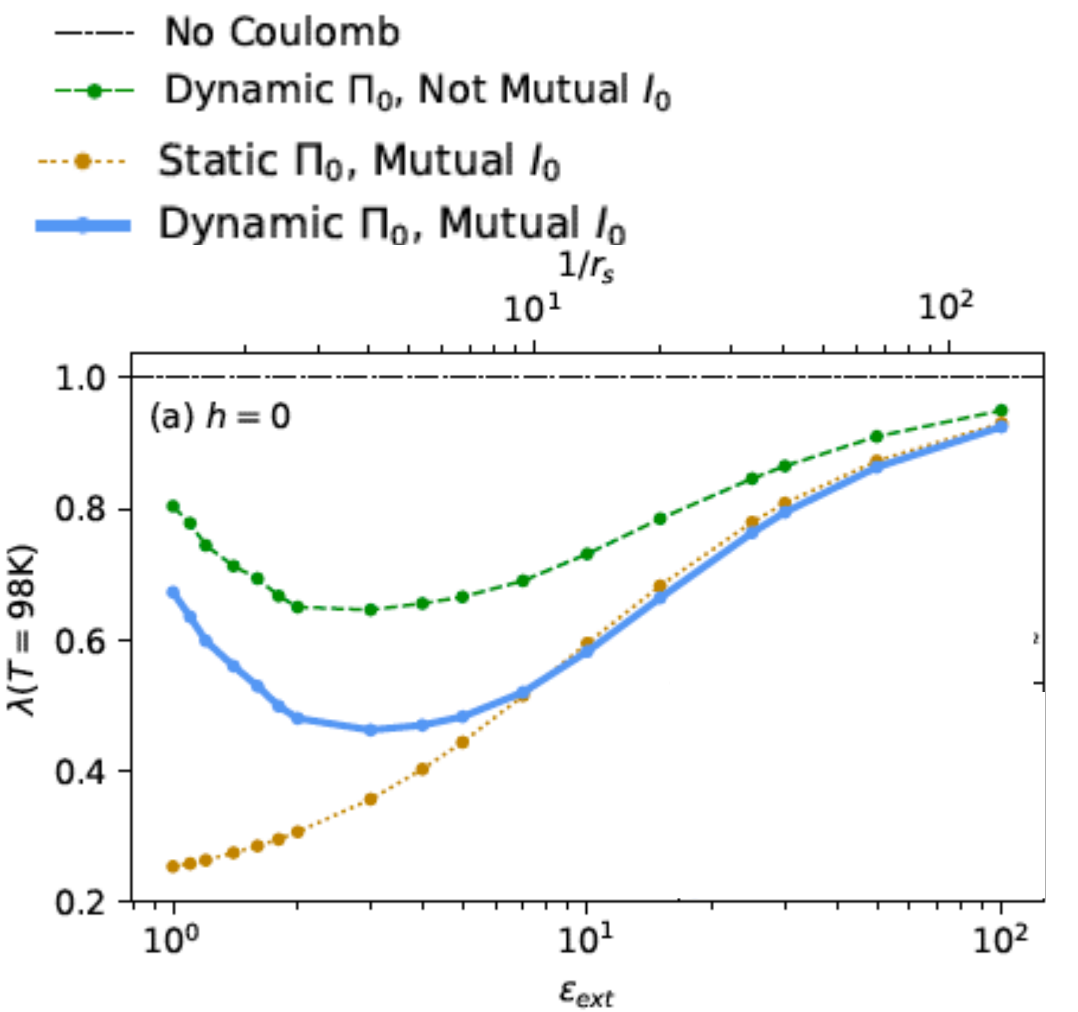
Pairing strength as function of r_S for different interaction models

Here: show leading eigenvalue of gap operator at because in 2023, getting to T=Tc is too expensive



Dynamical Coulomb interactions enhance Tc

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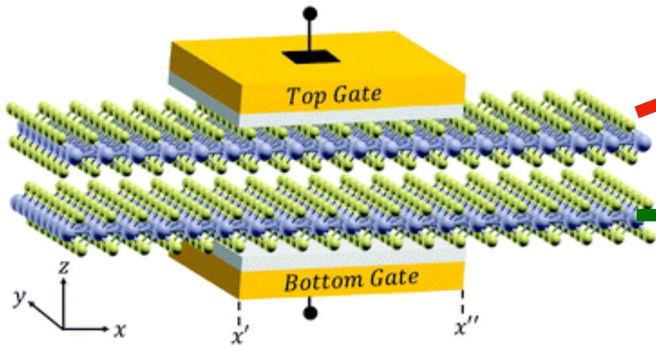
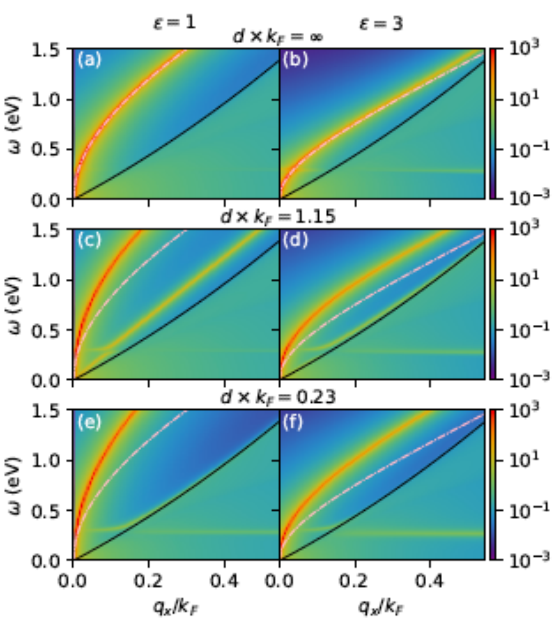
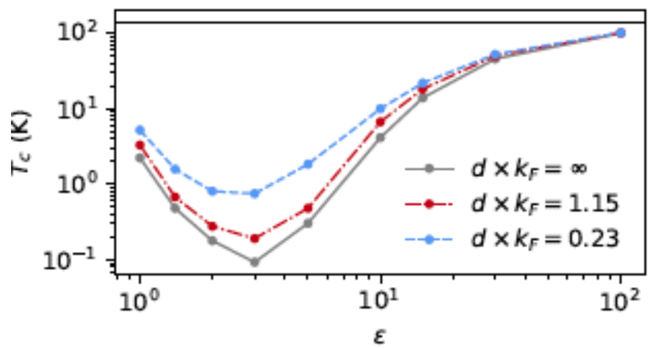
Double layers
(in preparation)

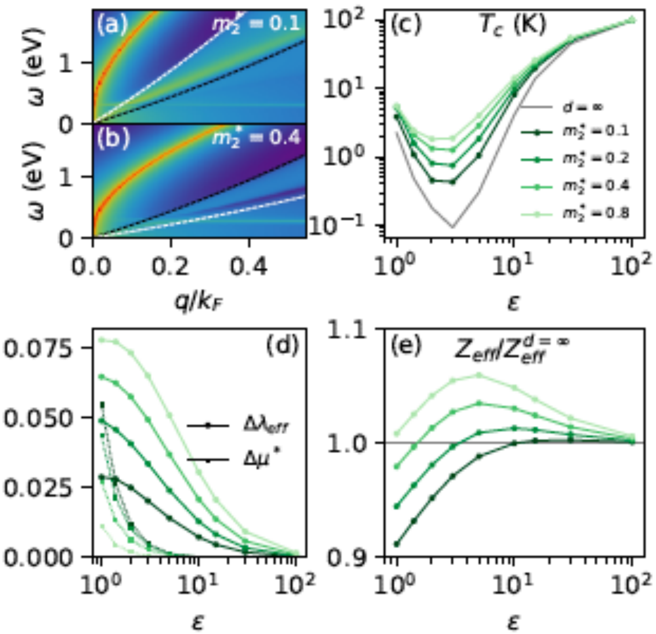


Screening layer

Superconducting layer







By tuning the properties of the screening layer, can enhance Tc.

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Essential technical step: compact low rank representations

Ex: Discrete Lehmann representation
J. Kaye, O. Parcollet et. Al; CCQ/Flatiron

$$G(i\nu_n) = \int_{-\infty}^{\infty} d\omega\, K(i\nu_n, \omega)\rho(\omega)$$

$$K(i\nu_n, \omega) \approx \sum_{l=1}^{r} K(i\nu_n, \omega_l)\pi_l(\omega).$$

$$G(i\nu_n) \approx \sum_{l=1}^{r} K(i\nu_n, \omega_l)\widehat{g}_l.$$

Interpolative decomposition=> controllable accuracy low rank representation of in terms of small number of frequencies and basis functions.

Generalized to 3 point vertices

Phys. Rev. B105, 235115 (2022)
Phys. Rev. B111, 035135 (2025)

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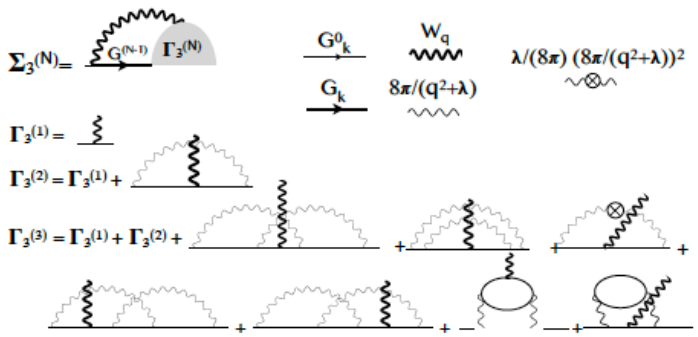
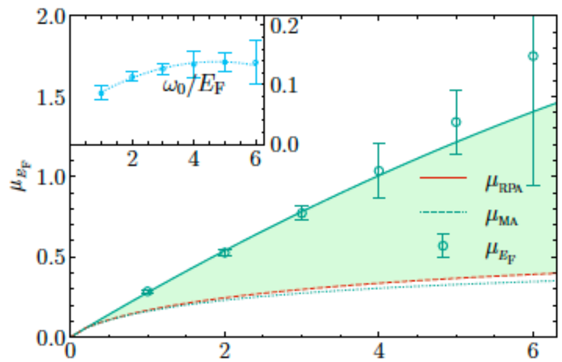
Beyond one loop approximation: Diagrammatic Monte Carlo for extended systems

Van Houcke; Prokof’ev and Svistunov; Kozik; Rossi; Ferrero; Haule, Chen…

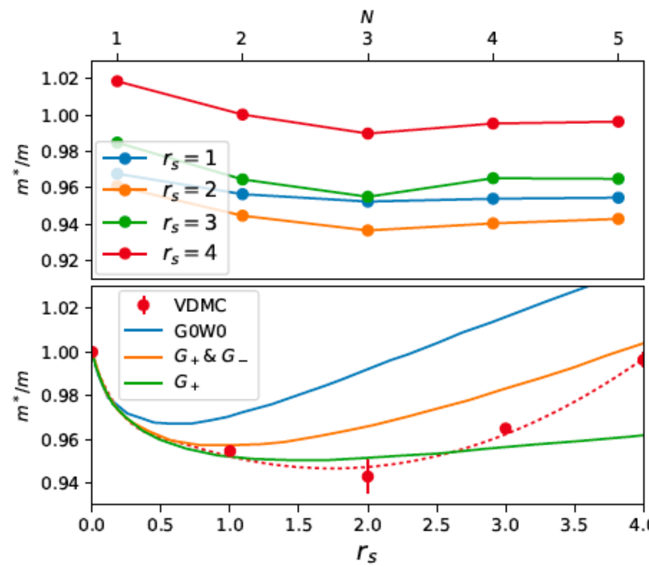
Evaluate Feynman Diagram series stochastically



Work in progress (Kun Chen): Coulomb repulsion term in gap equation: standard formula is a significant underestimate



Convergence w.r.t. order (for uniform electron gas)



Haule/Chen arXiv:20103146

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Beyond weak coupling: Bipolarons revisited

Sous, Zhang, Berciu, Prokof’ev, Reichman, Svistunov

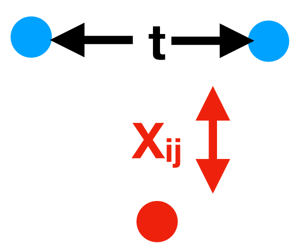
Polaron: self-trapped state of electron bound to lattice distortion

3 models:

Holstein: coupling to density $g\sum_{i\sigma} c^\dagger_{i\sigma} c_{i\sigma} X_i$

Peierls: coupling to hopping via bond-length modulation $g\sum_{i,j\sigma} c^\dagger_{i\sigma} c_{j\sigma}(X_i - X_j)$

Bond-Peierls: coupling to hopping via oscillator on-bond $g\sum_{i,j\sigma} c^\dagger_{i\sigma} c_{j\sigma} X_{ij}$



Linearization of $te^{-a-(X_i-X_j)}$

From interference of hopping pathways

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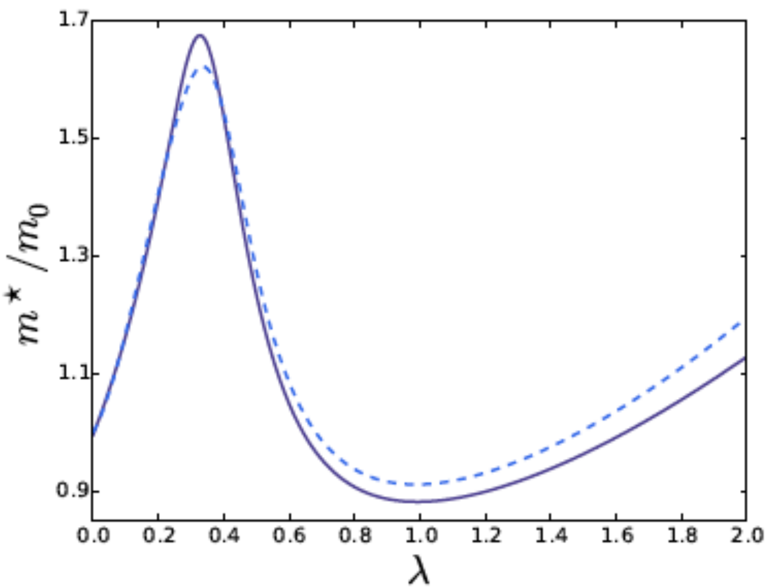
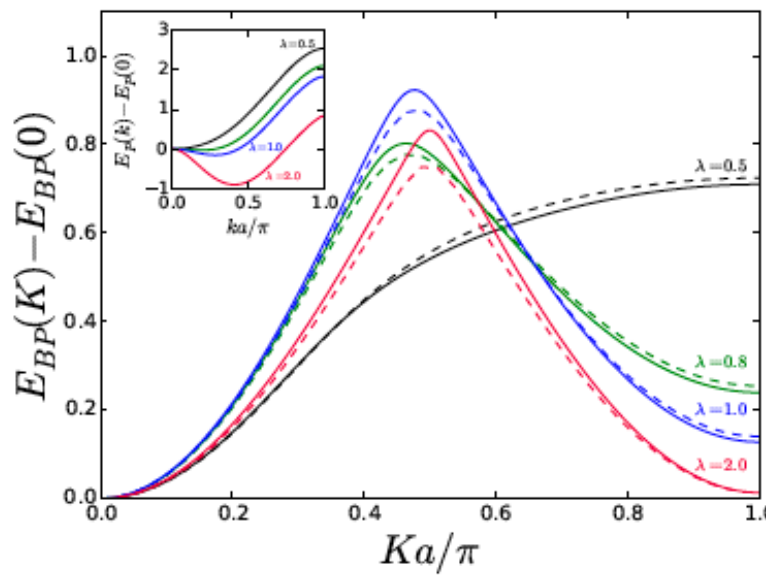
Sous, Chakraborty, Krems, Berciu: `Peierls' Bipolarons can be light

One dimensional Peierls model

$$\mathcal{H}_e = -t\sum_{i,\sigma}\left(c^\dagger_{i,\sigma}c_{i+1,\sigma} + h.c.\right) + \sum_{i\,\delta} U(\delta)\hat{n}_{i,\uparrow}\hat{n}_{i+\delta,\downarrow}$$

$$g\sum_{i,\sigma}\left(c^\dagger_{i,\sigma}c_{i+1,\sigma} + h.c.\right)\left(b^\dagger_i + b_i - b^\dagger_{i+1} - b_{i+1}\right)$$





Phys. Rev. Lett. 121, 247001 (2018)

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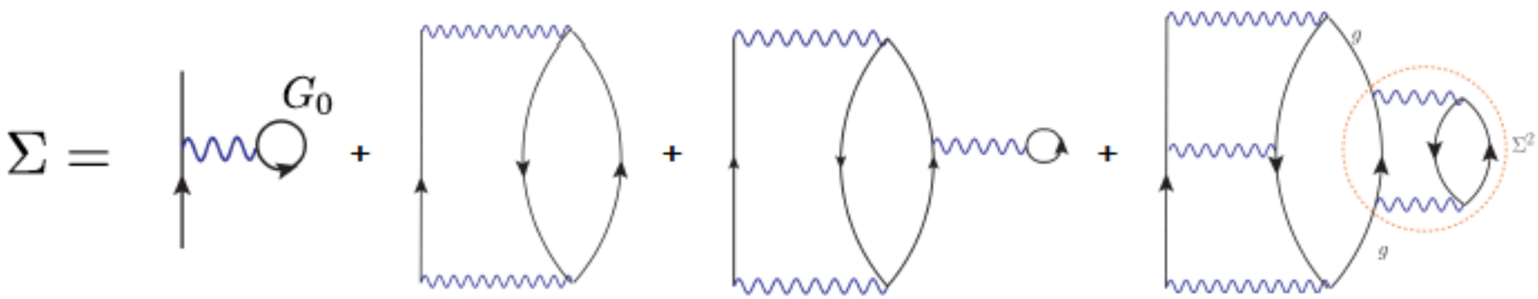
Diagrammatic Monte Carlo for Bipolaarons

Diag MC: evaluate terms in Feynman diagram series stochastically



N. V. Prokof'ev and B. V. Svistunov, PRL (2007)

sample series stochastically (diagram order, topology, internal integrals)

Zhang, Prokof’ev, Svistunov, Phys Rev B105 L020501(2022): for the special case of bipolarons (2 electrons +phonons) on a lattice this can be simplified, extended, and done in a sign-problem-free way. Can also do (with somewhat more computational effort) Holstein bipolarons

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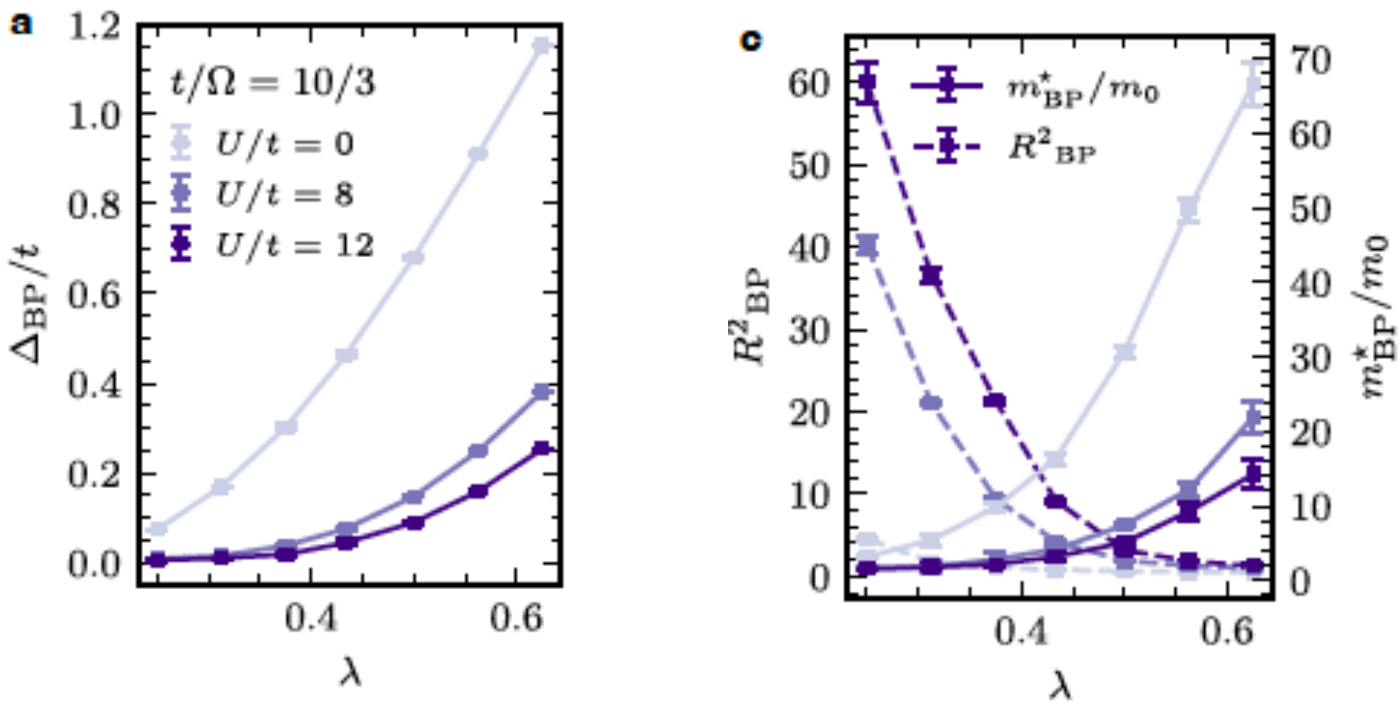
For Bond-Peierls Polarons in 2D

Phys. Rev. X 13, 011010



In 2D, up to ln[ln[corrections:

$$T_c \approx \frac{2n_{BP}}{m_{BP}}$$

Identify n_{BP} with $\dfrac{1}{\pi R^2_{BP}}$

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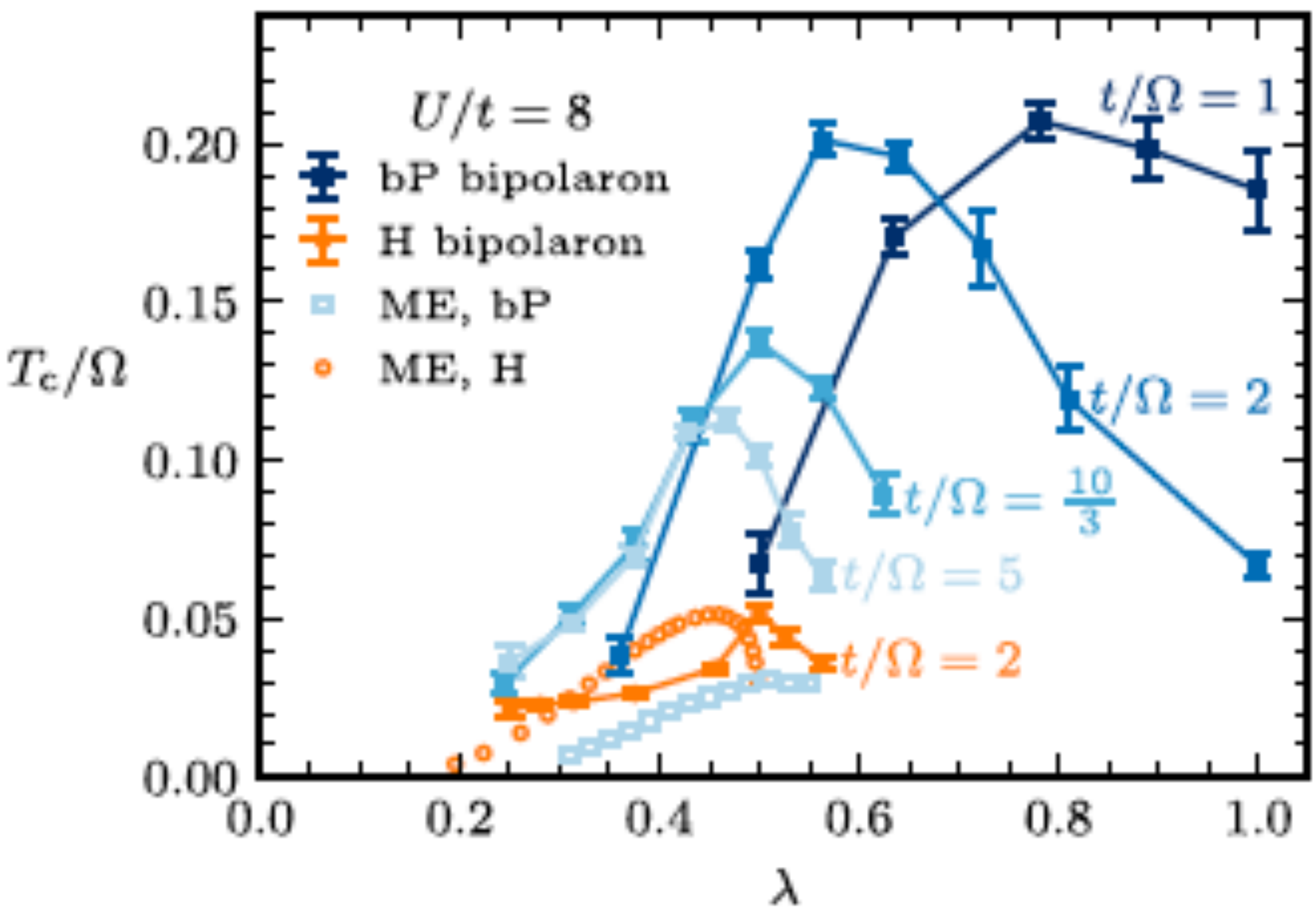
Maximum Transition temperature



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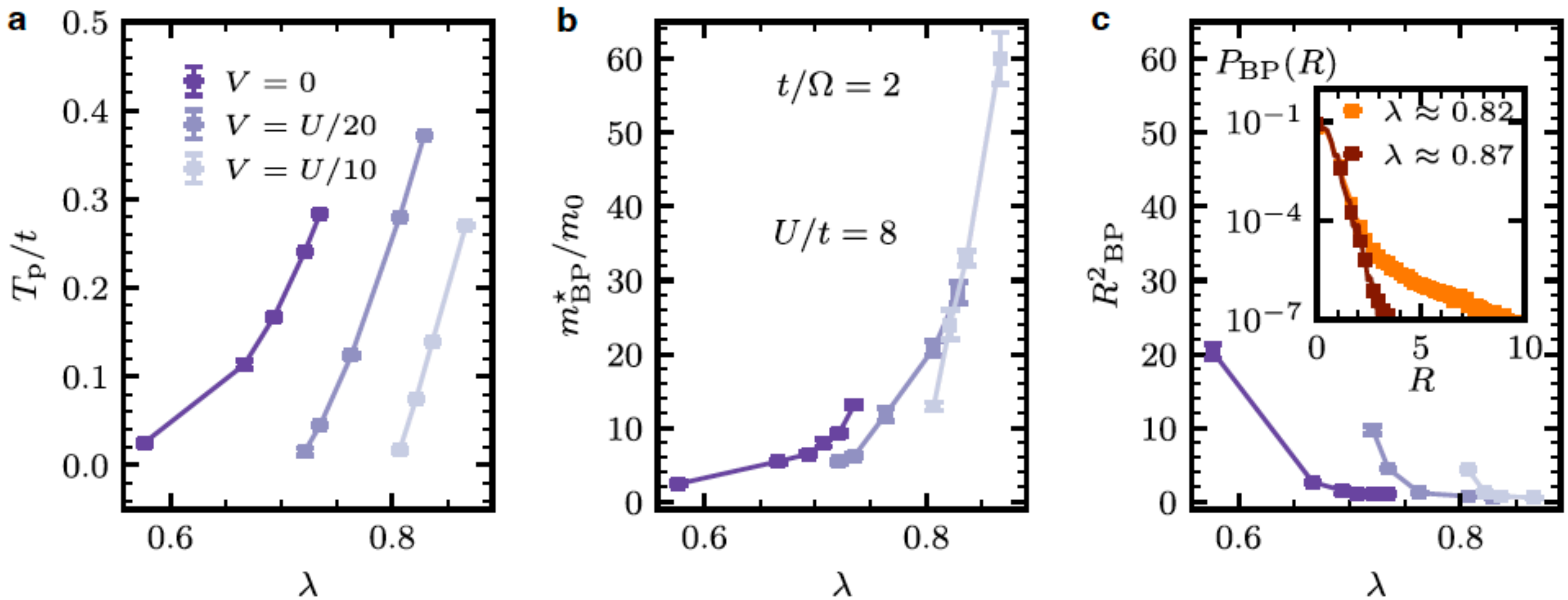
long ranged Coulomb interaction

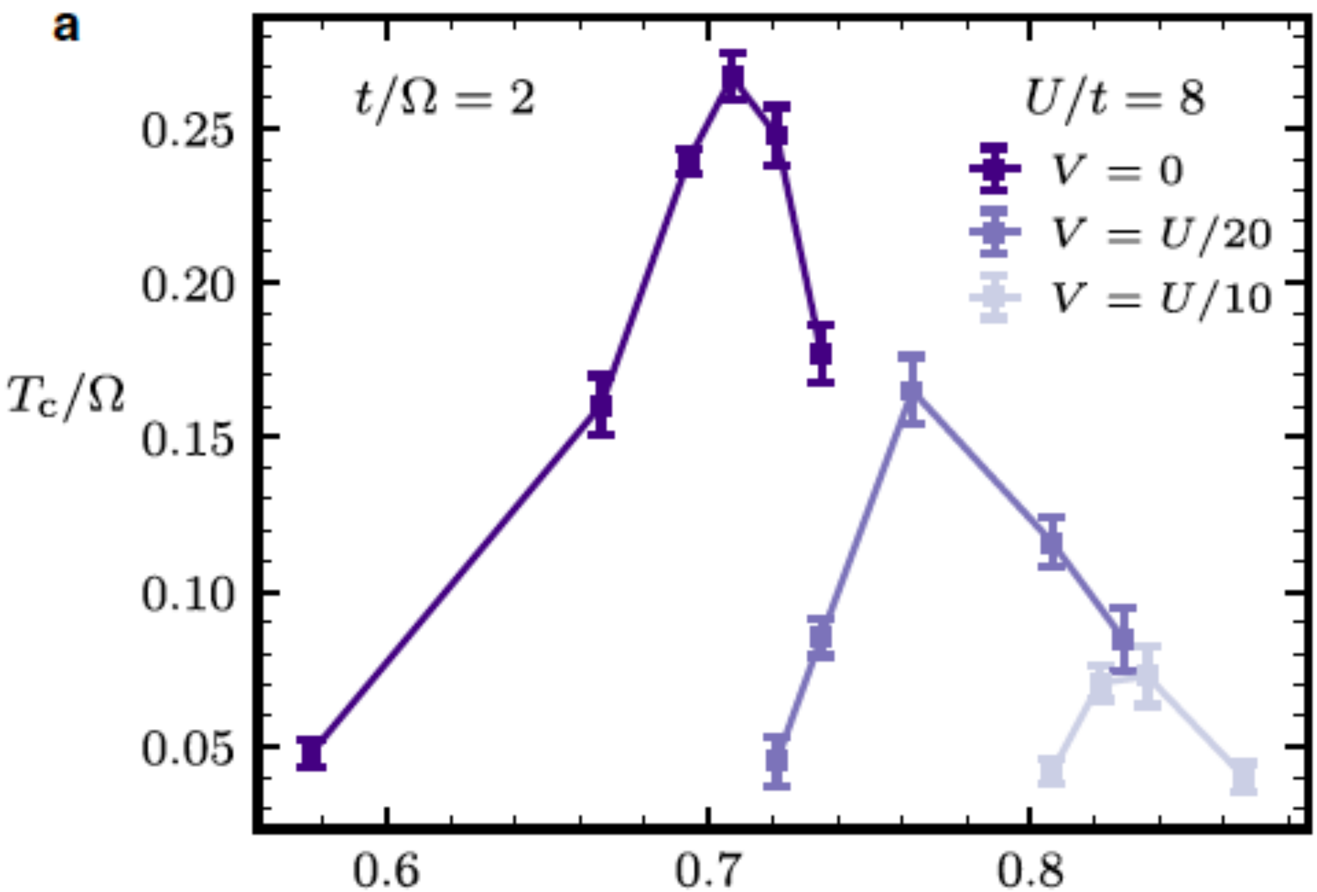
$$\mathcal{V}_{ij} = \frac{Va}{|r_i - r_j|}$$

arXiv:2210.14236



Long ranged interaction increases the coupling threshold

Effect of long ranged interaction on superconductivity



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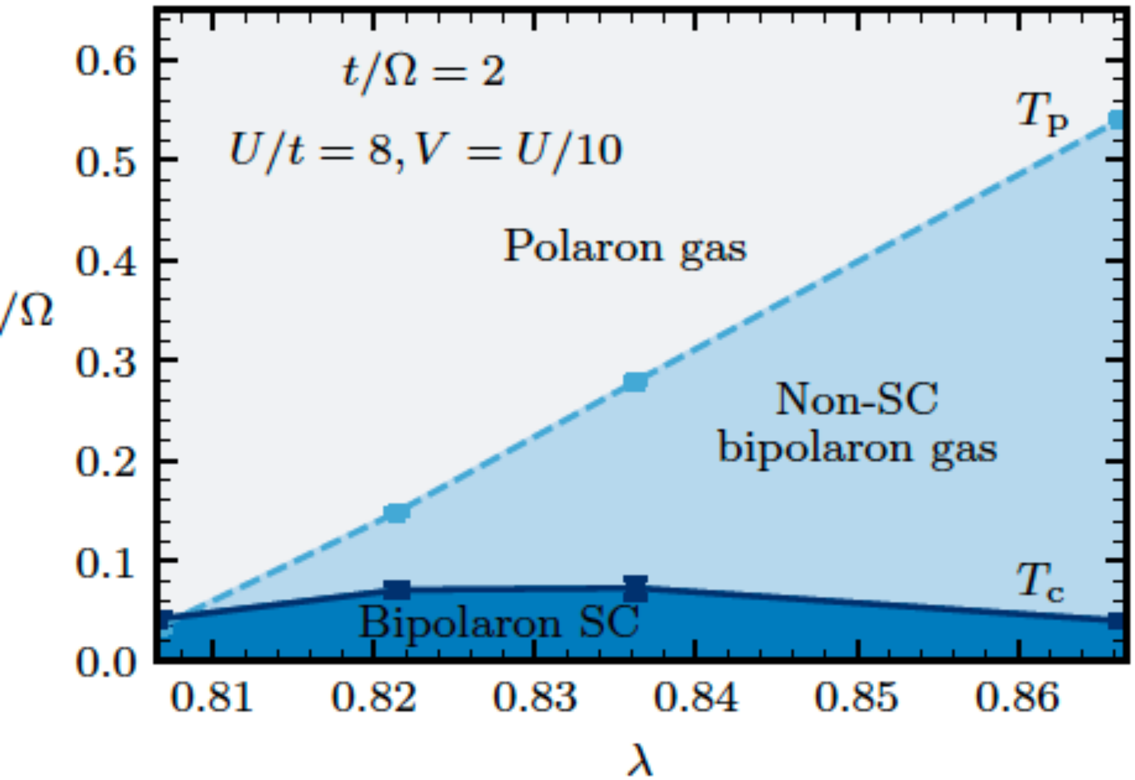
Bipolaron phase diagram

At fixed density



- Controlled calculations can be done!
- Bipolarons dont have to be heavy (at reasonable couplings)
- Long ranged part of Coulomb interaction is important but not disastrous

Open questions:
—identifying systems with strongly phonon-mediated hopping
—results beyond the dilute limit

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Beyond DFT:
Superconductivity in strongly correlated ("quantum") materials

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Families of superconductor

- ‘Conventional’: Hg, Pb, Nb3Sn, MgB2…
- Unconventional -conventional: A_3C_{60}; LaH_{10}
- Heavy fermion
- Organic: BEDT…
- Copper-Oxide
- Iron-pnictide
- ^3He
- Condensed atomic gasses
- Neutron Stars
- Quark matter

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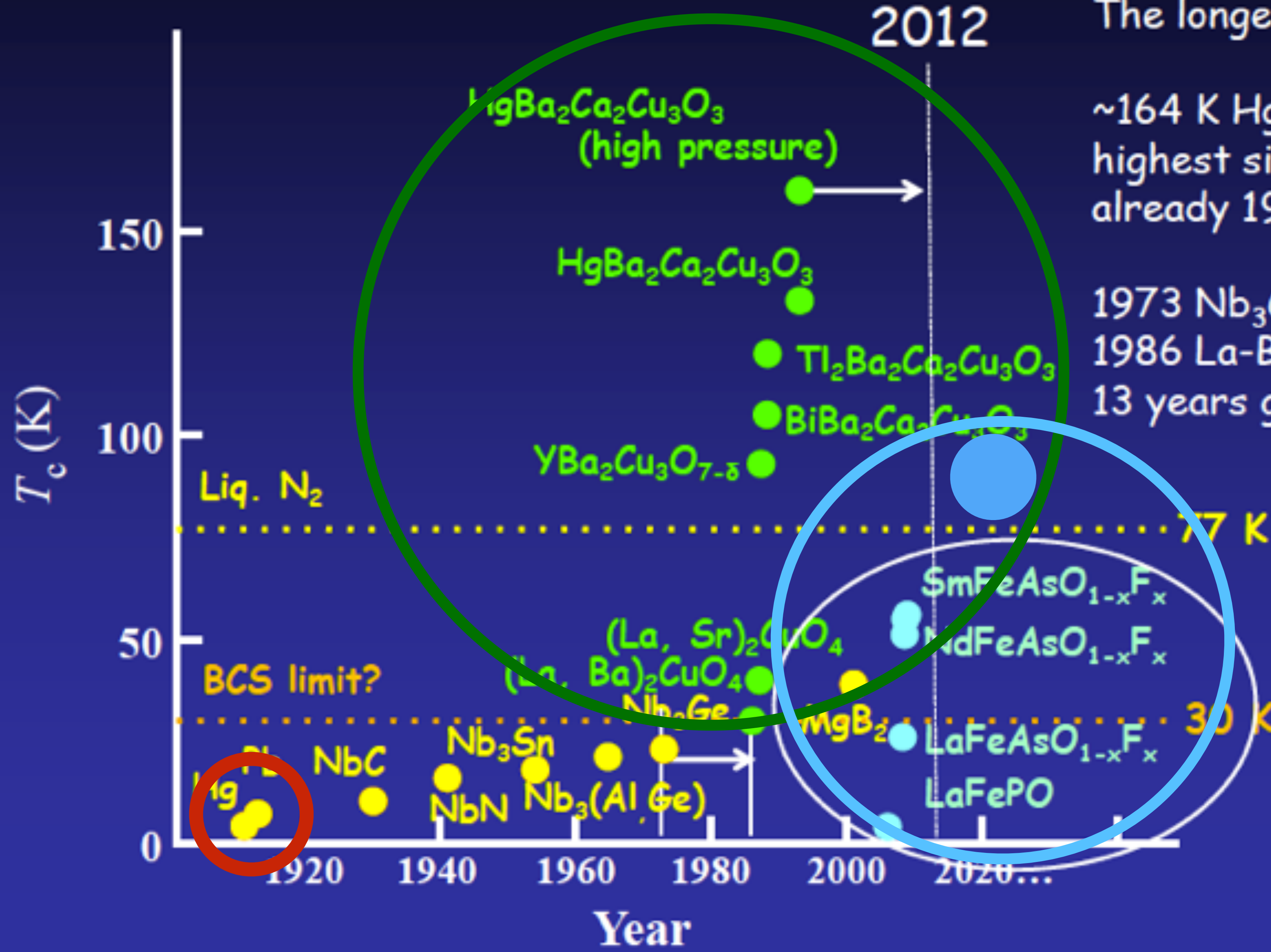
Where are we?

a history of the increase in T_C



The longest silence

~164 K Hg1223
highest since 1993
already 19 years gap

1973 Nb_3Ge (23 K)
1986 La-Ba-Cu-O (30 K)
13 years gap

Matthias' rules for finding superconductivity

1 Symmetric (pref. cubic) lattices
2 Avoid oxygen or similar elements
3 Avoid magnetism,
4 Avoid insulators,
5 Avoid theorists

- 'Conventional': Hg, Pb, Nb3Sn, MgB2…
- Unconventional -conventional: A_3C_{60}; LaH_{10}
- Heavy fermion
- Organic: BEDT…
- Copper-Oxide
- Iron-pnictide
- ^3He
- Condensed atomic gasses
- Neutron Stars
- Quark matter

All except first one on the list violate one or more of Matthias' rules

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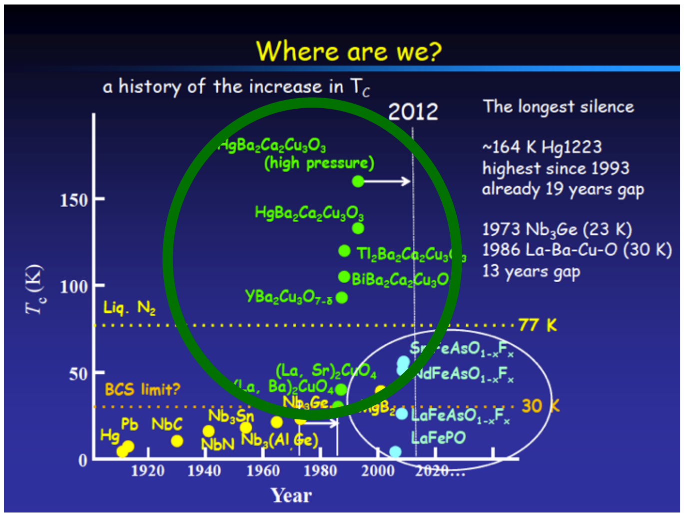
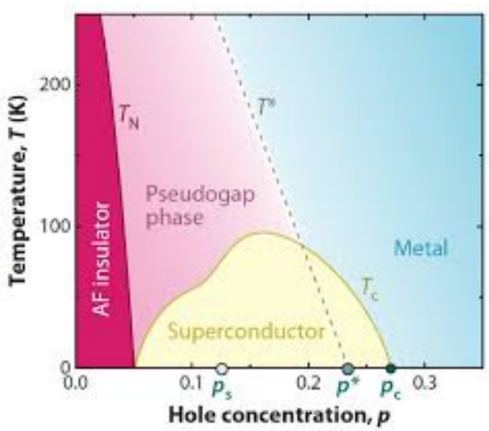
‘Novel’ superconductivity



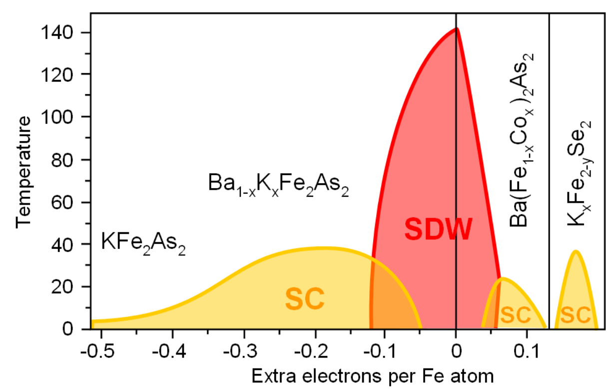
- **Emerges in proximity to other (often magnetic, often non-‘fermi liquid’) phase**

- **Apparently has unconventional (non s-wave) order parameter symmetry.**

- **Can have quite high transition temperature**



Phase_diagram_of_the_122_family_of_ferro-pnictides.png (PNG Image... https://upload.wikimedia.org/wikipedia/commons/f/f3/Phase_diagram_...



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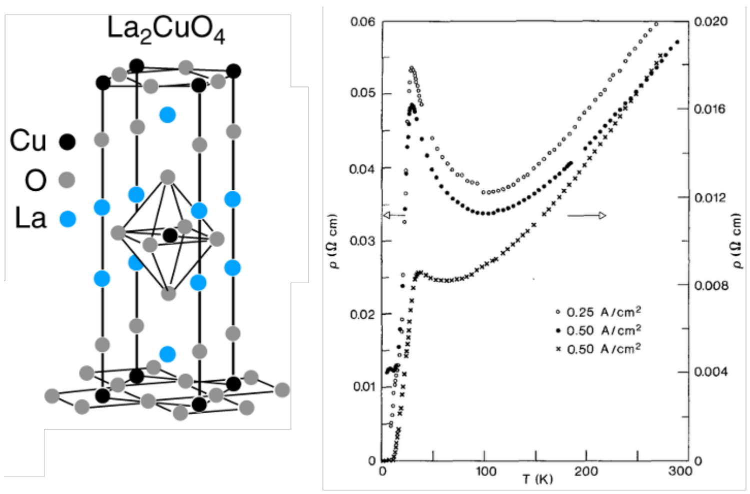
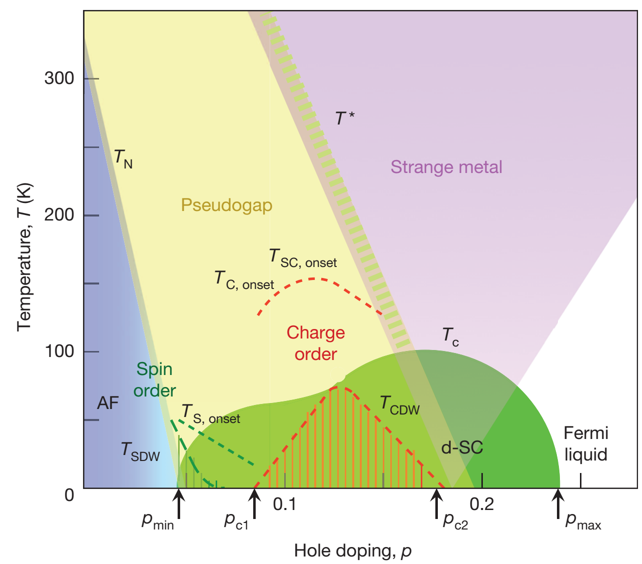
Copper -oxide superconductors

Adapted from H. Hwang slide



J. G. Bednorz & K. A. Müller,
Z. Phys. B - Condensed Matter **64**, 189 (1986)



B. Keimer, S. A. Kivelson, M. R. Norman, S. Uchida & J. Zaanen, *Nature* **518**, 179 (2015)

Superconductivity emerges in a material with a complicated but basically layered crystal structure, in proximity to a 'pseudogap' phase (or regime) with unusual properties

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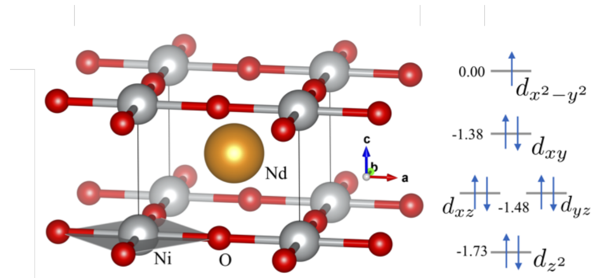
Is this specific to Cu-based materials

Adapted from H. Hwang slide

Question:

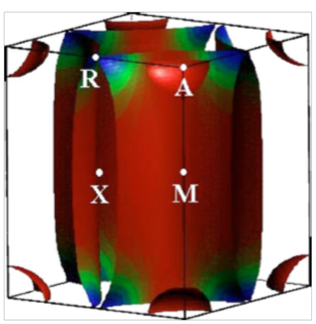
Can you make $Re^{3+}Ni^{1+}O_2$ look like $Ca^{2+}Cu^{2+}O_2$?



PHYSICAL REVIEW B **70**, 165109 (2004)

Infinite-layer $LaNiO_2$: Ni^{1+} is not Cu^{2+}

K.-W. Lee and W. E. Pickett
Department of Physics, University of California, Davis, California 95616, USA



Large Ni d_{x2-y2} hole band,
2 *Re* 5*d*-derived
electron pockets
– “self-doped” system

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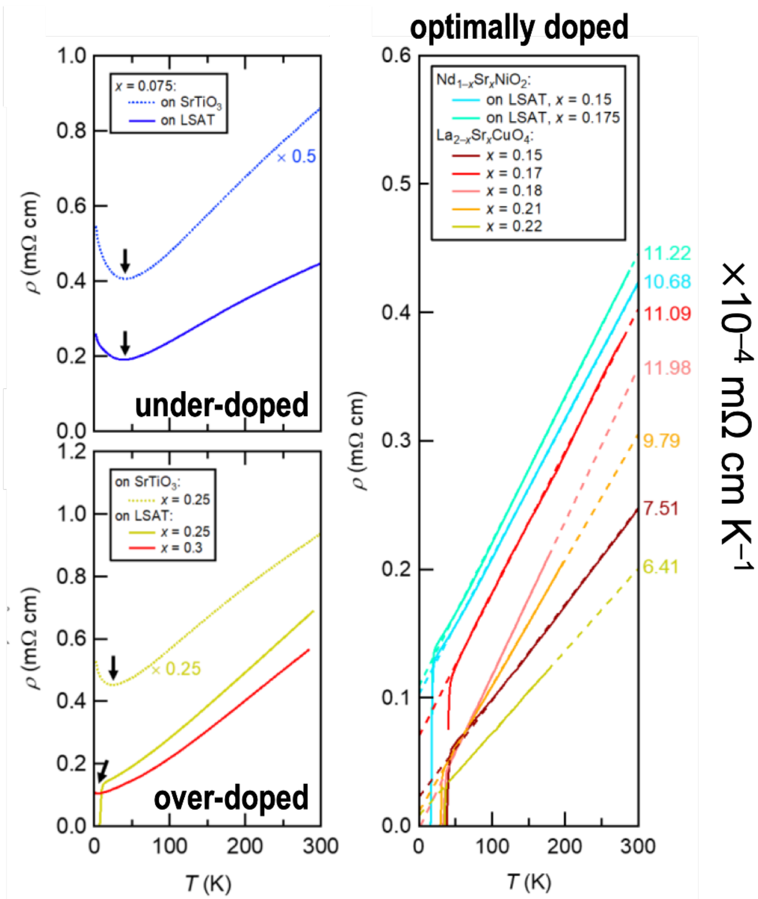
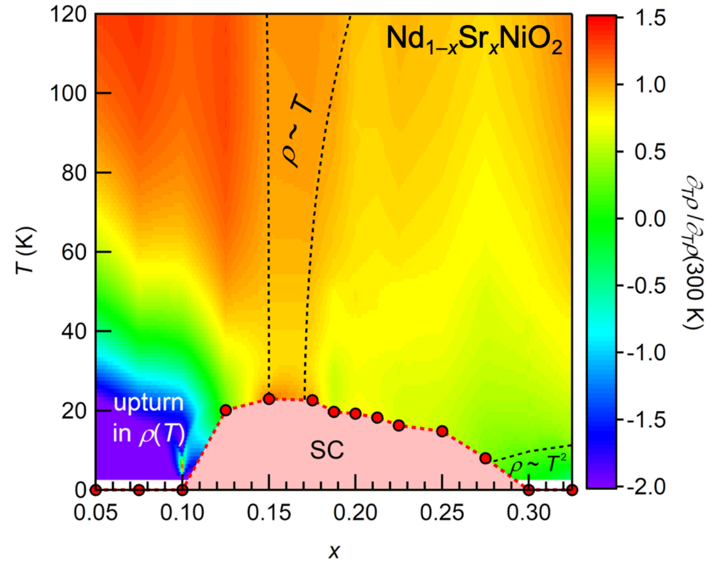
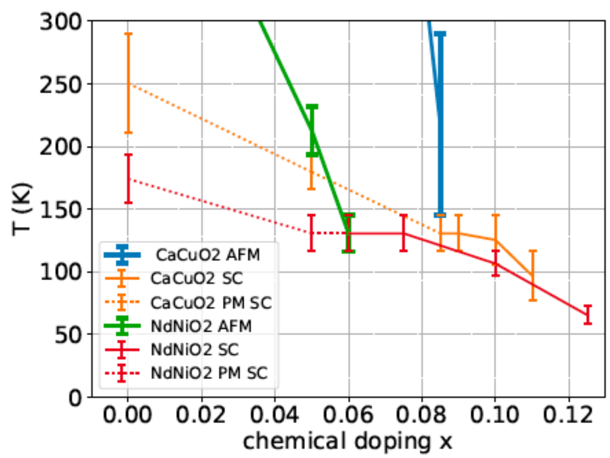
Answer: Yes

Kyuho Lee et al., *Nature* **619**, 288 (2023)

Adapted from H. Hwang slide





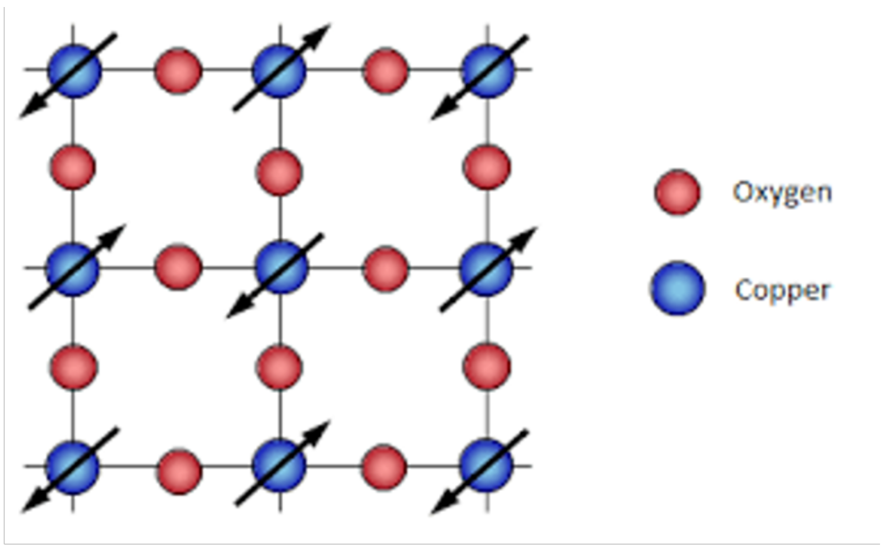


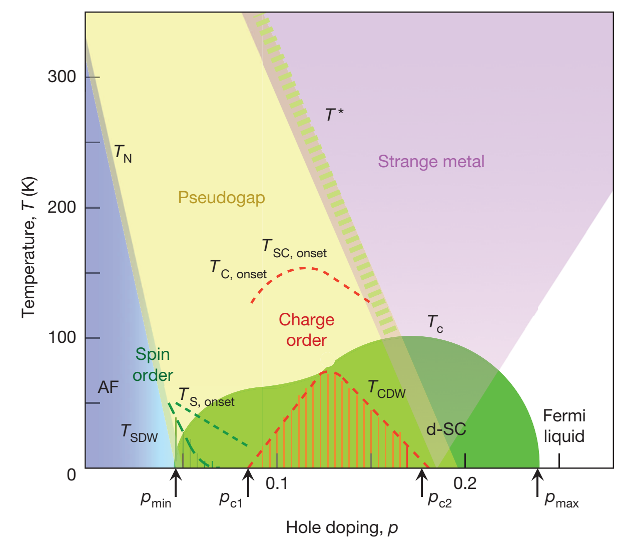
J. Karp, A. Hampel, AJM
Phys Rev B105 205131 (2022)

Copper and Ni -oxide superconductors

Adapted from H. Hwang slide

Theoretical model: the Hubbard model





$$H = -t \sum_{<i,j>} (c^{+}_{i,\sigma} c_{j,\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

After ~50 years, solution now in sight—key has been algorithm development and comparison of multiple methods. Cf A Georges talk Thursday

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Phase Diagram

U/t

T=0

Raise T above AF: 'Mott' insulator

Raise T above Stripe: 'Pseudogap'

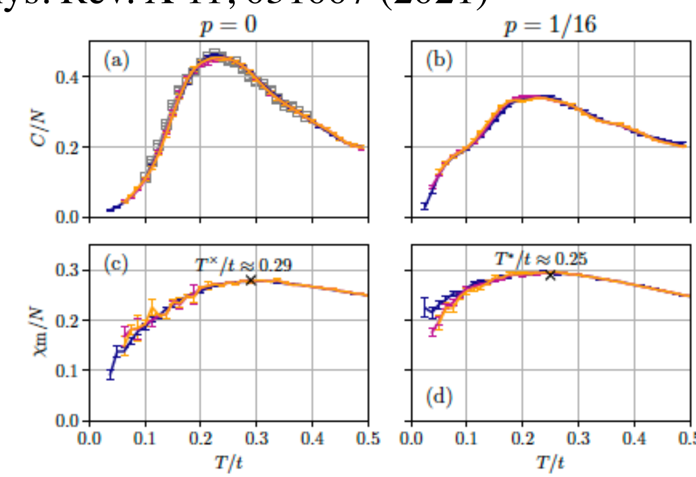
Raise T above SC/PG: 'Strange metal'

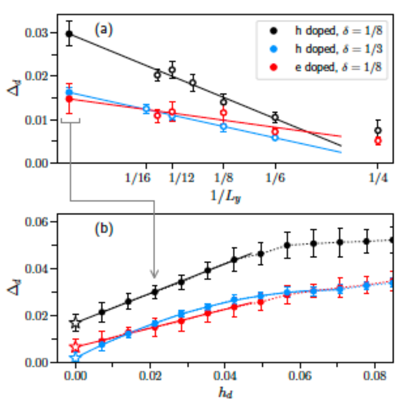
AF 'Stripe' AF SC Normal metal

p or d_{xy} s.c.; v. Low Tc

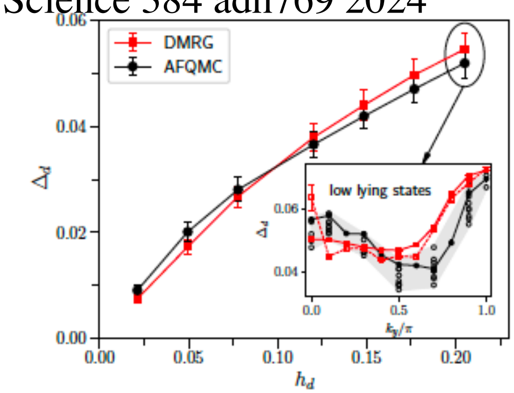
Carrier concentration x=1-n/site

Wietek He White Georges Stoudenmire
Phys. Rev. X 11, 031007 (2021)





Hu Chung Qin White Schollwoeck Zhang
Science 384 adh769 2024



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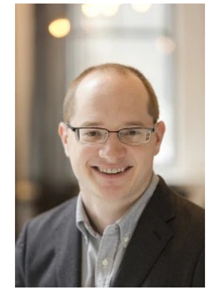
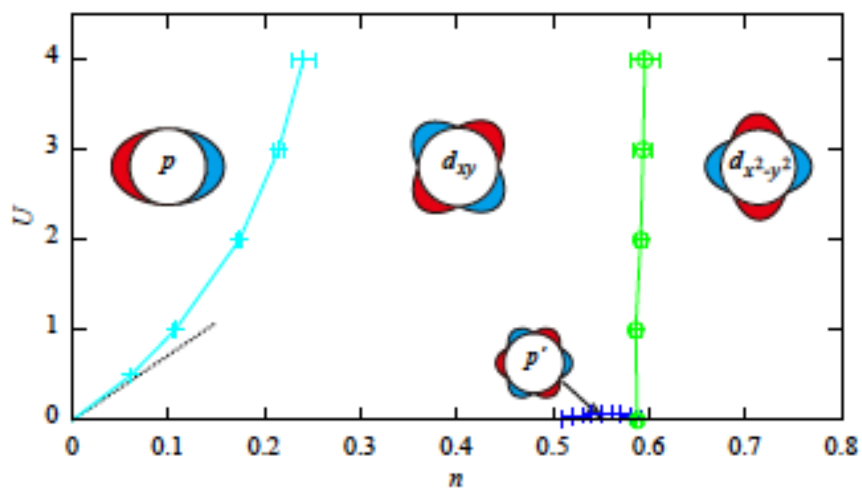
What causes superconductivity in the Hubbard Model



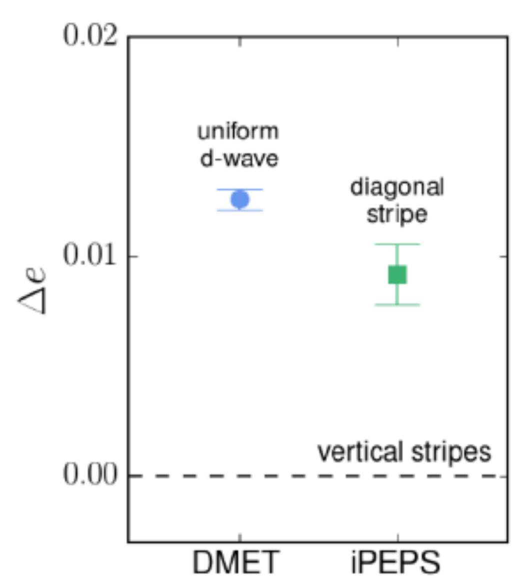
Xinyang Dong   Nat. Phys. 18 1293 (2022)   Emanuel Gull

$$\hat{H} = -\sum_{i,j;\,\sigma} t_{ij}\left(c^{\dagger}_{i\sigma}c_{j\sigma} + H.c.\right) + U\sum_{i}\hat{n}_{i\uparrow}n_{i\downarrow}$$

Weak coupling: SC (low TC)
Deng et al EPL 2015



Intermediate coupling, x=1/8
SC preempted by stripes
Chan et al Science 2017



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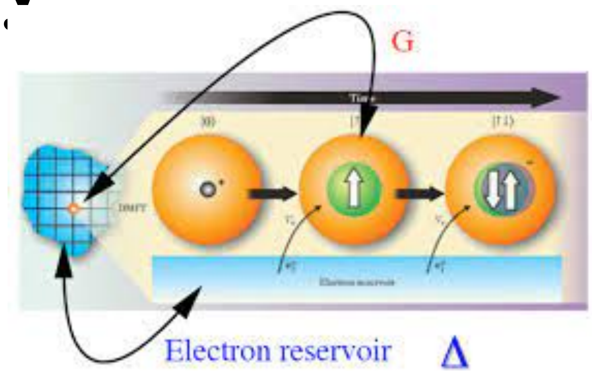
Dynamical Mean Field Theory



Yields an approximation to electron self energy by solving auxiliary problem of cluster of impurity sites coupled to a bath.

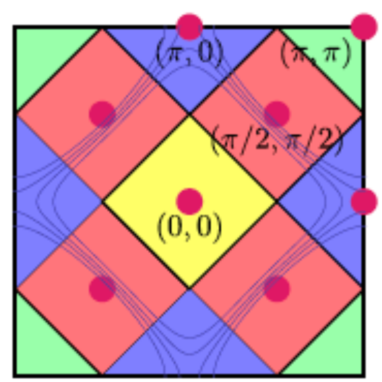
Broken symmetries treated as in usual mean field theory (ask—do you want to break this sym?)=>can see if stable sc phase exists

Not enough resolution to see stripes=>cant tell if stripes preempt sc

`DCA: piecewise constant self energy

$$\boldsymbol{\Sigma}(\mathbf{k},\omega) = \sum_{\mathbf{a}} \boldsymbol{\Theta}(\mathbf{k} \in \mathbf{a})\boldsymbol{\Sigma}_{\mathbf{a}}(\omega)$$

Example: 8 patches=>8 site cluster



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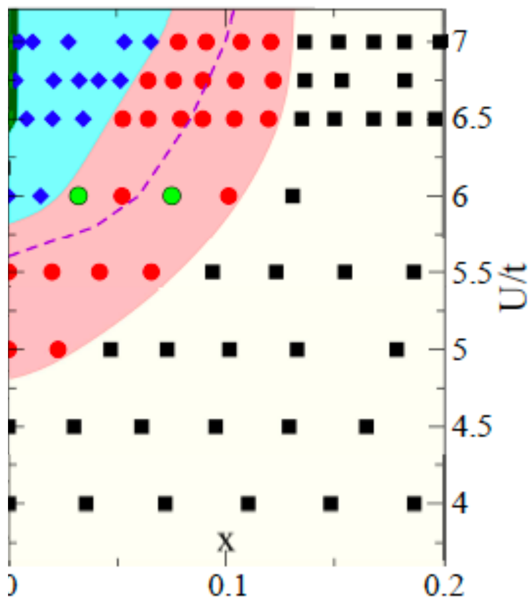
DMFT superconducting phase diagram



SC

PG

FL

Deng et al EPL 2015

Parcollet, Gull, AJM PRL 2013

d_{x2-y2} sc near insulator but cut off by pseudogap

Tc Max: t/20~250K

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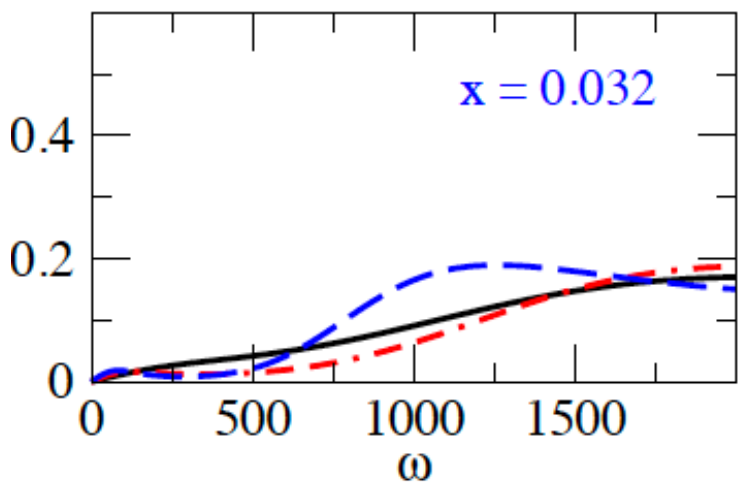
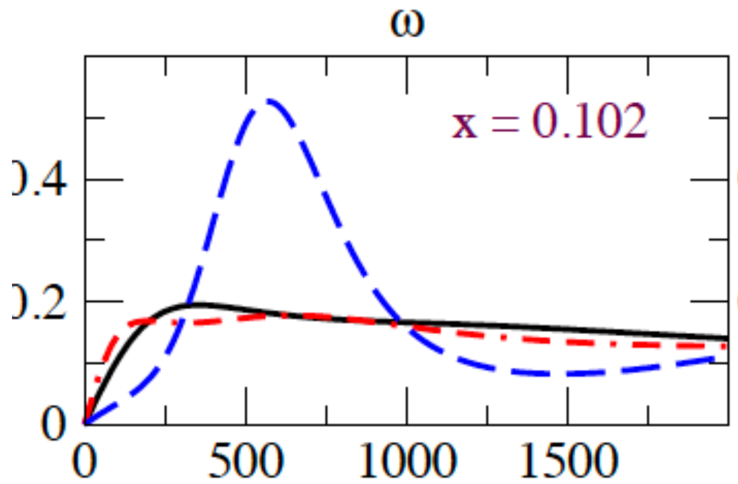
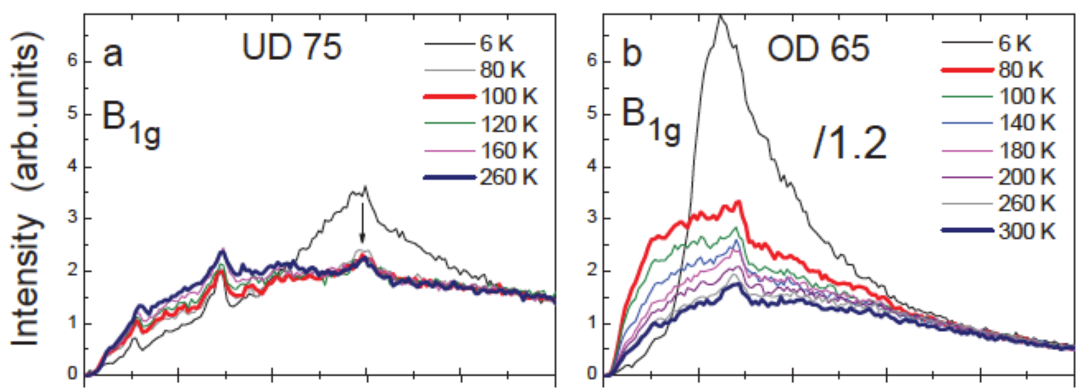
Superconducting /Pseudogap properties

Raman spectra in normal and SC state







Data

Sacuto et al

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DMFT finds a well defined SC state.
Is it from (low frequency) spin fluctuations?

Our approach:

- Independently compute spin fluctuation spectrum and normal and anomalous self energies
- Partition normal self energy into spin fluctuation and Mott parts. $\Sigma = \Sigma_{SF} + \Sigma_{rest}$
- Fix the electron-spin fluctuation coupling constant from SF part of normal component of self energy
- Solve Eliashberg equation for SC. Compare to directly calculated result

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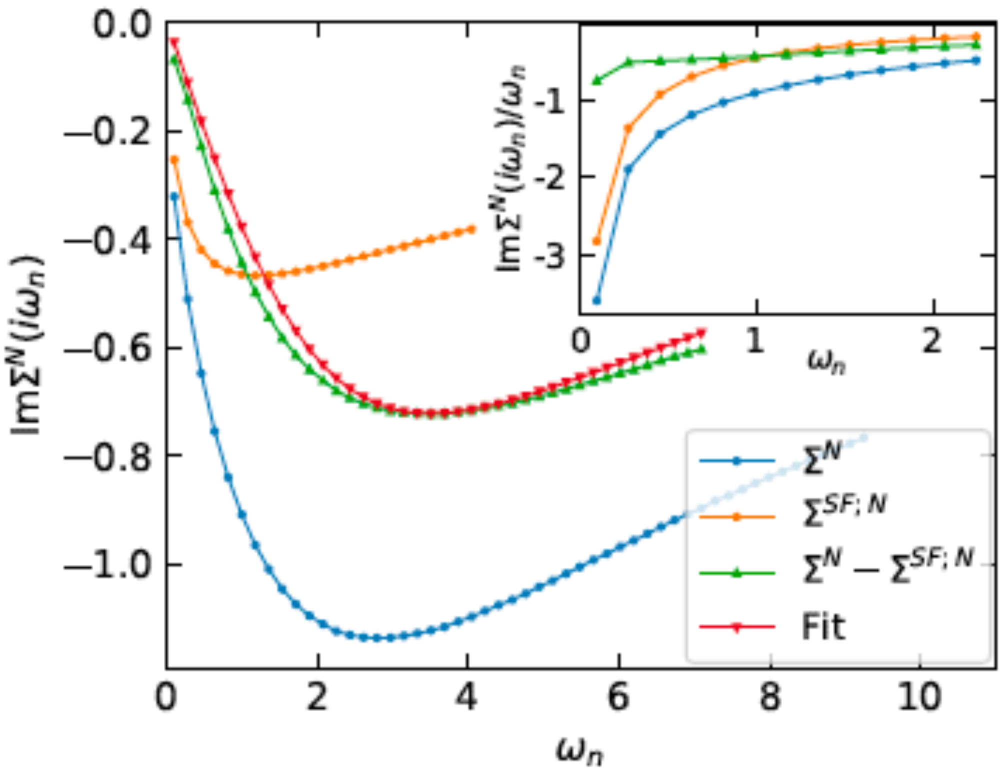
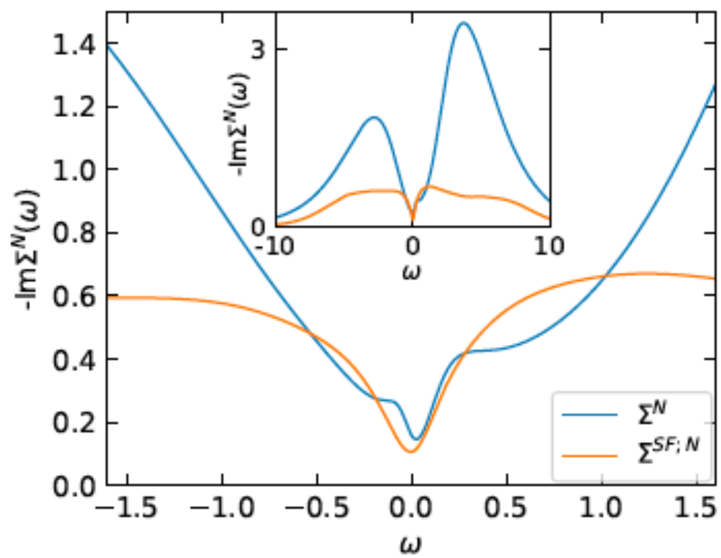
Normal State Analysis

$$\Sigma_K^{SF;N/A}(\omega) = g^2 \frac{1}{\beta N} \sum_{Q,\Omega} \chi_Q(\Omega) G_{K-Q}^{N/A}(\omega - \Omega).$$





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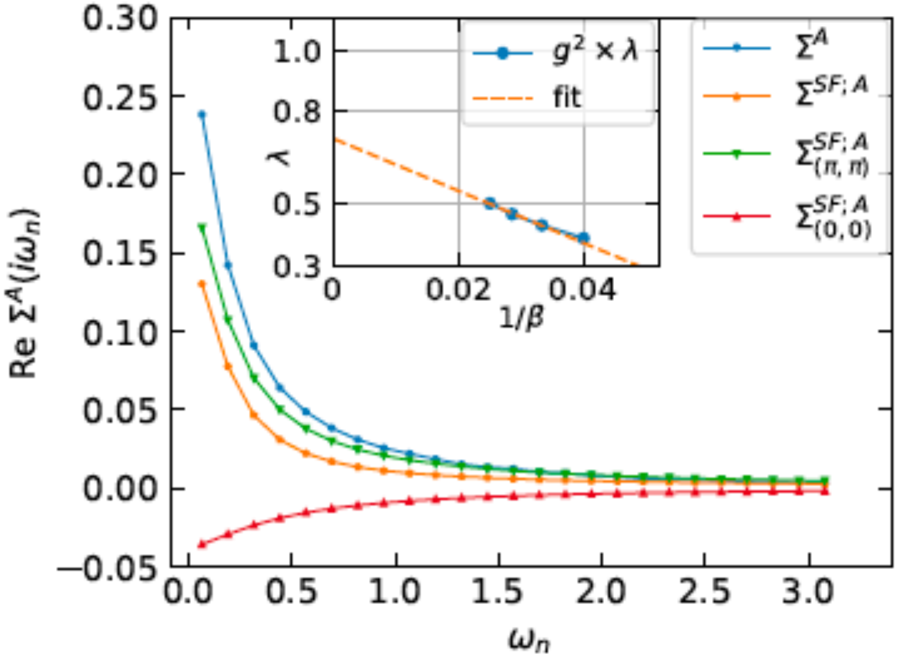
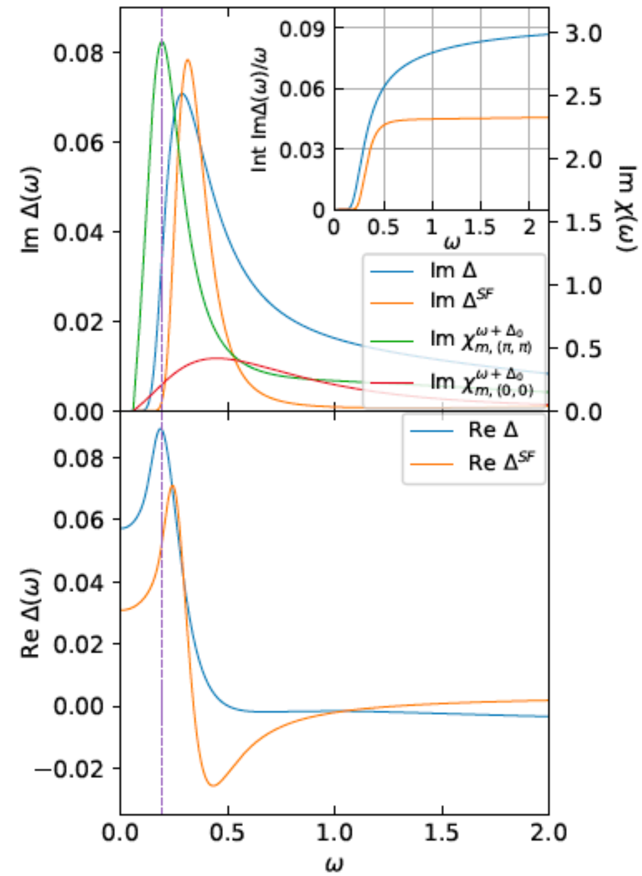
Now the superconducting part





Spin fluctuations account for at most half of the superconductivity

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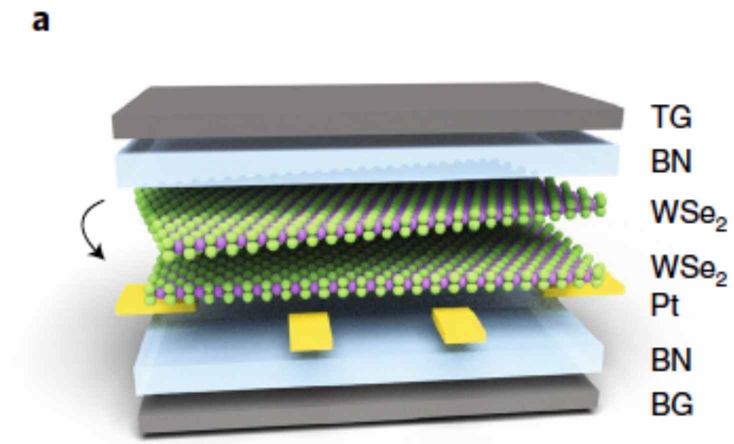
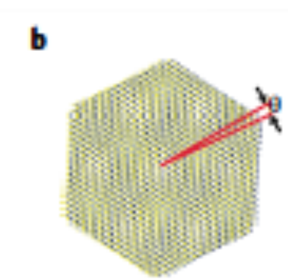
Implications

- Spin fluctuation as `pairing glue' is not the whole story—higher energy processes are important
- the coulomb interaction is not just a pseudopotential (or source of spin fluctuations)

‘Moire’ Materials

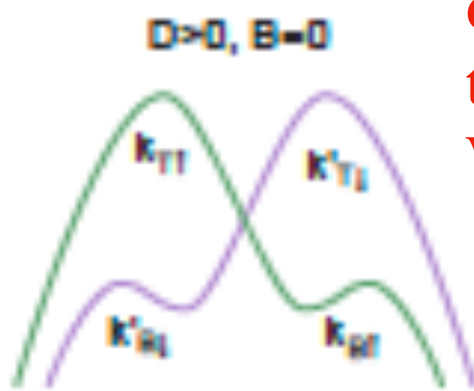
Homobilayer + Twist





- Moire=>large unit cell
- Kinetic/Interaction energy ratio controllable by twist angle, gates
- By adjusting top gate (TG) and bottom gate (BG) voltages independently, can control mean chemical potential and displacement field (<=> inversion symmetry, band structure, occupation of different layers)
- New feature: quantum geometry

Effect of displacement field



chemical potential above this line: `layer polarized’ Van Hove point appears

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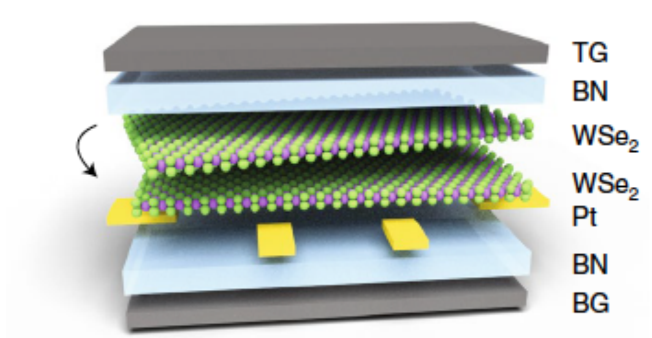
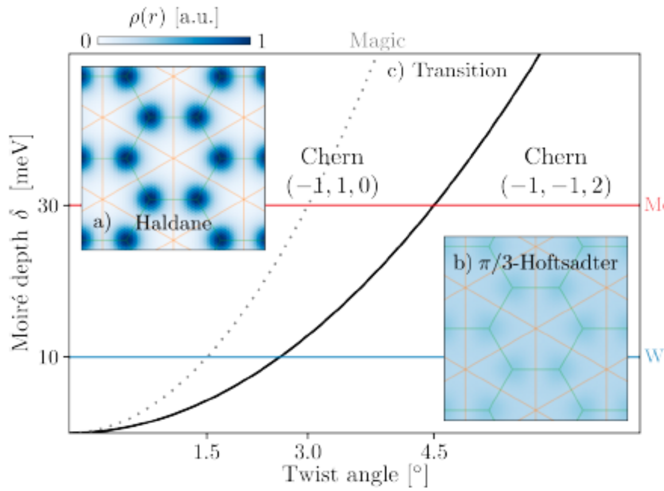
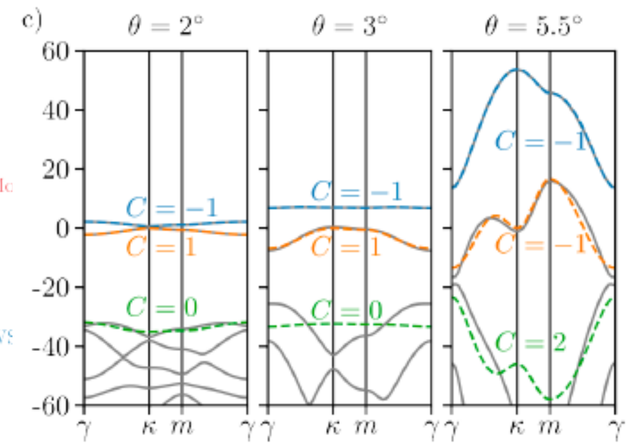
TMD Moire Bands:

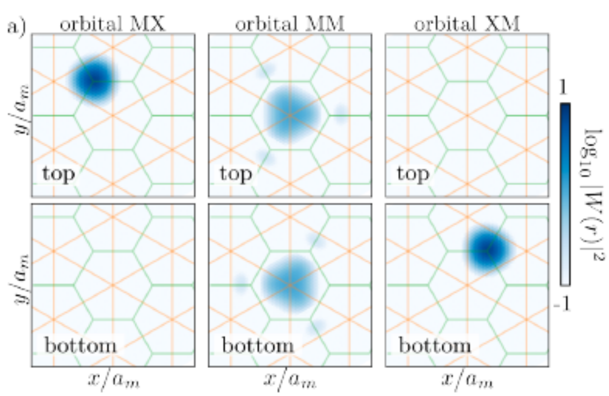
a







Topological bands; twist angle-tunable dispersion



Three band tight binding model reproduces dispersion and topology of upper two bands

Relative roles of the orbitals depend on displacement field

Interaction scales (project screened Coulomb onto Wannier orbitals): ~50 meV—LARGE

V. Crepel and AJM arXiv:2403.15546

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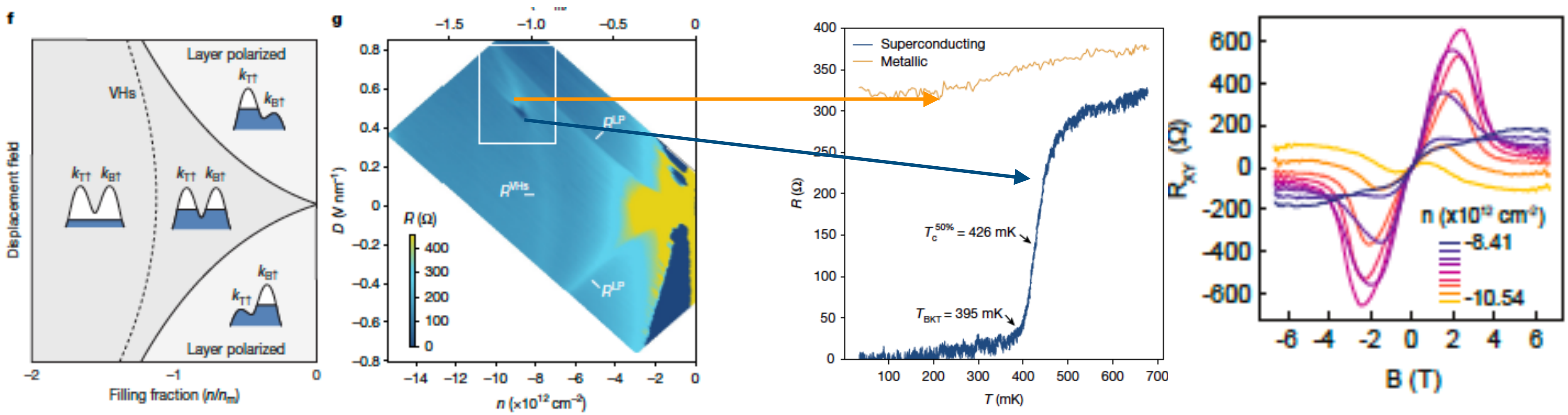
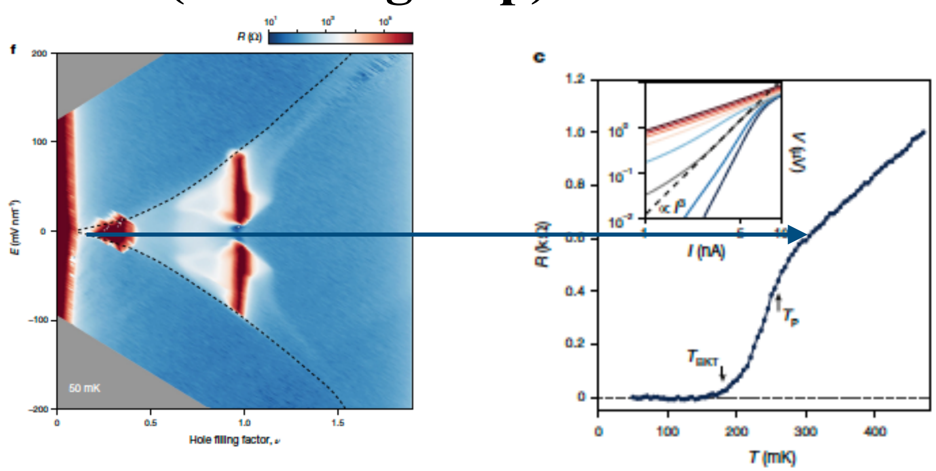
Superconductivity in Moire WSe2

Columbia and Cornell Experimental Groups

5^o twist (Columbia group)



3^o twist (Cornell group)



Superconductivity: at larger twist angle, found at "random" point in phase diagram. As twist angle decreases, superconductivity moves to commensurate filling.

SC always bounded by some kind of more resistive state

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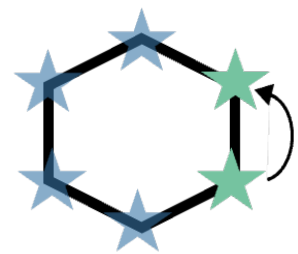
Consequences of nonzero quantum geometry: II
Superfluid weight and quantum metric

Slide text from P. Torma

Isolated flat band: $W \ll U \ll E_{\text{band gap}}$

Uniform pairing: $\Delta_{\text{orbital }\#} = \Delta$



$$\Rightarrow \quad [D_s]_{ij} = \frac{4e^2 U \nu(1-\nu)}{(2\pi)^{d-1} N_{\text{orb}}} \mathcal{M}_{ij}^{\text{R,min}}$$

$$\Delta = \frac{U}{N_{\text{orb}}} \sqrt{\nu(1-\nu)}$$

$$\mathcal{M}_{ij}^{\text{R}} = \frac{1}{2\pi} \int_{\text{B.Z.}} \mathrm{d}^d\mathbf{k}\, \underbrace{\mathrm{Re}\,\mathcal{B}_{ij}(\mathbf{k})}_{\text{quantum metric } g_{ij}}$$

$$[D_s]_{ij} = \frac{2e^2}{\pi\hbar^2} \frac{\Delta^2}{U N_{\text{orb}}} \mathcal{M}_{ij}^{\text{R,min}}$$

Mean-field → Peotta, Törmä, Nat Comm 2015

Mean-field → Huhtinen, Herzog-Arbeitman, Chew, Bernevig, Törmä, PRB 2022

Exact many-body → Herzog-Arbeitman, Chew, Huhtinen, Törmä, Bernevig, arXiv 2022

Quantum geometry enhances superfluid stiffness

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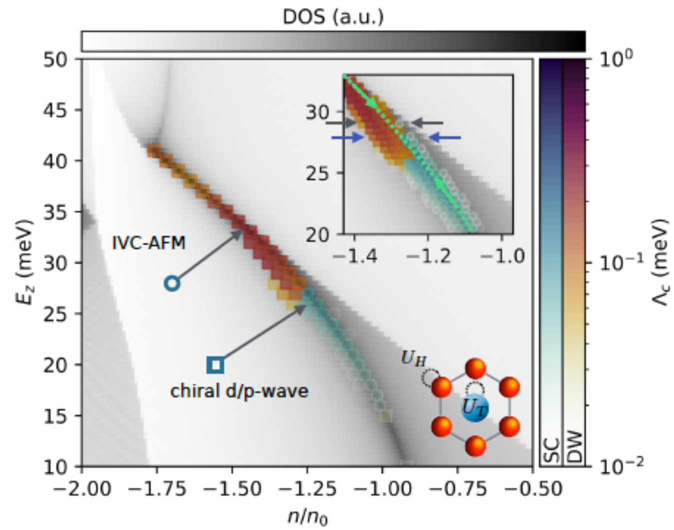
Theory: I

Kohn-Luttinger mechanism: arXiv:2408.16075



- **Interaction: screened Coulomb + form factors from projection onto top band. Similar spatial structure from spin fluct**

$$V_{\boldsymbol{g}\boldsymbol{g}'}(\boldsymbol{q}, i\omega) = \left[1 - V_0(\boldsymbol{q})\Pi(\boldsymbol{q}, i\omega)\right]^{-1}_{\boldsymbol{g}\boldsymbol{g}'} V_0(\boldsymbol{q}+\boldsymbol{g}')$$

$$\Pi_{\boldsymbol{g}\boldsymbol{g}'}(i\omega, \boldsymbol{q}) = \frac{1}{A}\sum_{\boldsymbol{k}\sigma} \frac{f(\xi_{\boldsymbol{k}\sigma}) - f(\xi_{\boldsymbol{k}+\boldsymbol{q}\sigma})}{i\omega + \epsilon_{\boldsymbol{k}\sigma} - \epsilon_{\boldsymbol{k}+\boldsymbol{q}\sigma}} \Lambda^{\boldsymbol{k},\boldsymbol{k}+\boldsymbol{q}}_{-\boldsymbol{g}\sigma} \Lambda^{\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}}_{\boldsymbol{g}'\sigma}$$

- **$\Lambda^{\mathbf{k,p}}_{\mathbf{g}\sigma}$ expresses quantum geometry, inversion symmetry breaking**

leading instability: 2D "E" irrep. Strong SOC + no inversion means SC state is of mixed singlet/triplet character. Chiral (d+id/p+ip, C=2 Altland-Zirnbauer A) over most of parameter range; small region of non-topological nematic SC. Chiral state is gapped, with Dirac edge modes

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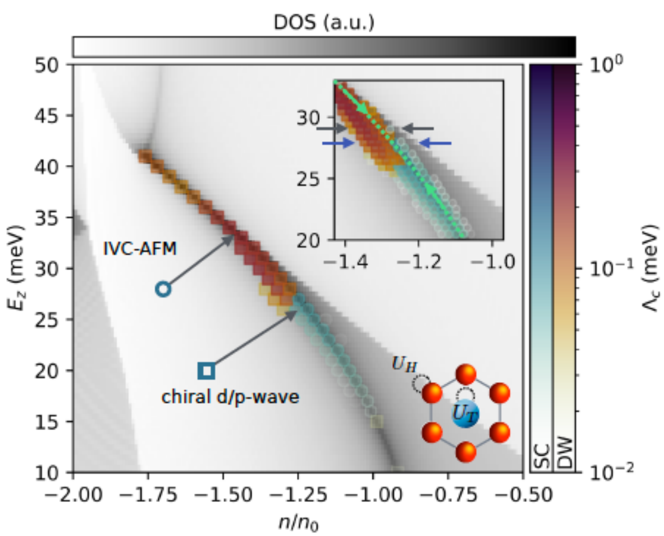
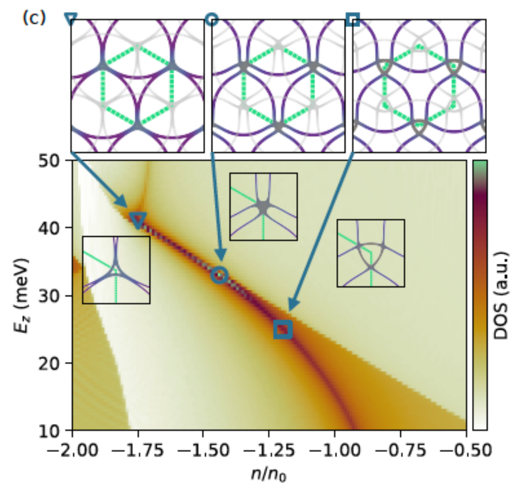
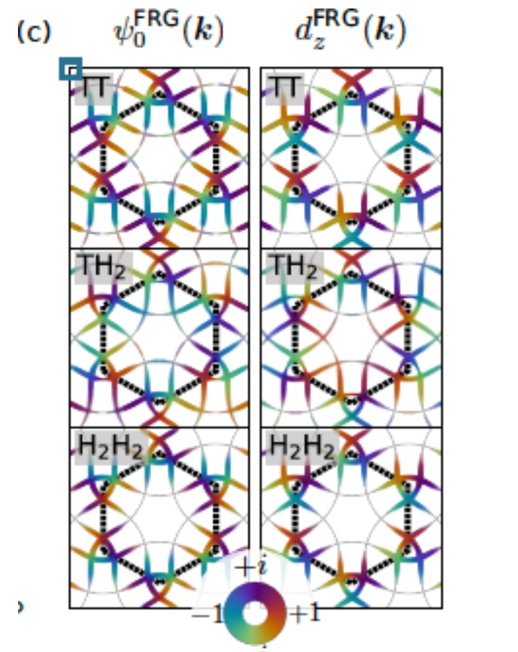
Theory

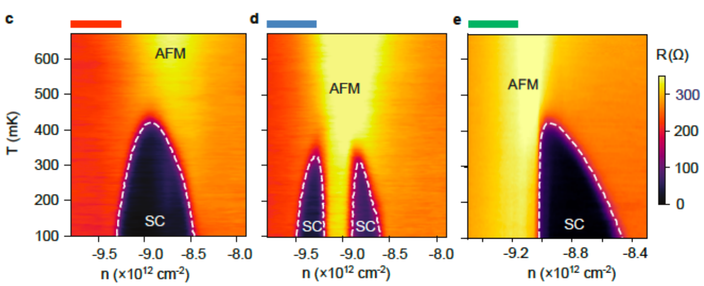
FRG: arXiv:2412.14296 based on arXiv:2403.15546

3 orbital t.b. model (V. Crepel): reproduces bands and topology.
Wannierizable=>purely local interactions.
Functional Renormalization Group calculations









Subtle relation of phase diagram to van Hove points

Complicated structure of phase of SC

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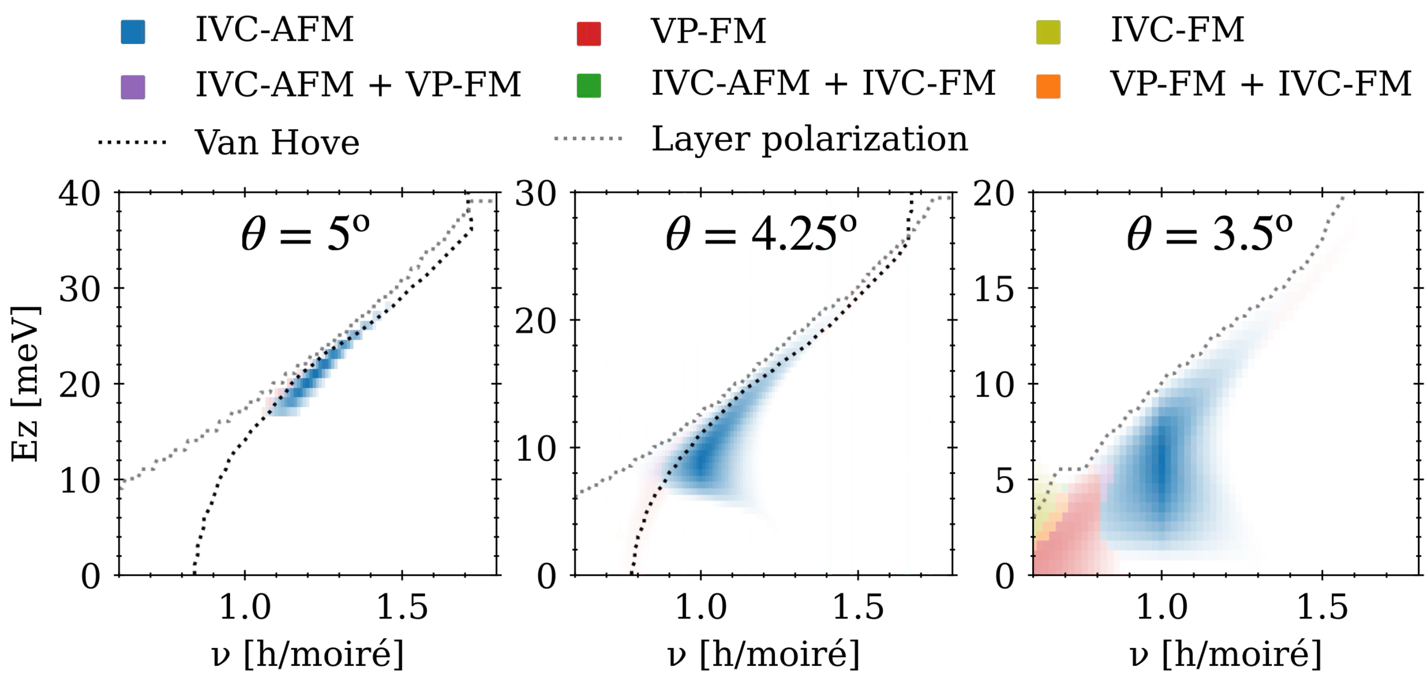
Theory: evolution of AF phase with twist angle

D.Munoz Segovia arXiv:2503.11763



At high twist angle, competing phase is associated with the van Hove singularity; at lower twist angle, detaches from van Hove and locks to n=1 and exists only at nonzero displacement field. ??Implications for superconductivity

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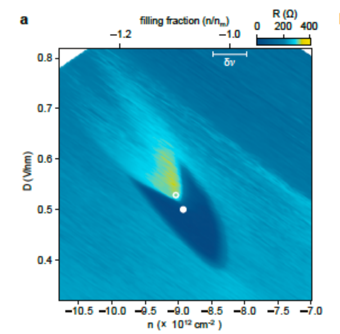
Twisted WSe2

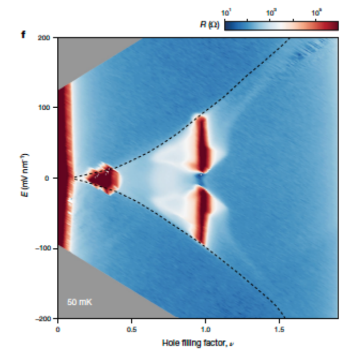
5^{o} twist angle: Superconductivity in proximity to fermi surface reconstructed phase at particular combination of displacement field and carrier concentration

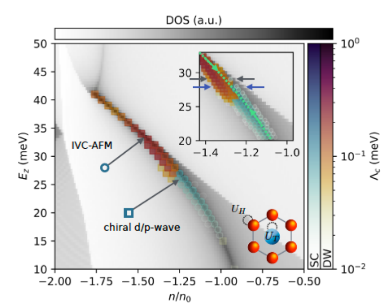
3.65^{o} twist angle: superconductivity at n=1 in proximity to insulating phase

Proposal: the superconductivity is topological and is understandable from spin fluctuations in proximity to magnetic phase

Role of quantum geometry to be determined







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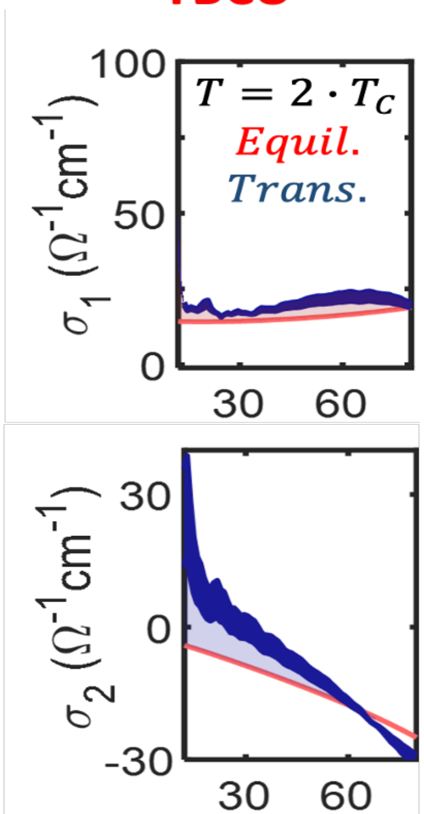
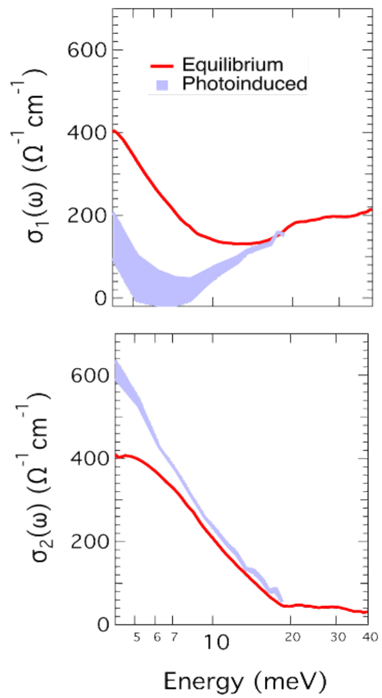
Inducing superconductivity with optical drive (A. Cavalleri and collaborators)

Images from A. Cavalleri

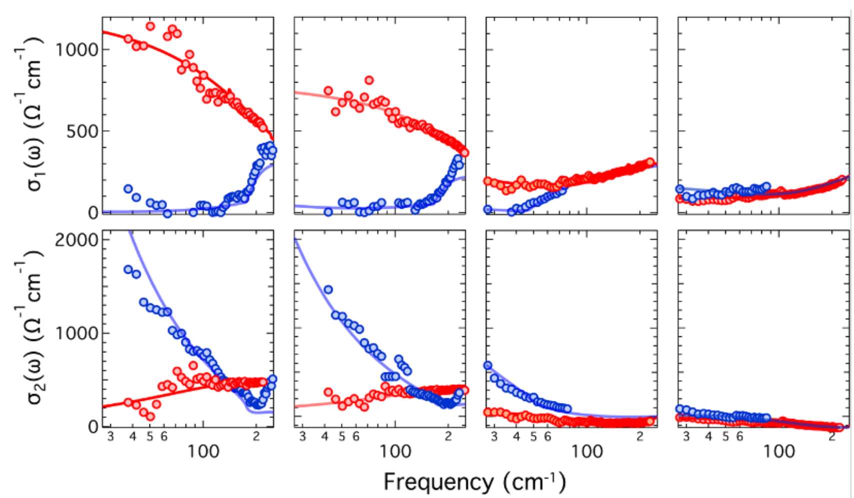
YBCO



K_3C_{60}



K-BEDT$_2$-Br



Fundamentally new questions in non equilibrium physics—see Demler talk

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Superconductivity:
The gift that keeps on giving

- **Understanding where the superconductivity comes from and using the knowledge to raise Tc: a continuing challenge in quantum many-body physics.**
- **Progress in theory of conventional, nearly conventional and unconventional superconductors comes from from new methods (machine learning, DMRG, DMFT, Diag-MC, AF-QMC..) working in tandem to cross compare results and allow 'handshake' between methods**
- **New experimental platforms allow new classes of systematic theory-experiment comparisons and raise new conceptual challenges**

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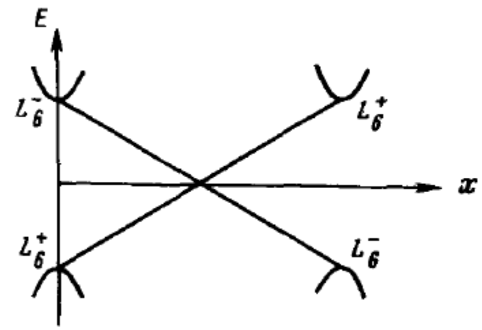
Consequences of nonzero quantum geometry

*If $C \neq 0$, 'Edge states' at interface between topological and vacuum or non-topological material

Volkov/Pankratov

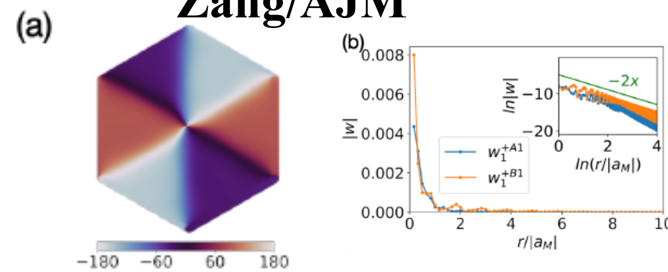


*Wannierization: $w_{n,R}(r) = \sum_k e^{i\vec{k}\cdot\vec{R}} M_k \psi_{mk}(r)$

Marzari/Vanderbilt: if C=0 can choose M so w's are exponentially localized around sites R

If $C \neq 0$, only power-law localization possible

Zang/AJM

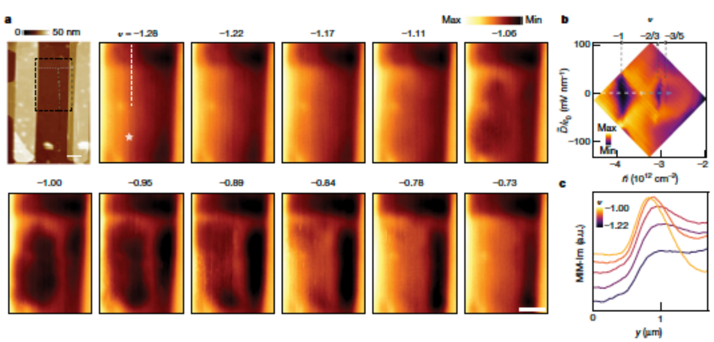


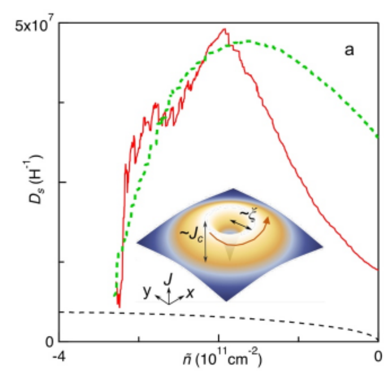
*Broken Galilean invariance, even in dilute limit=> Enhancement of stiffness (Torma), polarizability (Queiroz)

*(FCI) Quantized-Hall like states at 0 field

Bockrath

Xi, Xu, Shen, Nature



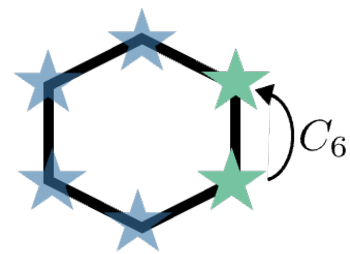


Consequences of nonzero quantum geometry: II
Superfluid weight and quantum metric

Slide text from P. Torma

Isolated flat band: $W \ll U \ll E_{\text{band gap}}$

Uniform pairing: $\Delta_{\text{orbital } \#} = \Delta$



$$\Rightarrow \quad [D_s]_{ij} = \frac{4e^2 U\nu(1-\nu)}{(2\pi)^{d-1} N_{\text{orb}}} \mathcal{M}_{ij}^{\text{R,min}}$$

$$\mathcal{M}_{ij}^{\text{R}} = \frac{1}{2\pi} \int_{\text{B.Z.}} \mathrm{d}^d\mathbf{k} \, \underbrace{\mathrm{Re}\, \mathcal{B}_{ij}(\mathbf{k})}_{\text{quantum metric } g_{ij}}$$

$$\Delta = \frac{U}{N_{\text{orb}}} \sqrt{\nu(1-\nu)}$$

$$[D_s]_{ij} = \frac{2e^2}{\pi\hbar^2} \frac{\Delta^2}{U N_{\text{orb}}} \mathcal{M}_{ij}^{\text{R,min}}$$

Mean-field → Peotta, Törmä, Nat Comm 2015
Mean-field → Huhtinen, Herzog-Arbeitman, Chew, Bernevig, Törmä, PRB 2022
Exact many-body → Herzog-Arbeitman, Chew, Huhtinen, Törmä, Bernevig, arXiv 2022

Quantum geometry enhances superfluid stiffness

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Theory II: Mott transition in an orbital field

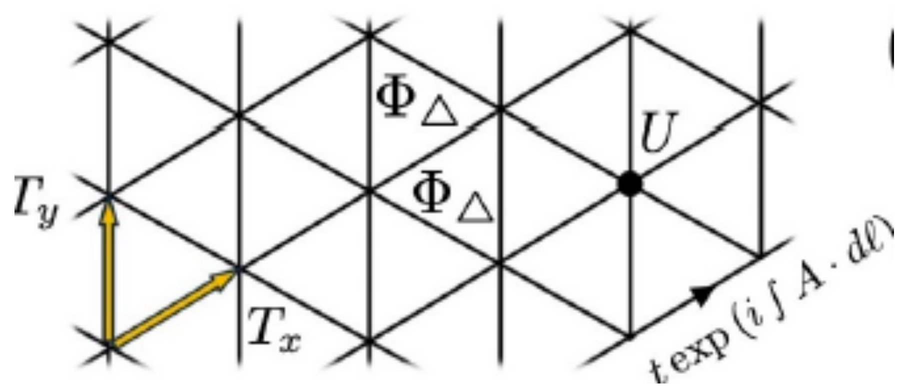
S. Divic, V. Crepel, M. Zalatel, AJM arXiv:2406.15348

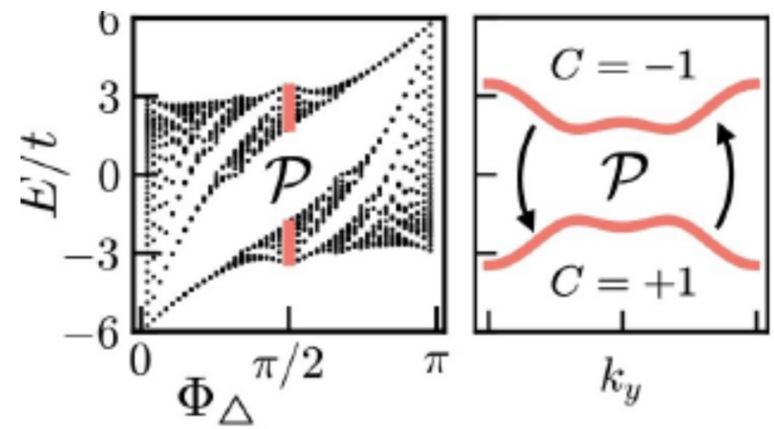
Tscheppe, Schaefer, AJM in preparation

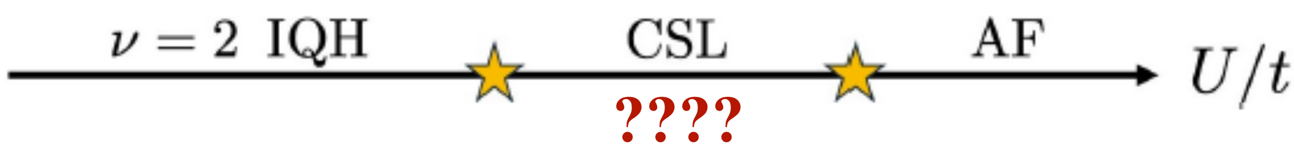
See also Kuhlenkamp, Kadow, Imamoglu, and Knap, Phys. Rev. X 14, 021013 (2024)

Triangular lattice Hubbard model
Flux $\Phi_\triangle$ per unit cell

Focus on $\Phi_\triangle = \frac{\pi}{2}$







Small U: integer quantized Hall effect
Very large U: conventional 120° antiferromagnet
?Is there an intermediate topological Mott insulator phase?

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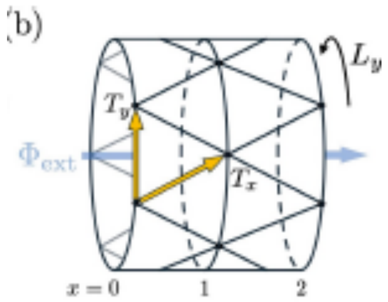
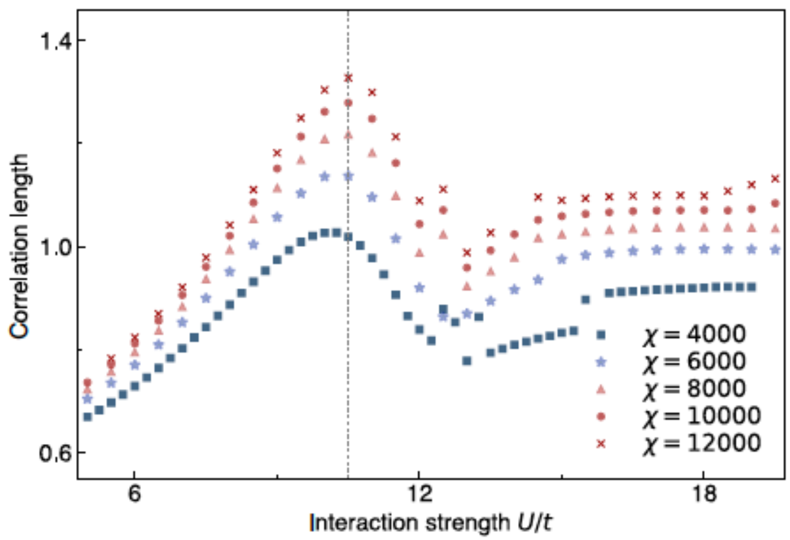
Theory: DMRG on small radius cylinders



Kuhlenkamp et al Phys. Rev. X 14, 021013 (2024): measure charge-charge correlator (averaged over cylinder circumference) $\langle n(x)n(0)\rangle$



Weak maximum at U~10t suggests transition to CSL

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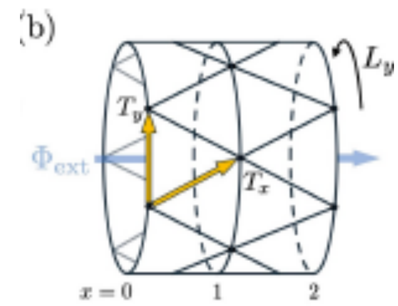
Theory: DMRG on small radius cylinders

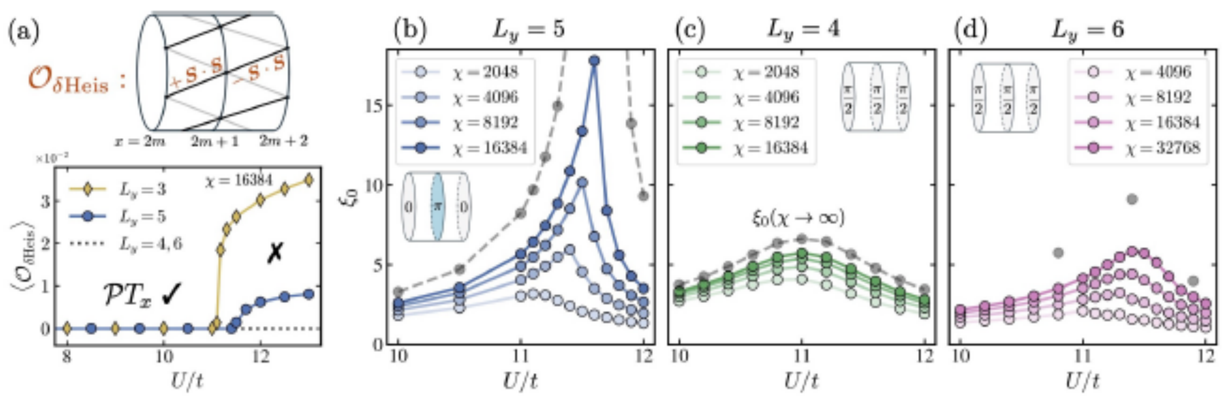


Stefan's idea:

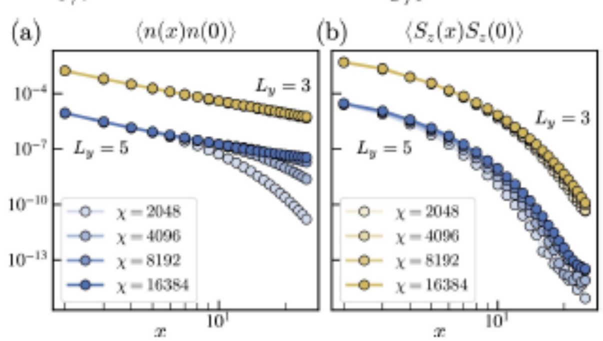
(1) Gauge choice: commuting translations T_x^2; T_y =>magnetic unit cell (2 segments)

(2) Gauss' law: $\Phi_{x+1} = \Phi_x - 2L_y\Phi_\triangle$ =>for even radius cylinders: flux invariant under T_x odd radius: T_x not a symmetry; PT_x is a symm.

(3) Spontaneously breaking of PT_x at CSL transition



Divergent correlation length



Power law charge correlations; exponentially decaying spin correlations

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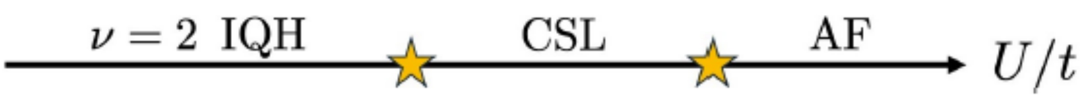
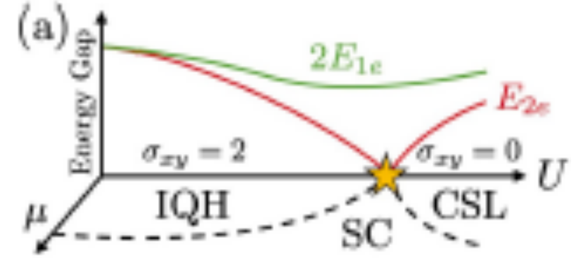
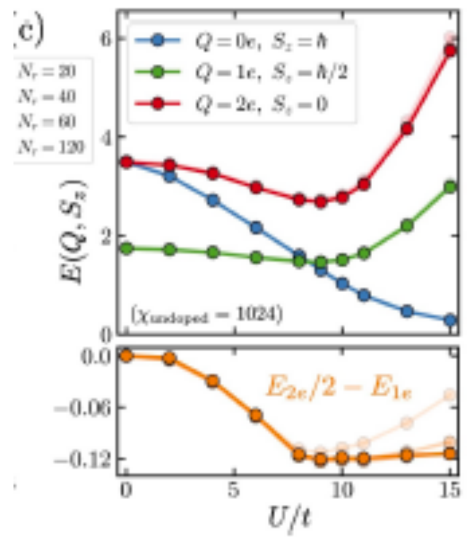
=>If proximal spin liquid phase, then interesting SC





Transition IQH->CSL: close and reopen a charge gap.
Transition CSL->120° AFM: close a spin gap



arXiv:2410.18175: Single particle gap remains non-zero=>gapless mode is a collective charge mode; in simple model, Yang ``eta-pairing" triplet. =>Doping leads to novel superfluid

(see also arXiv:2308.10935 (Sahay, … Divic…Zalatel)

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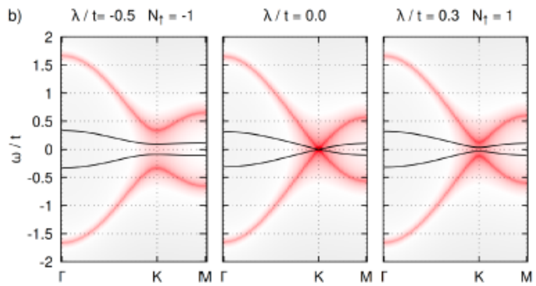
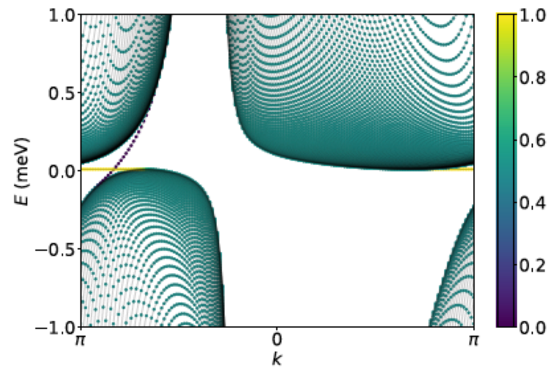
Summary
Interactions and topology: new physics

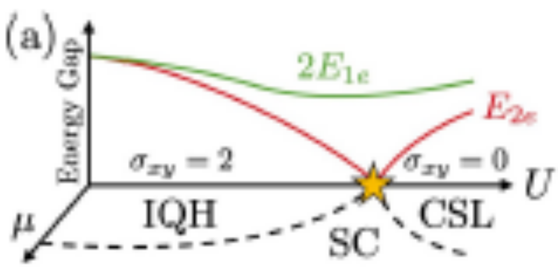
Topological Mott Insulators



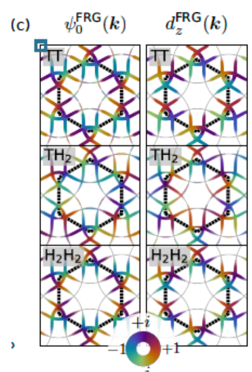
Topological Kondo Insulators

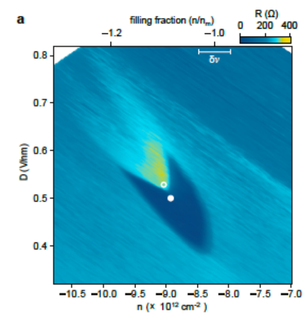


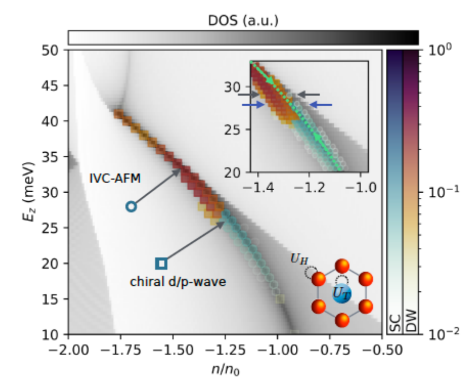
And (possibly) superconductors



Superconductors: Possibly Topological







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