Moments of nucleon parton distribution function on the lattice

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Outline

Introduction

Lattice QCD

Excitated states contamination

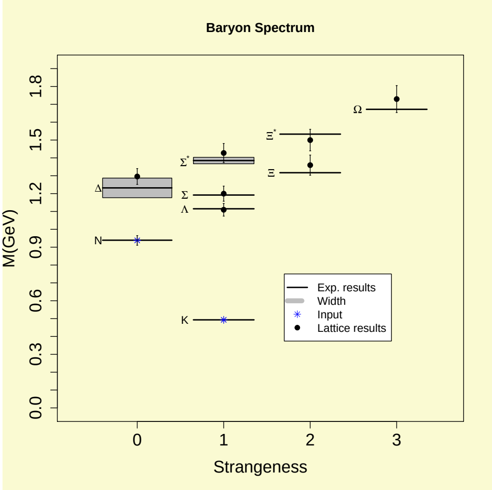
$< x >_{u+d-2s,\mu^2}$

Conclusion

Introduction

- Lattice QCD allows to compute *ab initio* :
  - ⋆ Moments of parton distribution function
  - ⋆ Form Factors (Electroweak, Generalized)
- Needs control over statistical errors :
  - ⋆ Finite statistics : $\sigma \sim 1/\sqrt{N}$
- Needs control over systematic errors :
  - ⋆ Finite Size effects : $V \longrightarrow \infty$
  - ⋆ Finite lattice spacing effects : $a \longrightarrow 0$
  - ⋆ "Chiral" limit : $m_q \longrightarrow m_q^{\text{phys}} \sim 0$
- Dynamical simulation with strange quark are now common ($N_f = 2 + 1$)
- First simulation with a doublet of non strange and charm quarks ($N_f = 2 + 1 + 1$)
  (Our setup)
- This talk : results from ETM collaboration

Baryon Spectroscopy



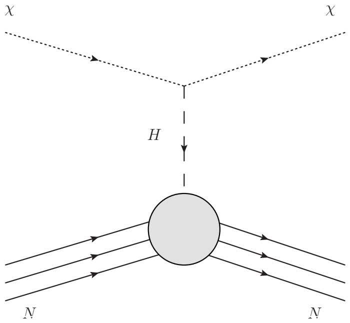
low lying baryon spectrum ($N_f = 2$)

Strange content of the Nucleon : Motivations

Motivations

- Experimental direct detection of dark matter put bounds on the WIMP-Nucleon cross section
- Results are interpreted using various models (including SUSY) : systematic uncertainty due to $\langle N(p)|\bar{q}q|N(p)\rangle$

  $\longrightarrow$ non-perturbative computation is required.



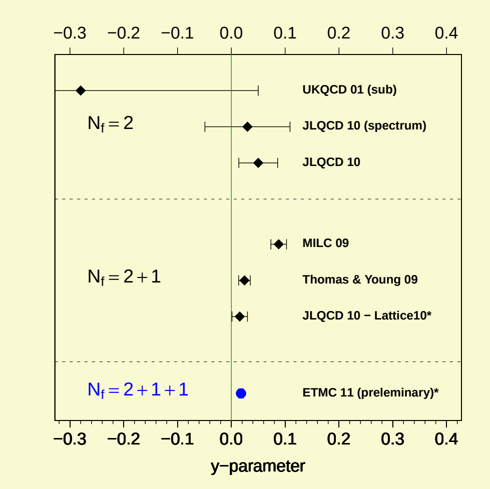
- sigma terms :

  $\sigma_{\pi N} \equiv m_q \langle N(p)|\bar{u}u + \bar{d}d|N(p)\rangle$

- dimensionless ratio :

  $y_N \equiv \frac{2\langle N(p)|\bar{s}s|N(p)\rangle}{\langle N(p)|\bar{u}u+\bar{d}d|N(p)\rangle}$

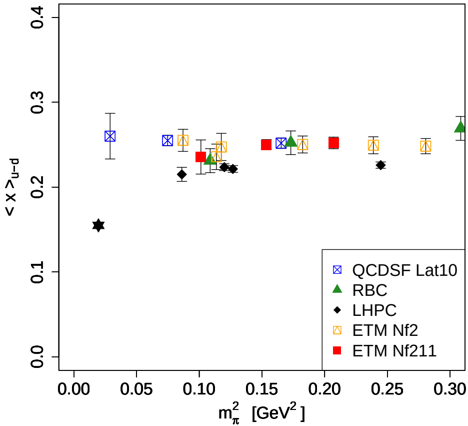
Preleminary results



- No systematic errors
- 5σ away from 0
- We use properties of the twisted mass formulation

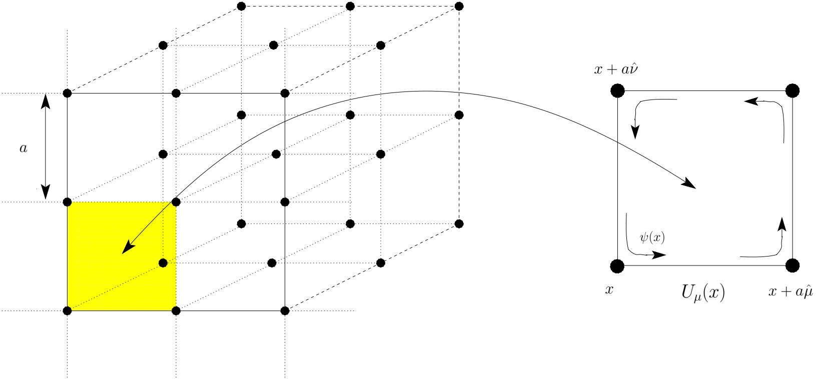
A long-standing puzzle

Up-to-date results for $\langle x\rangle_{u-d}$ ($\overline{\text{MS}}$-scheme $\mu = 2$ GeV)



- Discrepancy of 40%
- The same discrepancy is obtained for many other nucleon matrix elements (*e.g* : g_A, the axial coupling of the nucleon)

Discretization



- Discretize the QCD action on hypercubic lattice of lattice spacing a, and Volume $V = L^3 \times T$
- The fermionic part can be written : $S_{\text{fermion}} = \sum_x \overline{\psi}(x) D \psi(x)$
- Many choice possible for the Dirac operator D

Computation of correlation functions

- QCD in Euclidean space :

$$\langle O[\overline{\psi},\psi,U]\rangle = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}U\, e^{-S[\overline{\psi},\psi,U]}O[\overline{\psi},\psi,U]$$

exact integration of the fermionic fields

$$\rightsquigarrow\langle O[\overline{\psi},\psi,U]\rangle = \int \mathcal{D}U\, P[U]O[D^{-1},U]$$

- Use a supercomputer to generate $\{U_1,\ldots,U_N\}$
- Estimator

$$\langle O[\overline{\psi},\psi,U]\rangle = \frac{1}{N}\sum_i O[D^{-1}[U_i],U_i] + \mathcal{O}(1/\sqrt{N})$$

Twisted mass fermions (I)

Twisted mass action : $N_f = 2$ **(Frezzotti, Grassi, Sint, Weisz, 1999)**

- Let χ be a doublet of spinor field
- Starting from Wilson action and add a mass term "twisted " in flavour space *i.e* :

$$S^{tm}_{(m_0,\mu)} = a^4 \sum_x \bar{\chi}(x)\left[\gamma_\mu \tilde{\nabla}_\mu + m_0 - r\frac{a}{2}\nabla^*_\mu \nabla_\mu + i\mu\gamma_5\tau_3\right]\chi(x)$$

- m_0: bare Wilson mass, μ: bare twisted mass, τ_3: Pauli matrix, r: Wilson parameter
- Wilson fermions : $\mu = 0$
- In the continuum, twisted mass action reproduce the Dirac action with two degenerate flavours (isospin limit : $m_u = m_d$)

Twisted mass fermions (II)

Properties **(Frezzotti, Rossi, 2003)**

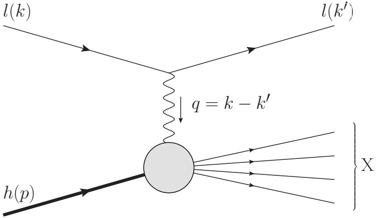
- parity-even correlators are $\mathcal{O}(a)$ improvement at maximally twisted
- tuning of only one parameter:
  the bare untwisted quark mass: $m_0 \to M_{\mathrm{cr}}$
- mixing pattern in the renormalization process can be simplified
- A non degenerate doublet to simulate (s,c) quarks can be added.
- explicit breaking of parity and isospin at finite lattice spacing

- Our setup :
  - ⋆ $N_f = 2 + 1 + 1$ simulation : light (u,d), strange (s), and charm (c) dynamical quarks
  - ⋆ Lighest pion mass $\sim$ 230 MeV ($m_\pi^{\mathrm{phys}} = 135$ MeV)
  - ⋆ 3 lattice spacings
  - ⋆ Several volume

Parton Distribution Functions (PDF)

- Deep inelastic scattering (DIS) : $l(k)N(p) \longrightarrow l(k')X$

$$Q^2 = -q^2$$
$$x = Q^2/2p \cdot q$$



- factorization :
  - ⋆ $\frac{d\sigma^{(lN)}}{d^3k'}(p,q) \sim \int_0^1 d\xi \sum_q \frac{d\sigma^{(lq)}}{d^3k'}(\xi p, q)\, q(\xi)$
  - ⋆ lepton-parton cross section is perturbative for large Q^2
  - ⋆ $q(\xi)$ encodes non perturbative dynamics

Parton Distribution Functions (PDF)

- Definition (unpolarized PDFs)

$$q(x,\mu) = \int \frac{d\lambda}{2\pi} e^{ixp\cdot\lambda n} \langle p,s|\bar{q}\left(-\frac{\lambda}{2}n\right) \not{n} W_n(-\frac{\lambda}{2}n, \frac{\lambda}{2}n) q\left(\frac{\lambda}{2}n\right)|p,s\rangle\Big|_{\mu^2}$$

with

$$W_n(-\frac{\lambda}{2}n, \frac{\lambda}{2}n) = \mathcal{P}\exp\left(\mathrm{ig}\int_{-\lambda/2}^{\lambda/2} \mathrm{d}\alpha\ \mathrm{A}(\alpha \mathrm{n})\cdot \mathrm{n}\right).$$

- PDFs involve quark and gluon fields seperated along the light-cone

$\rightsquigarrow$ difficult to construct explicitly in Euclidean space

Moments of PDF

- Definition :

$$\langle x^n \rangle_{q,\mu^2} = \int_{-1}^{1} dx\, x^n q(x,\mu^2) = \int_0^1 dx\, x^n \left\{ q(x,\mu^2) - (-1)^n \bar{q}(x,\mu^2) \right\}$$

- Forward matrix elements of twist-two operators :

$$\langle p,s|\bar{q}(0)\gamma^{\{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n\}} q(0)|p,s\rangle\Big|_{\mu^2} = 2\langle x^n \rangle_{q,\mu^2} p^{\{\mu_1} \cdots p^{\mu_n\}}$$

$T^{\{\mu_1 \cdots \mu_n\}}$: symmetrization and substraction of the traces

D^{μ} : covariant derivative

- Moments are related to local operators that can be calculated in Eucliean space.
- Benchmark quantity :

$$\langle p,s|\bar{\psi}\gamma^{\{\mu} iD^{\nu\}} \tau^3 \psi|p,s\rangle\Big|_{\mu^2} = 2\langle x \rangle_{u-d,\mu^2} p^{\{\mu} p^{\nu\}}, \quad \text{with} \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

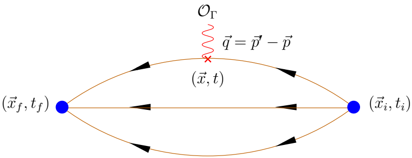
Non Perturbative renormalization

- General remarks :
  - ⋆ Non perturbative renormalization can be performed for multiplicatively renormalizable operators.
  - ⋆ $\langle x \rangle_{u-d,\mu^2}$ is multiplicatively renormalizable ($n = 1$ and isovector part)
- Non perturbative renormalization scheme (in the chiral limit) :

$$\mathcal{Z}^{RI-MOM}(\mu^2)\langle q, p^2 = \mu^2|\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\tau^3\psi|q, p^2 = \mu^2\rangle \equiv \langle q|\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\tau^3\psi|q\rangle\Big|_{\mathrm{tree},\mu^2}$$

- Conversion to $\overline{\mathrm{MS}}$ using analytical continuum perturbation theory (3-loops result for $\langle x \rangle_{u-d,\mu^2}$)

The ratio method



Nucleon matrix elements are extracted from a suitable ratio of correlation function which involve 2 time scales t and t_s :

$$\langle N(t_s)\mathcal{O}(t)\bar{N}(0)\rangle\rangle \propto \langle N|\mathcal{O}|N\rangle e^{-m_N t_s} + \dots$$

$$\langle N(t_s)\bar{N}(0)\rangle \propto e^{-m_N t_s} + \dots$$

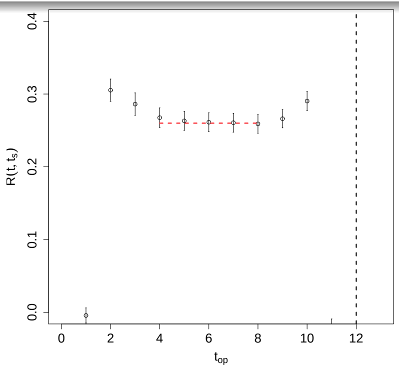
$$R(t, t_s) = \frac{\langle N(t_s)\mathcal{O}(t)\bar{N}(0)\rangle}{\langle N(t_s)\bar{N}(0)\rangle} = \langle N|\mathcal{O}|N\rangle + \text{terms that vanish in the limit } t, t_s \longrightarrow \infty$$

Basic concerns

In practice

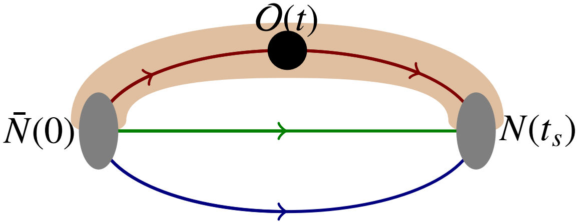
- finite t and t_s

  $\longrightarrow$ exponentially suppressed contribution of the excited states
- The "standard"' method requires to fix t_s to evaluate $R(t, t_s)$
- t_s cannot be chosen too large because statistical errors grow exponentially

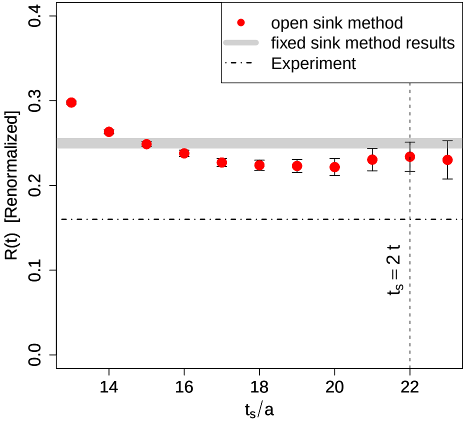


Dedicated investigation : the method

- Fix t instead of t_s
- Large statistic study on one $N_f = 2+1+1$ ensemble with $m_\pi \approx 380$ MeV ($L/a = 32$,$a = 0.078$ fm)
- Need to fix the operator : two benchmark quantities g_A and $\langle x \rangle_{u-d}$

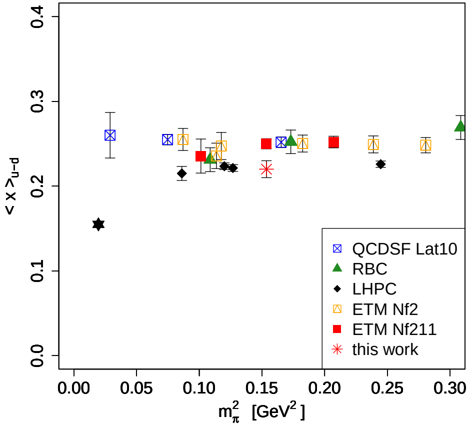


Dedicated investigation : results



- t/a fixed to 11
- statistics : $\sim$ 23000 (compare to $\sim$ 500 for the standard method)
- In general $t_s \sim 1$ fm, here $t_s \sim 2$ fm

Comparison with other results



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Comments & Perspectives

- Larger effects than finite volume and lattice spacing effects for a pion mass ~ 380 MeV
- if this persist at smaller pion mass excited states cannot be the single dominating effect responsible of the tension between lattice and phenomenology
- Demonstrate that a very careful and accurate analysis of systematic errors will be needed
- Conclusions hold for any discretization

⋆ Question : can we find a less expensive method to study excited states contamination at lower pion mass ?

⋆ Several study in this direction are under progress (in particular by S. Dinter) : Generalized Eigenvalue Problem (GEVP) ? , stochastic estimation ?

Motivations

Consider

$$\langle p,s|\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\lambda^8\psi|p,s\rangle\Big|_{\mu^2} = 2\langle x\rangle_{u+d-2s,\mu^2}p^{\{\mu}p^{\nu\}}, \quad \text{with} \quad \psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

where $\lambda_8 = \mathrm{diag}(1, 1, -2)$.

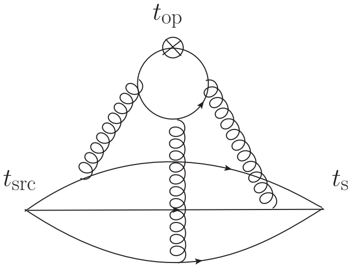
- $\langle x\rangle_{u+d-2s,\mu^2}$ has never been estimated on the lattice
- known experimentaly
- Cancelation of renormalizations factor in $\frac{\langle x\rangle_{u-d,\mu^2}}{\langle x\rangle_{u+d-2s,\mu^2}}$
- Problem : (quark)-disconnected diagrams

Disconnected diagrams

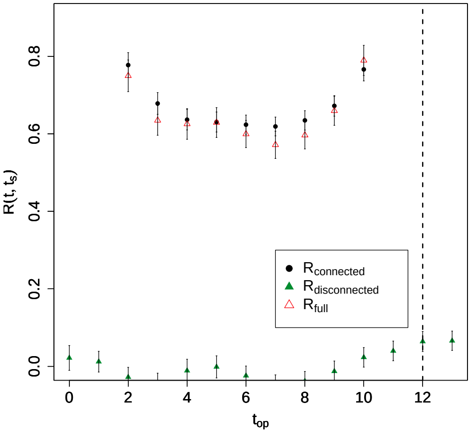
- (quark)-disconnected diagrams contribute to $\langle x \rangle_{u+d-2s,\mu^2}$
- Class of diagrams that is extremely noisy
- Disconnected contribution vanishes in the $SU(3)$ limit

  ⇝ expected to be small in our simulations
- Property of twisted mass fermions that is used to compute the strange content of the nucleon $\rangle N|\bar{s}s|N\rangle$ can be generalized and applied

$$R_{\text{full}}(t_{\text{op}}, t_s) = R_{\text{connected}}(t_{\text{op}}, t_s) + R_{\text{disconnected}}(t_{\text{op}}, t_s)$$

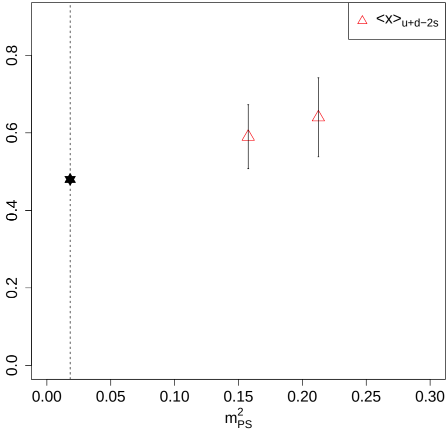


Results



$t_s = 12a$ fixed

Chiral behaviour : $< x >_{u+d-2s,\mu^2}$



(Phenomenological value from S. Alekhin)

Conclusion

- Lattice calculations have done many progress : spectrum, strange content of the nucleon
- Contamination of excited states is $\sim$ 10% for $\langle x \rangle_{u-d}$
- but $\langle x \rangle_{u-d}$ still show a discrepancy with respect to phenomenology
- Other sources of systematics have to be investigated to understand the remaining 30% discrepancy
- On going : provide a better method to control excited states contribution
- First calculation of $\langle x \rangle_{u+d-2s}$
- Disconnected diagrams are the main source of uncertainty
- Ultimately will provide a way to consider ratio of moments of PDFs that are renormalization free