Challenges ahead in
atomic, molecular, and nuclear few-body physics

Chris H Greene, Purdue University

Few-body reactive processes, bound states, and resonances

- Develop an analytic zero-range interaction solution to the four-body problem, analogous to the well-known exact solution to the zero-range 3-body problem
- Push numerical and/or analytical methods to treat more than 5 particles (e.g. atoms or nucleons), with or without spin, in their full Hilbert space including collisions

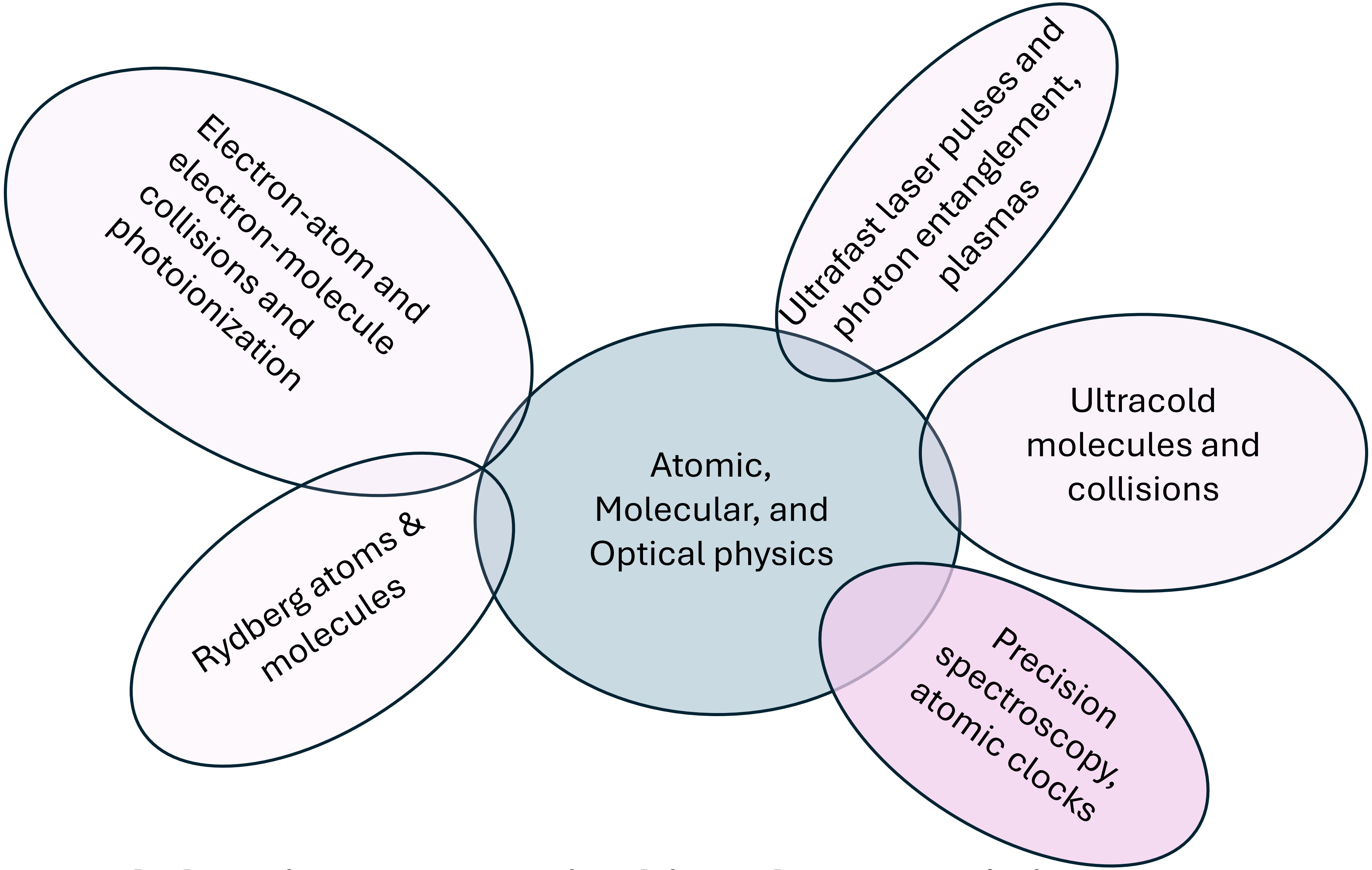
https://arxiv.org/a/greene_c_1.html

Challenges in the atomic realm

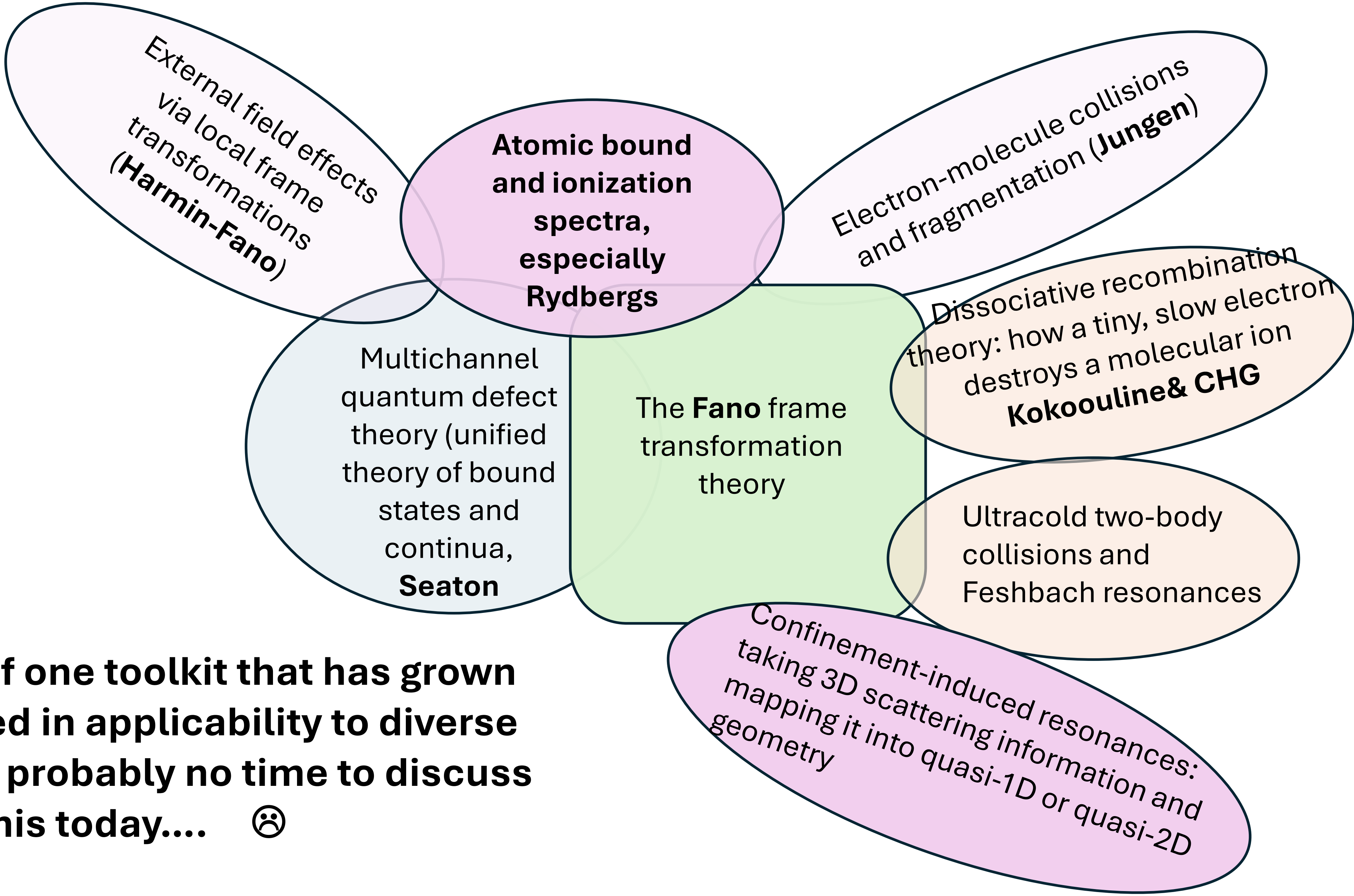
- Can the disastrously bad N^7 scaling of all-electron methods ever be overcome?
- Bring electron-atom (or ion) scattering theory to spectroscopic accuracy, comparable to the best treatments of bound state correlations?
- Compute electron-atom scattering lengths for any atom in the periodic table to within 10-20%?

Challenges in the molecular realm (diatomics & triatomics)

- Developing a theory and/or computational method that can determine the FULL electron-molecule scattering matrix including ALL fragmentation channels (e.g. in H_2: e + H_2+, $H(n_1\ l_1)$+$H(n_2\ l_2)$, p + H^-, etc.)
- Formulate a quantitatively accurate theory that can calculate the Born-Oppenheimer potential curves for two heavy open shell atoms, and their scattering lengths versus magnetic field



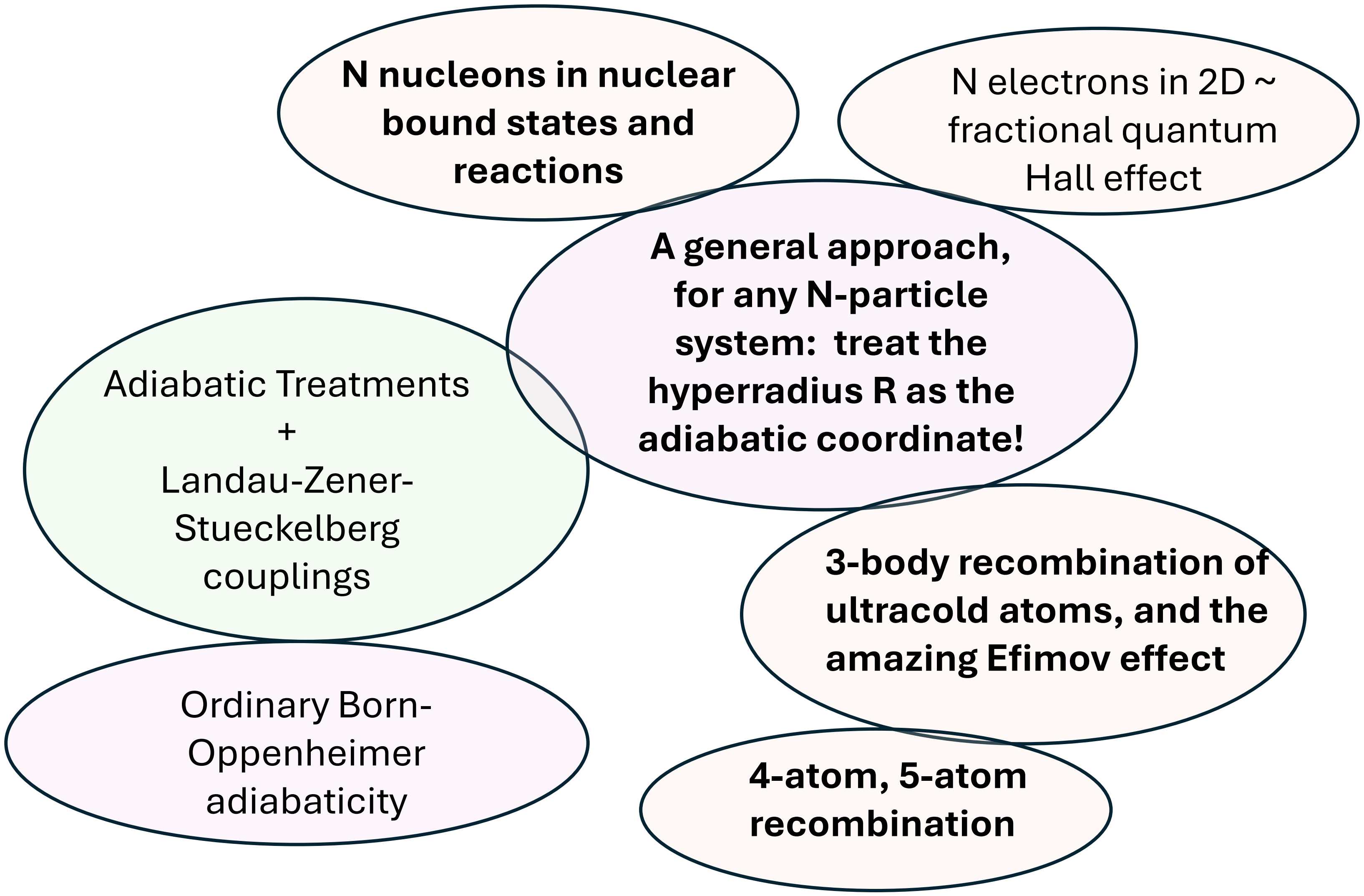
A theorist can organize his or her group's interest by physics subfields, as indicated here, AND/OR, as in the next slide, by toolkits



An example of one toolkit that has grown and broadened in applicability to diverse problems, but probably no time to discuss this today.... ☹

Here are some of the theory directions stimulated by sensing the power of an unconventional idea/toolkit, and exploring where it might yield valuable insights.

In this case, it is the adiabatic toolkit, broadly interpreted



Many Big Goals have been formulated for the field of ultracold atoms over the past 3 decades, such as:

- Realize the BCS-BEC crossover, by using tunable Fano-Feshbach resonances (accomplished!)
- Create a cold atom system that simulates high-Tc superconductivity, and helps to understand it. (not yet?)
- Realize the Fractional Quantum Hall effect with trapped atoms (perhaps only partially, in a sense)
- Simulate nuclear reactive processes, except with ultracold fermionic atoms instead of nucleons (not yet)

Challenges in the Realm of Few-Body Physics

**Current progress in the full treatment of N particles with short-range interactions has largely stalled at around 4 or 5 particles. Let's push this to more complex systems.

Limited bound state calculations for highly simplified Hamiltonians (e.g. Gaussian interactions) have occasionally treated more, e.g. up to around 8 or 10 particles. Many more particles than that have been treated by a variety of Monte Carlo methods, but usually only the ground state.

Treatments of collision/reactive processes for 5 particles are highly limited (examples shown below), and for 6 particles even fewer, and usually with extreme approximations.

Why is this a hard problem?
The N-body Schroedinger equation with no external forces has dimension 3N-3,
e.g. for 6 particles it is a 15-dimensional PDE.

Monte Carlo methods exist to solve for ground states in high dimensional problems,
but those methods have difficulty to describe collision processes, especially reactive ones

One of our long-time goals has been to improve our ability to understand reactivity of all kinds for systems having more and more particles, e.g. in chemical or nuclear or cold atom physics, reactions like:

A_2+BCD→AB+AC+D

or →A_2D+BC, etc.

This is the famous "N-body problem" in quantum mechanics

In FEW BODY PHYSICS, N might be 2 or 3 or 4 or 5 or ….

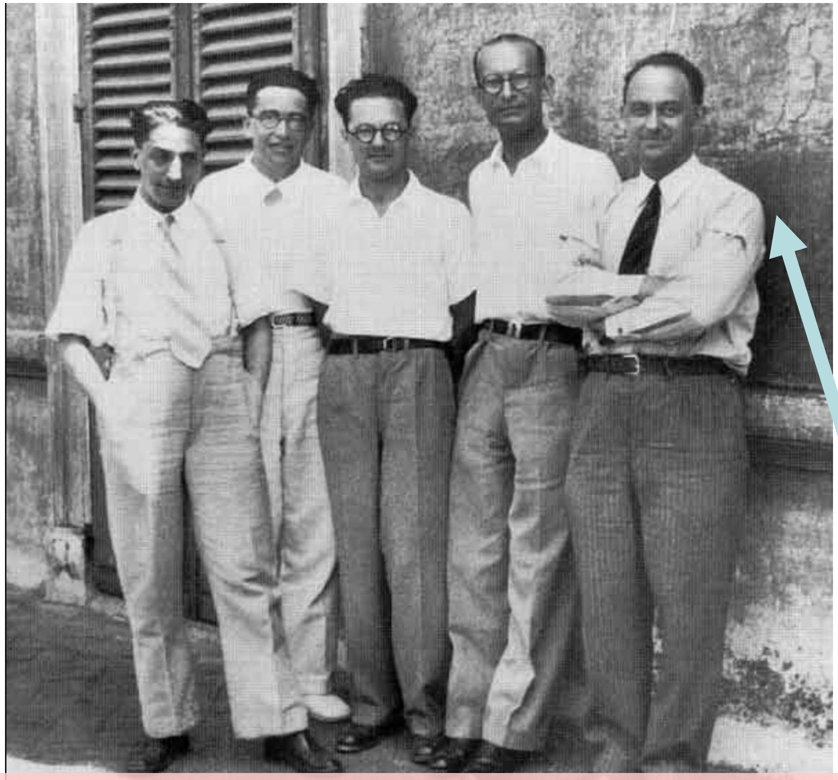
We have pursued this goal primarily by improving what we call the

"Adiabatic Hyperspherical Coordinate Toolkit",

both a mathematical technique and a way to gain intuition

One Key Idea: From Enrico Fermi – understanding the energy content of a single scattering event at low energies, i.e., what we now call

THE FERMI PSEUDOPOTENTIAL (models short range interactions at very low energy)



From left to right: Oscar D'Agostino, Emilio Segrè, Edoardo Amaldi, Franco Rasetti and Enrico Fermi, from Wikipedia, the „Panispera Boys“

$$U(\vec{r}) = \frac{2\pi a \hbar^2}{m_{\rm red}} \delta(\vec{r} - \vec{R})$$

$$a = -\mathrm{Lim}_{k\to 0} \tan \delta_0(k)/k$$

In the low energy limit of a particle colliding with another particle through only the S-wave, the energy of interaction is controlled by the scattering length *a* and can be represented by *U*.

The following discussion addresses a simple but fundamental question, for bosonic atoms:

We know that in 3D a certain minimum strength of attraction is needed in order to just barely bind two atoms into a stable molecular bound state.

One can ask, how much weaker does the attraction need to be in order to just barely bind 3 atoms together, and how much less still, to bind 4 atoms, or 5 atoms, etc…

…AND moreover, how can these molecules get formed in a gas of trapped free atoms?

Extensions of Universal Physics to N>3 bosonic particles in 3D --- Schroedinger's equation with pair-wise additive forces:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \sum_{i<j} U(r_{ij})$$

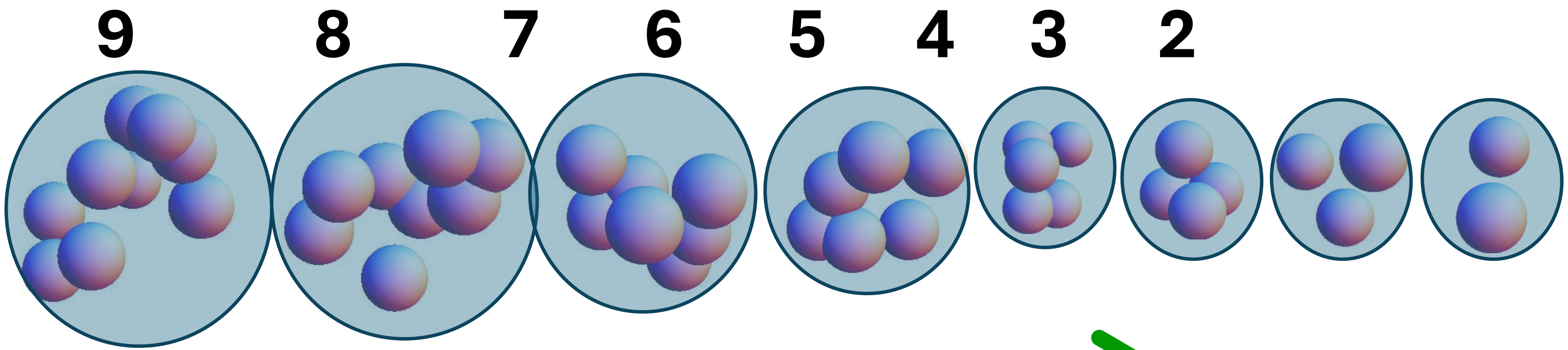
N repulsive terms >0

$\uparrow \frac{N(N-1)}{2}$ attractive terms <0

Can sometimes use:

$$U(\vec{r}) = \frac{2\pi a \hbar^2}{m_{\text{red}}} \delta(\vec{r} - \vec{R})$$

$$a = -\text{Lim}_{k \to 0} \tan \delta_0(k)/k$$



INCREASING ATTRACTION

(the interaction parameter *a* gets more negative) →

Atom-atom scattering length

J. Phys. B: At. Mol. Opt. Phys. 43 (2010) 101002 (5pp) doi:10.1088/0953-4075/43/10/101002

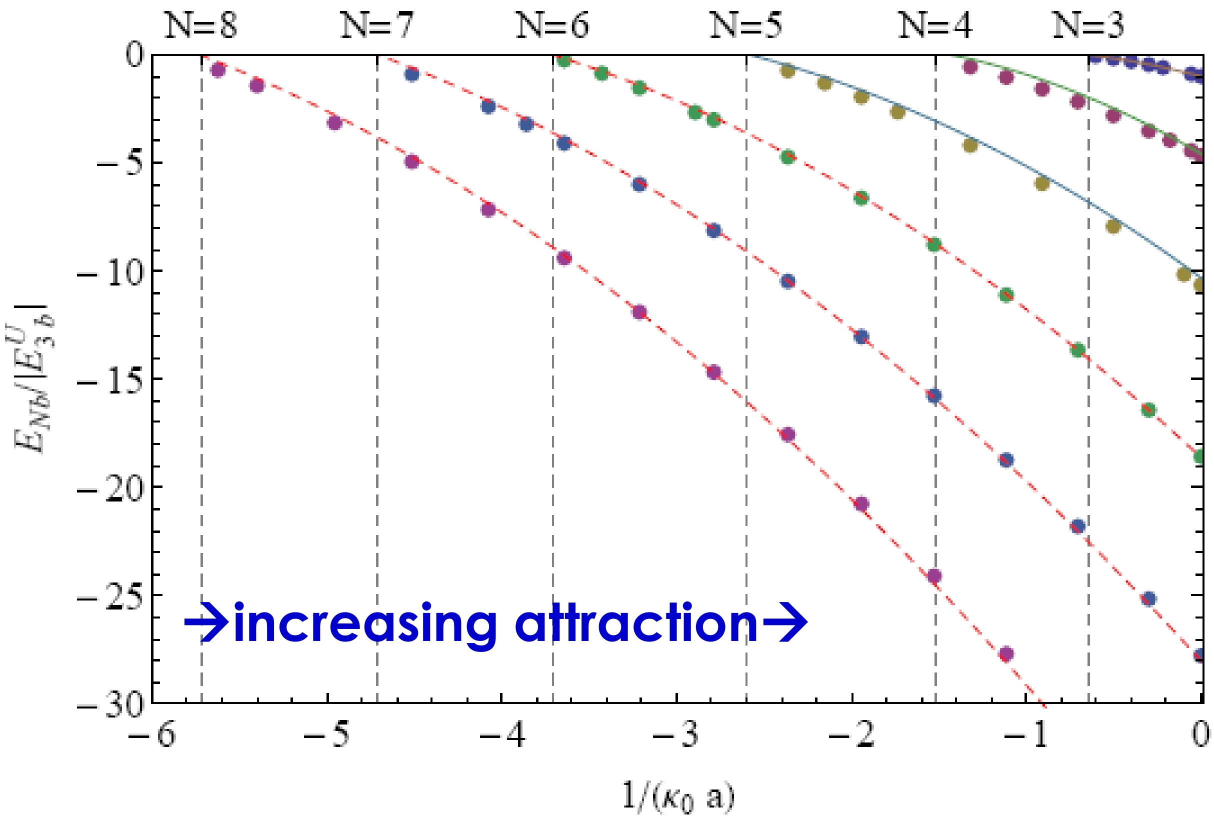
FAST TRACK COMMUNICATION

Weakly bound cluster states of Efimov character

MORE THAN 4 BOSONS: von Stecher's J. Phys. B article in 2010: combined study using correlated Gaussians, and diffusion Monte Carlo

Javier von Stecher

Clusters predicted up to N=13.



The idea of hyperspherical coordinates: replace the N independent-particle coordinates, namely:

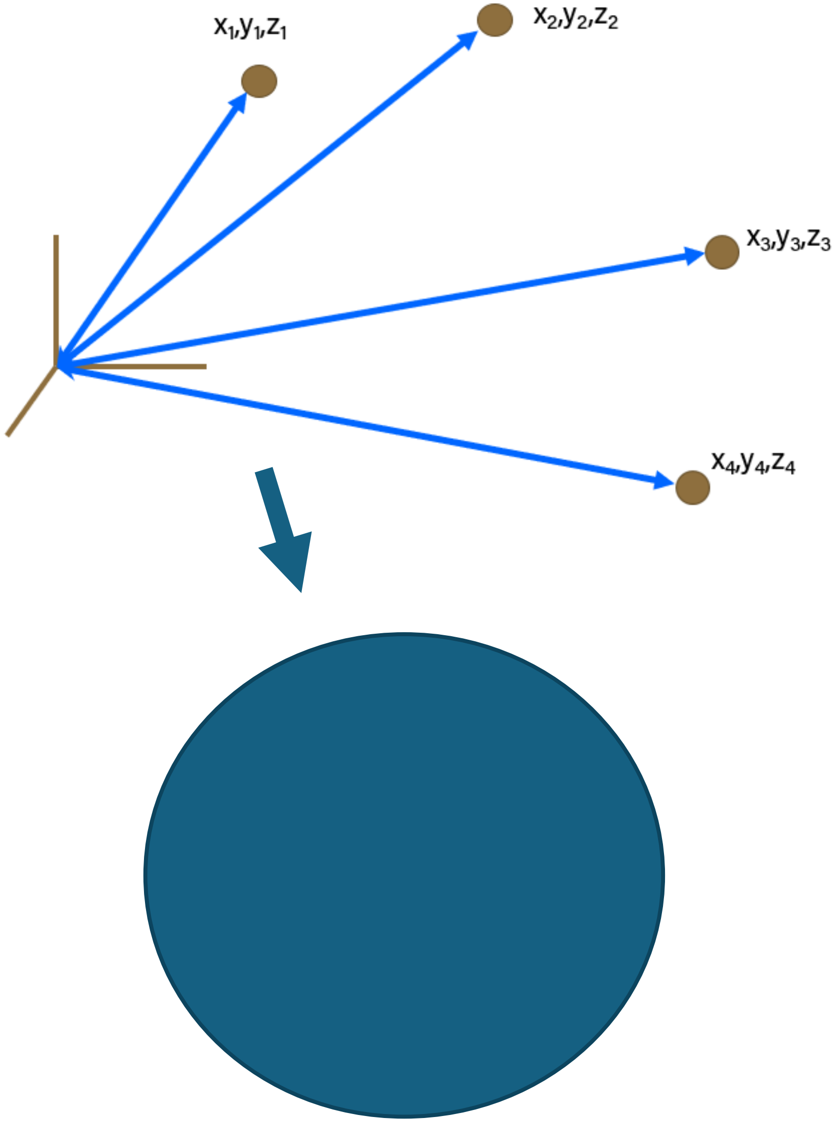
$$\{x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3, x_4, y_4, z_4, \ldots\}$$

by a representation in hyperspherical coordinates, which look like:

$$\{R, \theta_1, \theta_2, \theta_3, \theta_4, \ldots.\}$$

where now there is only ONE length-dimension coordinate, the hyperradius R, plus $(N-1)$ hyperangles.

And now, treat R as an adiabatic coordinate, implementing the very same math as in the Born-Oppenheimer approximation of physical chemistry, and calculate energy levels versus the hyperradius, $U_\nu(R)$, which gives us similar interpretive power as when we look at a diatomic molecule's potential energy curve.



To a mathematician, this would be called an "n-ball", i.e. a sphere in n-dimensions

How to tackle 5-body recombination for 5 free bosonic atoms with pairwise additive forces?

i.e. the reaction $A+A+A+A+A \rightarrow A_3+A_2$ or A_4+A or…

Start with the time-independent Schroedinger equation:

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} + \frac{p_4^2}{2m_4} + \frac{p_5^2}{2m_5}$$

$$+ V(r_{12}) + V(r_{13}) + V(r_{14}) + V(r_{15}) + V(r_{23}) + V(r_{24}) + V(r_{25}) + V(r_{34}) + V(r_{35}) + V(r_{45})$$

After eliminating the center-of-mass degree of freedom, we're left with a 12-dimensional PDE to solve, which can be reduced to **a mere 9 dimensions** for J=0 states after going to the body frame.

So we go to hyperspherical coordinates with only one distance **R** and 3N-4 hyperangles to represent the remainder

Strategy of the adiabatic hyperspherical representation: FOR ANY NUMBER OF PARTICLES, convert the partial differential Schroedinger equation into an infinite set of coupled ordinary differential equations:

To solve: →

$$\left[-\frac{1}{2\mu}\frac{\partial^2}{\partial R^2}+\frac{\Lambda^2}{2\mu R^2}+V(R,\theta,\varphi)\right]\psi_E=E\psi_E$$

First solve the fixed-R Schroedinger equation, for eigenvalues $U_n(R)$:

$$\left[\frac{\Lambda^2}{2\mu R^2}+\frac{15}{8\mu R^2}+V(R,\theta,\varphi)\right]\Phi_\nu(R;\Omega)=U_\nu(R)\Phi_\nu(R;\Omega)$$

Next expand the desired solution into the complete set of hyperangle eigenfunctions with unknowns F(R) →

$$\psi_E(R,\Omega)=\sum_\nu F_{\nu E}(R)\Phi_\nu(R;\Omega)$$

And the original T.I.S.Eqn. is transformed into the following set which can be truncated on physical grounds, with the eigenvalues interpretable as adiabatic potential curves, in the Born-Oppenheimer sense.

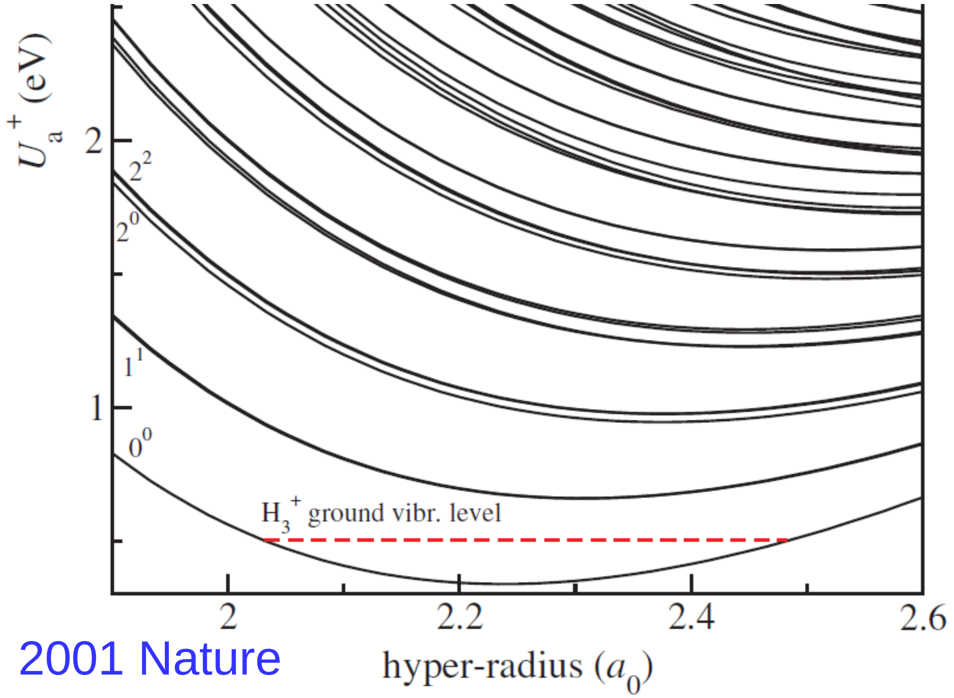
$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2}+U_\nu(R)\right]F_{\nu E}(R)-\frac{1}{2\mu}\sum_{\nu'}\left[2P_{\nu\nu'}(R)\frac{d}{dR}+Q_{\nu\nu'}(R)\right]F_{\nu' E}(R)=EF_{\nu E}(R)$$

This is a divide-and-conquer strategy: how to leverage bound state calculations to solve collision problems

Our group has advanced this approach to solving the many-particle Schroedinger equation in diverse contexts, from chemical physics to Bose-Einstein condensates in ultracold science to the fractional quantum Hall effect, the 4-neutron problem, and very recently, to the 5-boson problem

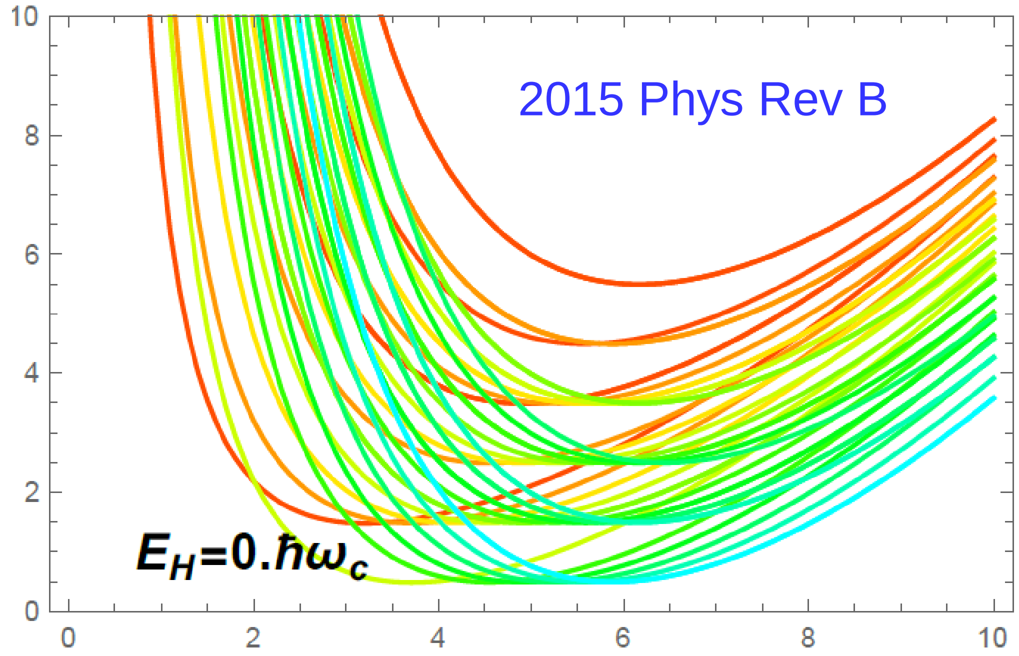
Hyperspherical Potential Energy Curve Examples

Simplest triatomic molecule (dissociative recombination of H_3^+)



2001 Nature

Integer and fractional quantum Hall effect (2D electron gas in a B-field)



Short history of the Efimov effect for three identical bosons:

In 1970, Efimov considered the problem of 3 particles with a scattering length "a" tuned to infinity. In this scenario, NO 2-body bound states exist in 3D. Nevertheless, Efimov predicted that an **infinite number** of 3-BODY bound states must exist.

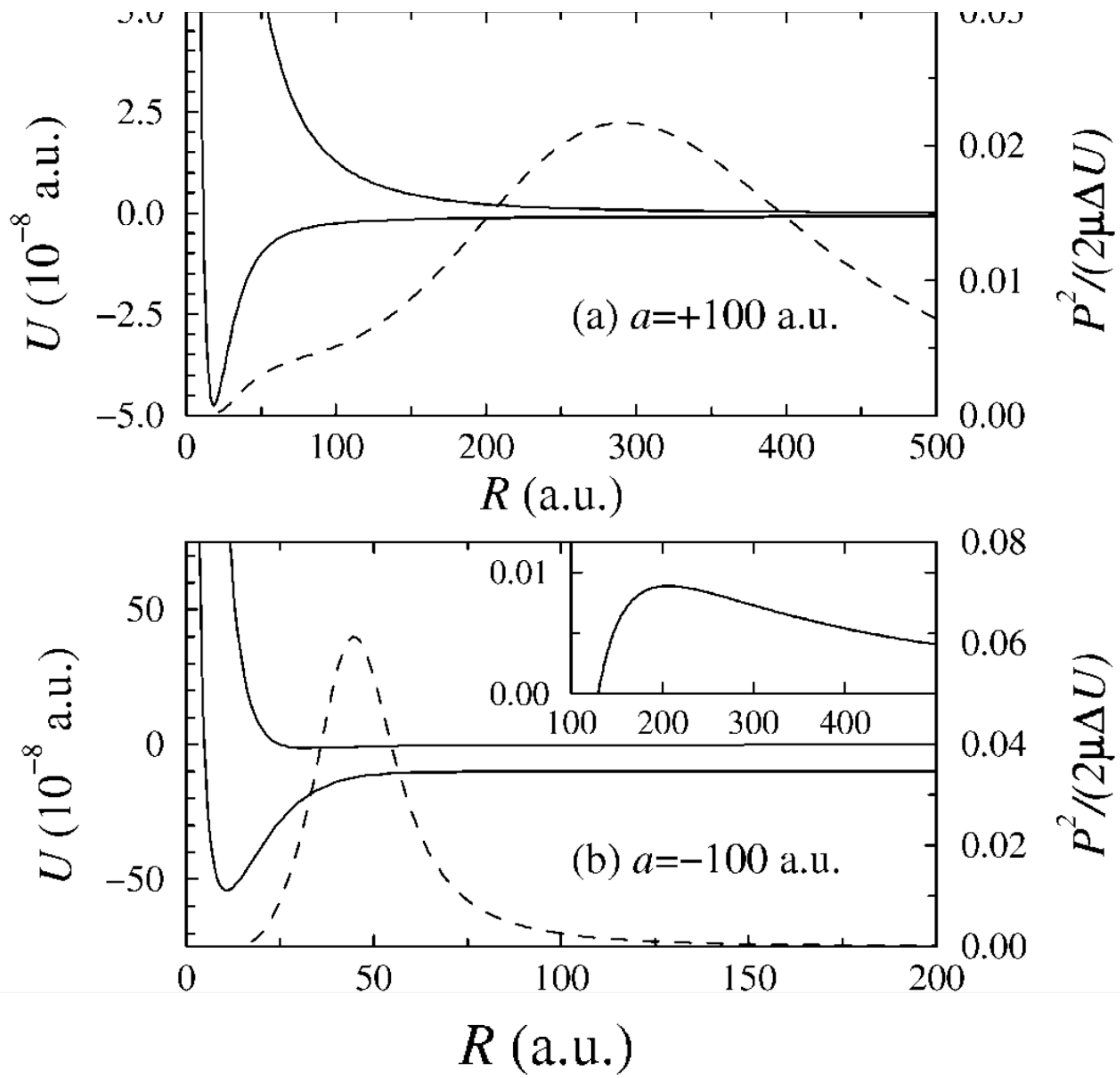
This was not observed experimentally for several decades, until the possibility to CONTROLLABLY CHANGE the atom-atom scattering length ***'a'*** became possible in an ultracold atomic gas, thanks to magnetic Feshbach resonances.

The spark that allowed the Efimov states to be observed in an ultracold gas came in a theory study that was studying something else, the 3-body recombination process in the gas, namely:

$A + A + A \rightarrow A_2 + A$ ...in late 1990s

which is the dominant loss process in a Bose-Einstein condensate (usually)

By 1999, our goal was to study 3-body recombination in an ultracold Bose gas, and find the systematics as well as possible ideas that might enable control of this dastardly loss process that was limiting the lifetime of BECs: **Esry, CHG, Burke, PRL 83, 1751 (1999), A+A+A→A_2+A**



Our first published predictions

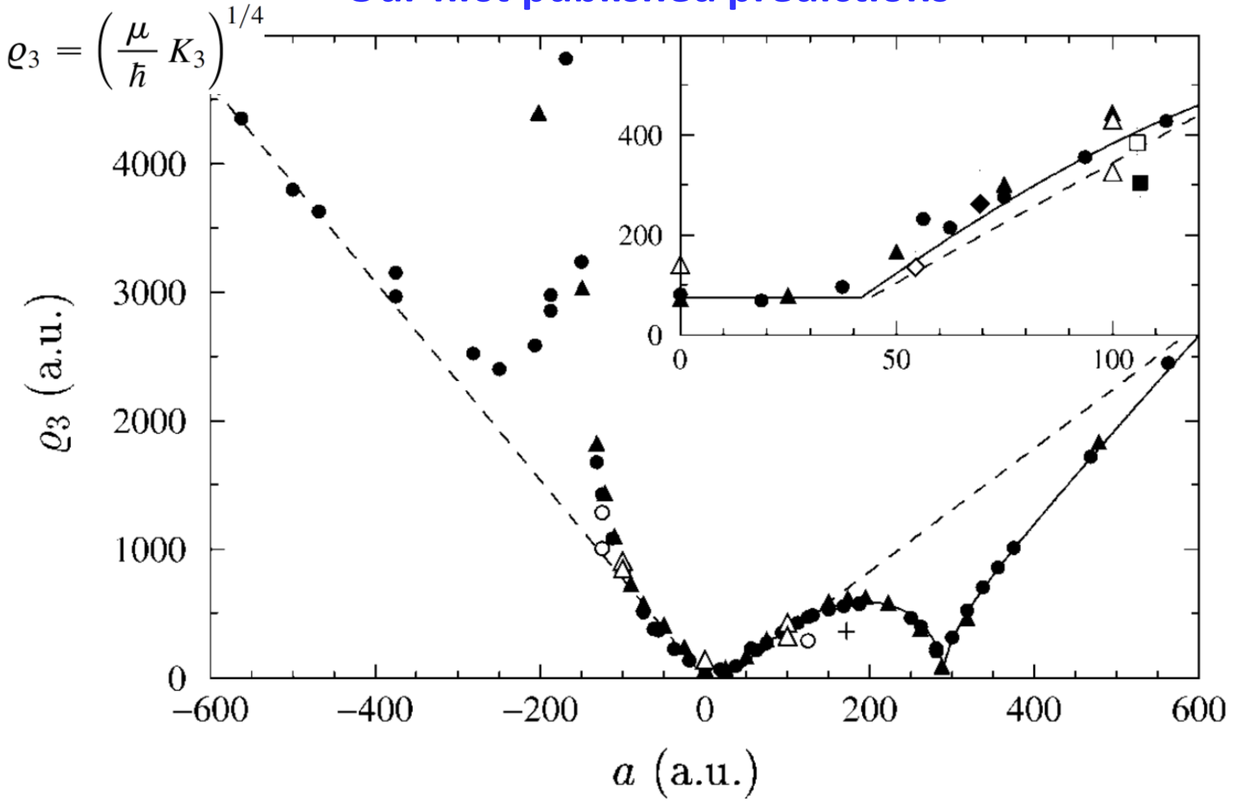


FIG. 2. Numerically calculated recombination lengths ϱ_3 for ultracold ^{4}He and spin-stretched ^{87}Rb are compared with

Three-body recombination rate plotted as a "recombination length", versus the atom-atom scattering length A, showing a prominent Efimov resonance in agreement with 3-body theory at A=-850 Bohr radii

Grimm group (Nature 2006) observation of an Efimov resonance in 3-body recombination

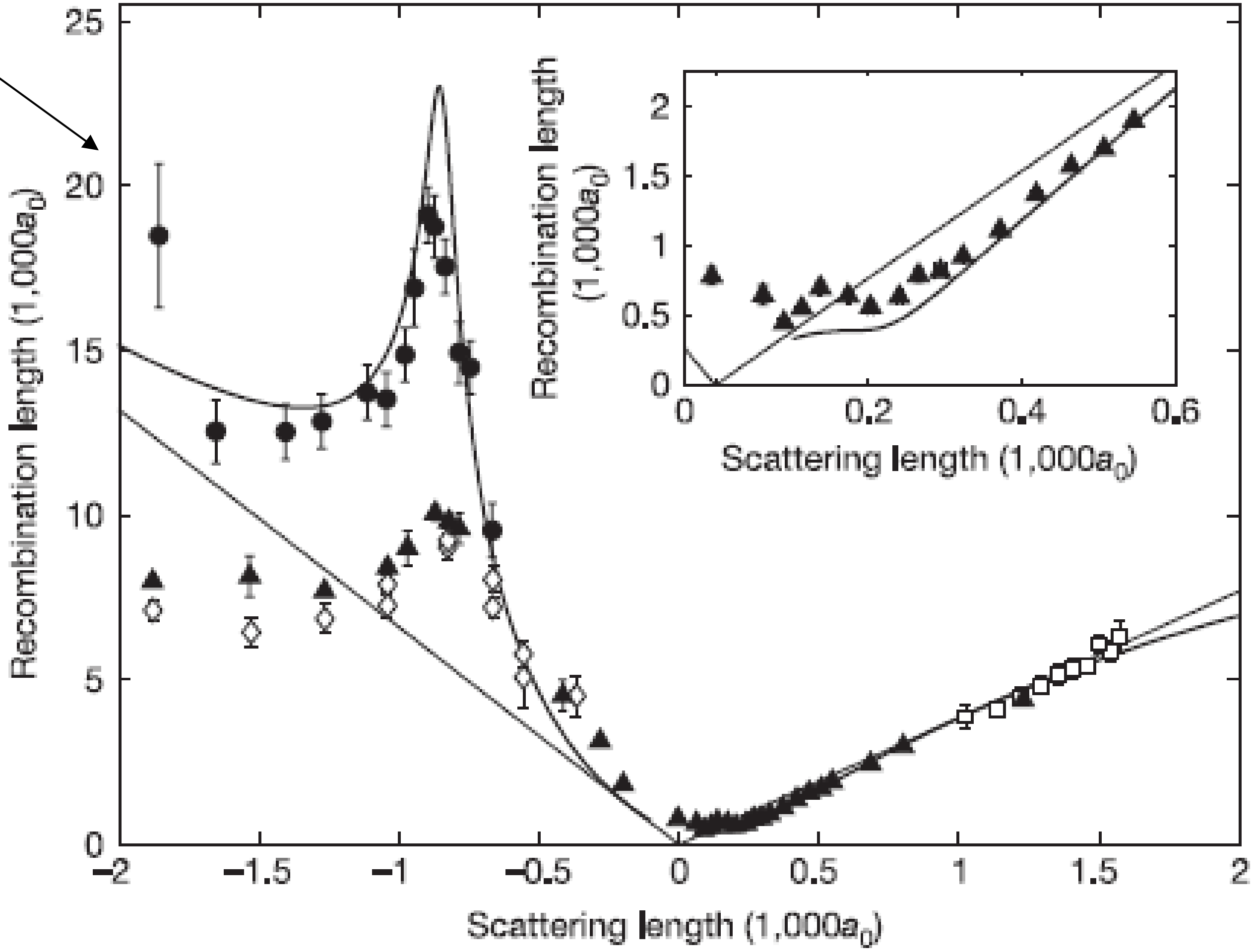
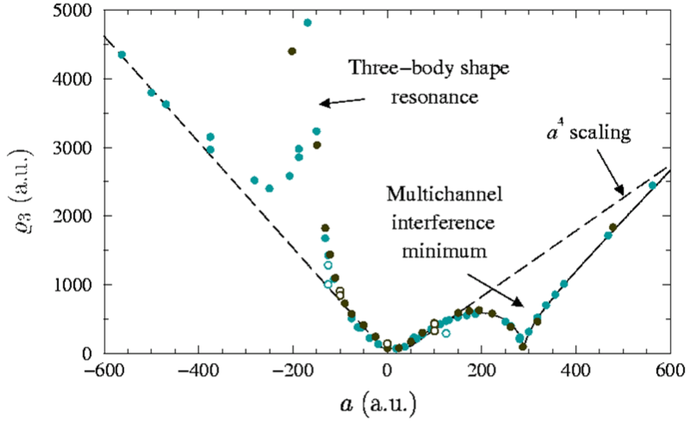


Figure 2 | Observation of the Efimov resonance in measurements of three-body recombination. The recombination length $\rho_3 \propto L_3^{1/4}$ is plotted as a function of the scattering length a. The dots and the filled triangles show

Theory prediction (1999)

General features of three-body recombination

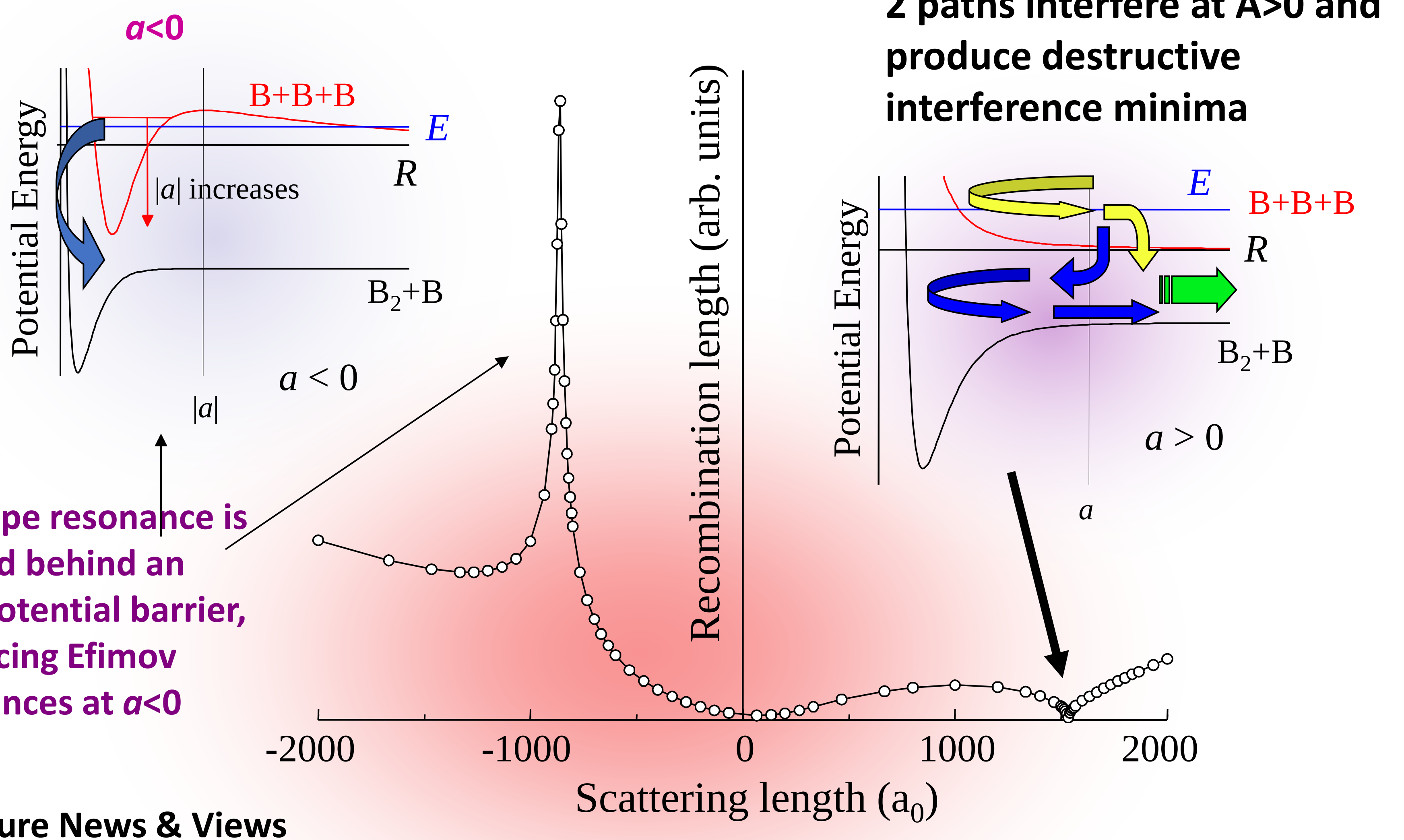


"Recombination length" defined as

$$\varrho_3 = \left(\frac{\mu}{\hbar} K_3\right)^{\frac{1}{4}} \propto a$$

Qualitative pictures of the 2-different quantum mechanisms for 3-body recombination

2 paths interfere at A>0 and produce destructive interference minima



Efimov shape resonance is trapped behind an effective potential barrier, producing Efimov resonances at *a*<0

Esry & CHG, 2006 Nature News & Views

After the 3-particle Efimov effect was proved correct, our attention turned to the 4-body problem next, as we realized that our divide-and-conquer hyperspherical strategy should work there too.

It took a major effort and a development of better ways to solve for the potential energy curves and nonadiabatic couplings, but we succeeded and were able to calculate 4-body recombination resonances, which had physics connected to Efimov states.

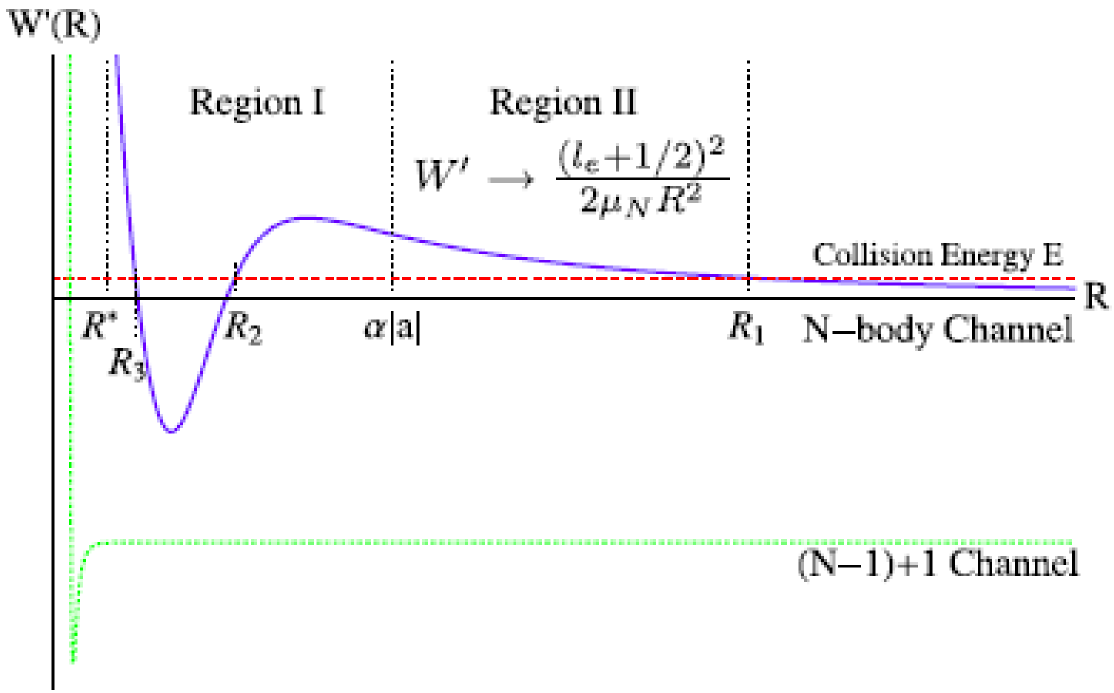
A very talented PhD student in my group, Javier von Stecher, spearheaded the treatment that solved the 4-body problem, and he was co-recipient of the DAMOP thesis award for that work.

Other theorists (especially Deltuva) have since confirmed that work and have carried out even much more accurate calculations of 4-boson recombination.

But there have been almost no calculations of 5-body recombination, until very recently.

How Efimov physics extends to more than 3 particles. This figure shows the schematic entrance channel **potential curve expected for N particles at negative 2-body scattering length**, From Mehta et al., 2009 PRL

Energy



Hyperradius,
R ~ (Sum r_{ij}^2)$^{1/2}$
For equal masses

FIG. 1 (color online). A schematic representation of the N-boson hyperradial potential curves is shown. When a metastable N-boson state crosses the collision energy threshold at $E = 0$, N-body recombination into a lower channel with $N - 1$ atoms bound plus one free atom is resonantly enhanced.

Before we could actually calculate the rate of 4-body recombination in an ultracold gas, we had to develop some scattering theory:

PRL 103, 153201 (2009)

A general theoretical description of N-body recombination

N. P. Mehta,[1,2] Seth T. Rittenhouse,[1] J. P. D'Incao,[1] J. von Stecher,[1] and Chris H. Greene[1]

[1] *Department of Physics and JILA, University of Colorado, Boulder, CO 80309*

[2] *Grinnell College, Department of Physics, Grinnell, IA 50112**

(Dated: March 24, 2009)

We present a formula for the cross section and event rate constant describing recombination of N particles in terms of general S-matrix elements. Our result immediately yields the generalized Wigner threshold scaling for the recombination of N bosons. We find that four-boson recombination is resonantly enhanced by the presence of metastable states in the entrance channel. Hence, recombination into a trimer-atom channel could be an effective mechanism for the formation of Efimov trimers.

And here it is, THE FORMULA for N-boson recombination, i.e. for the process:
A+A+A+….+..A → A_{N-1}+A or A_{N-2}+A+A +…etc.

$$K_N^{0^+} = \frac{2\pi\hbar}{\mu_N} N! \left(\frac{2\pi}{k}\right)^{(3N-5)} \frac{\Gamma\left((3N-3)/2\right)}{2\pi^{(3N-3)/2}} \left|S_{f0}^{0^+}\right|^2$$

Since 2006 – We initiated a concerted effort on the 4-body problem using hyperspherical coordinates

Resulting 2009 papers:

Correlated Gaussian Hyperspherical Method for Few-Body Systems

Javier von Stecher and Chris H. Greene

PRA **80**, 022504, (2009)

Universal Four-Boson States in Ultracold Molecular Gases: Resonant Effects in Dimer-Dimer Collisions

J. P. D'Incao,[1,2] J. von Stecher,[1] and Chris H. Greene[1]

PRL **103**, 033004, 2009

A general theoretical description of N-body recombination

N. P. Mehta,[1,2] Seth T. Rittenhouse,[1] J. P. D'Incao,[1,3] J. von Stecher,[1] and Chris H. Greene[1]

PRL **103**, 153201 (2009).

Signatures of universal four-body phenomena and their relation to the Efimov effect

J. von Stecher, J. P. D'Incao and Chris H. Greene*

Nature Phys. **5**, 417 (2009)

Dimer-dimer collisions at finite energies in two-component Fermi gases

J. P. D'Incao, Seth T. Rittenhouse, N. P. Mehta,* and Chris H. Greene

Phys. Rev. A **79**, 030501(R), (2009)

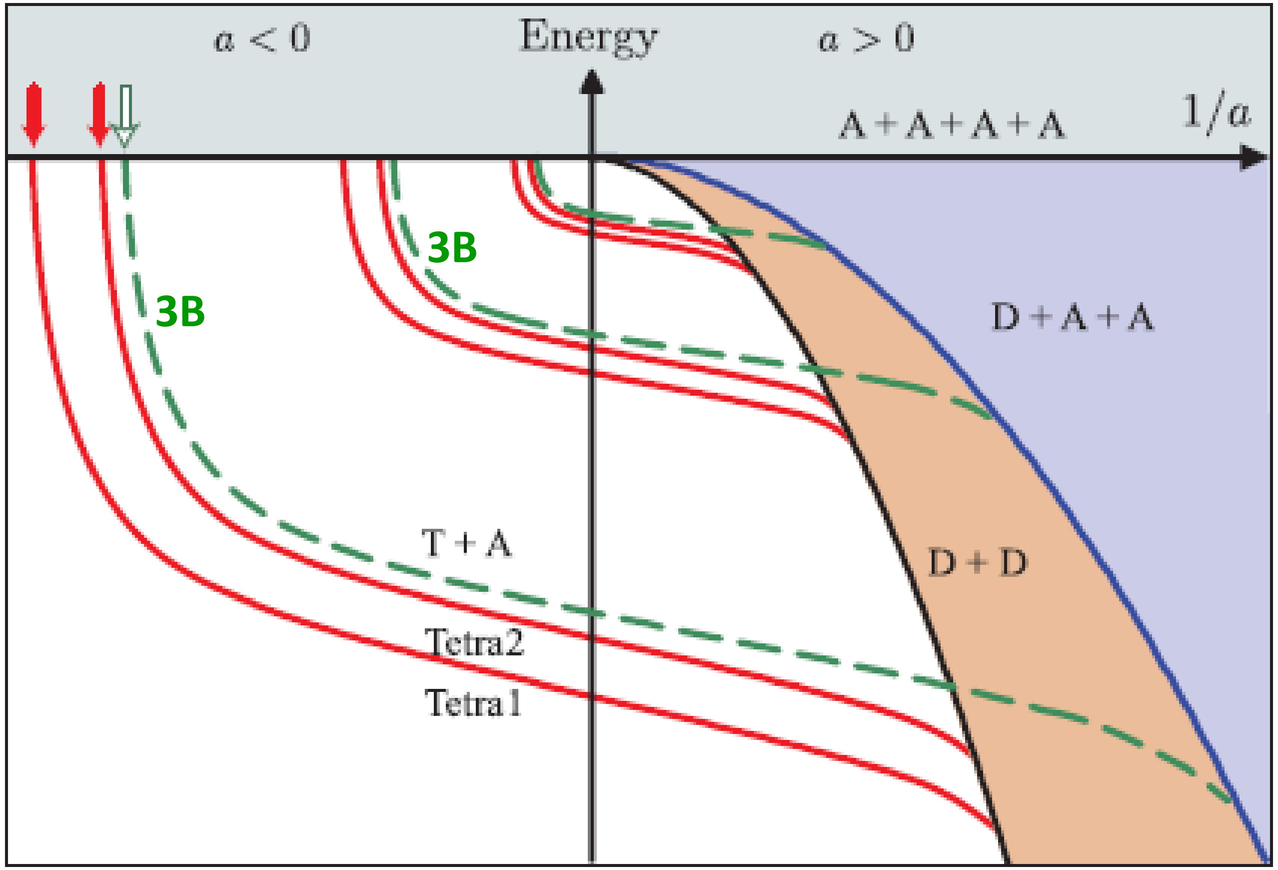
Technical Point: How to solve this high-dimensional PDE at many different values of the hyperradius, in order to determine the potential energy curves U(R) and their couplings: the correlated Gaussian hyperspherical method

$$\Phi_\nu(R,\Omega) = \hat{S} \sum_j c_j e^{-\frac{1}{2}\boldsymbol{x}^T \underline{A}_j \boldsymbol{x}} [\Theta_L^K(\boldsymbol{u},\boldsymbol{x}) \times \chi_S]_{JM_J}$$

This basis set has been the decisive tool, developed in Javier von Stecher's thesis work. Note that the Gaussian parameters in the basis set are determined stochastically.

The main difference from other Gaussian expansion calculations, e.g. by Suzuki and Varga, is that we have to perform all integrals at fixed hyperradius R, in the matrix elements of the fixed-R Hamiltonian. But we have found a way to do those integrals for an arbitrary number of particles and dimensions analytically, except for one final 1D integral in the complex plane that still must be done numerically.

Key finding: Two four-body states are found to lie between each successive pair of Efimov trimers - von Stecher et al. Nature Phys. 2009 – which confirms insightful work by Platter and Hammer (Eur. Phys. J. A. 2007), and extends it



Extended Efimov plot showing universal dimer, trimer, and tetramer states of four identical bosons with short-range interactions.

Figure from Ferlaino & Grimm. See also Ferlaino et al., PRL 102, 140401 (2009) for experimental confirmation, and later theoretical extensions by Deltuva.

Tetramer states predicted to hit zero energy at

a=0.43 a(Efimov)

and

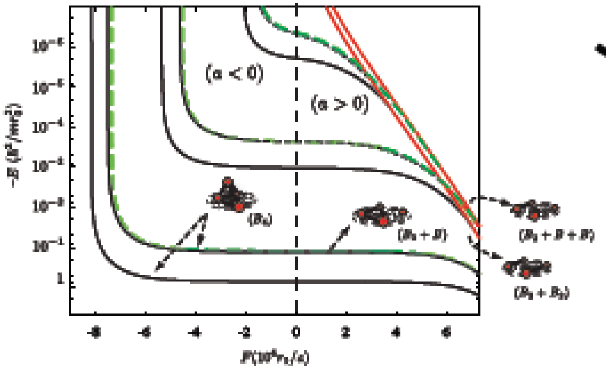
a=0.90 a(Efimov)

Four-boson Spectrum

Consistent with work of Platter and Hammer, we agree that there are:

Two four-body states per Efimov trimer !!!

2, 3, and 4-body energy levels



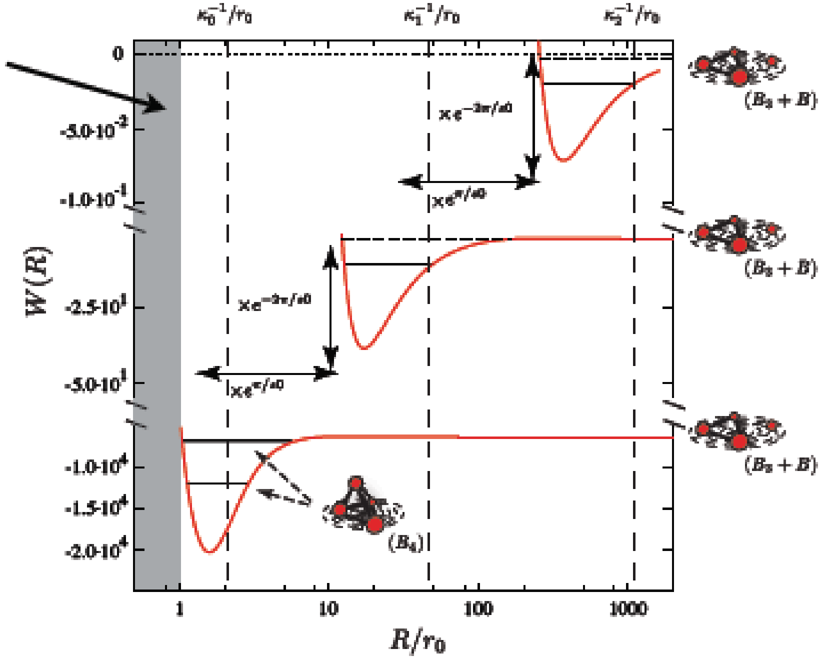
$(a = \infty)$

$$E_{4b}^{(n,m)} = c_m E_{3b}^{(n)} \qquad m = 1, 2$$
$$n = 1, 2, \ldots, \infty$$

$$(c_1 \approx 4.58,\ c_2 \approx 1.01)$$

(no four-body parameter)

details !



Hyperspherical 4-body potential curves converging asymptotically to Efimov trimer levels at a→ ∞

Four-body physics is truly Universal !!!
(geometric scaling: Efimov physics)

And, after our four-body recombination resonance predictions were verified experimentally, we felt that one should try eventually to tackle 5-atom recombination

The amount of attention to the 5-body problem in collision physics has not been very extensive. Here is one early reference.

Example of 5-body scattering theory (formalism only):

Tatuya SASAKAWA **Five-Body Equation**

Supplement of the Progress of Theoretical Physics, No. 61, 1977

→ Formulated the Fadeev-Yakubovsky coupled equations that would need to be solved.

Citations to date: 3 (all by Lazauskas and/or Carbonell)

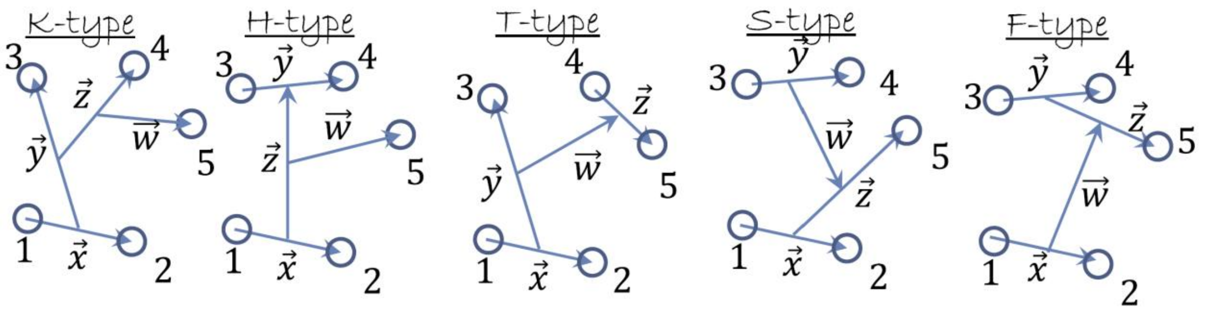


Fig. 91.1 Jacobi coordinate sets proper to 5-body Faddeev-Yakubovsky components. From left to right components $\mathcal{K}^4_{12,3}$, $\mathcal{H}^{34}_{12}$, $\mathcal{T}_{12,3}$, $\mathcal{S}^{34}_{12}$ and $\mathcal{F}^{34}_{12}$ are represented

$$
\begin{cases}
\mathcal{K}^l_{ij,k} = G_{ij}V_{ij}\left(\mathcal{K}^l_{ik,j} + K^l_{jk,i} + \psi^{ikl}_{ik} + \psi^{jkl}_{jk} + \psi^{ik,jl}_{ik} + \psi^{jk,il}_{jk}\right), \\
\mathcal{H}^{kl}_{ij} = G_{ij}V_{ij}\left(\mathcal{H}^{ij}_{kl} + \psi^{jkl}_{kl} + \psi^{ikl}_{kl}\right), \\
\mathcal{T}_{ij,k} = G_{ij}V_{ij}\left(\mathcal{T}_{ik,j} + T_{jk,i} + \psi^{lm}_{ik} + \psi^{lm}_{jk}\right), \\
\mathcal{S}^{lm}_{ij} = G_{ij}V_{ij}\left(\mathcal{F}^{ij}_{lm} + \psi^{jk}_{lm} + \psi^{ik}_{lm}\right), \\
\mathcal{F}^{lm}_{ij} = G_{ij}V_{ij}\left(\mathcal{S}^{ij}_{lm} + \psi^{klm}_{lm}\right).
\end{cases}
\quad (91.3)
$$

This has only been applied to solve the simplest 5-body scattering problem, namely neutron+ ^{4}He elastic scattering

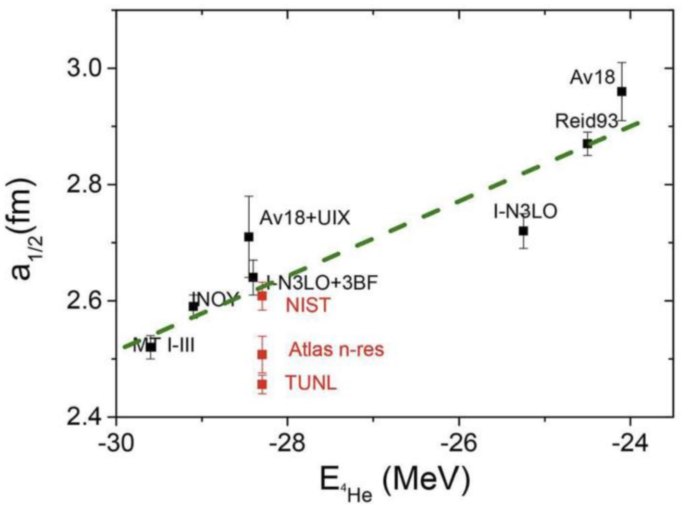


FIGURE 10 | Apparent correlation between the calculated ^{4}He binding energies and n-^{4}He scattering length. Calculated values are compared with those adapted from the experiment by NIST [100], Atlas n-res [99], and TUNL [25].

Interesting result, a correlation between the ^{4}He binding energy and the n-^{4}He elastic scattering length

Example calculation: Probably THE most impressive and IMPORTANT 5-body reaction computed to date is the **d+t → α +n nuclear fusion reaction**, accomplished using the no-core shell model approach, plus huge computational resources

(i.e. from the Lawrence Livermore National Laboratory (LLNL) institutional **Computing Grand Challenge** program)

As far as I am aware, this is probably the ONLY realistic computation of this 5-nucleon reaction carried out to date, and the reference below is the latest study, showing that by polarizing the colliding fragments, a large increase in efficiency results

nature COMMUNICATIONS

2019

https://doi.org/10.1038/s41467-018-08052-6

OPEN

Ab initio predictions for polarized deuterium-tritium thermonuclear fusion

Guillaume Hupin[1,2,3], Sofia Quaglioni[3] & Petr Navrátil[4]

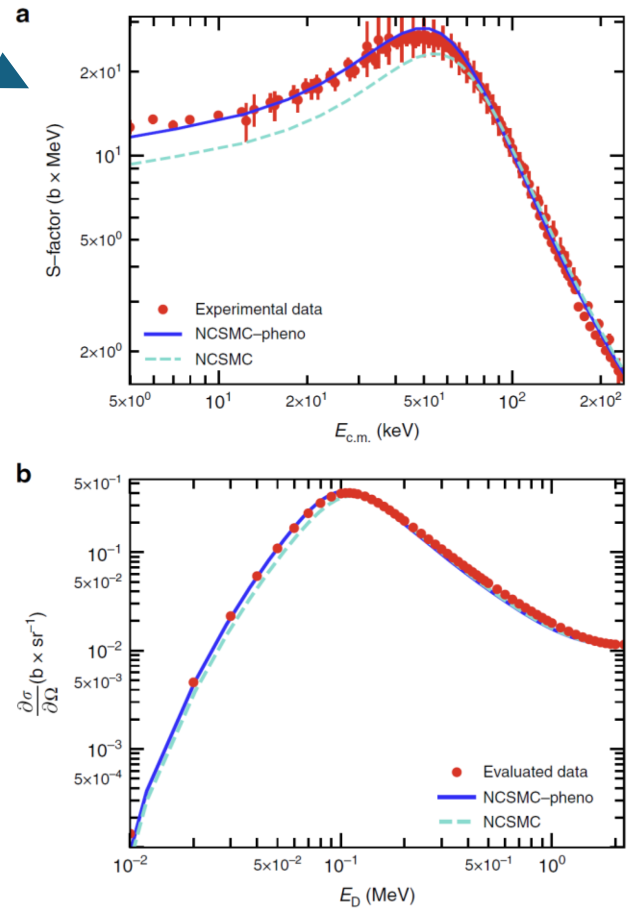


Fig. 1 Unpolarized DT cross sections. **a** Astrophysical S-factor as a function of the energy in the center-of-mass (c.m.) frame, $E_{c.m.}$, compared to

Why is there ONLY ONE full 5-nucleon calculation to date that includes 3-nucleon interactions? It is a high dimensional Hilbert space AND it is a problem in the scattering continuum.

Our very recent article on a 5-atom reactive process

PNAS 2025 Vol. 122 No. 18 e2503390122

Five-body recombination of identical bosons

Michael D. Higgins[a,1] and Chris H. Greene[a,b,1,2]

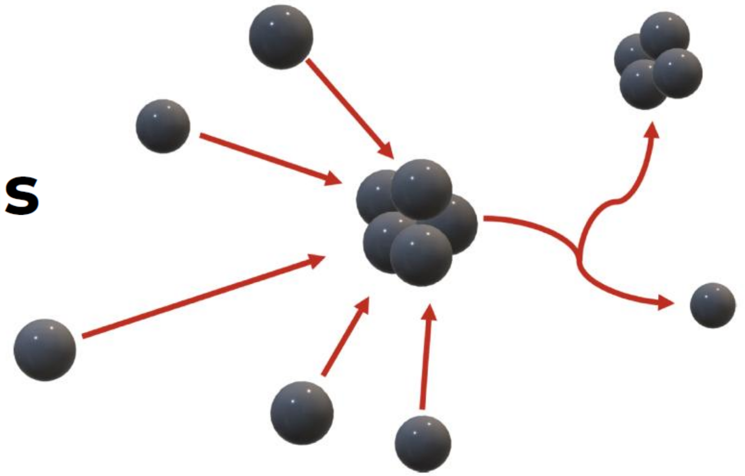
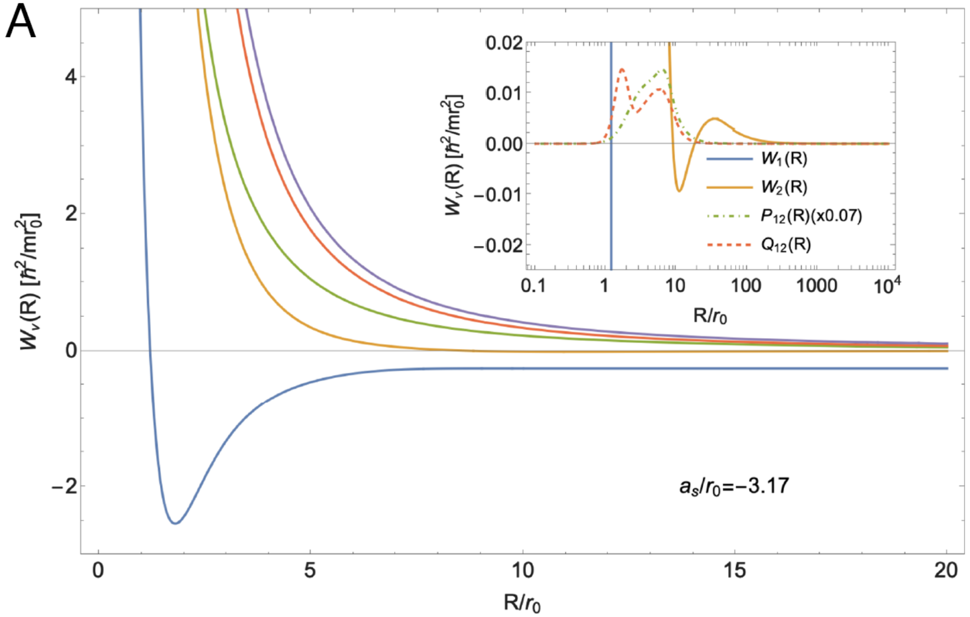


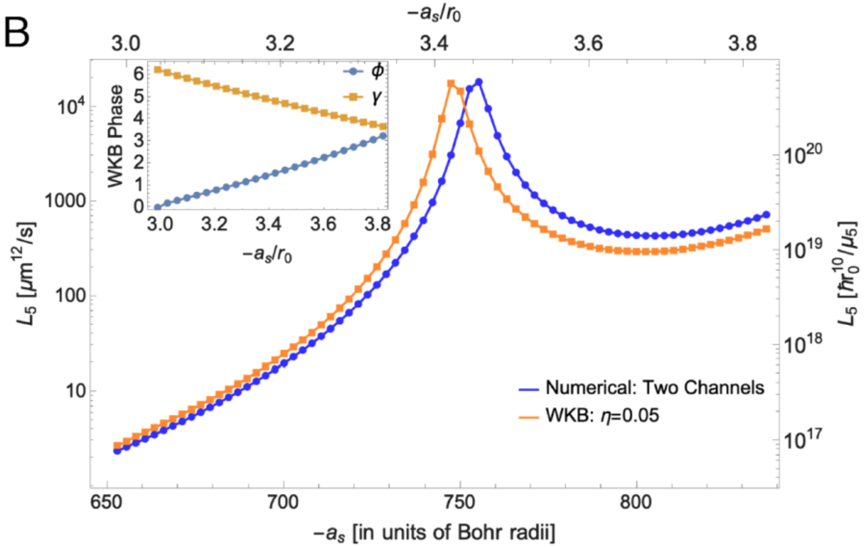
Fig. 1. Conceptual picture of a 5-body recombination event, where 5 free atoms collide and exchange energy, resulting in 4 of those atoms bound together as a tetramer and the fifth atom flying away with the majority of the kinetic energy released.

A



Adiabatic hyperspherical potential curves for 5 cesium atoms, showing the entrance 5-body continuum channels at W>0, and a bound 4+1 recombination channel that is negative beyond about R = 2 r_0.

B



Five-body recombination rate coefficient versus two-body scattering length, showing a resonance in Cs_5*

Comparison of 3, 4, and 5-body recombination loss rates, expressed as effective 3-body rates, for a realistic experimental cold Cs atom density.

Somewhat surprisingly, there are clear regions of the two-body scattering length ***a*** where the dominant loss process is actually 5-body recombination

Note: there are three 5-body recombination resonances per Efimov cycle

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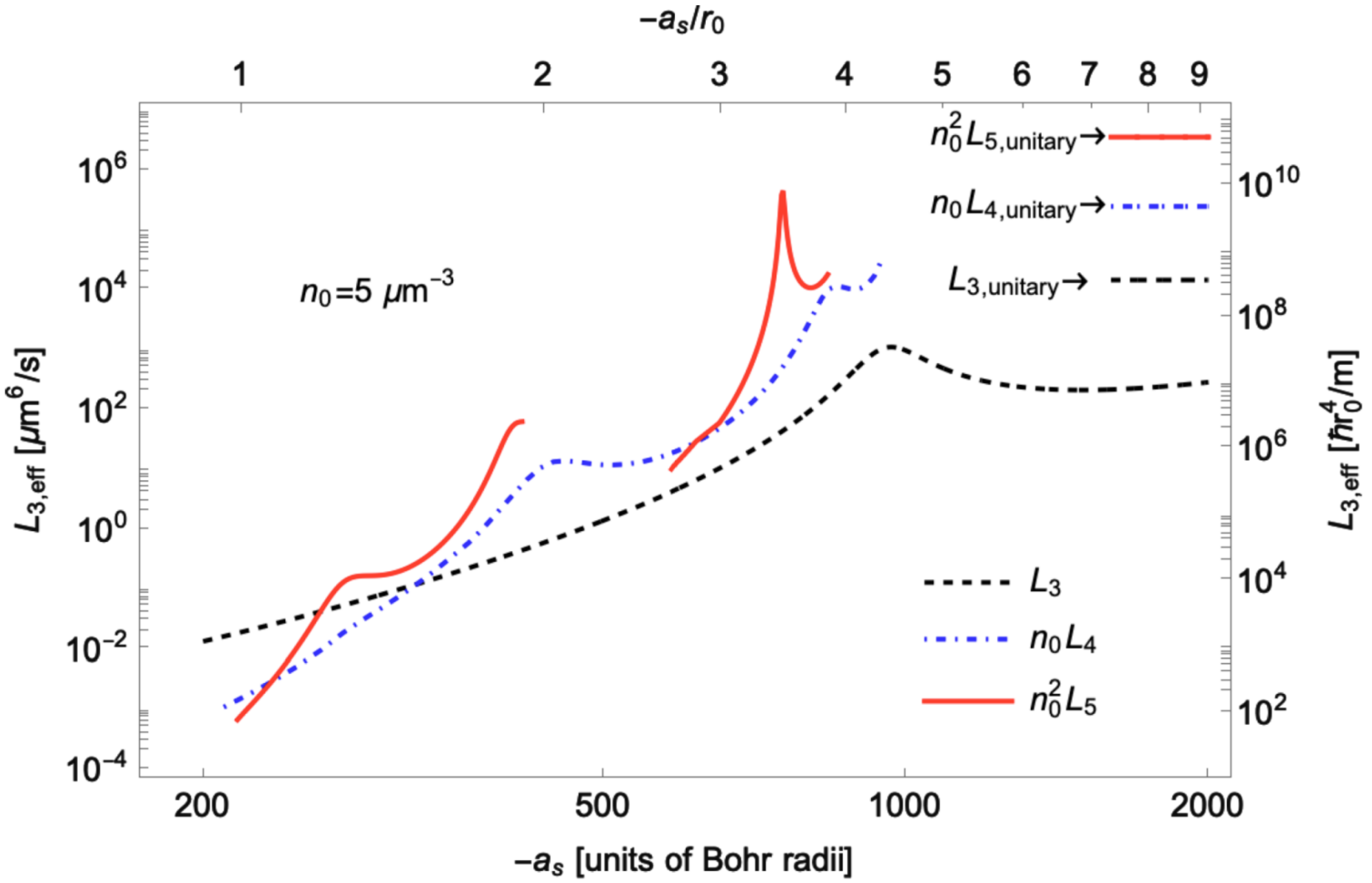

Fig. 7. Effective 3-body loss rates, displayed as L_3, $n_0 L_4$, and $n_0^2 L_5$ for a

Our article with the Innsbruck group in 2013. The first and only experimental observation of 5-body recombination (as far as I'm aware).

Resonant Five-Body Recombination in an Ultracold Gas

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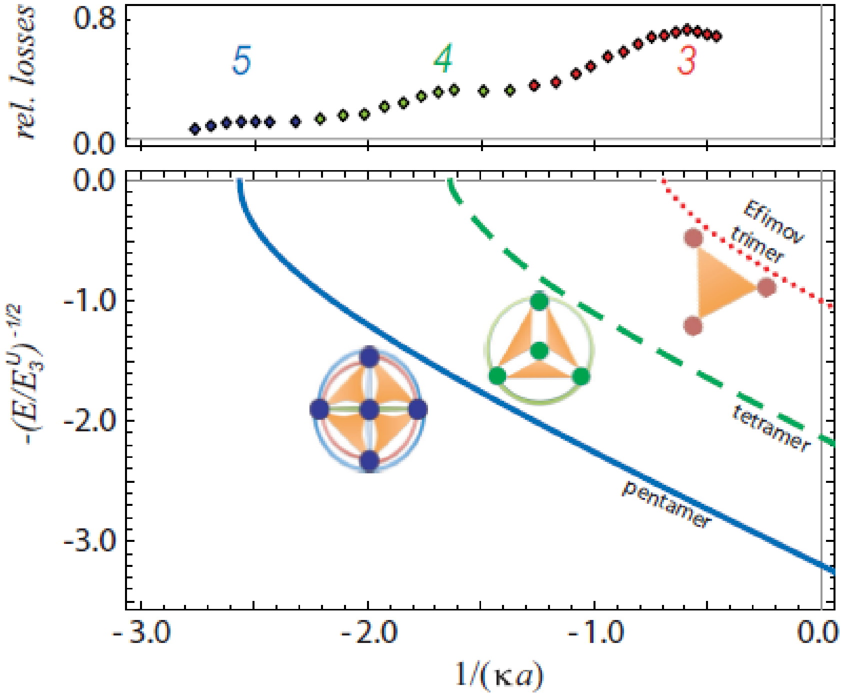

FIG. 1. (color online) N-body scenario in the region of negative two-body scattering length a. The lower panel shows the N-body binding energies as a function of the inverse scattering length. $E_3^U = (\hbar\kappa)^2/m$ is the trimer binding energy for resonant interaction. The dotted,

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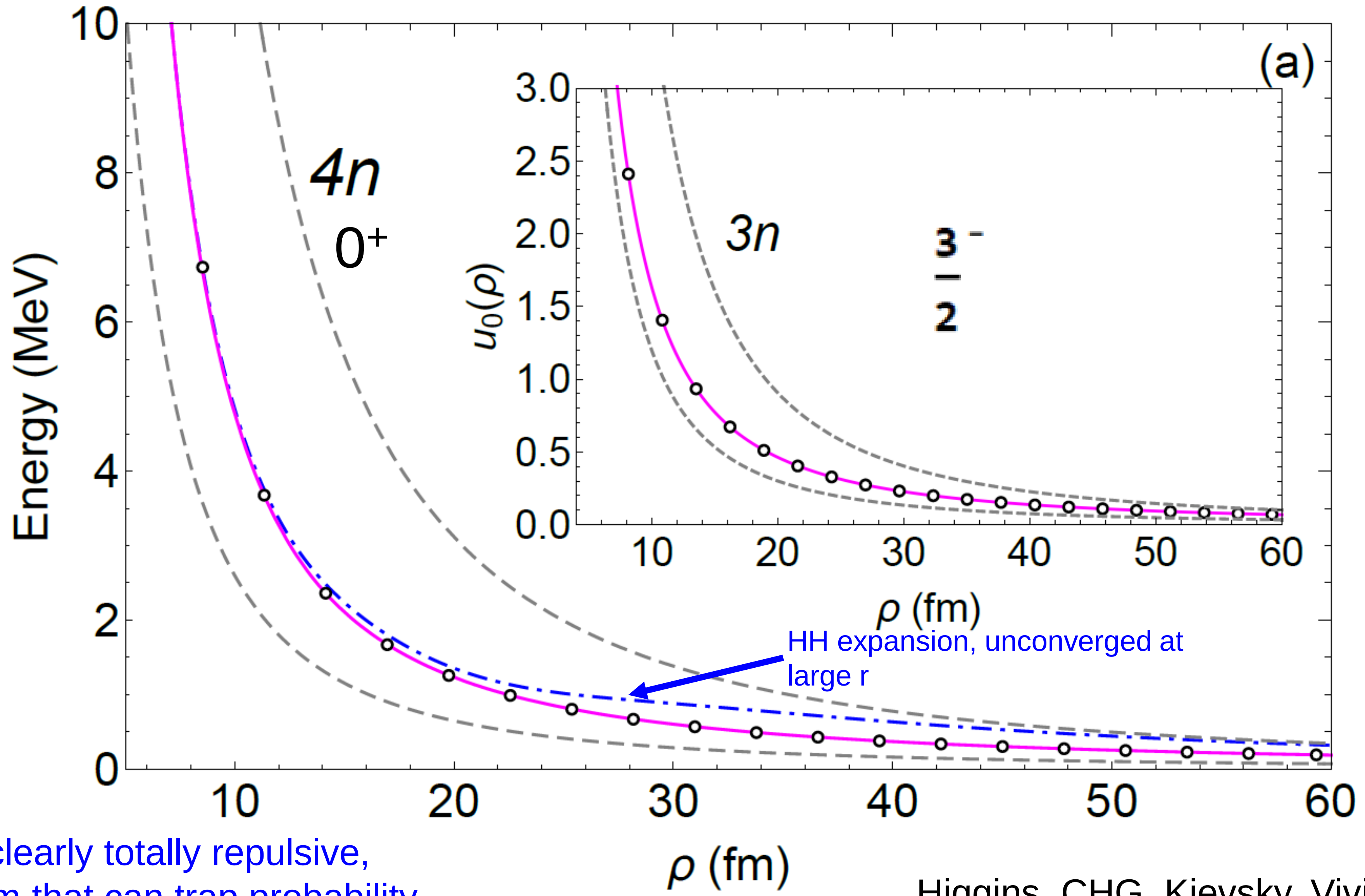
The most attractive hyperspherical potential curves for the 4n and 3n systems, obtained using realistic n-n interaction potentials

Nuclear physics: Does the tetraneutron exist? Controversy!

Totally repulsive potential energy curves we computed for the 3-neutron and 4-neutron systems, that we argue are a DISPROOF of any possible existence of a tri-neutron or a tetra-neutron bound state or resonance.



The converged potentials are clearly totally repulsive, with <u>no sign of a local maximum</u> that can trap probability in a resonance.

Higgins, CHG, Kievsky, Viviani
PHYSICAL REVIEW LETTERS 125, 052501 (2020)

PHYSICAL REVIEW C 111, 014001 (2025), Michael Higgins & CHG

Resonances and collisional properties of neutron-rich helium isotopes in the adiabatic hyperspherical representation

Our first 5-body calculation in nuclear physics: the ^{8}He nucleus

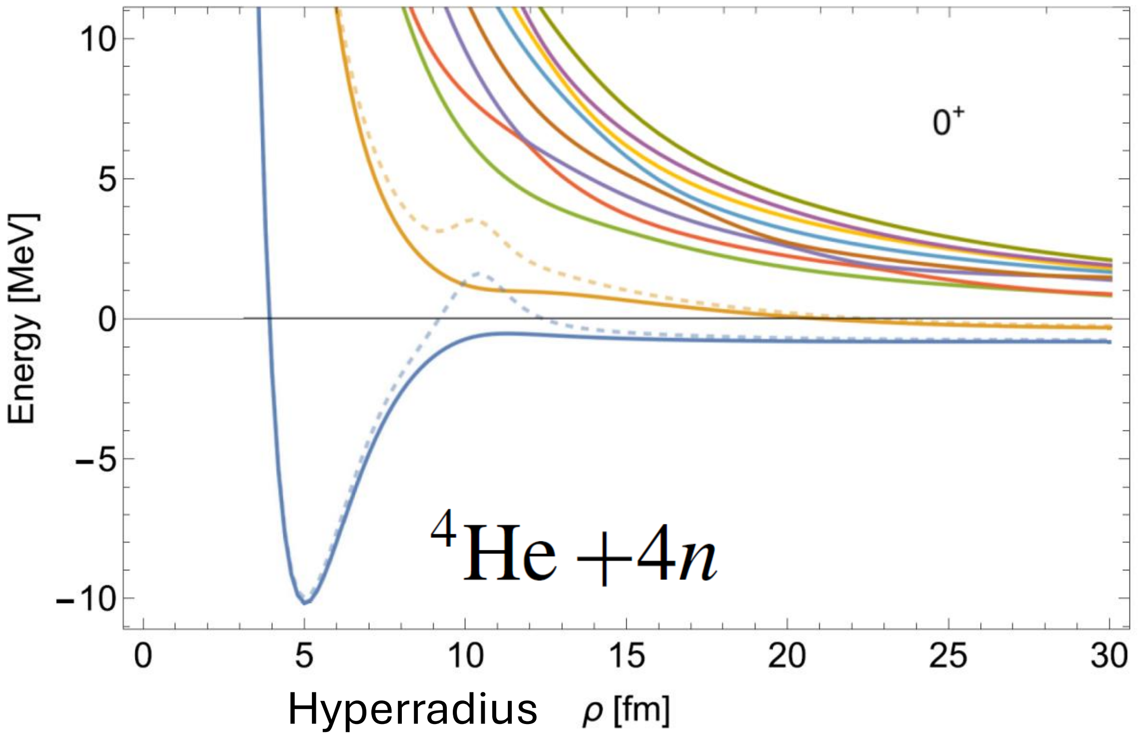


FIG. 9. The lowest few Born-Oppenheimer potential curves for the ^{4}He $+4n$ system in the $(L^{\pi}, S)J^{\pi} = (0, 0)0^{+}$ symmetry. The solid curves are the Born-Oppenheimer potentials and the dashed curves are the adiabatic potentials, which includes the diagonal second-derivative coupling term. Only the lowest two adiabatic potentials are shown here.

Adiabatic hyperspherical potential energy curves, energy versus hyperradius, for the 0+ symmetry of the ^{8}He nucleus.

A goal for the future is to compute such potential energy graphs for the lowest few symmetries, for all nuclei up to A=8, as a way to visualize and compute reaction pathways in energy and position.

Future 6-body challenges for theory:

- Collision between two Efimov trimers at ultracold temperature (the best to date is a very approximate treatment by Naidon, Endo, and Garcia-Garcia 2016)

- Triton-Triton fusion reaction, t+t → α + n + n

- Note that there is a fully analytical solution of the 3-body hyperangular problem with zero-range (S-wave) interactions, but a similar analytical solution has never been found for the 4-body problem. This would be a desirable goal for theory to solve, if possible.

Conclusion:

Progress is underway to extend the predictive and analysis power of the adiabatic hyperspherical representation to handle more particles and more complex few-particle scenarios in both nuclear and atomic systems.

But these are hard problems and each bit of progress is hard-earned....

Thanks for listening!