Engineering Gravity via CFTs

Alejandra Castro, University of Cambridge

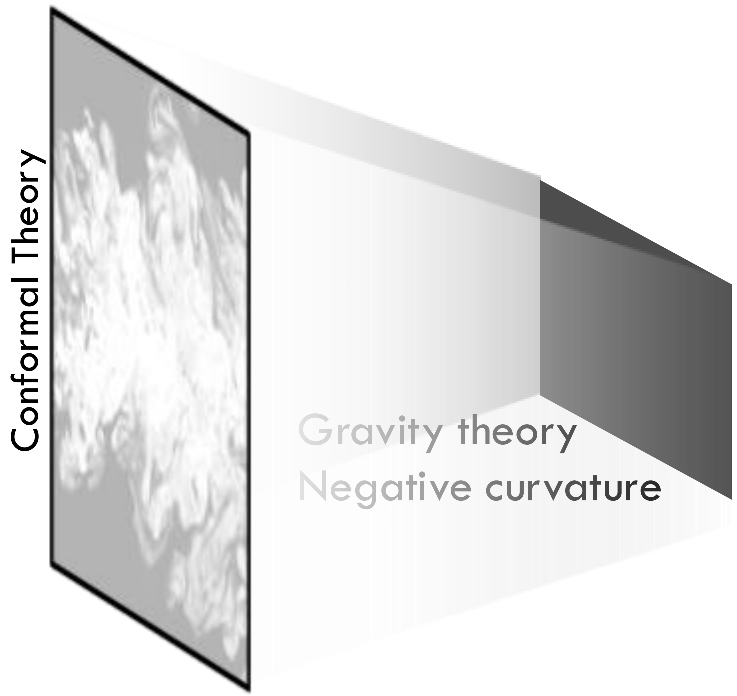
WPC Theoretical Physics Symposium 2025

DESY, Hamburg

$\text{AdS}_{d+1}/\text{CFT}_d$

The most influent result from string theory in the XXI century.

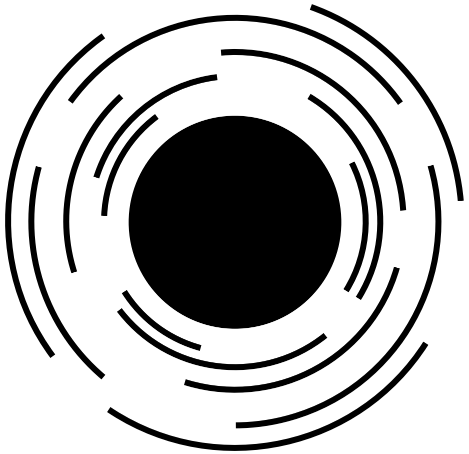
The AdS/CFT correspondence argues that a theory of quantum gravity in negatively curved spaces (AdS) is equivalent a conformal field theory (CFT).



It is an equivalence between two systems that live in different number of dimensions.

$\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

Inspired by
quantum black holes

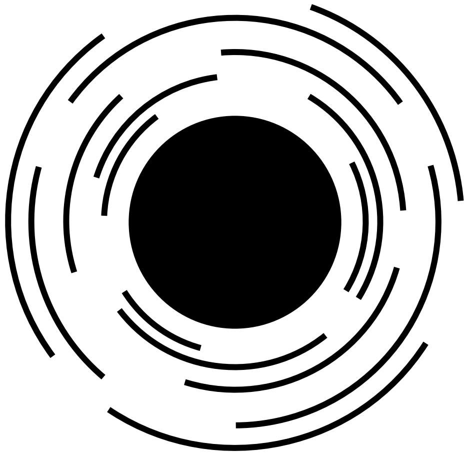


Bekenstein-Hawking entropy

$$S_{BH} = k_B \frac{c^3}{G_N \hbar} \frac{A_H}{4}$$

$\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$

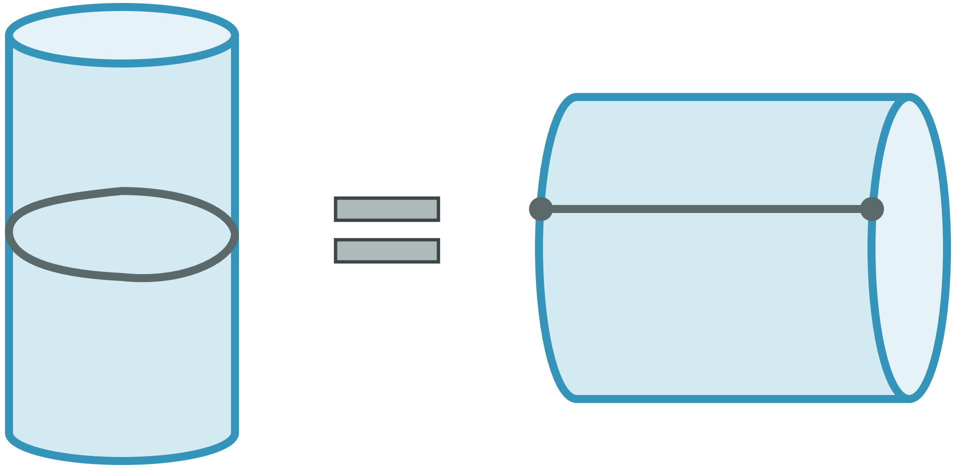
Inspired by
quantum black holes



Bekenstein-Hawking entropy

$$S_{BH} = k_B \frac{c^3}{G_N \hbar} \frac{A_H}{4}$$

Concrete realizations
in string theory



Closed String

Excitations include gravity

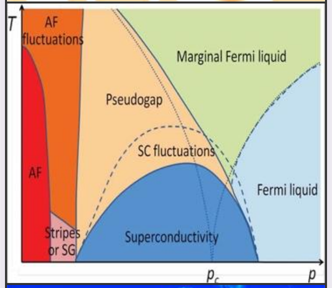
Open String

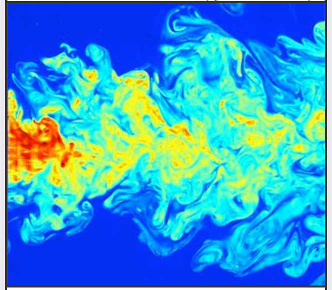
Excitations include gluons

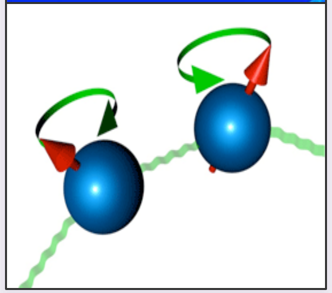
AdS/CFT: Practical uses

- Quark-gluon plasma
- High Tc superconductors
- Relativistic hydrodynamics
- Non-relativistic strongly coupled systems
- Quantum information

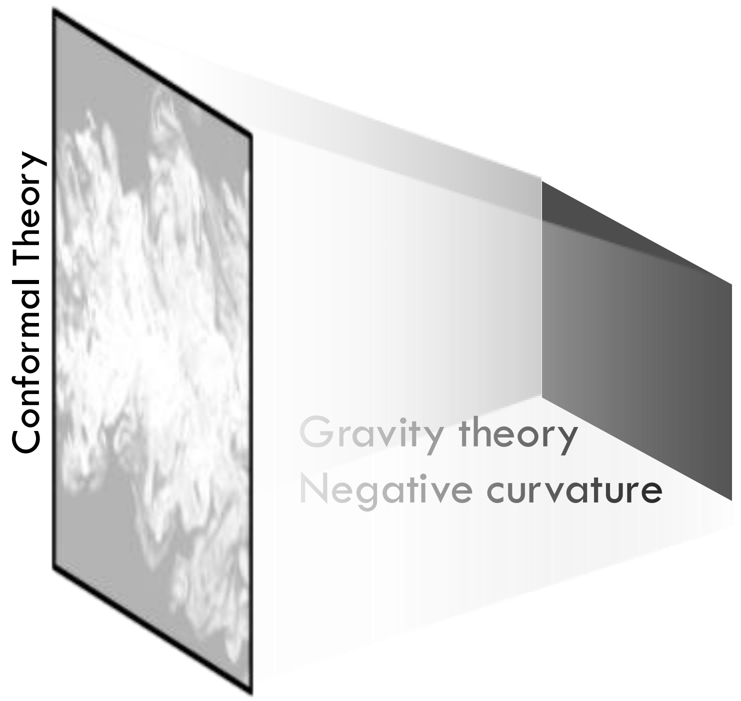
Gravity teaches us about quantum phenomena!







AdS/CFT & Quantum Gravity



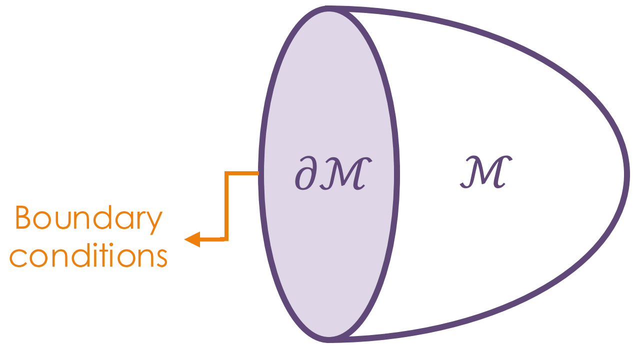
A strategy: engineering quantum gravity in AdS via the Conformal Field Theory.

This requires a fundamental understanding of the duality,
and a new perspective on CFTs.

AdS/CFT: lessons on Quantum Gravity

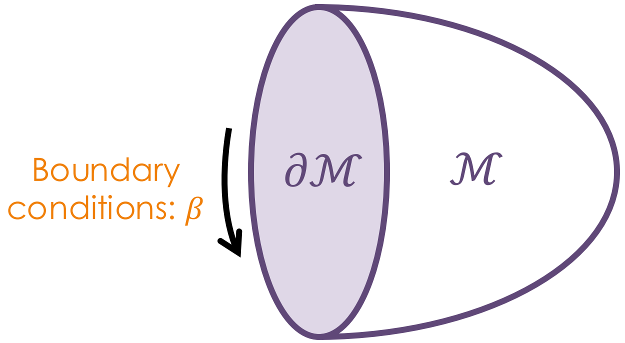
A centerpiece in recent years: the Gravitational Path Integral (GPI)

$$\mathcal{Z} = \int_{\mathcal{M}} \underbrace{\mathcal{D}g}_{\text{Geometries +topologies}} \; \underbrace{\mathcal{D}\phi_i}_{\text{Fields}} \; e^{-\underbrace{I(g,\phi_i)}_{\text{Action for gravity+fields}}}$$



A centerpiece in recent years: the Gravitational Path Integral (GPI)

$$\mathcal{Z} = \int_{\mathcal{M}} \underbrace{\mathcal{D}g}_{\text{Geometries +topologies}} \, \underbrace{\mathcal{D}\phi_i}_{\text{Fields}} \, e^{-\underbrace{I(g,\phi_i)}_{\text{Action for gravity+fields}}}$$

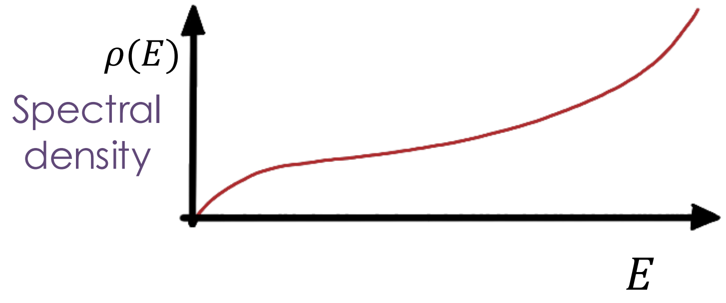


$$S(\beta) = \left(1 - \beta \frac{\partial}{\partial\beta}\right) \log \mathcal{Z}(\beta)$$

Applied to black holes in various dimensions, yields quantities that are deeply rooted in properties of an underlying microstate description, e.g., black hole entropy and the Page curve.

A centerpiece in recent years: the Gravitational Path Integral (GPI)

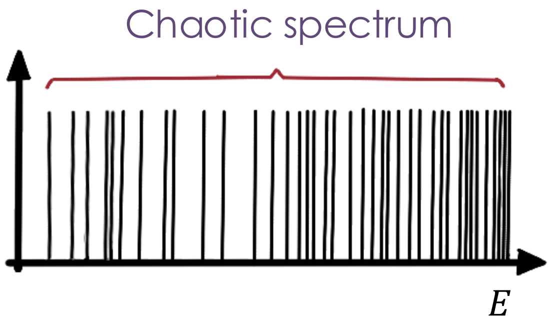
$$\mathcal{Z} = \int_{\mathcal{M}} \underbrace{\mathcal{D}g}_{\text{Geometries +topologies}} \; \underbrace{\mathcal{D}\phi_i}_{\text{Fields}} \; e^{-\underbrace{I(g,\phi_i)}_{\text{Action for gravity+fields}}}$$



Based on explicit results, the emerging lessons are:

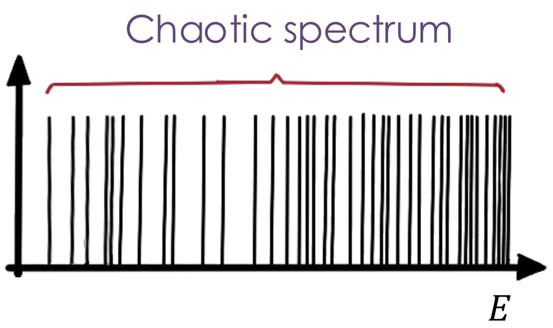
- The low-energy description of quantum gravity in terms of a metric and matter fields needs to be understood as a coarse-graining.
- Classical results can change drastically due to quantum effects.
- Observables need to be organized according to new principles: quantum chaos, algebraic structures, …

Suppose we have an underlying UV framework for quantum gravity

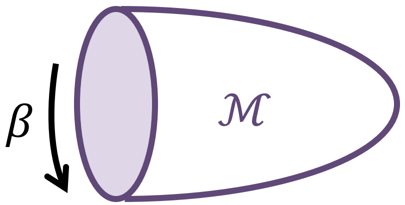


$$Tr\left(e^{-\beta H}\right)$$

Suppose we have an underlying UV framework for quantum gravity

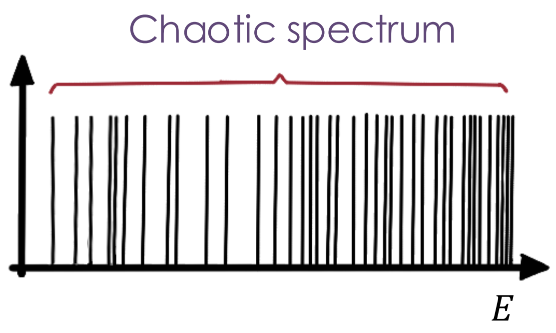


$$Tr\left(e^{-\beta H}\right) \overset{?}{=} \mathcal{Z}(\beta) = \int_{\mathcal{M}} \mathcal{D}g\, \mathcal{D}\phi_i\, e^{-I(g,\phi_i)}$$

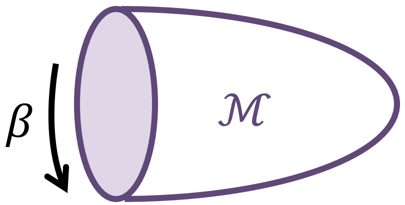


Can we derive in detail the GPI description from the UV?

Suppose we have an underlying UV framework for quantum gravity

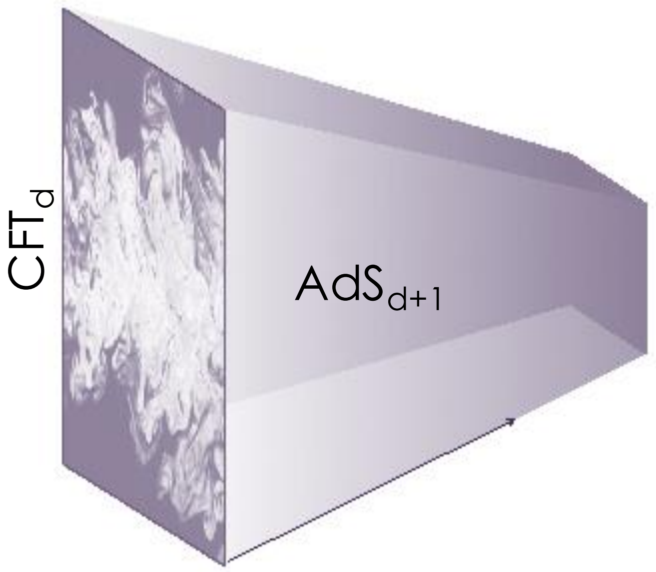


$$Tr\left(e^{-\beta H}\right) \overset{?}{=} \mathcal{Z}(\beta) = \int_{\mathcal{M}} \mathcal{D}g\, \mathcal{D}\phi_i\, e^{-I(g,\phi_i)}$$

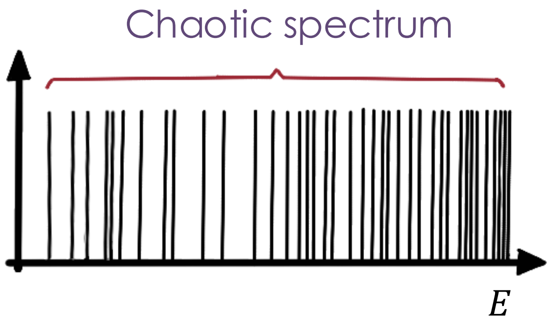


Can we derive in detail the GPI description from the UV?

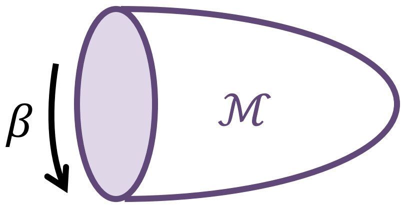
For specific AdS_{d+1} backgrounds in string theory, the duality involves a single Hamiltonian with a discrete spectrum.



Suppose we have an underlying UV framework for quantum gravity



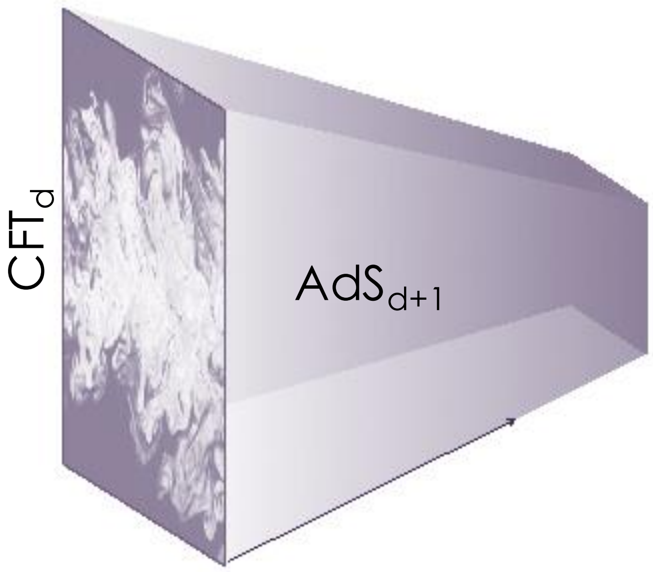
$$Tr\left(e^{-\beta H}\right) \stackrel{?}{=} \mathcal{Z}(\beta) = \int_{\mathcal{M}} \mathcal{D}g\, \mathcal{D}\phi_i\, e^{-I(g,\phi_i)}$$



Can we derive in detail the GPI description from the UV?

For specific AdS_{d+1} backgrounds in string theory, the duality involves a single Hamiltonian with a discrete spectrum.

How the GPI arises from this fundamental description is unclear.



Present two sharper questions that could address part of this puzzle.

$$Tr\left(e^{-\beta H}\right) \stackrel{?}{=} \mathcal{Z}(\beta) = \int_{\mathcal{M}} \mathcal{D}g\, \mathcal{D}\phi_i\, e^{-I(g,\phi_i)}$$

Present two sharper questions that could address part of this puzzle.

$$Tr\left(e^{-\beta H}\right) \overset{?}{=} \mathcal{Z}(\beta) = \int_{\mathcal{M}} \mathcal{D}g\, \mathcal{D}\phi_i\, e^{-I(g,\phi_i)}$$

1) Given a CFT, how to organize it such that quantum gravity is manifest?

2) Which CFTs capture classical (geometric) properties of gravity?

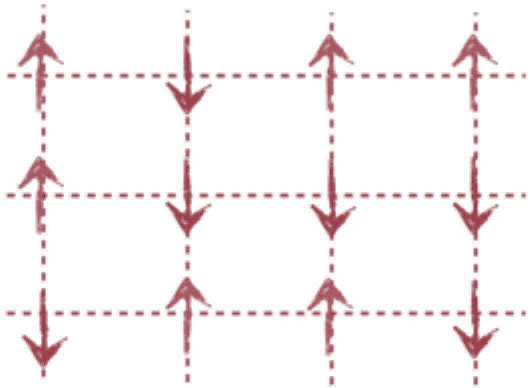
1) Given a CFT, how to organize it such that quantum gravity is manifest?

Simple situation based on CFT_2 that are well known:

1) Given a CFT, how to organize it such that quantum gravity is manifest?

Simple situation based on CFT_2 that are well known:

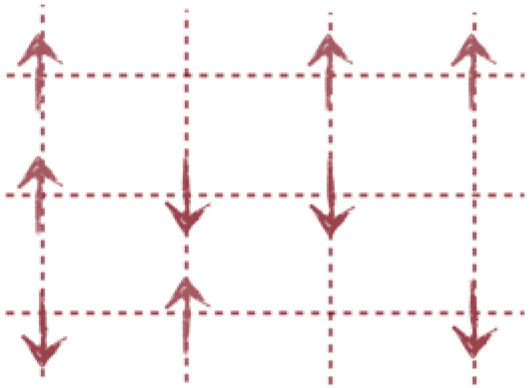
Ising Model



Virasoro Minimal Model (3,4)

$$c = \frac{1}{2}$$

Tricritical Ising Model



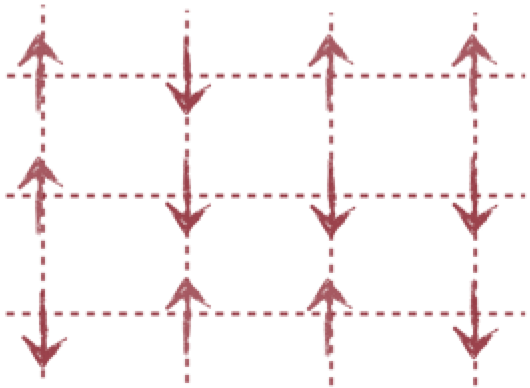
Virasoro Minimal Model (4,5)

$$c = \frac{7}{10}$$

1) Given a CFT, how to organize it such that quantum gravity is manifest?

Simple situation based on CFT_2 that are well known:

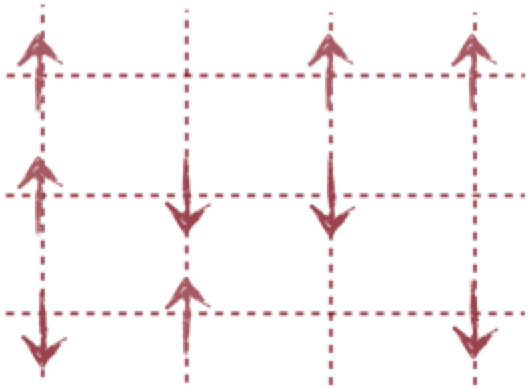
Ising Model



Virasoro Minimal Model (3,4)

$$c = \frac{1}{2}$$

Tricritical Ising Model



Virasoro Minimal Model (4,5)

$$c = \frac{7}{10}$$

Partition function: $\mathcal{Z}_{CFT}(\tau, \bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau) \chi_h(\bar{\tau})$

Organized in terms of primary operators $(h, \bar{h})$ and descendants.

1) Given a CFT, how to organize it such that quantum gravity is manifest?

$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau})$$

$$\Big\updownarrow ?$$

$$\mathcal{Z}_{Grav}(\tau,\bar{\tau}) = \int_{\mathcal{M}} \mathcal{D}g\; e^{-I(g)}$$

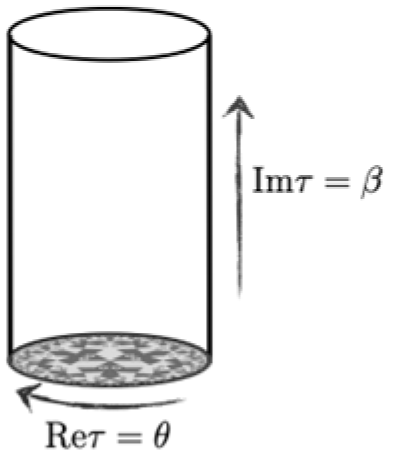
A. Lepage-Jutier, A. Maloney, AC, 2010
T. Hartman, A. Maloney, AC, 2011
M. Gaberdiel, T. Hartman, A. Maloney, R. Volpato, AC, 2011

1) Given a CFT, how to organize it such that quantum gravity is manifest?

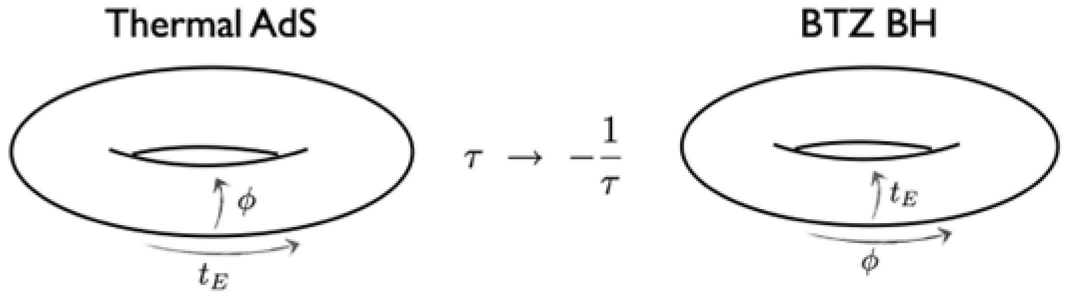
$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau})$$

$$? \Big\updownarrow$$

$$\mathcal{Z}_{Grav}(\tau,\bar{\tau}) = \int_{\mathcal{M}} \mathcal{D}g\; e^{-I(g)}$$



Euclidean saddles



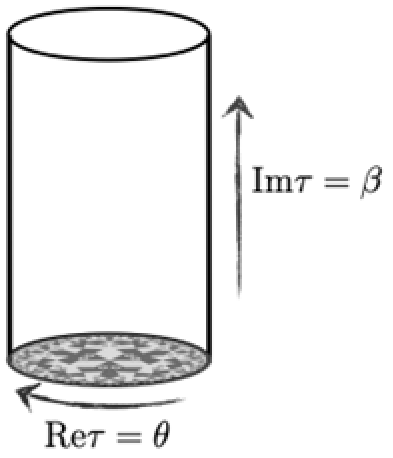
A. Lepage-Jutier, A. Maloney, AC, 2010
T. Hartman, A. Maloney, AC, 2011
M. Gaberdiel, T. Hartman, A. Maloney, R. Volpato, AC, 2011

1) Given a CFT, how to organize it such that quantum gravity is manifest?

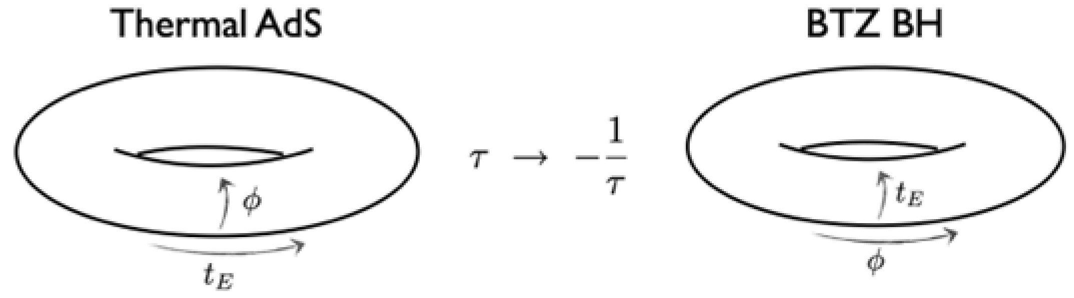
$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau}\,)$$

$$\Big\updownarrow\ ?$$

$$\mathcal{Z}_{Grav}(\tau,\bar{\tau}) = \int_{\mathcal{M}} \mathcal{D}g\; e^{-I(g)} = \sum_{\gamma\in\Gamma_c\backslash SL(2,\mathbb{Z})} Z_{vac}(\gamma\tau,\bar{\gamma}\bar{\tau})$$



Euclidean saddles



A. Lepage-Jutier, A. Maloney, AC, 2010
T. Hartman, A. Maloney, AC, 2011
M. Gaberdiel, T. Hartman, A. Maloney, R. Volpato, AC, 2011

1) Given a CFT, how to organize it such that quantum gravity is manifest?

$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau})$$

Ising Model

Tricritical Ising Model

YES! $\updownarrow$

$$\mathcal{Z}_{Grav}(\tau,\bar{\tau}) = \int_{\mathcal{M}} \mathcal{D}g\; e^{-I(g)} = \sum_{\gamma\in\Gamma_c\backslash SL(2,\mathbb{Z})} Z_{vac}(\gamma\tau,\bar{\gamma}\bar{\tau})$$

A. Lepage-Jutier, A. Maloney, AC, 2010
T. Hartman, A. Maloney, AC, 2011
M. Gaberdiel, T. Hartman, A. Maloney, R. Volpato, AC, 2011

1) Given a CFT, how to organize it such that quantum gravity is manifest?

$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_{h}(\bar{\tau})$$

Ising Model

Tricritical Ising Model

YES! $\updownarrow$

$$\mathcal{Z}_{Grav}(\tau,\bar{\tau}) = \int_{\mathcal{M}} \mathcal{D}g\; e^{-I(g)} = \sum_{\gamma\in\Gamma_c\backslash SL(2,\mathbb{Z})} Z_{vac}(\gamma\tau,\bar{\gamma}\bar{\tau})$$

Important remarks:

- These are the only 2 minimal models that admit this very specific form.
- Alternative proposal by R. Janik [2502.19015], where $\mathcal{Z}_{Grav}$ is defined differently to accommodate an approximate definition of chaos in the Ising Model.
- Gravitational theory is very quantum (low+fixed central charge). We have no access to classical properties of the theory on AdS.

1) Given a CFT, how to organize it such that quantum gravity is manifest?

Moving forward, re-think how on should recast CFT observables.

1) Given a CFT, how to organize it such that quantum gravity is manifest?

Moving forward, re-think how on should recast CFT observables.
For example, the partition function in

$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau}\,) = \sum_{\gamma\in\Gamma_c\backslash SL(2,\mathbb{Z})} Z_{light}(\gamma\tau,\bar{\gamma}\bar{\tau}) \ + Z_{spec}(\tau,\bar{\tau})$$

One guiding principle could be the aspects of the GPI that lead to coarse/chaotic features: single those features out in the CFT.

G. Di Ubaldo, E. Perlmutter, 2023
F. Haehl, C. Marteau, W. Reeves, M. Roshali, 2023
N. Benjamin, S. Collier, A.L. Fitzpatrick, A. Maloney, E. Perlmutter,2021

1) Given a CFT, how to organize it such that quantum gravity is manifest?

Moving forward, re-think how on should recast CFT observables.
For example, the partition function in

$$\mathcal{Z}_{CFT}(\tau,\bar{\tau}) = \sum_{h,\bar{h}} N_{h,\bar{h}}\, \chi_h(\tau)\chi_h(\bar{\tau}\,) = \sum_{\gamma\in\Gamma_c\backslash SL(2,\mathbb{Z})} Z_{light}(\gamma\tau,\bar{\gamma}\bar{\tau}) \;+ Z_{spec}(\tau,\bar{\tau})$$

One guiding principle could be the aspects of the GPI that lead to coarse/chaotic features: single those features out in the CFT.

G. Di Ubaldo, E. Perlmutter, 2023
F. Haehl, C. Marteau, W. Reeves, M. Roshali, 2023
N. Benjamin, S. Collier, A.L. Fitzpatrick, A. Maloney, E. Perlmutter,2021

This has two motivations behind it:

- Interplay of quantum gravity with quantum matter in the GPI.
- We are terrible at understanding the space of CFTs. This could give an interesting perspective of what is a “generic” CFT.
- Challenges:
  - Definition of Quantum Chaos in CFTs (tension with integrability).
  - Implementation of chaos in classification/construction of CFTs.

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

A few (but not all) properties associated to them are:

- Large central charge (large-N)
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.
- …

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

A few (but not all) properties associated to them are:

- Large central charge (large-N) $N \sim \frac{(\ell_{AdS})^{d-2}}{G_N} \gg 1$
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.
- …

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

A few (but not all) properties associated to them are:

- Large central charge (large-N)
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.
- …

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

A few (but not all) properties associated to them are:

- Large central charge (large-N)
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.
- …

How many conditions do I need to impose?
How stringent are the conditions?

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

2) Which CFTs capture classical (geometric) properties of gravity?

Holographic CFT: A CFT whose dual gravity theory has a low-energy EFT description.

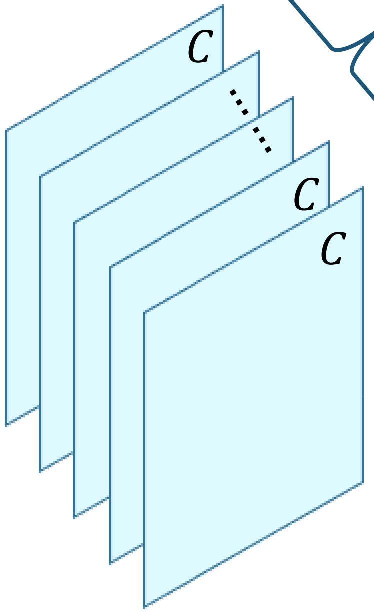
A few (but not all) properties associated to them are:

- Large central charge (large-N)
- Sparse spectrum (degeneracy of light operators are not controlled by N).
- Factorization of correlation functions, i.e., Generalized Free Fields.
- …

How many conditions do I need to impose?
How stringent are the conditions?
Can we engineer CFTs that satisfy these criteria?

See, for example:
Heemskerk, J. Penedones, J. Polchinski, and J. Sully 2009
El-Showk and Papadodimas 2011

To engineer, start by selecting the material: CFT_2 based on Symmetric Product Orbifolds

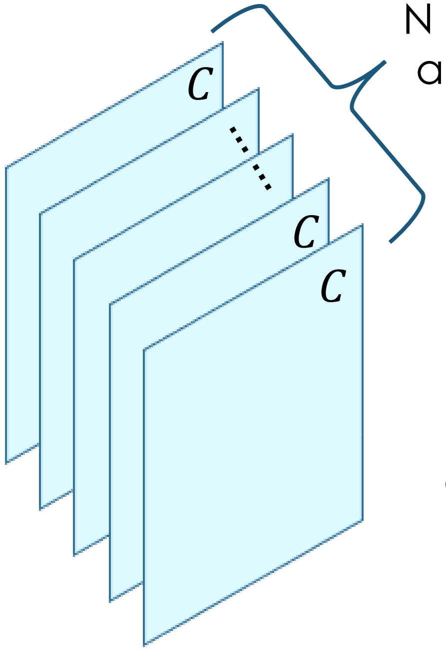


N copies of a unitary and compact CFT_{2}.
$\mathcal{C}$ = seed theory

Orbifold by the permutation group S_N

$$Sym^N(\mathcal{C}) = \frac{\mathcal{C}^{\otimes N}}{S_N}$$

To engineer, start by selecting the material: CFT_2 based on Symmetric Product Orbifolds



$$Sym^N(\mathcal{C}) = \frac{\mathcal{C}^{\otimes N}}{S_N}$$

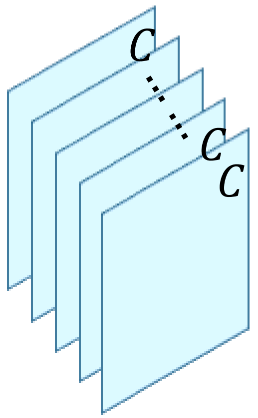
Orbifold by the permutation group S_N

The orbifold introduces two class of states:

- untwisted sector: it keeps states that are invariant under S_N.
- twisted sectors: new states labelled by conjugacy classes of S_N.

Symmetric Product Orbifolds are excellent material since

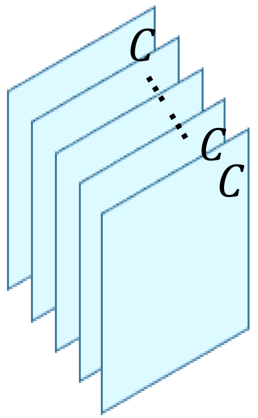
- Appeal: Mathematical and analytic control, e.g., DMVV formula.
- Familiarity: D1D5 CFT.
- Utility: compelling features for AdS/CFT.



$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

Symmetric Product Orbifolds are excellent material since

- Appeal: Mathematical and analytic control, e.g., DMVV formula.
- Familiarity: D1D5 CFT.
- Utility: compelling features for AdS/CFT.



$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

What does $Sym^N(C)$ teach us about sufficient conditions for a holographic CFT?

Differences from low-energy gravity

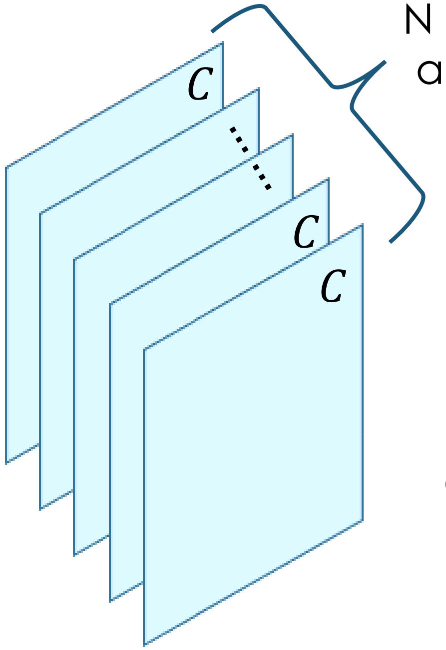
Similarities with low-energy gravity

Classification of Symmetric Product orbifolds

Differences from low-energy gravity

Similarities with low-energy gravity

Classification of Symmetric Product orbifolds



N copies of a unitary and compact CFT_2.
C = seed theory

$$Sym^N(C) = \frac{C^{\otimes N}}{S_N}$$

Orbifold by the permutation group S_N

Universality: Several properties of $Sym^N(C)$ are independent of the seed theory

Universal Properties of Symmetric Product Orbifolds

Similarities with low-energy gravity

- Factorization of correlation functions
- Phase structure and entropy
- Thermal correlation functions of single trace operators
- …

Differences with low-energy gravity

- Higher spin currents
- Hagedorn growth of light states
- …

Lesson: not all necessary conditions are sufficient to make a CFT holographic.

Conclusion

Engineering tasks continues…

1) Given a CFT, how to organize it such that quantum gravity is manifest?

2) Which CFTs capture classical (geometric) properties of gravity?

1) Given a CFT, how to organize it such that quantum gravity is manifest?

- Determine observables that are better adapted to chaos and coarse-graining.
  - Use these to understand the classification of CFTs.

2) Which CFTs capture classical (geometric) properties of gravity?

- What are possible theories of quantum gravity that can be designed?
  - What are the materials needed to assemble them?

Thank you!