Determination of the nonperturbative form factor in QCD transverse momentum resummation for vector boson production

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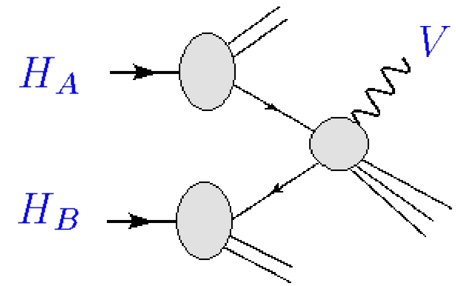
Introduction

◆ Hadroproduction of vector bosons

Drell, Yan ('70)

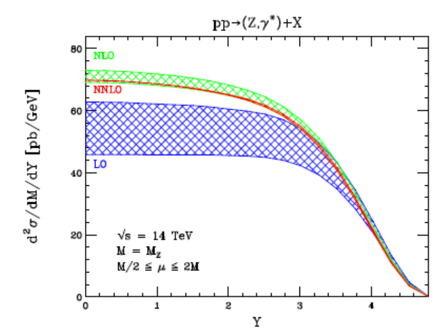


$$H_A + H_B \to V(Q, Q_T, y, \cdots) + X, \quad V = Z, W, \gamma$$

- "Benchmark" process at the LHC
  (detector calibration, test of parton shower, etc.)
- Constraints for PDFs.
- Background for new physics search.
- Indirect search for new physics via Mw, A_{FB} measurements

→ Precise theoretical predictions are mandatory.

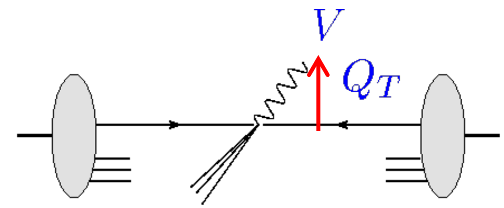
- QCD corrections up to NNLO:
  - Total cross section
    Hamberg, Matsuura, van Neerven ('91), Harlander, Kilgore ('02)
  - Rapidity distribution
    Anastasiou, Dixon, Melnikov, Kilgore ('02)
  - Fully exclusive calculation
    Melnikov, petriello ('02), Catani, Cielri, Ferrera, de Florian, Grazzini ('09)



Anastasiou et al.

- NLO EW corrections Dittmaier, Kramer ('02), Bauer, Wackeroth ('02), Caloni Calame et al. ('06)

◆ Transverse momentum (Q_T) distribution

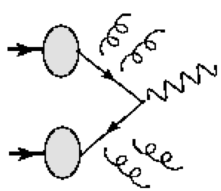


- Fixed order corrections known up to NLO

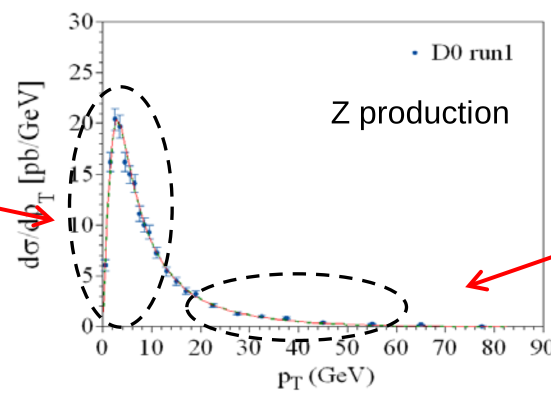
  Ellis, Martinelli, Petronzio (‘83), Arnold Reno (’89), Gonzalvas, Pawlowski, Wei(‘89)

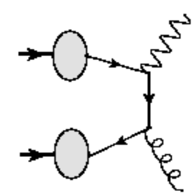
- Due to the soft gluon singularity, the fixed-order perturbation breaks down as $Q_T \to 0$.

$$\left.\frac{d\hat{\sigma}}{dQ^2 dQ_T^2}\right|_{Q_T \ll Q} = \sum_{n=0} \alpha_s^n \left[c_n \delta(Q_T^2) + \sum_{k=0}^{2n-1} d_{nk} \frac{\ln^k (Q^2/Q_T^2)}{Q_T^2} \right]$$



soft gluon emission





hard gluon emission

- A formalism for the all-orders resummation of the enhanced log corrections at small Q_T is known. Collins, Soper (’81,82) , Collins, Soper, Sterman (‘85)

- The nonperturbative effects (≈ primordial k_T) would also be relevant In the small Q_T region, which are implemented in the resummation formalism as the “nonperturbative form factor”.

Resummation

◆Sudakov form factor

– Elastic form factor of quarks with large momentum transfer Q

- Leading logs of soft photon effects exponentiate

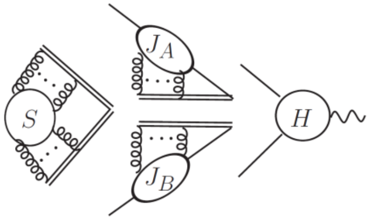
Sudakov ('56), Yennie, Frautchi, Suura ('61), Jackiew ('68) , Fishbane, Sullivan ('70), etc.

- All-logs treatment of QCD corrections → prototype of QCD resummation.

Mueller ('79), Collins ('80), Sen('81,'83) , Korchemsky ('89), etc.

Factorization: $F = J_A(Q, m, \lambda) \times J_B(Q, m, \lambda) \times S(m, \lambda) \times H(Q)$

$m, \ \lambda :$ quark, gluon mass



Evolution eq.: $$\frac{\partial \ln F}{\partial \ln Q} = \frac{\partial \ln J_A}{\partial \ln Q} + \frac{\partial \ln J_B}{\partial \ln Q} + \frac{\partial \ln H}{\partial \ln Q} = K(\lambda, m, g, \mu) + G(Q/\mu; g)$$

RG eq. for K & G: $$\frac{\partial K}{\partial \ln \mu} = -\gamma_K = \frac{\partial G}{\partial \ln \mu}$$

$$\to \quad \frac{\partial \ln F}{\partial \ln Q} = -\left[\int_\mu^Q \frac{d\mu}{\mu} \gamma_K(g(\mu')) + K(\lambda, m, g(\mu), \mu) + G(1; g(Q))\right]$$

Resummation of the double & single logs.

Q_T resummation (Collins-Soper-Sterman formalism)

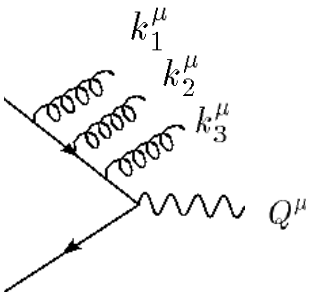
Collins, Soper, Sterman ('85)

◆Impact parameter b space

Parisi, Petronzio ('79)

- Transverse momentum conservation:

$$\delta^{(2)}(Q_T - k_{T1} - k_{T2} - \cdots - k_{Tn}) \to \int d^2 b e^{ibQ_T} \prod_n e^{-ib\cdot k_T}$$



Large logarithm $\ln\left(Q^2/Q_T^2\right) \to \ln(Q^2 b^2)$

◆Resummed cross section

$$\left.\frac{d\sigma}{dQ^2 dQ_T^2 dy}\right|_{Q_T \ll Q} = \mathcal{N} \int d^2 b e^{ib\cdot Q_T} \sum_j e_j^2 W_j(b, Q, x_A, x_B)$$ + non-singular terms

j = flavor

$$x_{A,B} = \frac{Q}{\sqrt{S}} e^{\pm y}$$

Short distance region $(b < 1/\Lambda_{\rm QCD})$

Evolution eq.:

$$\frac{\partial}{\partial \ln Q^2} W_j(b; Q, x_A, x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A_j(\alpha_s(\bar{\mu}^2)) + B_j(\alpha_s(\bar{\mu}^2))\right] W_j(b; Q, x_A, x_B)$$

$\downarrow$ Solution

$$W_j(b,Q,x_A,x_B) = e^{S(b,Q)} \sum_j e_j^2 \bar{\mathcal{P}}_{j/A}(x_A,b;b_0)\bar{P}_{\bar{j}/B}(x_B,b;b_0)$$

Transverse momentum dependent (TMD) distribution

Sudakov factor $\exp\{S(b,Q)\} = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A_j(\alpha_s(\bar{\mu}^2)) + B_j(\alpha_s(\bar{\mu}^2))\right]\right\}$ $\quad b_0 = e^{-\gamma_E}$

- TMD distributions can be expressed in terms of the ordinary PDFs in this region.

PDF $\quad (b < 1/\Lambda_{\rm QCD})$

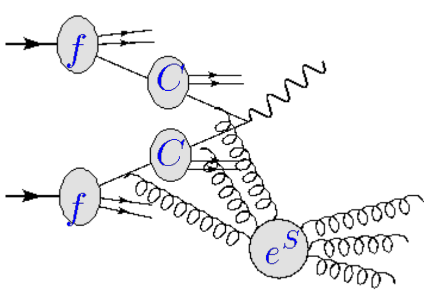
$$\bar{\mathcal{P}}_{j/A}(x_A,b;b_0) = \sum_a C_{j/A} \otimes f_{a/A}(x;b_0^2/b^2) + \mathcal{O}((b\Lambda))$$

Collins, Soper ('81)

$\nwarrow$ Coeff. function

- Final expression

$$W_j(b,Q,x_A,x_B) \simeq e^{S(b,Q)} C_{ja} \otimes f_{a/A}(x_A, b_0^2/b^2) \cdot C_{\bar{j}b} \otimes f_{b/B}(x_B, b_0^2/b^2)$$
$$\equiv W_j^{pert}(b,Q,x_A,x_B)$$



$\alpha_s^n \ln^{2n}(bQ) \leftrightarrow A^{(1)}$ Leading Log (LL)

Dokshitzer,D'yakonov,Troyan ('78)

$\alpha_s^n \ln^{2n-1}(bQ),\ \alpha_s^n \ln^{2n-2}(bQ) \leftrightarrow A^{(2)},\ B^{(1)},\ C^{(1)}$ NLL

Kodaira, Trentadue ('81), Davies, Stirling ('84)

Long distance region $(b \geq 1/\Lambda_{\rm QCD})$

— Prescription to handle Landau singularity has to be be specified.

(1) b_* prescription

Collins, Soper, Sterman ('85), Davies, Stirling ('84), and many others.

$$\int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b, Q, x_A, x_B) \to \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b_*, Q, x_A, x_B) F_*^{\rm NP}(b)$$

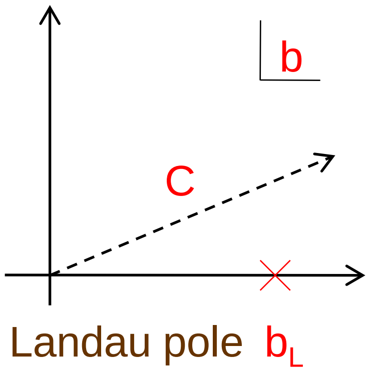
– $b_* = \dfrac{b}{\sqrt{1 + b^2/b_{\rm max}^2}} \ll 1/\Lambda_{\rm QCD}$ α_s frozen for $b \geq b_{\rm max} \simeq 0.5{\rm GeV}^{-1}$

– Non-perturbative form factor $F_*^{\rm NP}(b) \equiv \dfrac{W(b, Q, x_A, x_B)}{W^{\rm pert}(b_*, Q, x_A, x_B)}$

(2) Minimal prescription (MP)

Leanen, Sterman, Vogelsang ('00), Kulesza, Sterman, Vogelsang ('02,'04)
Bozzi, catani, Ferrera, de Florian, Grazzini ('03-'11), Bozzi, Fuks, Krasen ('08)
Koike, Nagashima, Vogelsang ('06), Kodaira, Tanaka, HK ('06-'08)

Contour deformation



$$\int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b, Q, x_A, x_B) \qquad J_0 = \frac{1}{2}[H_0^{(1)} + H_0^{(2)}]$$

$$\to \int_C db \frac{b}{2} H_0^{(1)}(bQ_T) W(b, Q, x_A, x_B) F_{\rm MP}^{\rm NP}(b) + ({\rm c.c.})$$

(2) Minimal prescription (cont'd)

$$\int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b,Q,x_A,x_B) \to \int_C db \frac{b}{2} H_0^{(1)}(bQ_T) W(b,Q,x_A,x_B) F_{\rm MP}^{\rm NP}(b) + ({\rm c.c.})$$

Pros

- No additional parameter b_{max}
- By expanding in α_s (taking F^{NP}=1), the fixed order results are reproduced order by order.
  $\to$ matching with the fixed order result is simple.
- $\ln(Q^2 b^2/b_0^2) \to \ln(Q^2 b^2/b_0^2 + 1)$ renders the large logs vanish at b=0.

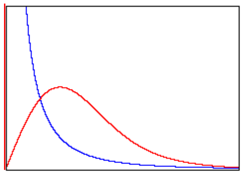
"Unitarity relation" $\int_0^\infty dQ_T \frac{d\sigma}{dQ_T} = \sigma^{\rm tot}$

Caveat: Terms proportional to $\delta(Q_T)$ remain in $\frac{d\sigma^{\rm res}}{dQ_T} - \left.\frac{d\sigma^{\rm res}}{dQ_T}\right|_{\rm f.o.} + \frac{d\sigma^{\rm f.o.}}{dQ_T}$ if you go to NNLL+NLO and higher.

Kodaira,Tanaka,HK ('07)

Cons

- Calculation is much more involved than b* prescription because PDF at complex scales are needed.



Non-perturbative form factor

(1) b_* prescription $$F_*^{\rm NP}(b) \equiv \frac{W(b,Q,x_A,x_B)}{W^{\rm pert}(b_*,Q,x_A,x_B)}$$

- Has been determined by fitting to the experimental data.

Ladinsky & Yuan ('94), Landry et al. ('01, '03), Kulesza, Stirling ('04), Konychev, Nadolsky ('06), etc.

ex. $$F_*^{\rm NP}(b) = \exp\left\{-g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 b^2\right\} \quad Q_0 = 1.6{\rm GeV}$$ Davies, Stirling, Webber ('85)

$$g_1 = 0.016{\rm GeV}^2, \quad g_2 = 0.54{\rm GeV}^2 \ ({\rm CTEQ3M})$$ Landry, Brock, Nadolsky, Yuan ('03)

$$g_1 = 0.08{\rm GeV}^2, \quad g_2 = 0.67{\rm GeV}^2 \ ({\rm MRST01})$$ Kulesza, Sterling ('03)

- Implemented in a code ResBos (unpol. & pol)

(2) MP $$F_{\rm MP}^{\rm NP}(b) \equiv \frac{W(b,Q,x_A,x_B)}{W^{\rm pert}(b,Q,x_A,x_B)}$$

- Not well-known. A smaller value ($0.6{\rm GeV}^2$) than b_* case prefered for Z production.

Kulesza, Sterling , Vogelsang ('02)

- We have performed a fit with experimental data with different Q values,

Fitting pricedure

– Functional form: DSW-type (2 parameter) Davies, Stirling, Webber ('85)

$$\exp\left[-\left(g_1+g_2\ln\left(\tfrac{Q}{2Q_0}\right)\right)b^2\right],\ \left(Q_0=m_c\right)$$

– PDF: CTEQ6.6 & MSTW08

– χ^2 with normalization errors

Normalization   Data   Theory

$$\chi^2=\sum_{i=1}^{N_{\exp}}\left[\left(\frac{1-N_i}{\delta N_i}\right)^2+\sum_{j=1}^{N_{data}(i)}\left(\frac{N_jD_j-T_j(\{g_k\})}{\delta D_j}\right)^2\right]$$

Normalization error

- Determin $(g_1,\ g_2,\ N_i)$ at the same time.

Experimental data

- Q_T distribution (integrated over rapidity Y)
  - R209 [D. Antreasyan et al., PRL47, 12 (1981), from H-L. Lai]

    Drell-Yan process
  - CDF run-0 [F. Abe et al., PRL67, 2937 (1991)]
  - CDF run-1 [T. Affolder et al., PRL84, 845 (2000)]
  - D0 run-1 [B. Abbott et al., PRD61, 032004]

    Z-boson production

Exp	√s (GeV)	Target	pT range (GeV)	M range (GeV)	# of data (pT < 22 GeV)	δN_{exp}
R209	62	P-P	0.2 – 1.8	5.0 - 11.0	10	10%
CDF run-0	1800	P-Pbar	0.0 – 22.8	75 - 105	7	-
CDF run-1	1800	P-Pbar	0.0 – 22.0	66 - 116	33	3.9%
D0 run-1	1800	P-Pbar	0.0 – 22.0	75 - 105	15	4.4%

Result of 2-parameter fit

$$g_{\rm NP} = g_1 + g_2 \ln\left(\frac{Q}{2Q_0}\right), \quad Q_0 = 1.3{\rm GeV}$$

PDF set	DWS-G (p_T cut=22GeV)			
	g_1		g_2	
CTEQ6.6M	0.241	$^{+0.026}_{-0.028}$	0.121	$^{+0.041}_{-0.038}$
MSTW08	0.330	$^{+0.024}_{-0.026}$	0.066	$^{+0.039}_{-0.037}$

– PDFdependence large（Same as b* prescription）

Results (χ^2 values)

CTEQ6.6M

$$\chi^2/d.o.f = 0.77$$

Exp	# of data (pT < 22 GeV)	DWS-G		
		χ^2_{Data}	χ^2_{Norm}	N_{fit}
R209	10	11.88	2.74	0.834
CDF run-0	7	4.61	-	1(fixed)
CDF run-1	33	13.58	5.77	0.906
D0 run-1	15	6.04	0.56	0.967
Total	65	45.20		

MSTW08

$$\chi^2/d.o.f = 0.78$$

R209	10	15.29	1.09	0.895
CDF run-0	7	4.74	-	1(fixed)
CDF run-1	33	13.99	4.63	0.916
D0 run-1	15	5.77	0.29	0.976
Total	65	45.81		

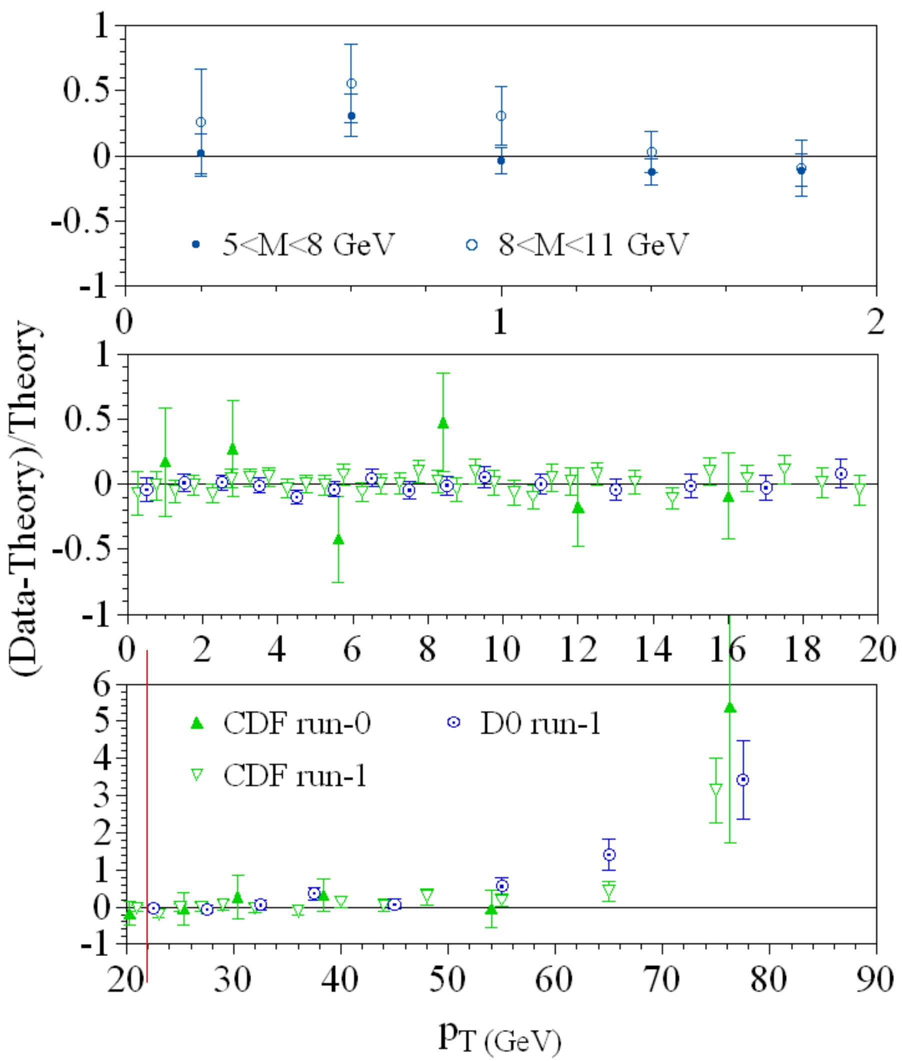
Data vs. Theory

CTEQ6.6

R209

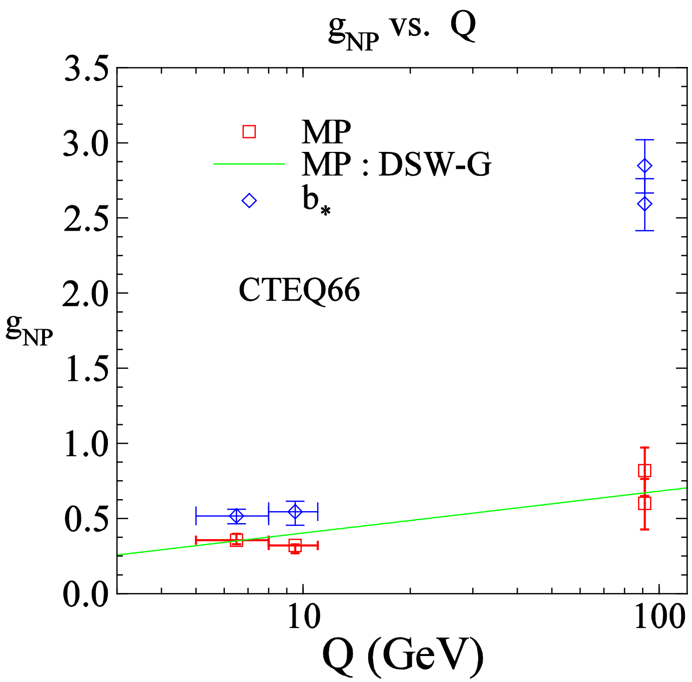
D0, CDF

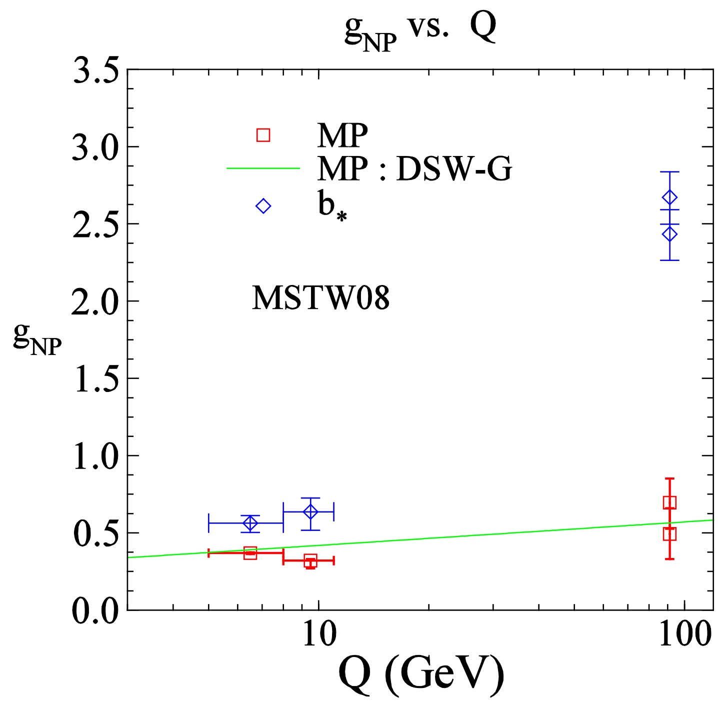
$Q_T < 22\mathrm{GeV}$



MP vs. b_* comparison（1 parameter fit）

• 1 parameter Gaussian fit for each experiment. $F_*^{NP}(b) = \exp\left(-g_{NP} b^2\right)$





- g_{NP} as much as 2.5 GeV for Z-production for b_* prescription.
- For MP, Q dependence is significantly small.

  → Separation of perturbative and non-perturbative region is done well when MP is taken.

MP vs. b_* comparison

NP form factors are not calculable, but their ratio can be expressed perturbatively.

$$\frac{F^{\rm NP}_{\rm MP}(b)}{F^{\rm NP}_{*}(b)} \equiv \exp\left\{ -\ln\left(\frac{Q^2}{Q_0^2}\right)\delta g_1(b, b_{\rm max}) - \delta g_{j/A}(x_A, b, b_{\rm max}, Q_0) - \delta g_{\bar{j}/B}(x_B, b, b_{\rm max}, Q_0) \right\}$$

- Q dependent term

$$\delta g_1(b, b_{\rm max}) = \int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A\left(\alpha_s(\bar{\mu}^2)\right) = \int_{b_0^2/b^2 + b_0^2/b_{\rm max}^2}^{b_0^2/b^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A\left(\alpha_s(\bar{\mu}^2)\right)$$

Rough estimation (for $b < b_{max} = 0.5$GeV)

$$\delta g_1(b, b_{\rm max}) \simeq -\frac{b_0^2}{b_{\rm max}^2} \frac{1}{\left(\frac{b_0^2}{b^2}\right)} A\left(\alpha_s(b_0^2/b^2)\right) \simeq -\frac{1}{b_{\rm max}} A\left(\alpha_s(b_0^2/b^2)\right) b^2 \simeq -0.5 b^2$$

Q dependence in MP is supposed to be much smaller than in b_* prescription.

cf. LBNY $g_2 = 0.54 {\rm GeV}^2$

- Q independent term

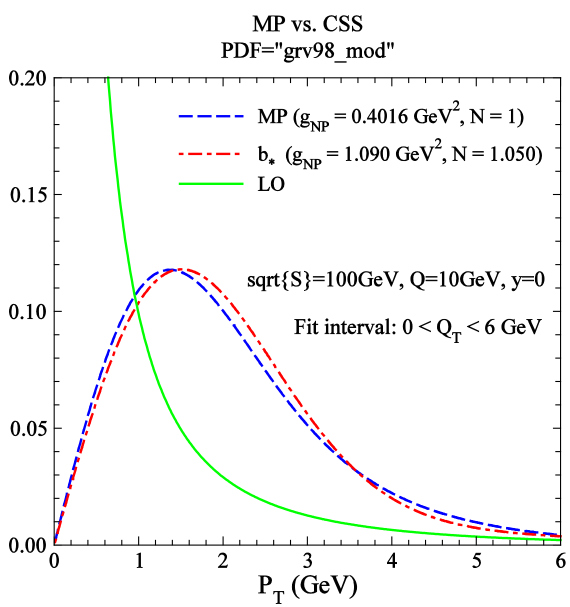
$$\delta g_{j/A}(x_A, b, b_{\rm max}, Q_0) = \ln\left(\frac{C_{ja} \otimes f_{a/A}(x_A, b)}{C_{ja} \otimes f_{a/A}(x_A, b_*)}\right) + \frac{1}{2}\left\{ \int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln\left(\frac{Q_0^2}{\bar{\mu}^2}\right) A\left(\alpha_s(\bar{\mu}^2)\right) + B\left(\alpha_s(\bar{\mu}^2)\right) \right] \right\}$$

MP vs. b_* comparison (shape)

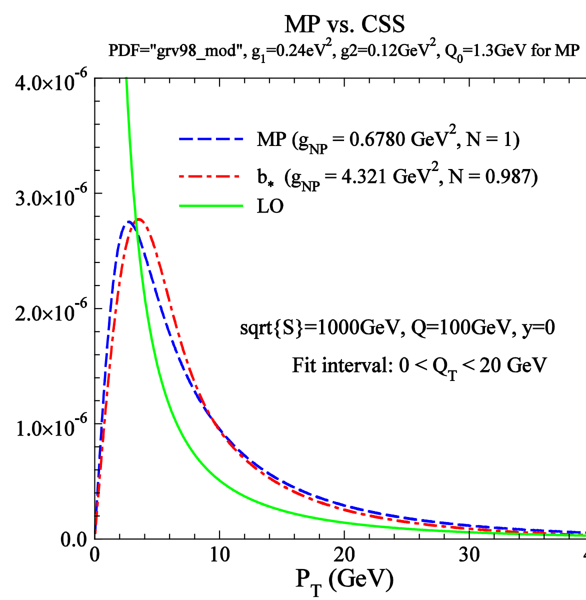
- To compare the difference of shapes, we set the normalization and Gaussian parameter such that the average Q_T and Q_T-integrated cross section are same.

$$\langle Q_T \rangle \qquad \int dQ_T \frac{d\sigma}{dQ_T}$$

Q = 10 GeV



Q = 100 GeV



Only virtual photon

- Difference seems to be more significant for larger Q, we still cannot tell which describes the data better at the moment.

Summary

- Vector boson production is a benchmark process at the LHC.

- Soft gluon resummation is crucial for making a reliable prediction for Q_T distribution of vector bosons at small Q_T .

  — Prescriptions for the region: $b \geq 1/\Lambda_{\rm QCD}$
  b* prescription & "minimal prescription" (new approach)

  — NP function is important for Q_T ≈ (several) GeV.

- We determined the NP form factor in MP by fitting the experimental data.

$$F^{\rm NP}(b) = \exp\left[\left\{g_1 + g_2 \ln\left(\frac{Q}{2Q_0}\right)\right\} b^2\right]$$

$g_1 = 0.24{\rm GeV}^2,\ g_2 = 0.12{\rm GeV}^2$ CTEQ6.6

$g_1 = 0.33{\rm GeV}^2,\ g_2 = 0.06{\rm GeV}^2$ MSTW08

- The results are useful for calculation of diboson producition, squark production, etc.

- Q dependence of $F^{NP}_{MP}(b)$ is much smaller than that of $F^{NP}_{*}(b)$, which can be understood by a rough (but plausible) discussion.

- More detailed analyses (x dependence etc.) is left for future work.