Hadron scattering, resonances and exotics from lattice QCD

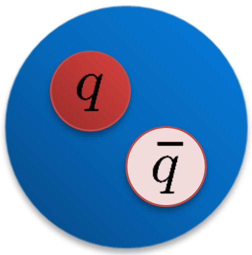
Christopher Thomas, University of Cambridge

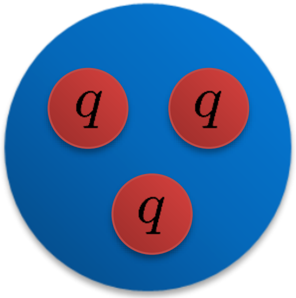
c.e.thomas@damtp.cam.ac.uk

Berlin, Germany
20 January 2025

Hadron spectroscopy

Hadron spectroscopy



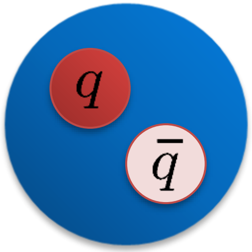


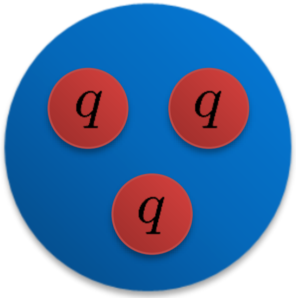
Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4230)$, $Z_c^+(4430)$, $Z_c^+(3900)$, $X(6900)$, $T_{cc}(3875)$, $D_{s0}(2317)$, $T_{cs}(2900)$, Z_b^+, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

KLOE(-2)

CLAS12





Hadron spectroscopy

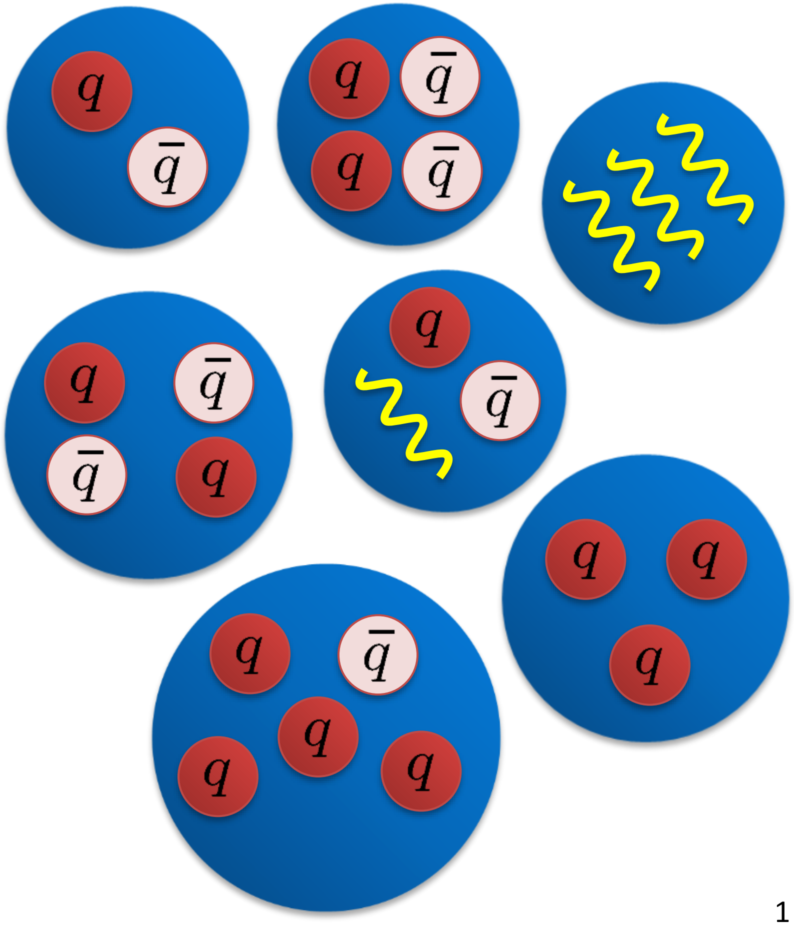
Intriguing observations, e.g. $X(3872)$, $Y(4230)$, $Z_c^+(4430)$, $Z_c^+(3900)$, $X(6900)$, $T_{cc}(3875)$, $D_{s0}(2317)$, $T_{cs}(2900)$, Z_b^+, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

Belle II, BESIII, ATLAS, PANDA, LHCb, CMS, COMPASS

KLOE(-2)

CLAS12

GlueX

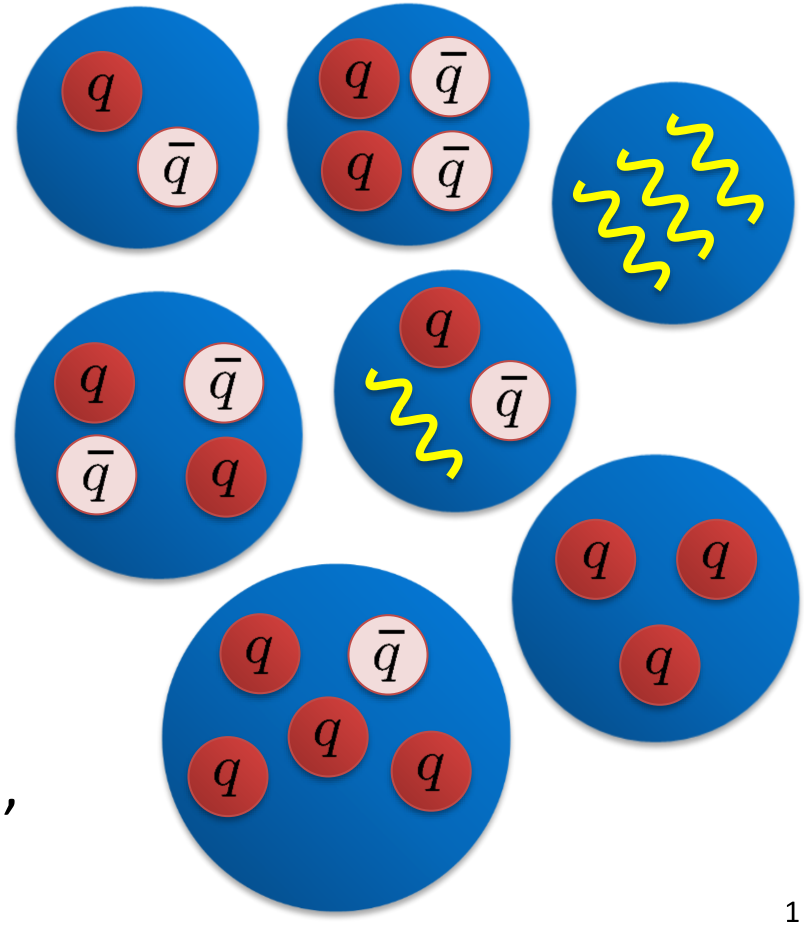


Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4230)$, $Z_c^+(4430)$, $Z_c^+(3900)$, $X(6900)$, $T_{cc}(3875)$, $D_{s0}(2317)$, $T_{cs}(2900)$, Z_b^+, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

KLOE(-2)

CLAS12



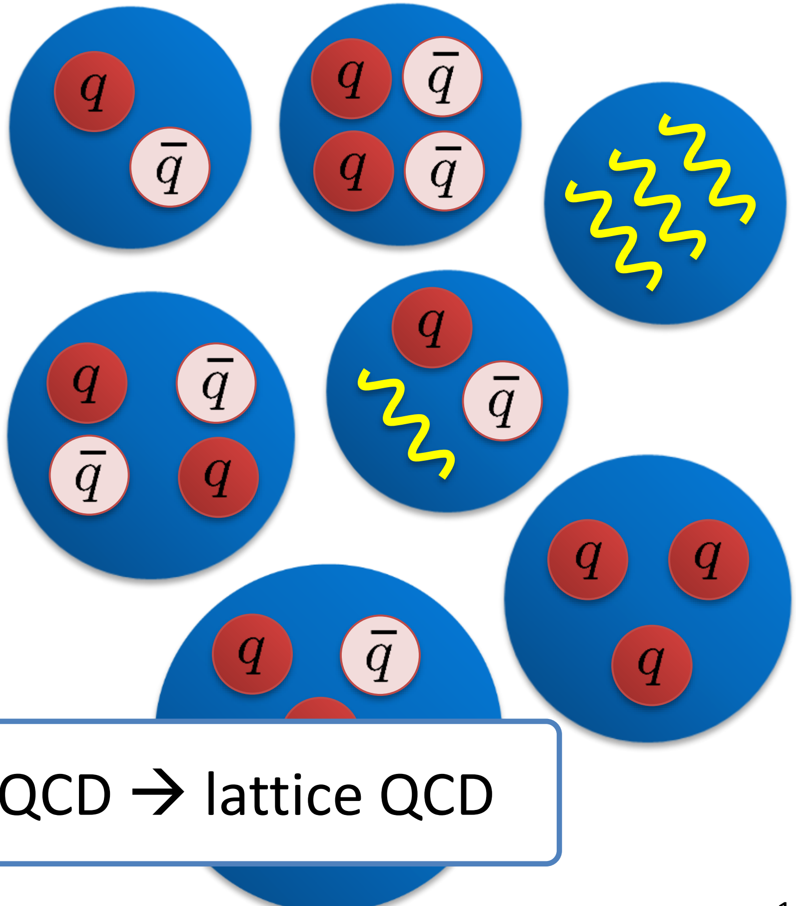
Exotic quantum numbers are particularly interesting, can't be just $\bar{q}q$, e.g. flavour or $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$

Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4230)$, $Z_c^+(4430)$, $Z_c^+(3900)$, $X(6900)$, $T_{cc}(3875)$, $D_{s0}(2317)$, $T_{cs}(2900)$, Z_b^+, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

KLOE(-2)

CLAS12



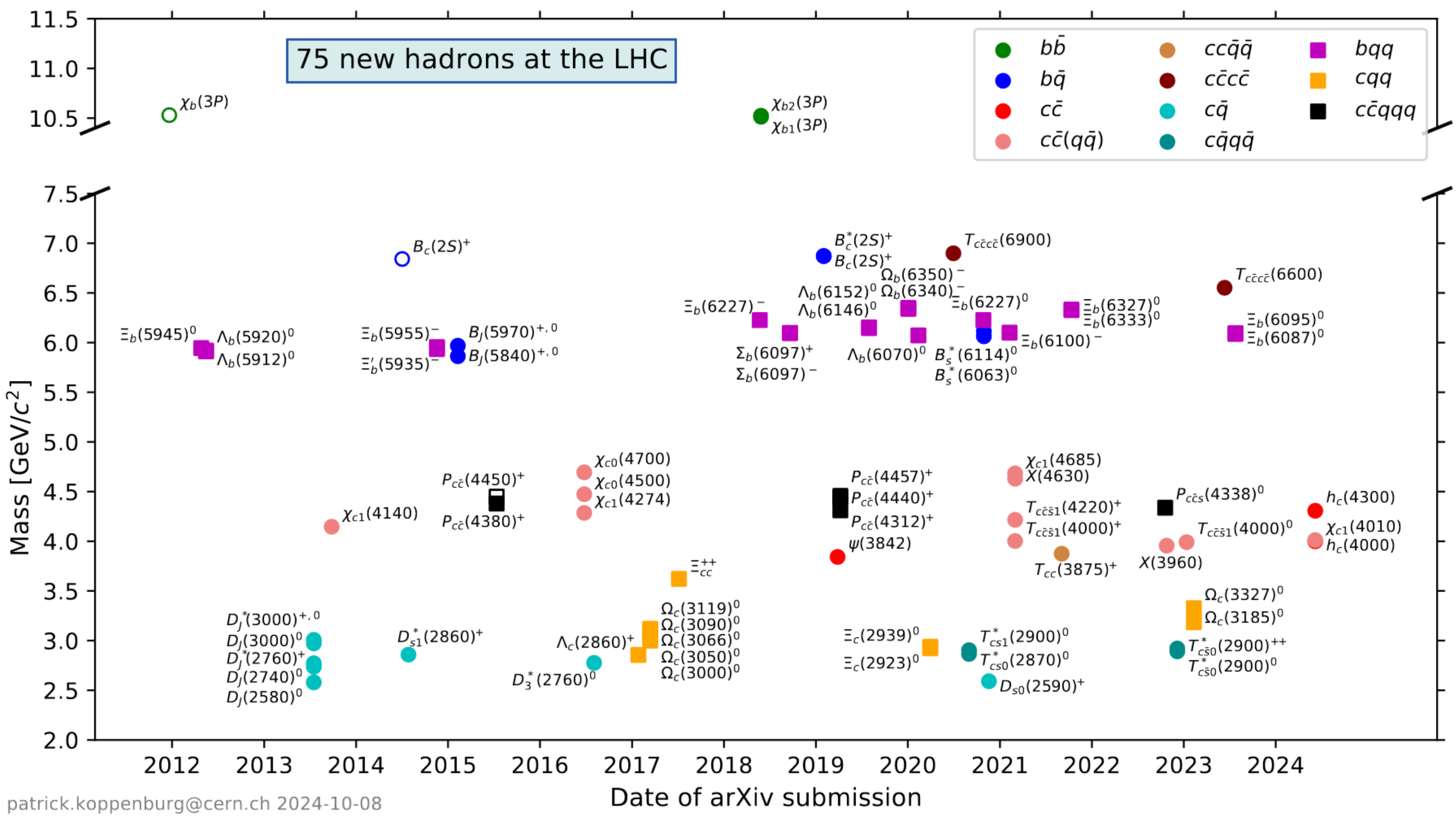
Exotic quantum numbers are

particu

e.g. flavour or J = 0 , 0 , 1 , 2

First-principles calculations in QCD → lattice QCD

Hadron spectroscopy

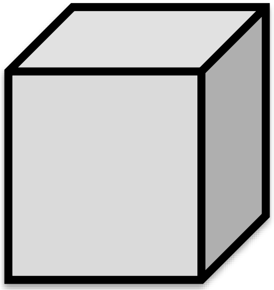


patrick.koppenburg@cern.ch 2024-10-08

Outline

- Introduction
- Some examples of recent HadSpec work:
  - T_{cc} and T_{cc} in coupled DD^*, D^*D^* scattering
  - Scalar and tensor charmonium resonances
  - [DK/π scattering – dependence on m_π]
- Summary

Lattice QCD spectroscopy



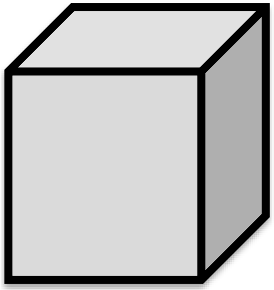
Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Lower-lying hadrons in each flavour sector are well determined (including isospin breaking, QED).

Lattice QCD spectroscopy



Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0|\mathcal{O}_i(0)|n\rangle\langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

Excited states: in each symmetry channel compute matrix of correlators for **large bases of interpolating operators** with appropriate variety of structures.

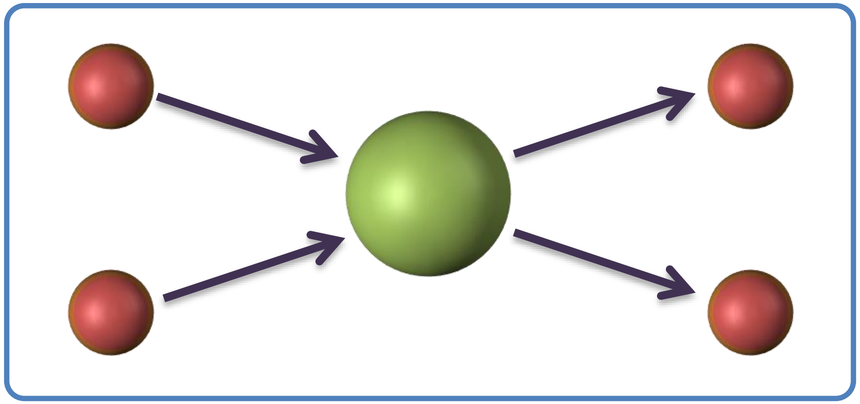
Variational method (generalised eigenvalue problem) → $\{E_n\}$

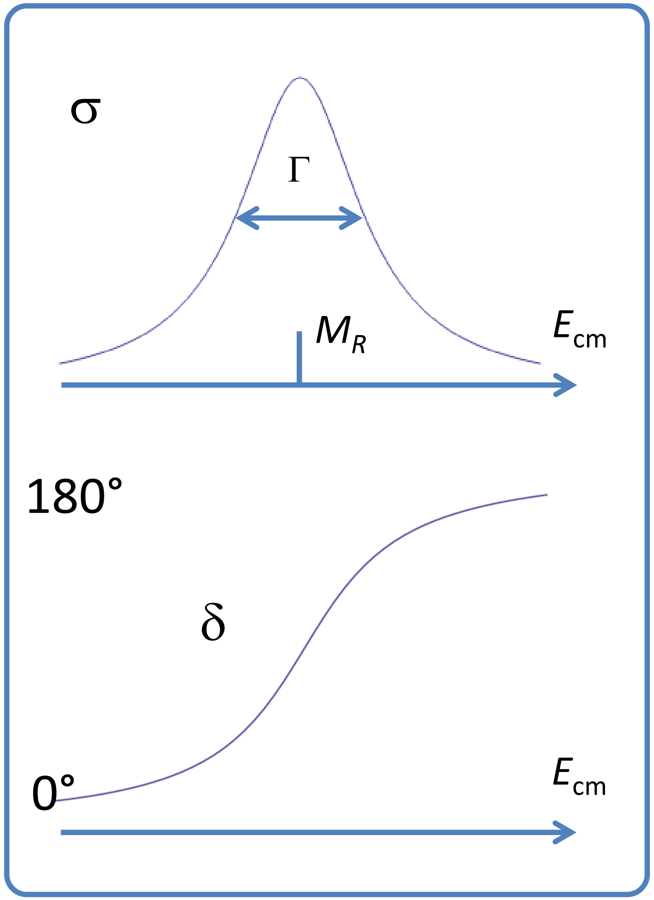
$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)} \qquad \lambda^{(n)}(t) \sim e^{-E_n(t-t_0)}$$

$$v_i^{(n)} \to Z_i^{(n)} \equiv \langle 0|\mathcal{O}_i|n\rangle \qquad \Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

Scattering and resonances

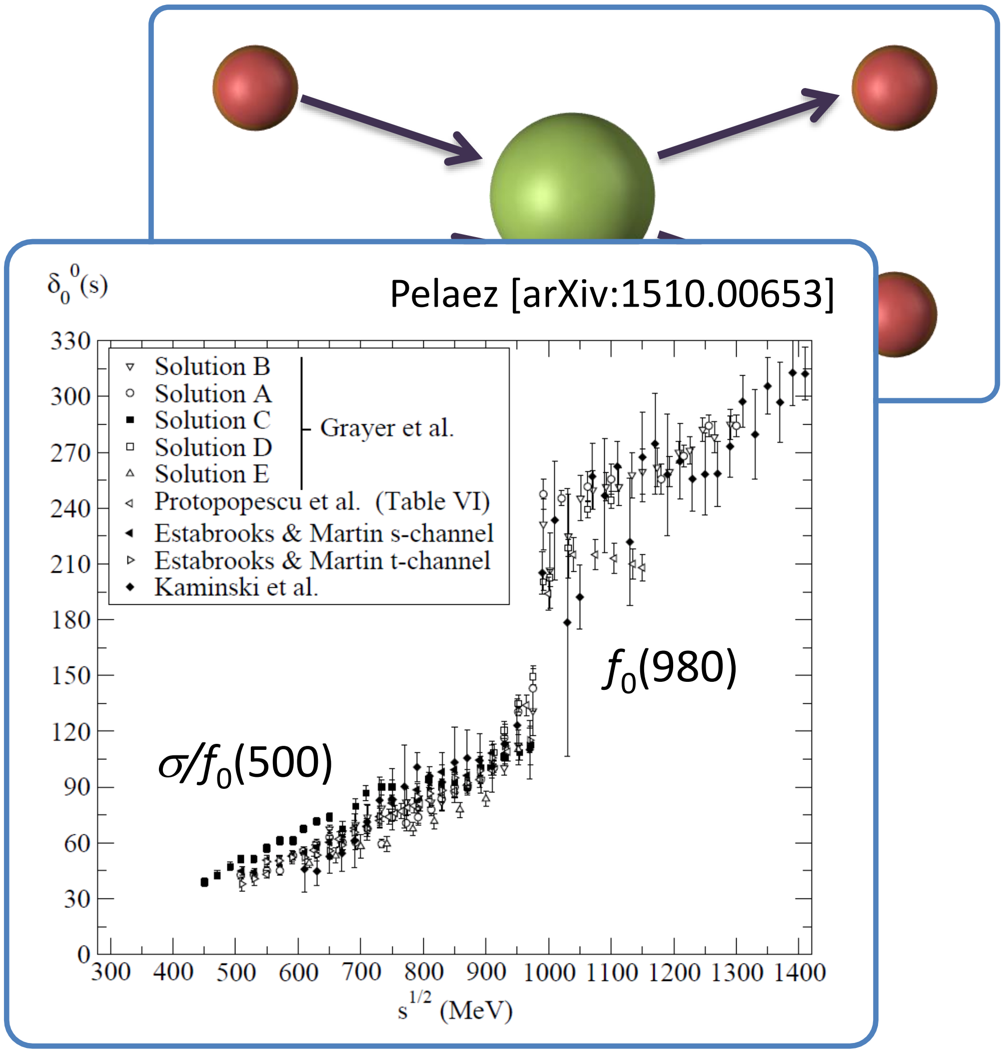
Most hadrons are resonances and decay strongly to lighter hadrons

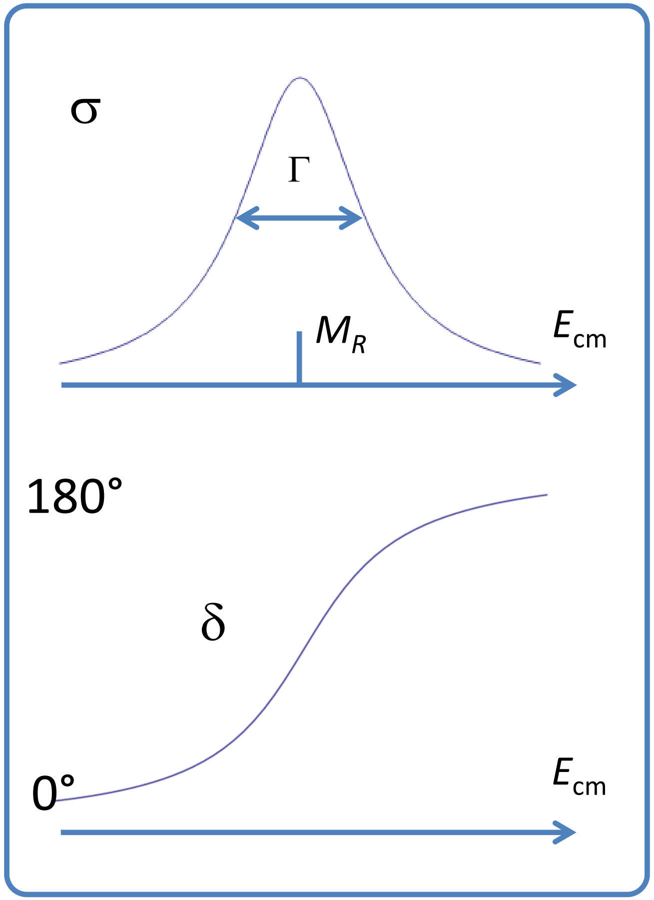




Scattering and resonances

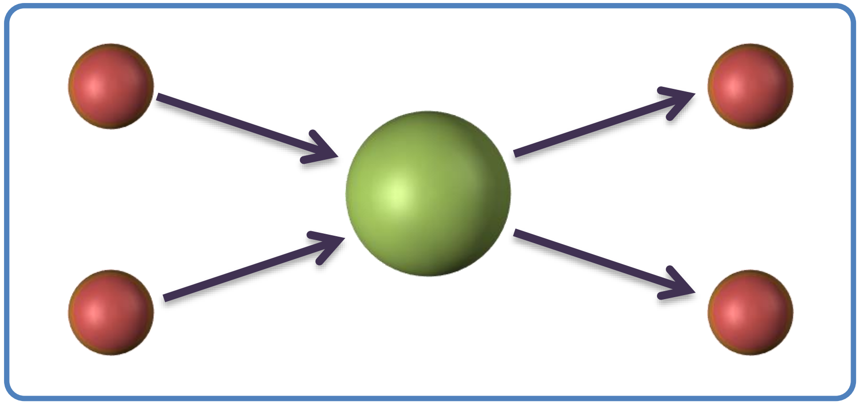
Most hadrons are resonances and decay strongly to lighter hadrons



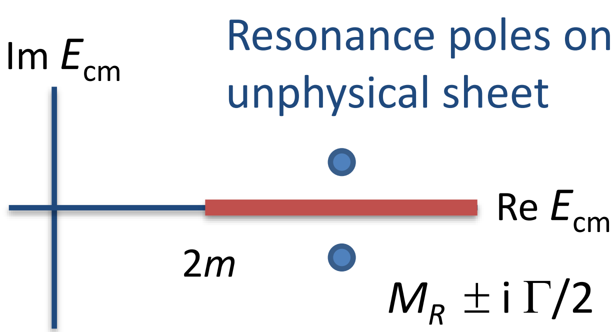


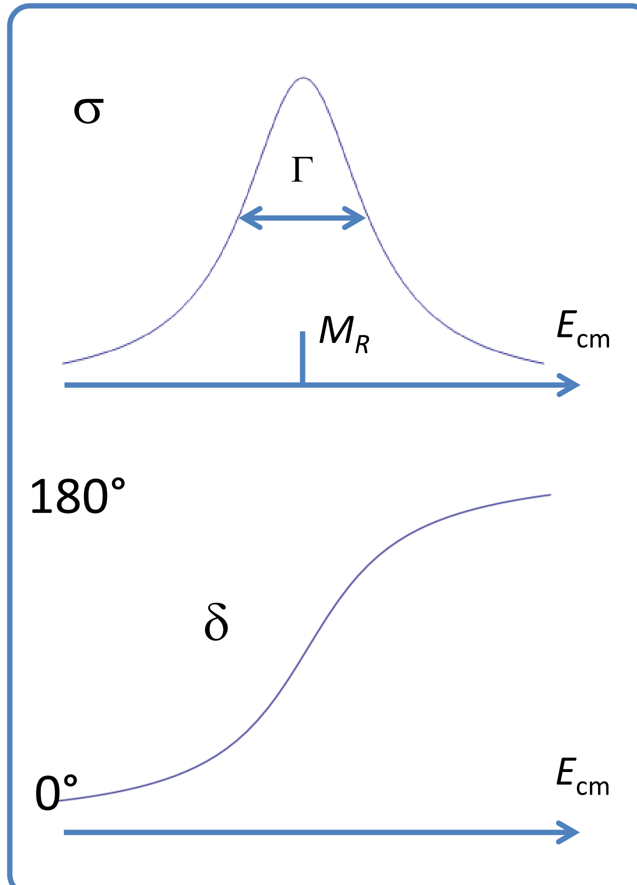
Scattering and resonances

Most hadrons are resonances and decay strongly to lighter hadrons



Singularity structure of scattering matrix (poles → state content)

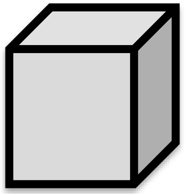




Scattering and resonances in lattice QCD

Can’t directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.

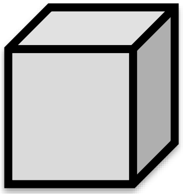


$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{cm}\}$ to **infinite-volume scattering *t*-matrix**.



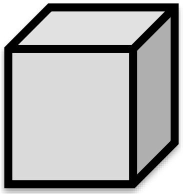
$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\boldsymbol{\rho}(E_{\text{cm}})\,\boldsymbol{t}(E_{\text{cm}})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

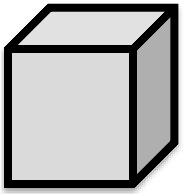
Infinite-volume scattering *t*-matrix

Effect of finite volume (including reduced sym.)

Scattering and resonances in lattice QCD

Can’t directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{\mathrm{cm}})\,\boldsymbol{t}(E_{\mathrm{cm}})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\mathrm{cm}}, L)\right)\right] = 0$$

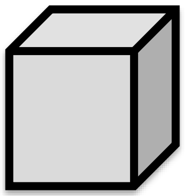
Elastic scattering: one-to-one mapping $E_{\mathrm{cm}} \leftrightarrow t(E_{\mathrm{cm}})$

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{cm}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{cm})\,\boldsymbol{t}(E_{cm})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{cm}, L)\right)\right] = 0$$

Elastic scattering: one-to-one mapping $E_{cm} \leftrightarrow t(E_{cm})$

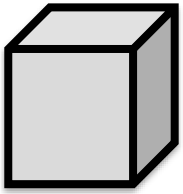
Coupled channels: under-constrained problem
(each E_{cm} constrains t-matrix at that E_{cm})
Param. $t(E_{cm})$ using various forms (K-matrix forms, …)
[see e.g. review Briceño, Dudek, Young, Rev. Mod. Phys. 90, 025001 (2018)]

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\rm cm}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{\rm cm})\,\boldsymbol{t}(E_{\rm cm})\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\rm cm}, L)\right)\right] = 0$$

Elastic scattering: one-to-one mapping $E_{\rm cm} \leftrightarrow t(E_{\rm cm})$

Coupled channels: under-constrained problem
(each $E_{\rm cm}$ constrains t-matrix at that $E_{\rm cm}$)
Param. $t(E_{\rm cm})$ using various forms (K-matrix forms, …)

Analytically continue $t(E_{\rm cm})$ in complex $E_{\rm cm}$ plane, look for poles.

Demonstrated in calcs. of ρ, light scalars, b_1, charm mesons, …

The ρ resonance in $\pi\pi$ scattering

($J^{PC} = 1^{--}$, I = 1)

Experimentally

$$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$$

The ρ resonance in $\pi\pi$ scattering

($J^{PC} = 1^{--}$, I = 1)

Experimentally

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

Anisotropic lattices,
$a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm,
L ≈ 4 fm ($m_\pi L \approx 4$)

$N_f = 2+1$,
Wilson-clover fermions,
$m_\pi \approx 236$ MeV

Used *distillation* to compute correlation fns.
[PR D80 054506 (2009)]

Use many different operators

$$\bar{\psi}\Gamma D \ldots \psi$$

$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)\pi(\vec{p}_1)\pi(\vec{p}_2)$$

$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)K(\vec{p}_1)\bar{K}(\vec{p}_2)$$

built from optimised π & *K* ops

Wilson *et al* (HadSpec) [PR D92, 094502 (2015)] and Dudek, Edwards, CT (HadSpec) [PR D87, 034505 (2013)]

The ρ resonance in $\pi\pi$ scattering

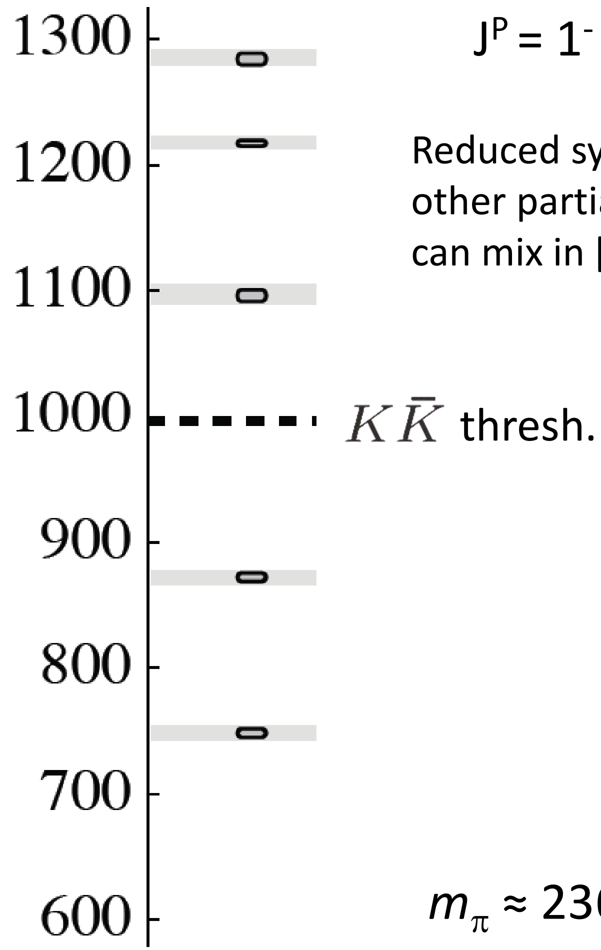
($J^{PC} = 1^{--}$, I = 1)

E_{cm} / MeV

P = [0,0,0]

$J^P = 1^-$ [ℓ = 1]

Reduced sym. → other partial waves can mix in [T_1^-]



$m_\pi \approx 236$ MeV

Experimentally

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

Use many different operators

$$\bar{\psi}\Gamma D \dots \psi$$

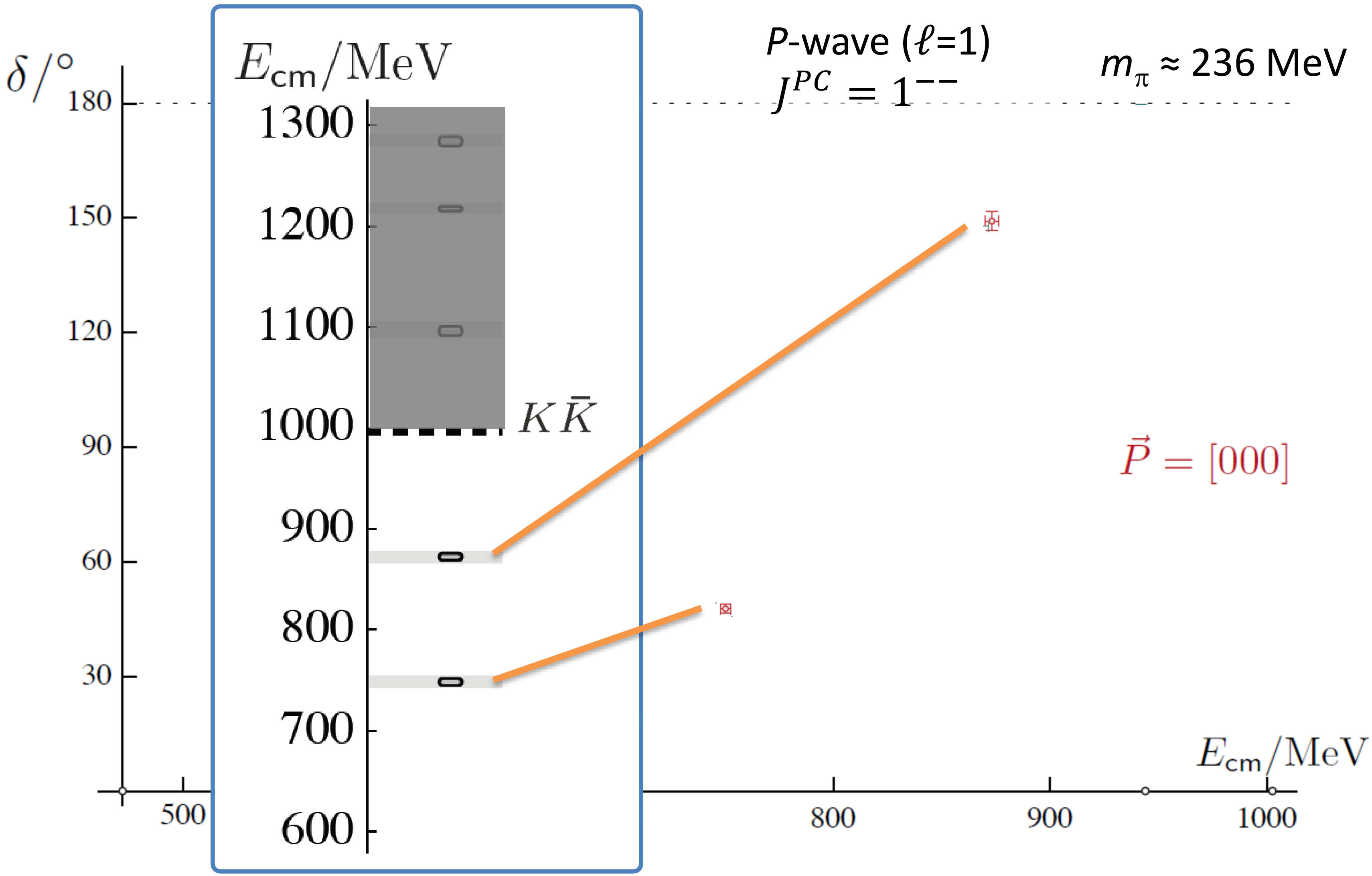
$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)\pi(\vec{p}_1)\pi(\vec{p}_2)$$

$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)K(\vec{p}_1)\bar{K}(\vec{p}_2)$$

built from optimised π & *K* ops

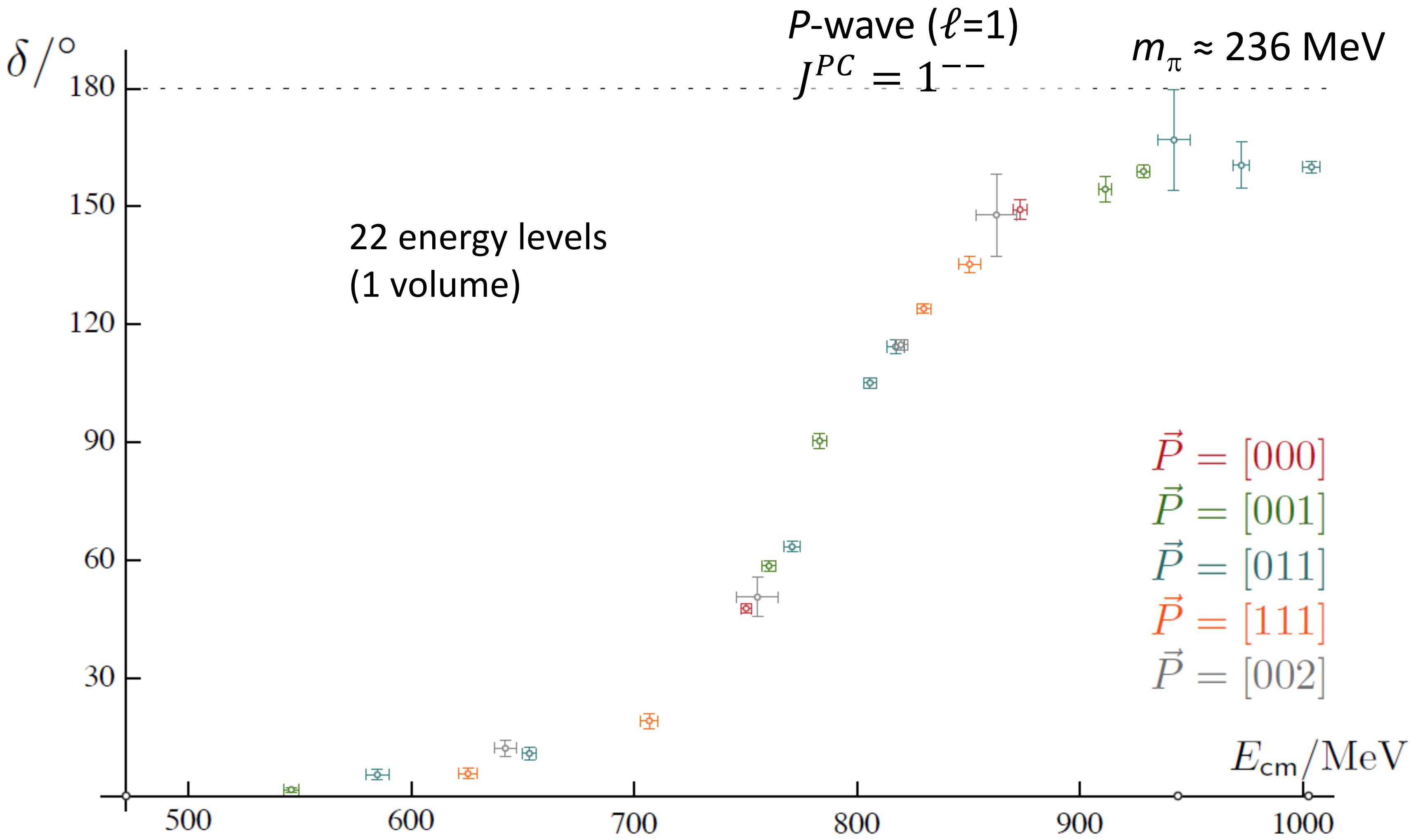
Wilson *et al* (HadSpec) [PR D92, 094502 (2015)] and Dudek, Edwards, CT (HadSpec) [PR D87, 034505 (2013)]

The ρ resonance: **elastic** $\pi\pi$ scattering



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

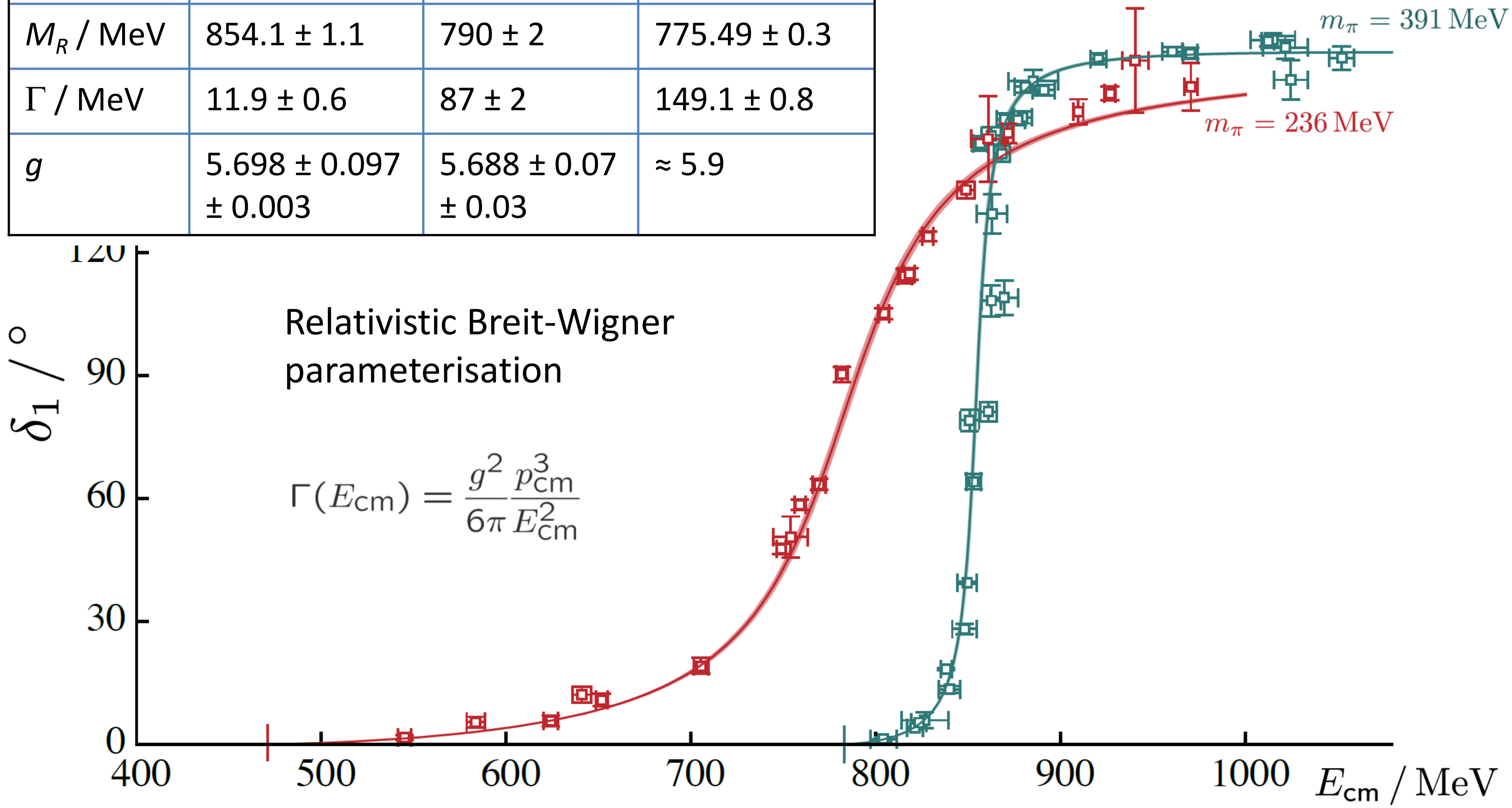
The ρ resonance: **elastic** $\pi\pi$ scattering



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** $\pi\pi$ scattering

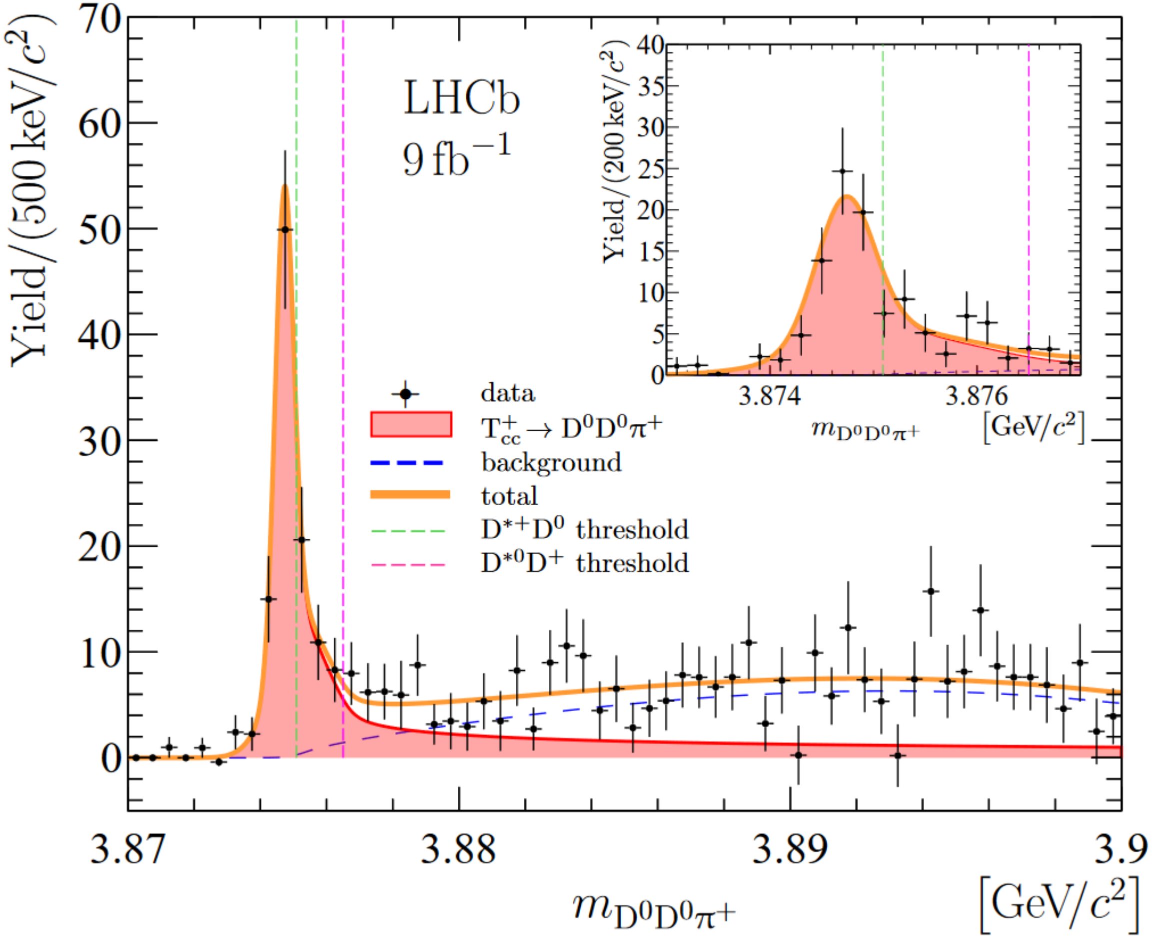
m_π / MeV	391	236	Experimental
M_R / MeV	854.1 ± 1.1	790 ± 2	775.49 ± 0.3
Γ / MeV	11.9 ± 0.6	87 ± 2	149.1 ± 0.8
g	5.698 ± 0.097 ± 0.003	5.688 ± 0.07 ± 0.03	≈ 5.9



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

T_{cc}

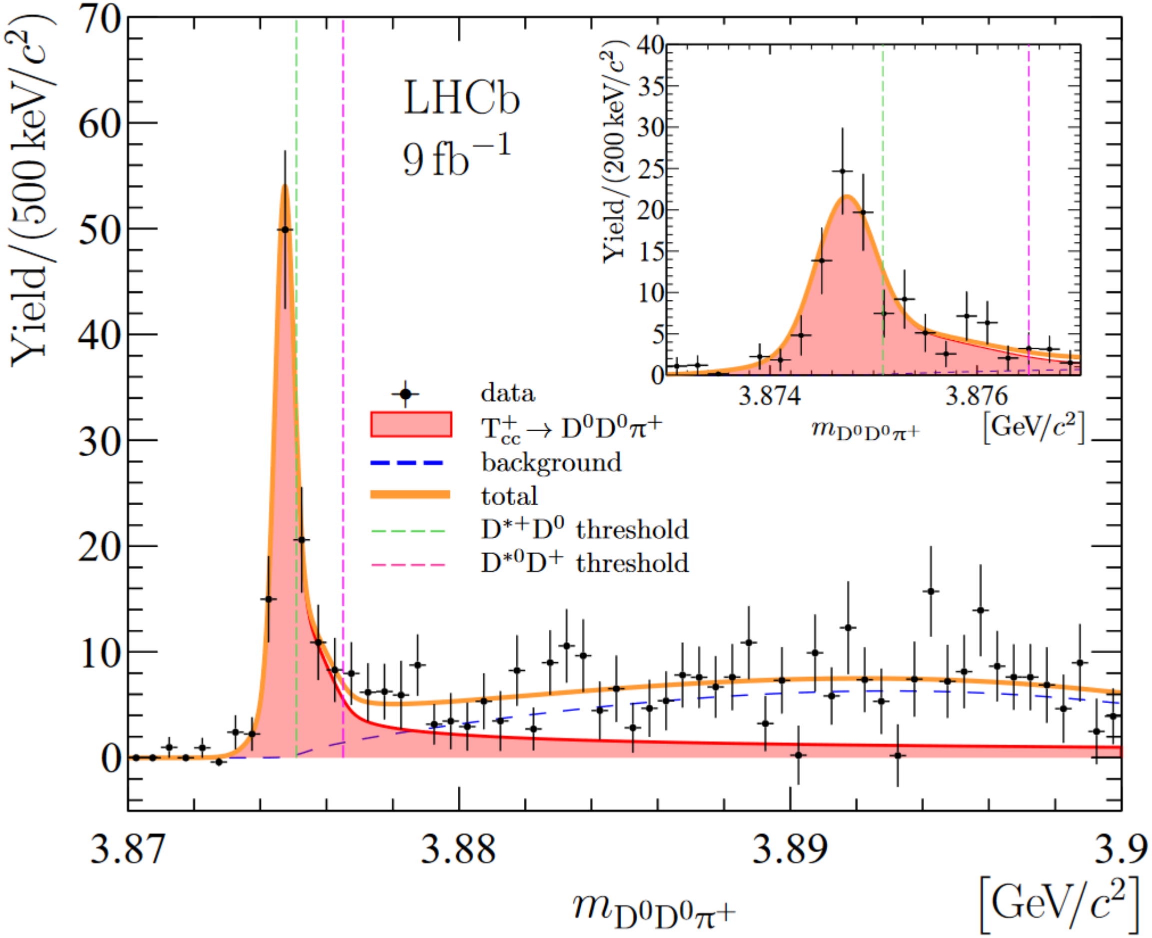
T_{cc}^+ seen in $D^0D^0\pi^+$ at LHCb [2109.01038, 2109.01056]
Close to DD^* threshold, J^P=1$^+$, I=0, **exotic flavour** $(cc\bar{u}\bar{d})$



T_{cc}

T_{cc}^+ seen in $D^0 D^0 \pi^+$ at LHCb [2109.01038, 2109.01056]

Close to DD^* threshold, J^P=1^+, I=0, **exotic flavour** $(cc\bar{u}\bar{d})$



What about higher energies (coupled DD^*, D^*D^*)?

Other lattice calcs:

- Padmanath & Prelovsek [2202.10110, PRL];
- Chen *et al* [2206.06185, PLB];
- Lyu *et al* (HAL QCD) [2302.04505, PRL];
- Collins *et al* [2402.14715, PRD];
- Meng *et al* [2411.06266];

See also:

- Du *et al* [2303.09441, PRL];
- Meng *et al* [2312.01930, PRD].
- Gil-Domínguez & Molina [2409.15141].
- Dawid *et al* [2409.17059, JHEP].

Coupled DD^*, D^*D^* scattering

First lattice QCD calculation of coupled DD^*, D^*D^* scattering

$m_\pi \approx 391$ MeV (D^* is stable),
3 lattice volumes ($L \approx 2 - 3$ fm)

Use many meson-meson-like DD^* and D^*D^* ops (I = 0) with optimised D, D^* ops

[**Whyte**, Wilson, Thomas (HadSpec), 2405.15741]

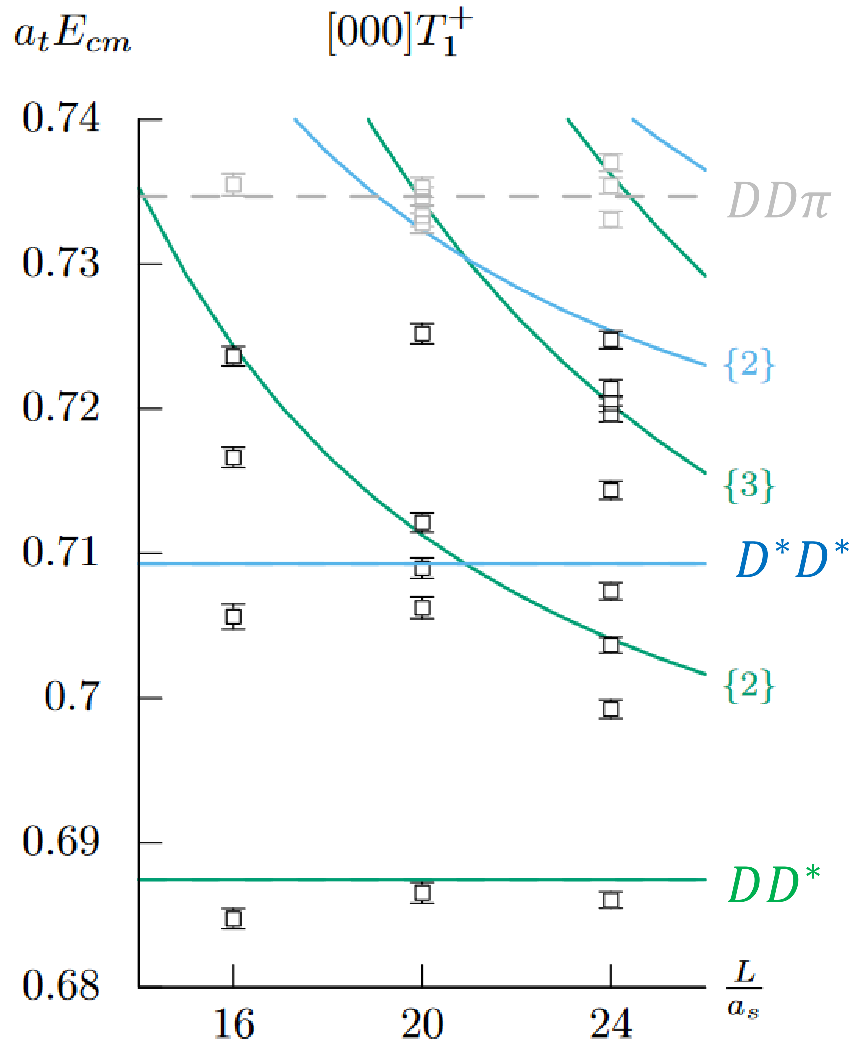
Coupled DD^*, D^*D^* scattering

First lattice QCD calculation of coupled DD^*, D^*D^* scattering

$m_\pi \approx 391$ MeV (D^* is stable),
3 lattice volumes ($L \approx 2 - 3$ fm)

Use many meson-meson-like DD^* and D^*D^* ops (I = 0) with optimised D, D^* ops

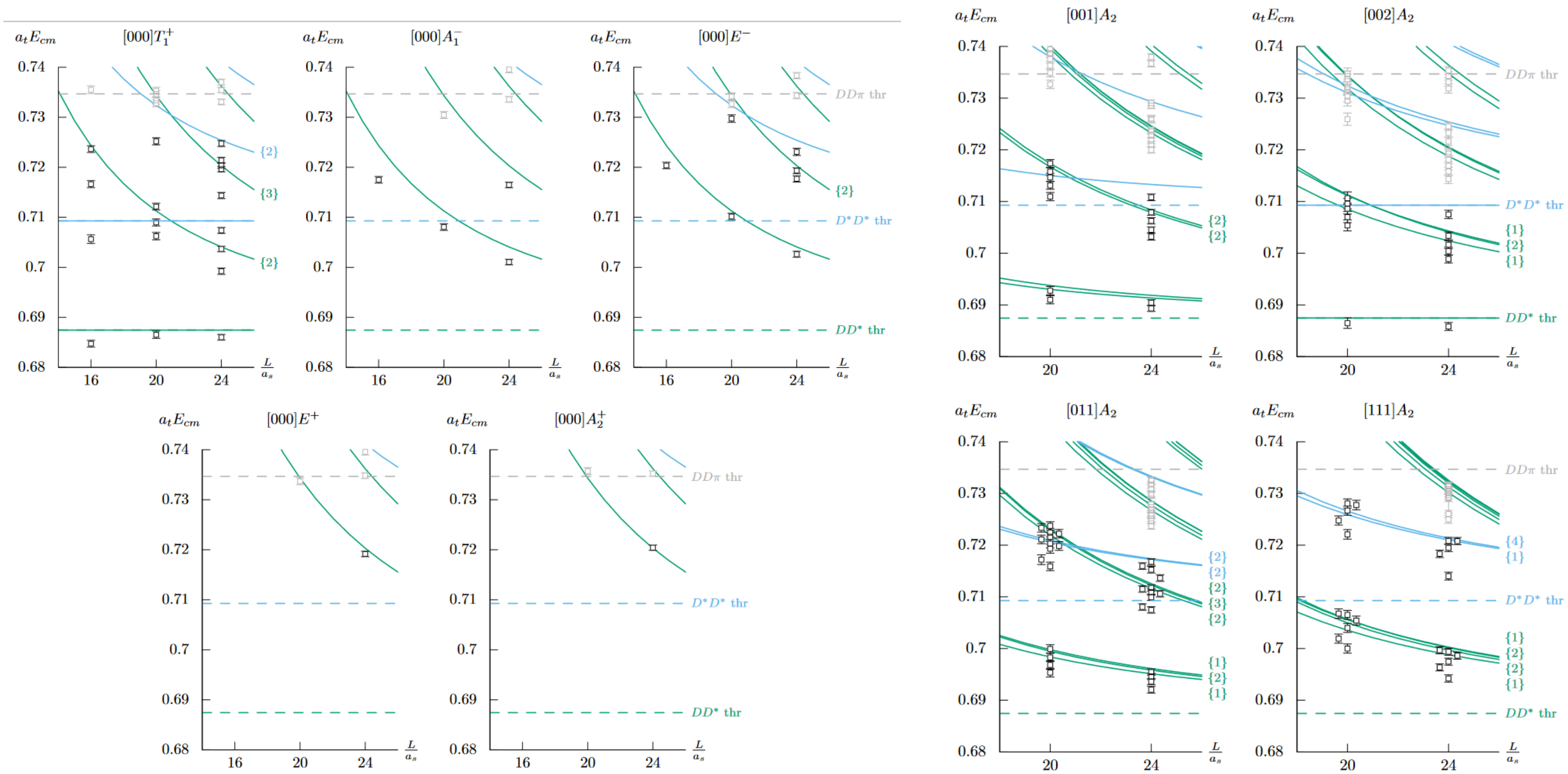
P = [0,0,0] $J^P = 1^+$, ...



[**Whyte**, Wilson, Thomas (HadSpec), 2405.15741]

Coupled DD^*, D^*D^* scattering

[2405.15741]



Use 109 energy levels

Coupled DD^*, D^*D^* scattering

[2405.15741]

Partial wave amplitudes for $J^P = 1^+$:

$DD^*\ l = 0, 2; S = 1$

$D^*D^*\ l = 0, 2; S = 1$

and 'background' partial waves

Coupled DD^*, D^*D^* scattering

[2405.15741]

Partial wave amplitudes for $J^P = 1^+$:

DD^* $l = 0, 2; S = 1$

D^*D^* $l = 0, 2; S = 1$

and 'background' partial waves

K-matrix param. – respects unitarity (conserve prob.) and flexible

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)$$

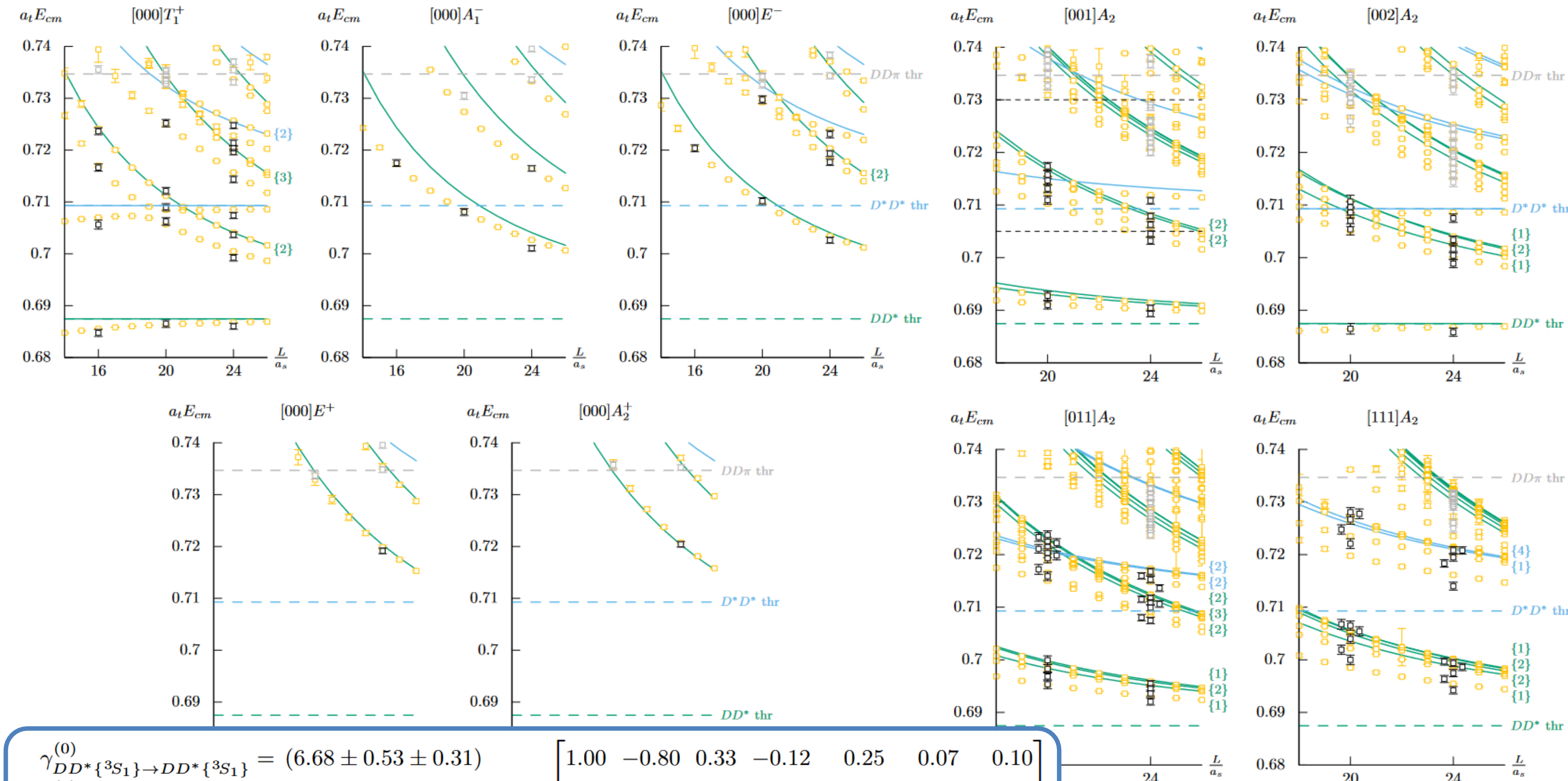
$$\mathrm{Im}[I_{ij}(s)] = -\delta_{ij}\rho_i(s)$$

$$\rho_i(E_{\mathrm{cm}}) = \frac{2k_i}{E_{\mathrm{cm}}}$$

In this work $K(s)_{\ell S J a, \ell' S' J b} = \sum_n \gamma^{(n)}_{\ell S J a, \ell' S' J b} s^n,$

Coupled DD^*, D^*D^* scattering

[2405.15741]

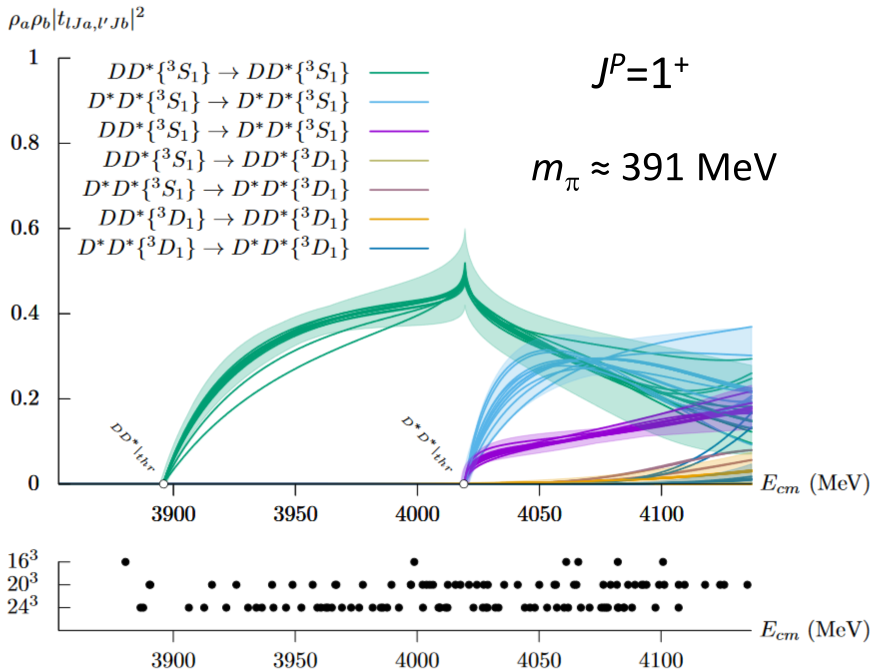


$$
\begin{aligned}
\gamma^{(0)}_{DD^*\{^3S_1\}\to DD^*\{^3S_1\}} &= (6.68 \pm 0.53 \pm 0.31) \\
\gamma^{(1)}_{DD^*\{^3S_1\}\to DD^*\{^3S_1\}} &= (-59 \pm 13 \pm 21)\cdot a_t^2 \\
\gamma^{(0)}_{DD^*\{^3S_1\}\to D^*D^*\{^3S_1\}} &= (4.11 \pm 0.51 \pm 0.94) \\
\gamma^{(0)}_{D^*D^*\{^3S_1\}\to D^*D^*\{^3S_1\}} &= (12.7 \pm 2.5 \pm 2.8) \\
\gamma^{(1)}_{D^*D^*\{^3S_1\}\to D^*D^*\{^3S_1\}} &= (-122 \pm 53 \pm 71)\cdot a_t^2 \\
\gamma^{(0)}_{DD^*\{^3D_1\}\to DD^*\{^3D_1\}} &= (149 \pm 63 \pm 42)\cdot a_t^4 \\
\gamma^{(0)}_{D^*D^*\{^3D_1\}\to D^*D^*\{^3D_1\}} &= (-504 \pm 178 \pm 1095)\cdot a_t^4
\end{aligned}
\quad
\begin{bmatrix}
1.00 & -0.80 & 0.33 & -0.12 & 0.25 & 0.07 & 0.10 \\
 & 1.00 & 0.19 & 0.23 & -0.27 & -0.00 & -0.04 \\
 & & 1.00 & 0.27 & -0.04 & 0.06 & 0.09 \\
 & & & 1.00 & -0.95 & 0.04 & 0.06 \\
 & & & & 1.00 & -0.03 & -0.02 \\
 & & & & & 1.00 & 0.03 \\
 & & & & & & 1.00
\end{bmatrix}
$$

$$\chi^2/N_{\rm dof} = \frac{121.3}{109-14} = 1.28\,.$$

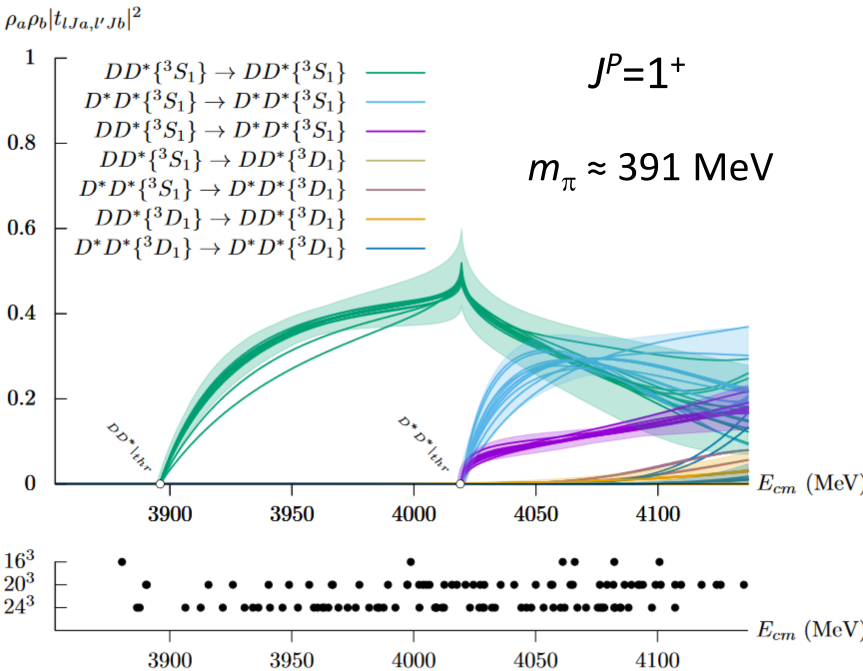
T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering

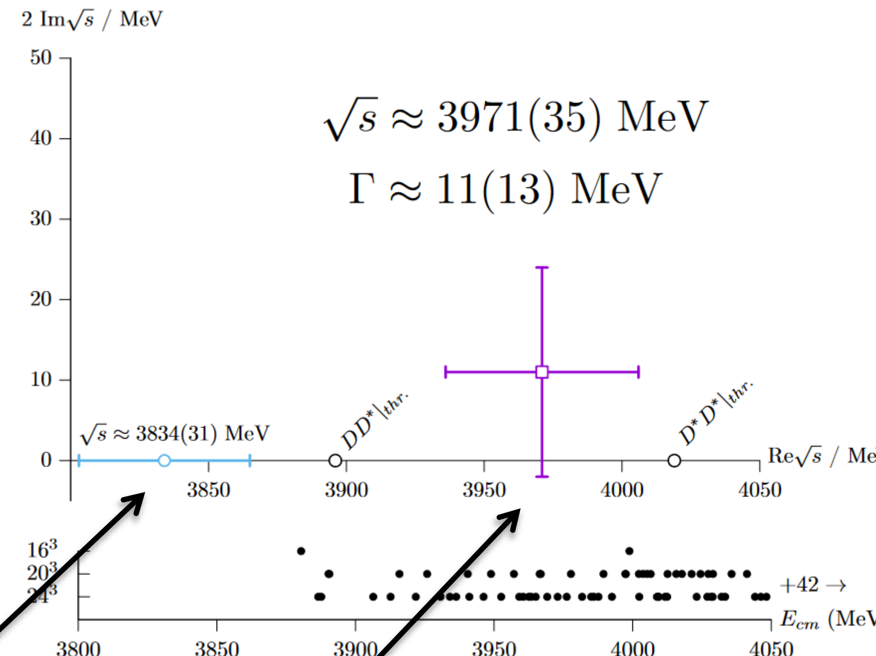
[2405.15741]



T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering

[2405.15741]





Virtual bound state (T_{cc}) below DD^* threshold

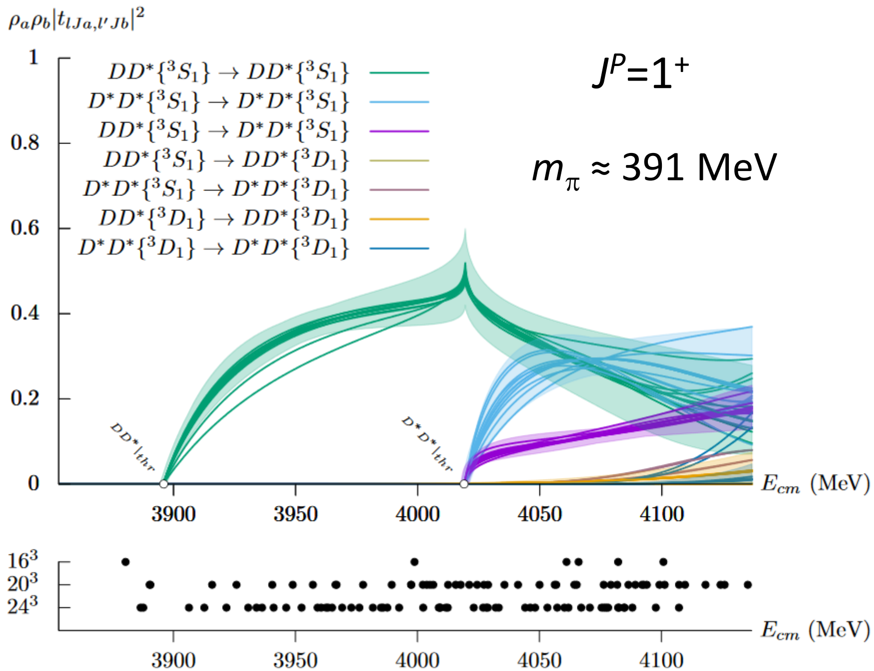
$\sqrt{s} \approx 3834(31)$ MeV

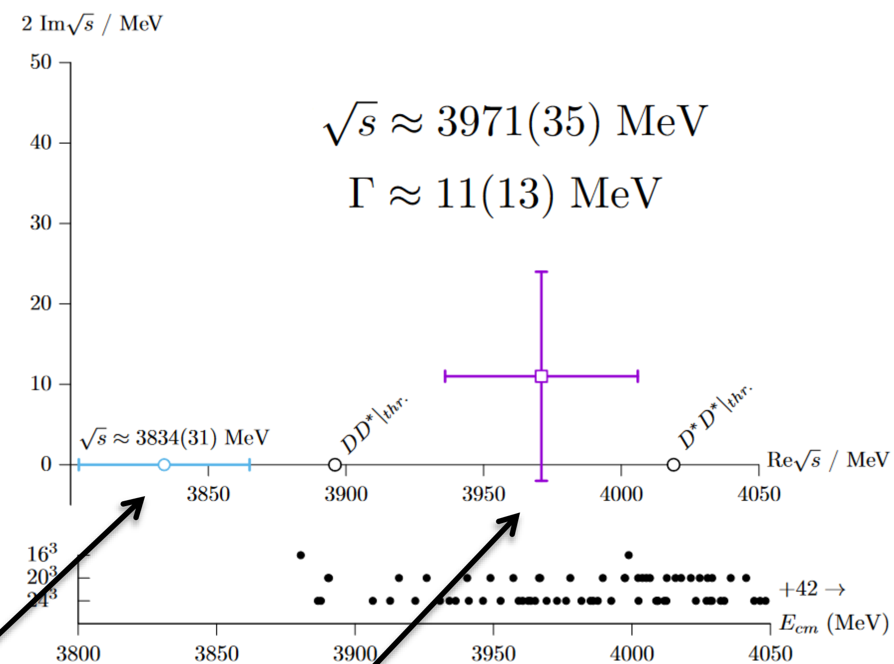
Resonance (T'_{cc}) pole on (+ , –) sheet below D^*D^* threshold (might be seen in DD, $DD\pi$)

– prediction of new state

T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering

[2405.15741]





Virtual bound state (T_{cc}) below DD^* threshold

$\sqrt{s} \approx 3834(31)$ MeV

Resonance (T'_{cc}) pole on (+ , –) sheet below D^*D^* threshold (might be seen in DD, $DD\pi$)

– prediction of new state

Dependence on m_π?

Effect of left hand cut from π exchange (≈18 MeV below DD^* thresh)?

Charmonium scalar (0^{++}) and tensor (2^{++}) resonances

Experimental situation:

- Ground state $\chi_{c0}(1P)$ (0^{++}) and $\chi_{c2}(1P)$ (2^{++}) below $D\bar{D}$ threshold. Above that it is less clear…

Charmonium scalar (0^{++}) and tensor (2^{++}) resonances

Experimental situation:

- Ground state $\chi_{c0}(1P)$ (0^{++}) and $\chi_{c2}(1P)$ (2^{++}) below $D\bar{D}$ threshold. Above that it is less clear…

- $\chi_{c0}(3860) \to D\bar{D}$ (Belle). Not seen in $B^+ \to D^+D^-K^+$ (LHCb). Theoretical reanalyses: may be from pole below $D\bar{D}$ thresh.
- $\chi_{c0}(3930) \to D\bar{D}$ (LHCb)
- $\chi_{c0}(3960) \to D_s\bar{D}_s$ (LHCb)
- $X(3915) \to J/\psi\omega$ (Belle)
- $\chi_{c2}(3930) \to D\bar{D}$ (Belle, BABAR, LHCb)

Charmonium 0^{++} and 2^{++} resonances

$m_\pi \approx 391$ MeV,
3 lattice volumes ($L \approx 2 - 3$ fm)
No c - $\bar{c}$ annihilation.

Use many
fermion-bilinear ($\bar{c}\, \Gamma\, \mathrm{D} \ldots\, c$)
and meson-meson-like ops
($\eta_c \eta$, $D\bar{D}$, $\eta_c \eta'$, $D_s \bar{D}_s$, $D\bar{D}^*$,
$D_s \bar{D}_s^*$, $\psi\omega$, $D^* \bar{D}^*$, $\psi\phi$, $\eta_c \sigma$,
$\chi_{c0,2}\sigma$, …)

First ‘complete’ lattice study
of this energy region.

[**Wilson**, Thomas, Dudek, Edwards (HadSpec),
2309.14070 (PRL), 2309.14071 (PRD)]

Charmonium 0^{++} and 2^{++} resonances

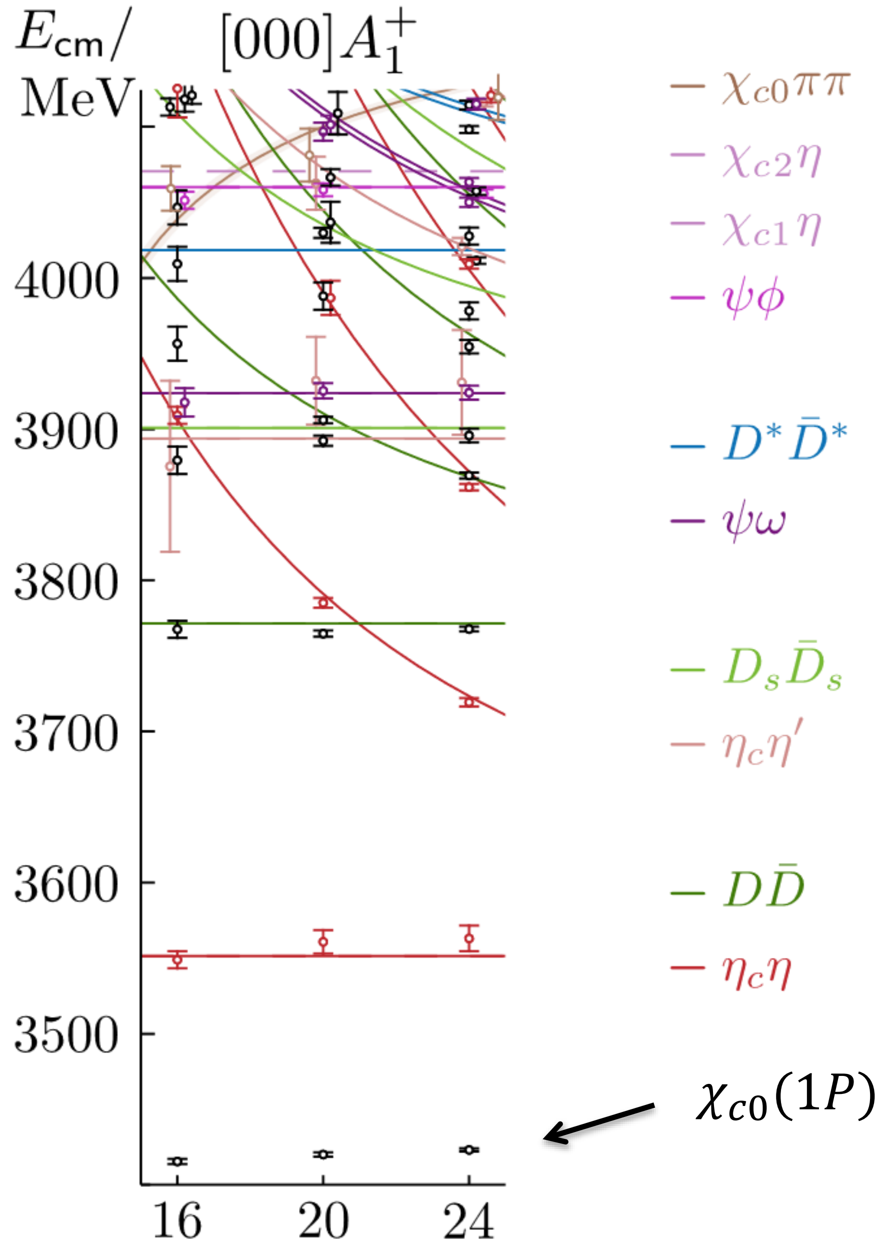
$m_\pi \approx 391$ MeV,
3 lattice volumes ($L \approx 2 - 3$ fm)
No c - $\bar{c}$ annihilation.

Use many fermion-bilinear ($\bar{c}\,\Gamma\,\mathrm{D}\,\ldots\,c$) and meson-meson-like ops ($\eta_c\eta$, $D\bar{D}$, $\eta_c\eta'$, $D_s\bar{D}_s$, $D\bar{D}^*$, $D_s\bar{D}_s^*$, $\psi\omega$, $D^*\bar{D}^*$, $\psi\phi$, $\eta_c\sigma$, $\chi_{c0,2}\sigma$, ...)

First 'complete' lattice study of this energy region.

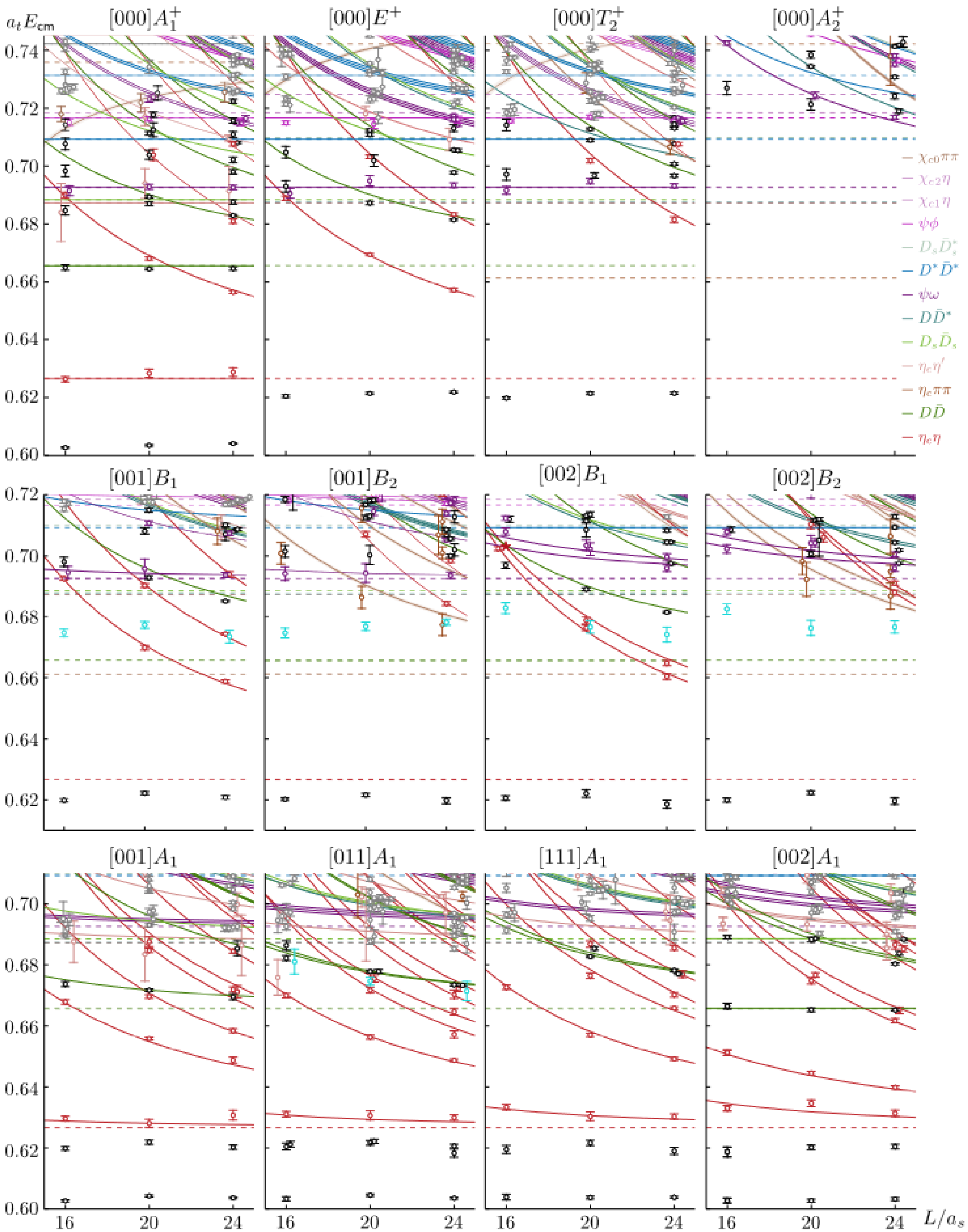
[**Wilson**, Thomas, Dudek, Edwards (HadSpec), 2309.14070 (PRL), 2309.14071 (PRD)]

P = [0,0,0] $J^P = 0^+, (4^+, \ldots)$



Charmonium 0^{++} and 2^{++} resonances

Use more than 200 energy levels



[2309.14070, 2309.14071]

Scattering amplitudes

0^{++}	$\eta_c\eta, D\bar{D}, \eta_c\eta', D_s\bar{D}_s, \psi\omega, D^*\bar{D}^*, \psi\phi\{^1S_0\}$
2^{++}	$\eta_c\eta, D\bar{D}, [\eta_c\eta'], D_s\bar{D}_s\{^1D_2\};\ D\bar{D}^*, [D_s\bar{D}_s^*]\{^3D_2\}$
	$\psi\omega, D^*\bar{D}^*, \psi\phi\{^5S_2\}$
3^{++}	$D\bar{D}^*, \psi\omega, D_s\bar{D}_s^*, \psi\phi\{^3D_3\};\ [\eta_c\sigma\{^1F_3\}]$
	$\psi\omega, D^*\bar{D}^*, [\psi\phi, D_s^*\bar{D}_s^*]\{^5D_3\}$

and 'background' 1^{-+}, 2^{-+}, 3^{-+} amplitudes

[2309.14070, 2309.14071]

Scattering amplitudes

$$K_{ij} = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^2 - s} + \sum_a \gamma_{ij}^{(a)} s^a$$

$J^P = 2^+$

$$\begin{aligned}
a_t m &= (0.7025 \pm 0.0012 \pm 0.0007) \\
g_{D\bar{D}^*\{^3D_2\}} &= (-37.9 \pm 5.0 \pm 3.94) \cdot a_t \\
g_{D_s\bar{D}_s\{^1D_2\}} &= (-3.3 \pm 4.3 \pm 2.5) \cdot a_t \\
g_{D^*\bar{D}^*\{^1S_2\}} &= (1.58 \pm 0.15 \pm 0.22) \cdot a_t^{-1} \\
\gamma_{\eta_c\eta\{^1D_2\}\to\eta_c\eta\{^1D_2\}} &= (16.3 \pm 23.1 \pm 7.5) \cdot a_t^4 \\
\gamma_{D\bar{D}\{^1D_2\}\to D_s\bar{D}_s\{^1D_2\}} &= (-81 \pm 129 \pm 100) \cdot a_t^4 \\
\gamma_{\psi\omega\{^5S_2\}\to\psi\omega\{^5S_2\}} &= (0.55 \pm 0.72 \pm 0.81) \\
\gamma_{\psi\phi\{^5S_2\}\to\psi\phi\{^5S_2\}} &= (2.19 \pm 0.77 \pm 0.11) \\
g_{D\bar{D}\{^1D_2\}} &= 10 \cdot a_t \text{ (fixed)} \\
\chi^2/N_{\text{dof}} &= \tfrac{62.8}{86-8-23} = 1.14\,,
\end{aligned}$$

1.00	−0.04	−0.00	−0.23	−0.14	−0.03	−0.02	−0.03
	1.00	−0.08	−0.28	0.13	0.10	−0.07	−0.12
		1.00	0.00	−0.02	−0.51	0.25	0.08
			1.00	−0.02	−0.15	−0.03	−0.06
				1.00	0.03	0.06	0.02
					1.00	−0.23	−0.01
						1.00	0.27
							1.00

$J^P = 0^+$

$$\begin{aligned}
a_t m &= (0.7065 \pm 0.0015 \pm 0.0004) \\
a_t g_{D\bar{D}\{^1S_0\}} &= (0.1174 \pm 0.0226 \pm 0.0039) \\
a_t g_{D_s\bar{D}_s\{^1S_0\}} &= (0.189 \pm 0.046 \pm 0.026) \\
a_t g_{\psi\omega\{^1S_0\}} &= (-0.127 \pm 0.069 \pm 0.230) \\
a_t g_{D^*\bar{D}^*\{^1S_0\}} &= (0.330 \pm 0.095 \pm 0.023) \\
\gamma_{\eta_c\eta\{^1S_0\}\to\eta_c\eta\{^1S_0\}} &= (0.144 \pm 0.097 \pm 0.038) \\
\gamma_{D\bar{D}\{^1S_0\}\to D_s\bar{D}_s\{^1S_0\}} &= (-0.974 \pm 0.301 \pm 0.027) \\
\gamma_{\eta_c\eta'\{^1S_0\}\to\eta_c\eta'\{^1S_0\}} &= (2.55 \pm 1.03 \pm 0.73) \\
\gamma_{\psi\phi\{^1S_0\}\to\psi\phi\{^1S_0\}} &= (1.36 \pm 0.90 \pm 0.26) \\
\gamma_{\psi\omega\{^5D_4\}\to\psi\omega\{^5D_4\}} &= (162 \pm 254 \pm 43) \cdot a_t^8 \\
\chi^2/N_{\text{dof}} &= \tfrac{91.0}{90-10-16} = 1.42\,,
\end{aligned}$$

1.00	-0.05	-0.17	0.02	-0.33	-0.26	0.23	-0.06	-0.03	-0.23
	1.00	0.52	-0.44	0.64	0.08	-0.07	0.04	-0.04	0.01
		1.00	-0.55	0.69	0.05	-0.23	-0.03	0.00	0.02
			1.00	-0.54	0.05	0.33	0.02	-0.03	0.05
				1.00	0.09	-0.24	0.02	-0.08	0.03
					1.00	-0.07	0.22	0.01	0.08
						1.00	0.05	-0.03	-0.04
							1.00	-0.00	0.03
								1.00	-0.01
									1.00

(Constrain 3^{++}, 1^{-+}, 2^{-+}, 3^{-+} separately and use as inputs here)

[2309.14070, 2309.14071]

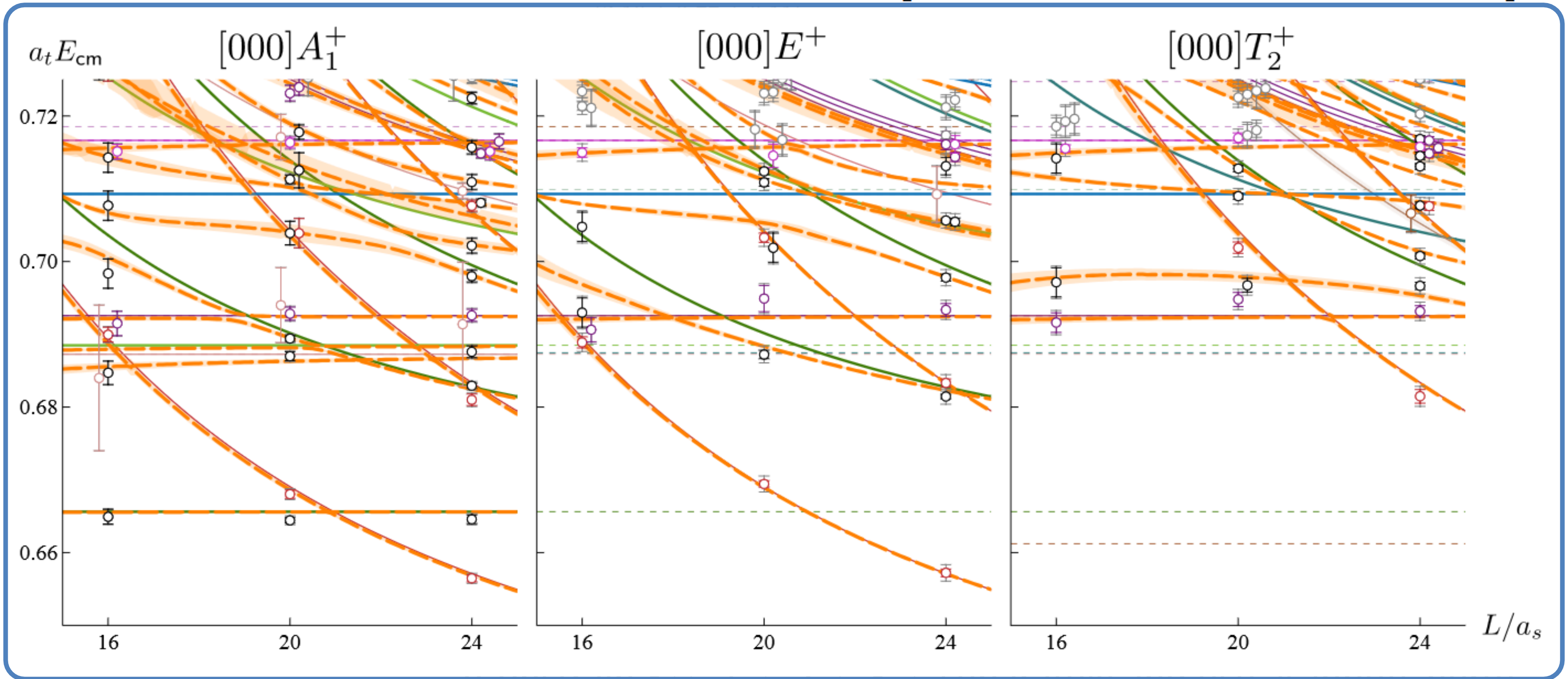
Scattering amplitudes

$$K_{ij} = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^2 - s} + \sum_a \gamma_{ij}^{(a)} s^a$$

$J^P = 2^+$

$$\begin{aligned}
a_t m &= (0.7025 \pm 0.0012 \pm 0.0007) \\
g_{D\bar{D}^*\{^3D_2\}} &= (-37.9 \pm 5.0 \pm 3.94) \cdot a_t \\
g_{D_s\bar{D}_s\{^1D_2\}} &= (-3.3 \pm 4.3 \pm 2.5) \cdot a_t \\
g_{D^*\bar{D}^*\{^1S_2\}} &= (1.58 \pm 0.15 \pm 0.22) \cdot a_t^{-1} \\
\gamma_{\eta_c\eta\{^1D_2\}\to\eta_c\eta\{^1D_2\}} &= (16.3 \pm 23.1 \pm 7.5) \cdot a_t^4 \\
\gamma_{D\bar{D}\{^1D_2\}\to D_s\bar{D}_s\{^1D_2\}} &= (-81 \pm 129 \pm 100) \cdot a_t^4 \\
\gamma_{\psi\omega\{^5S_2\}\to\psi\omega\{^5S_2\}} &= (0.55 \pm 0.72 \pm 0.81)
\end{aligned}$$

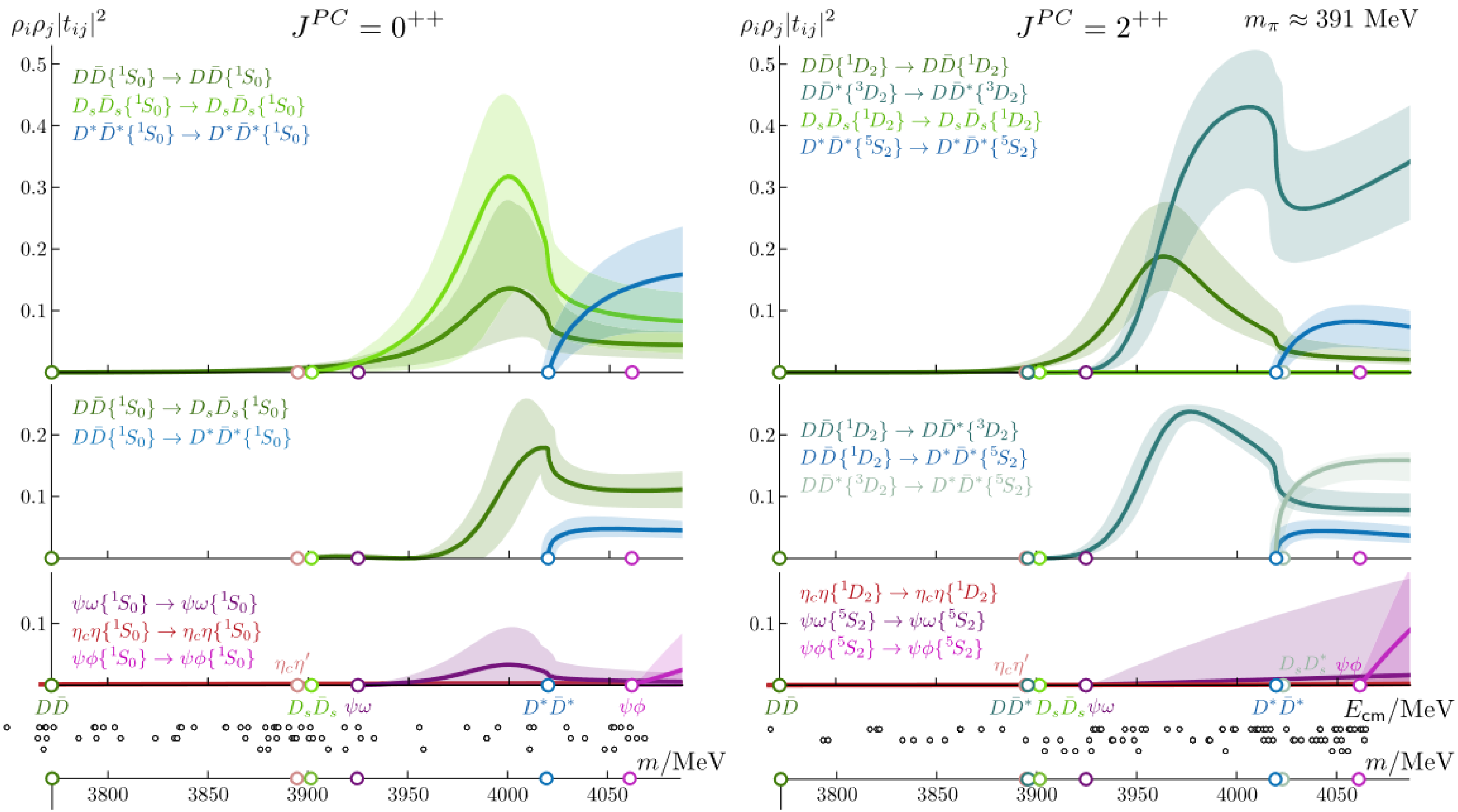
$$\begin{bmatrix}
1.00 & -0.04 & -0.00 & -0.23 & -0.14 & -0.03 & -0.02 & -0.03 \\
 & 1.00 & -0.08 & -0.28 & 0.13 & 0.10 & -0.07 & -0.12 \\
 & & 1.00 & 0.00 & -0.02 & -0.51 & 0.25 & 0.08 \\
 & & & 1.00 & -0.02 & -0.15 & -0.03 & -0.06 \\
 & & & & 1.00 & 0.03 & 0.06 & 0.02 \\
 & & & & & 1.00 & -0.23 & -0.01 \\
 & & & & & & 1.00 & 0.27 \\
 & & & & & & & 1.00
\end{bmatrix}$$



(constrain ... separately and use as inputs here)

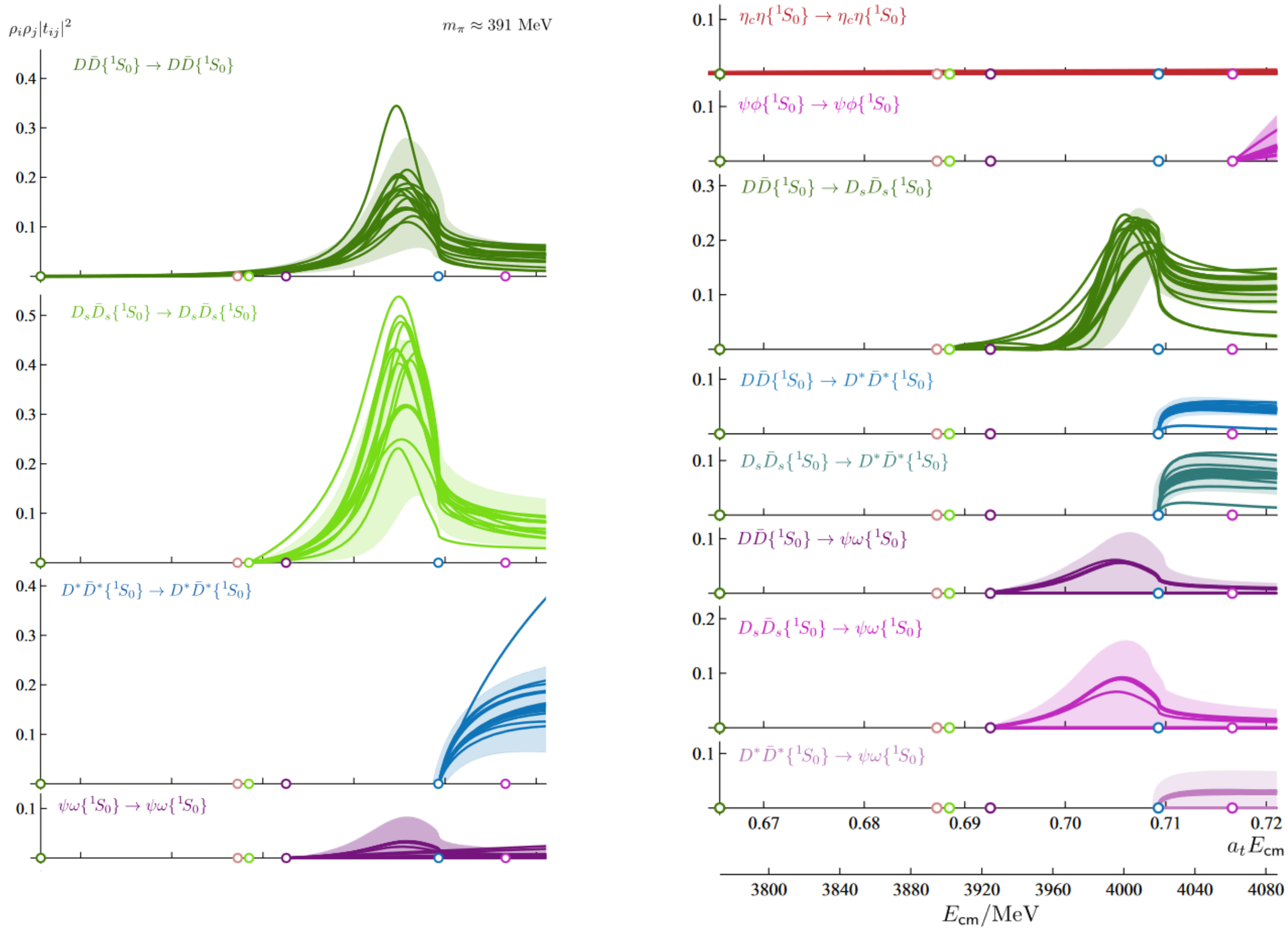
[2309.14070, 2309.14071]

0^{++} and 2^{++} scattering amplitudes



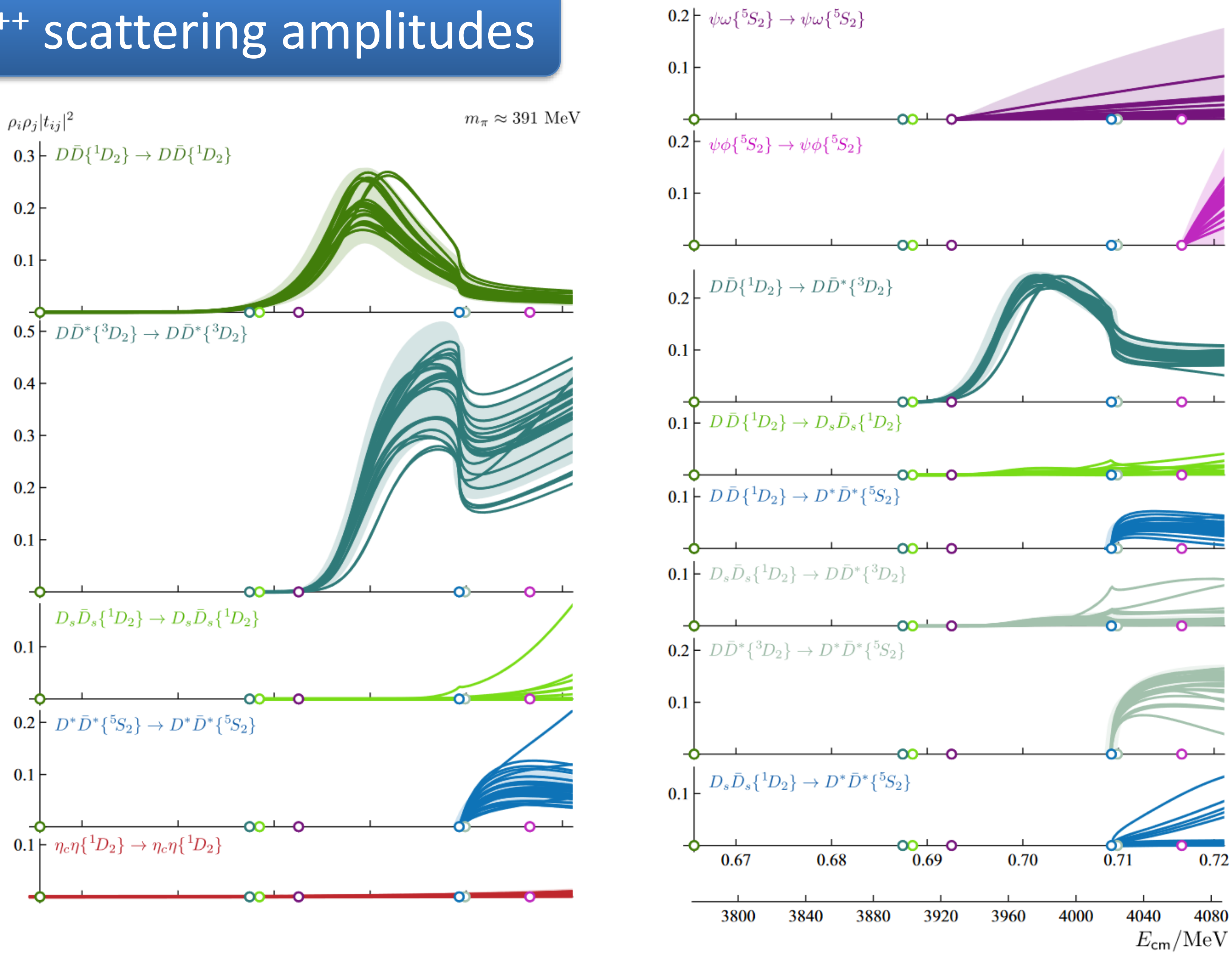
[2309.14070, 2309.14071]

0^{++} scattering amplitudes



[2309.14070, 2309.14071]

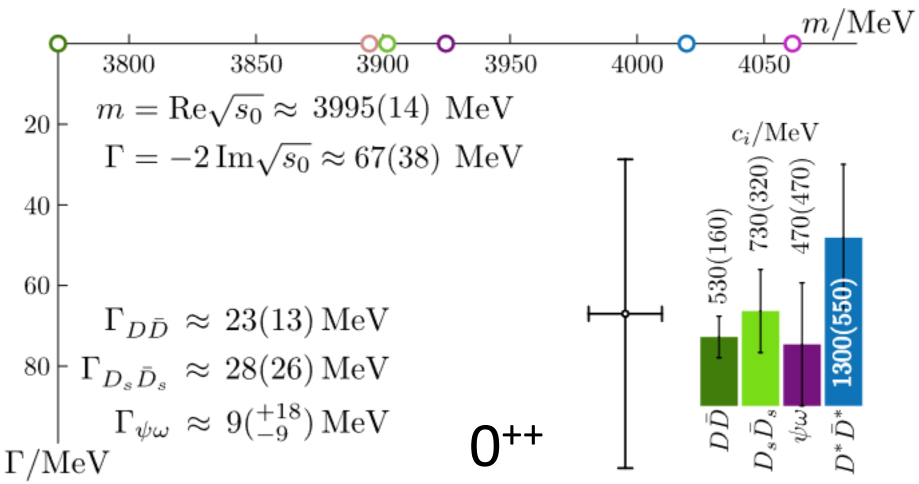
2^{++} scattering amplitudes



[2309.14070, 2309.14071]

Charmonium 0^{++} and 2^{++} resonances

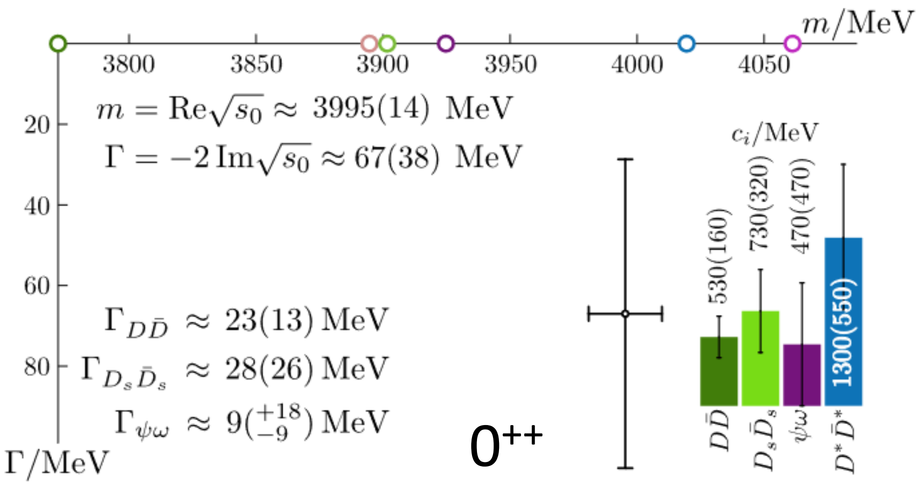
[2309.14070, 2309.14071]

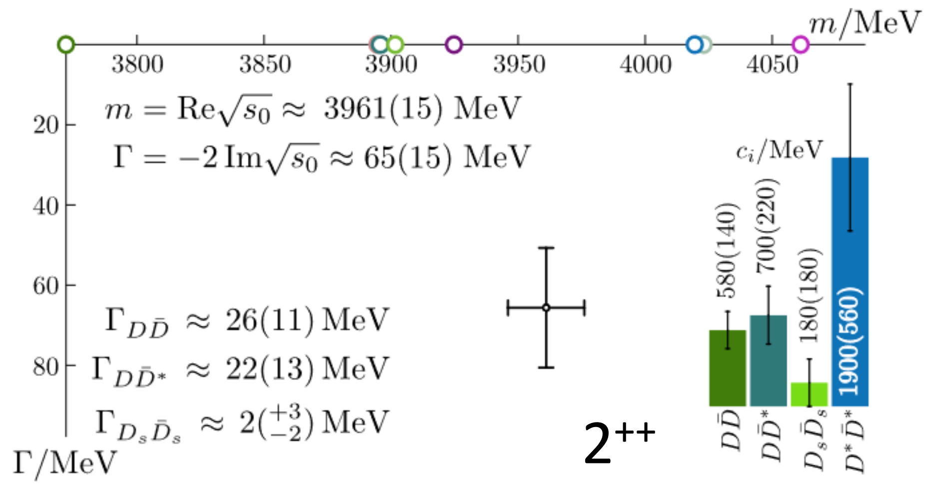


$$t_{ij} \sim \frac{c_i c_j}{(s_0 - s)}$$

Charmonium 0^{++} and 2^{++} resonances

[2309.14070, 2309.14071]

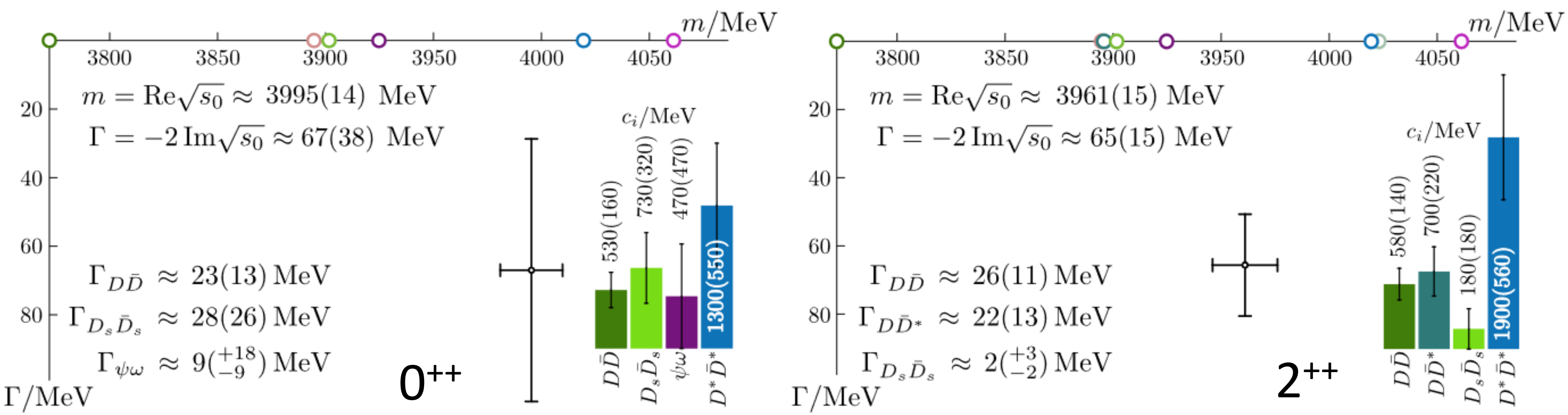




$$t_{ij} \sim \frac{c_i c_j}{(s_0 - s)}$$

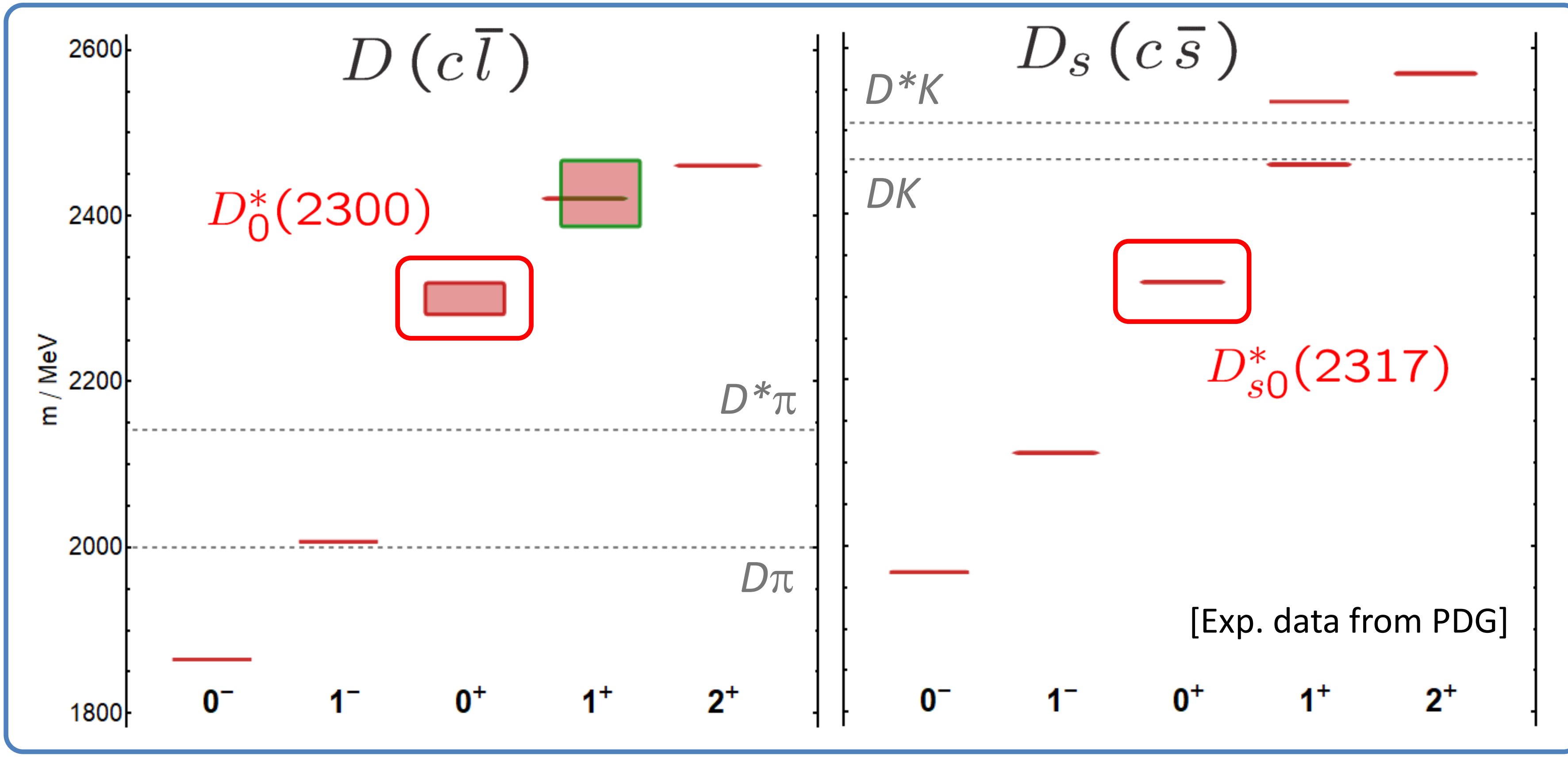
Charmonium 0^{++} and 2^{++} resonances

[2309.14070, 2309.14071]

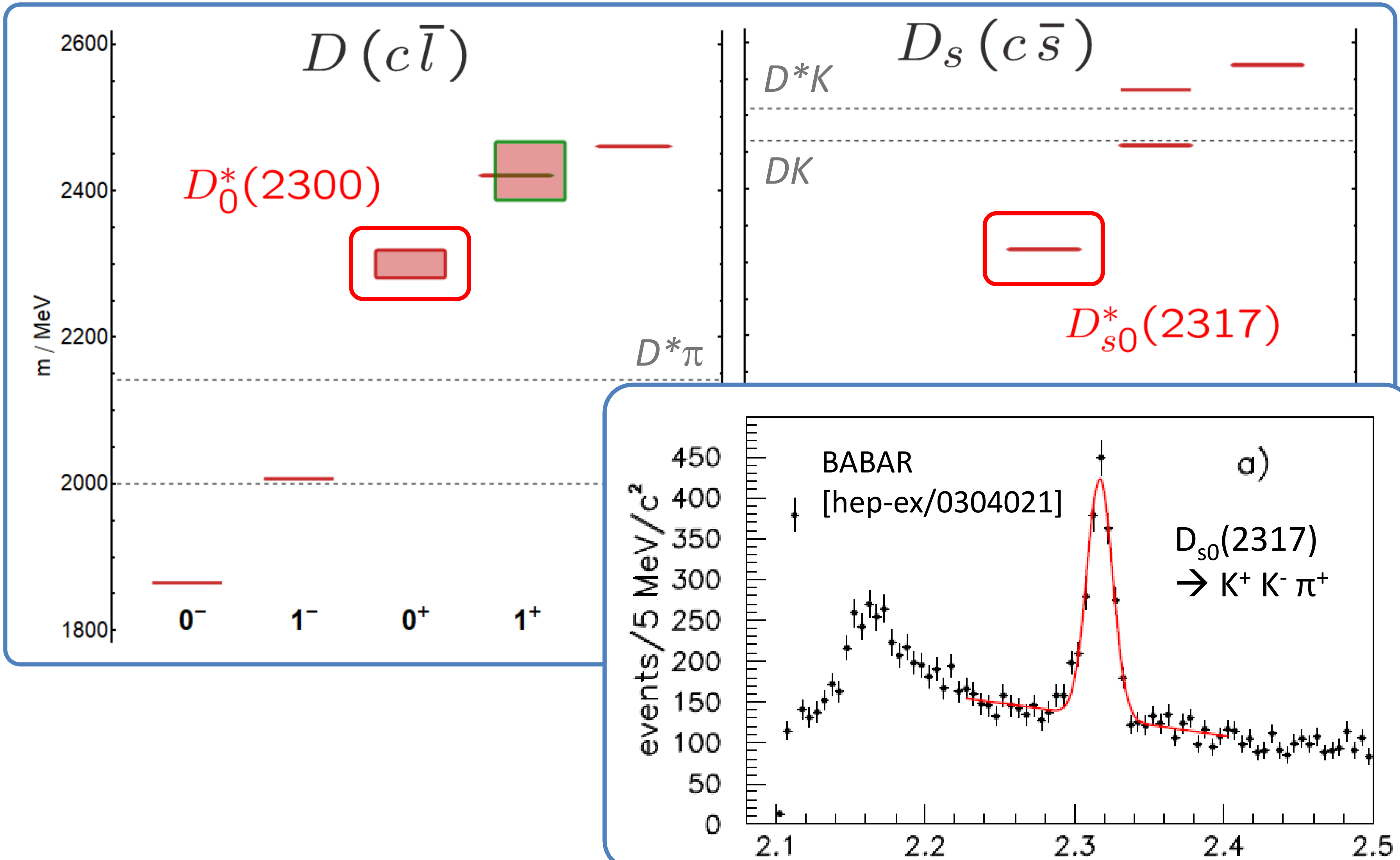


- Only one 0^{++} and one 2^{++} resonance up to ≈ 4100 MeV.
- No large scattering amps in channels with $\bar{c}c$ + light meson (OZI)
- Above ground state χ_{c0} no other 0^{++} bound states or near-$D\bar{D}$ / $D_s\bar{D}_s$ threshold resonances.
  c.f. claims for an additional $\chi_{c0}(3860)$ by Belle [1704.01872], lattice calculation by Prelovsek *et al* [2011.02542], some models and some reanalysis of experimental data.
- (Also bound state in 2^{-+} and narrow resonance in 3^{++}.)

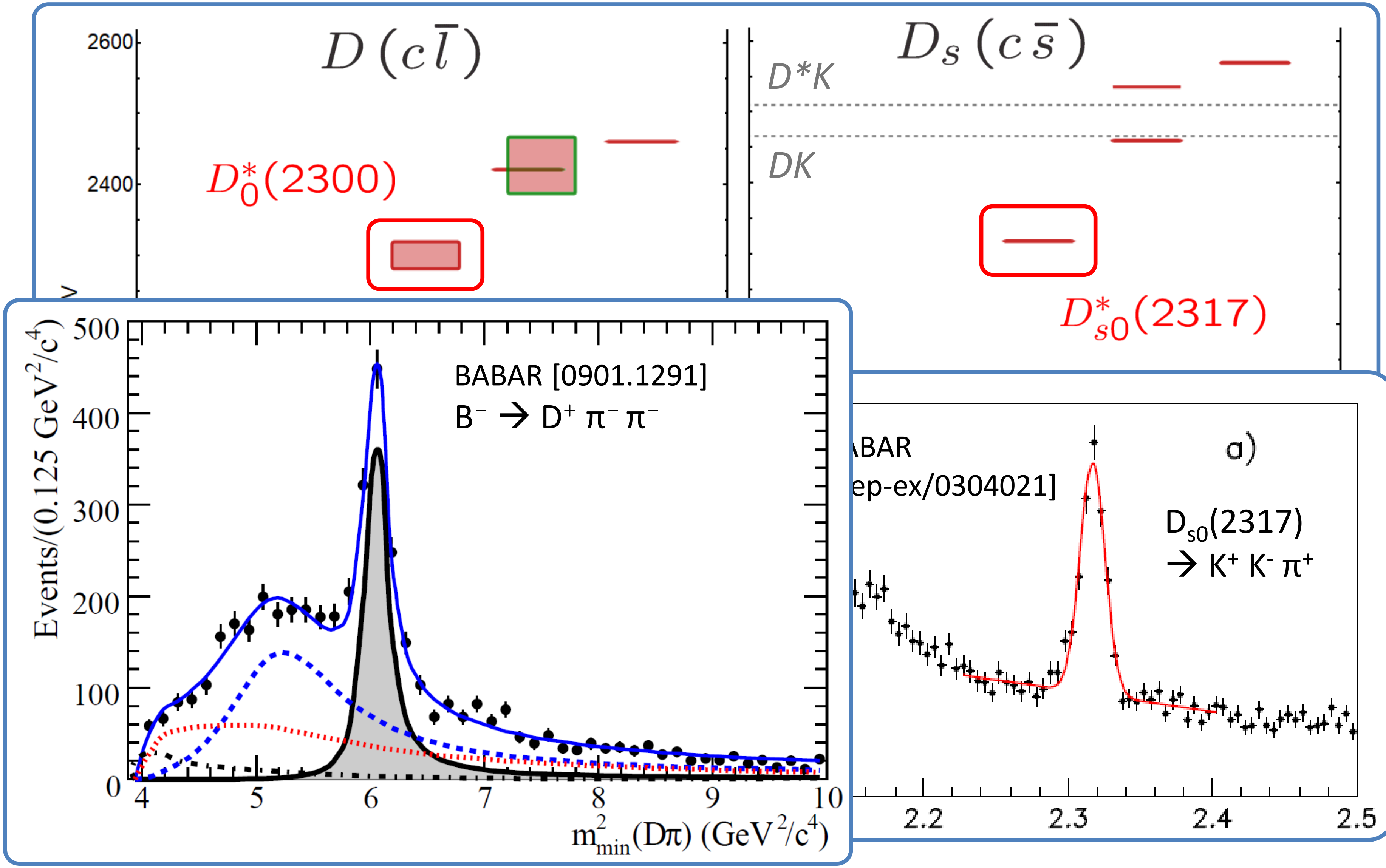
Charm (D) and charm-strange (D_s) mesons



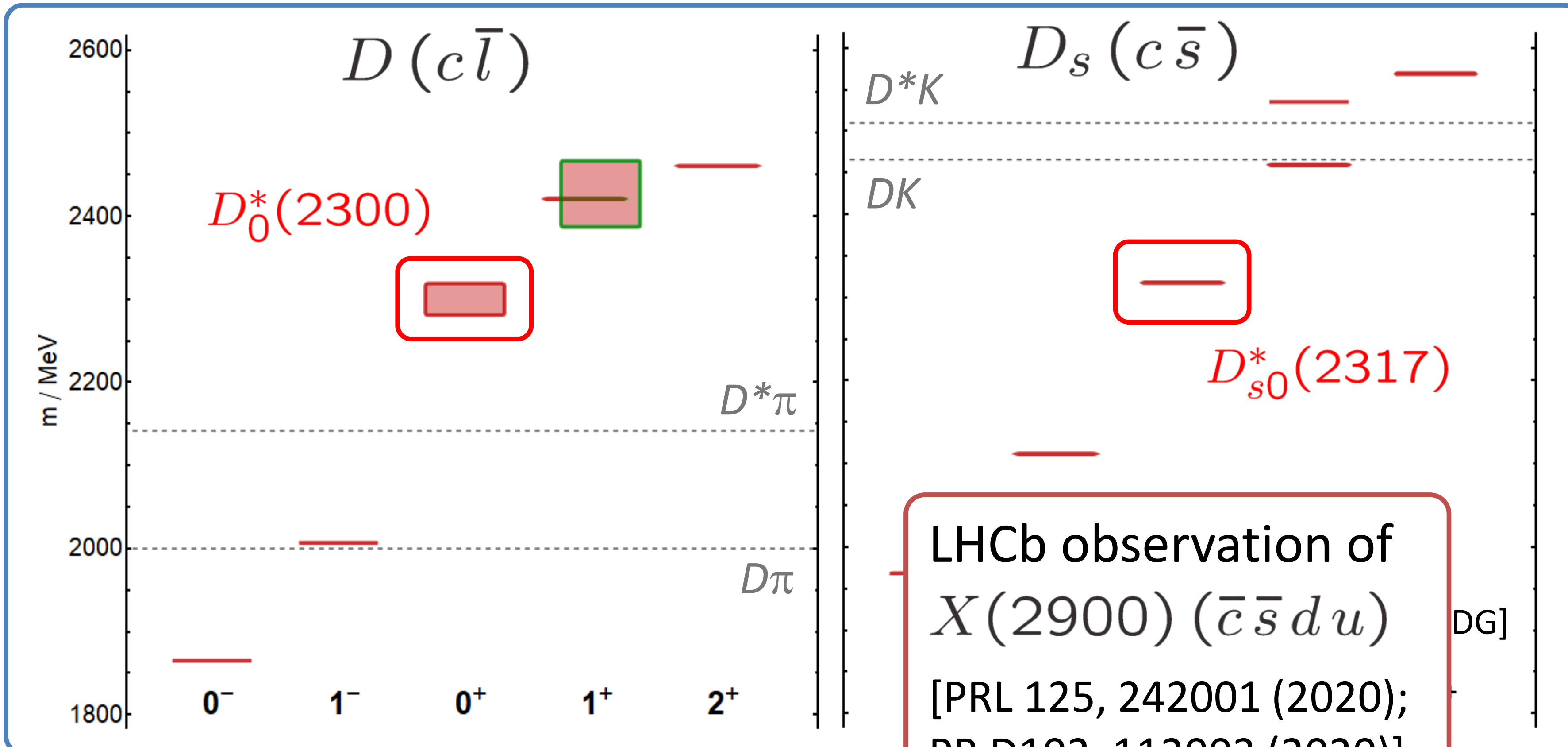
Charm (D) and charm-strange (D_s) mesons



Charm (D) and charm-strange (D_s) mesons



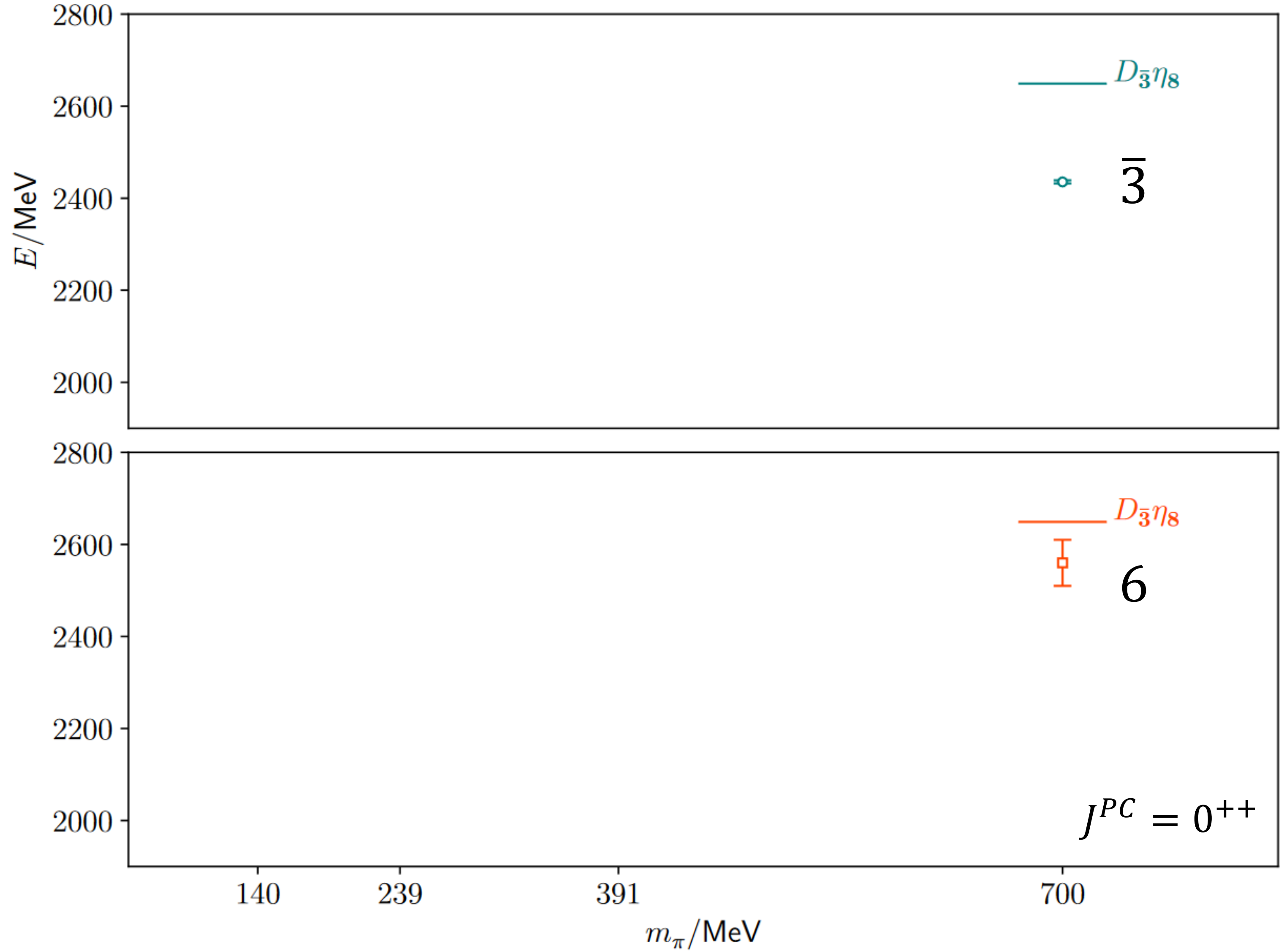
Charm (D) and charm-strange (D_s) mesons



LHCb observation of
$X(2900)\ (\bar{c}\,\bar{s}\,d\,u)$
[PRL 125, 242001 (2020);
PR D102, 112003 (2020)]

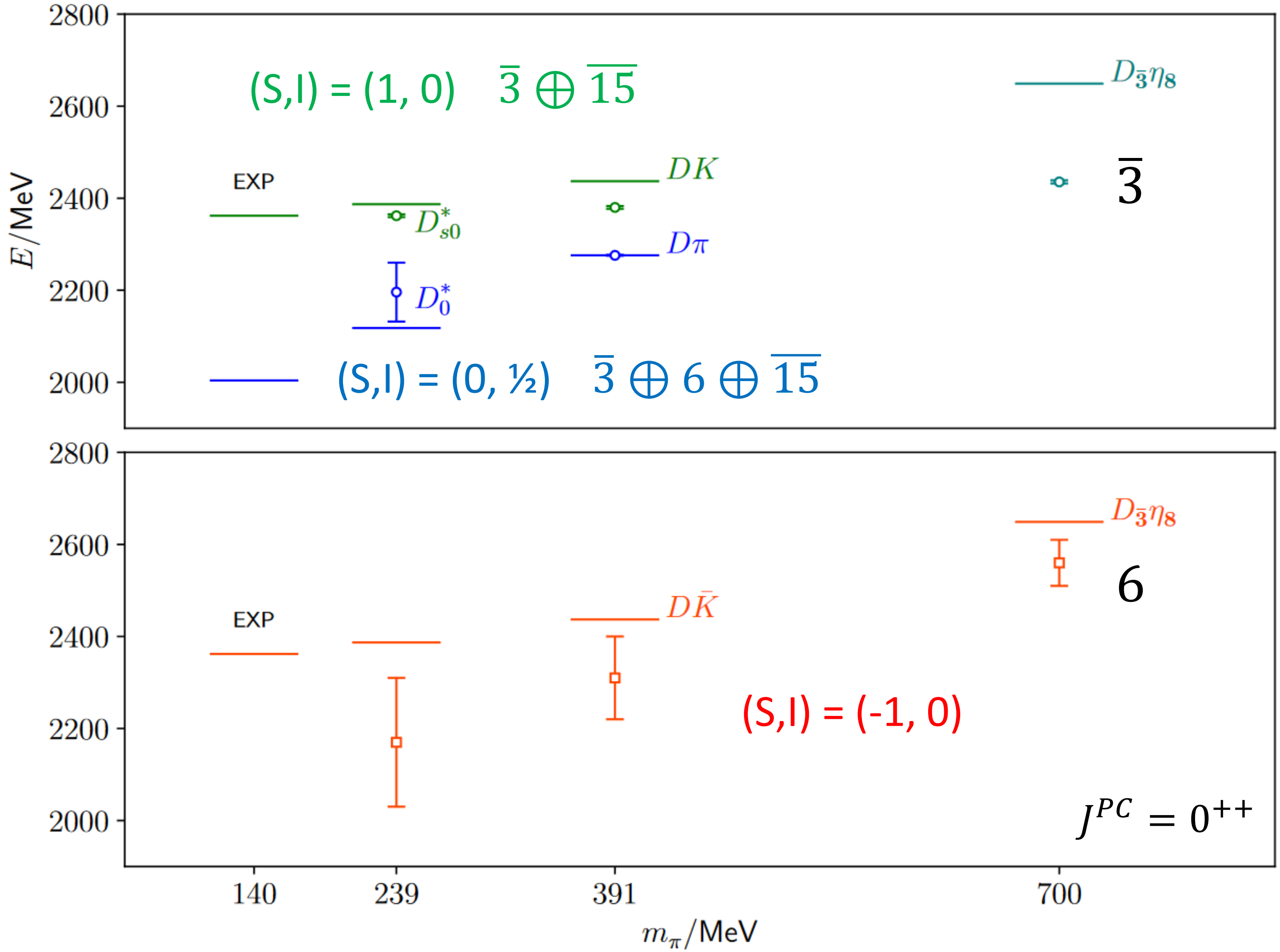
DK/π – dependence on m_π

[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



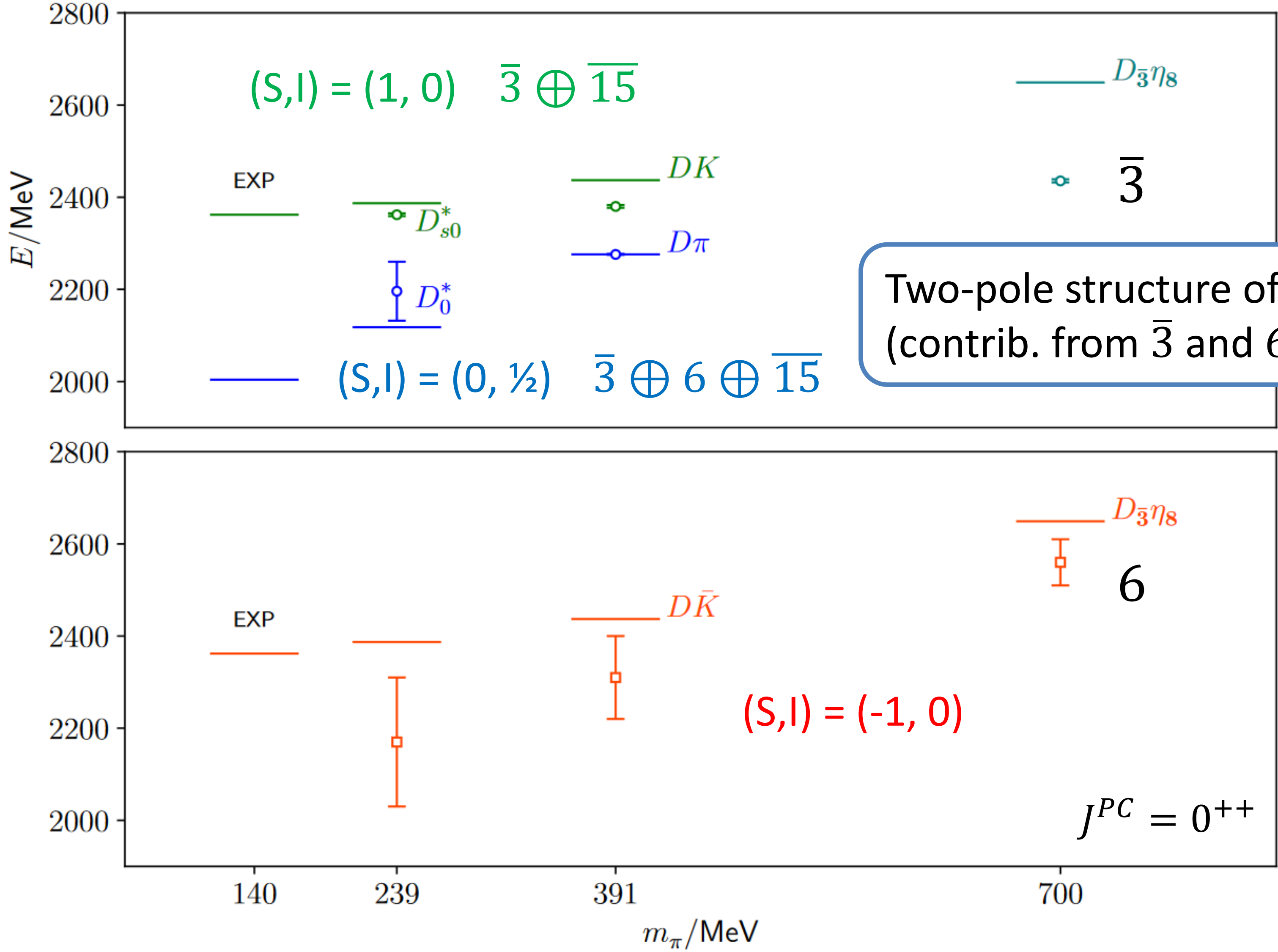
DK/π – dependence on m_π

[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



DK/π – dependence on m_π

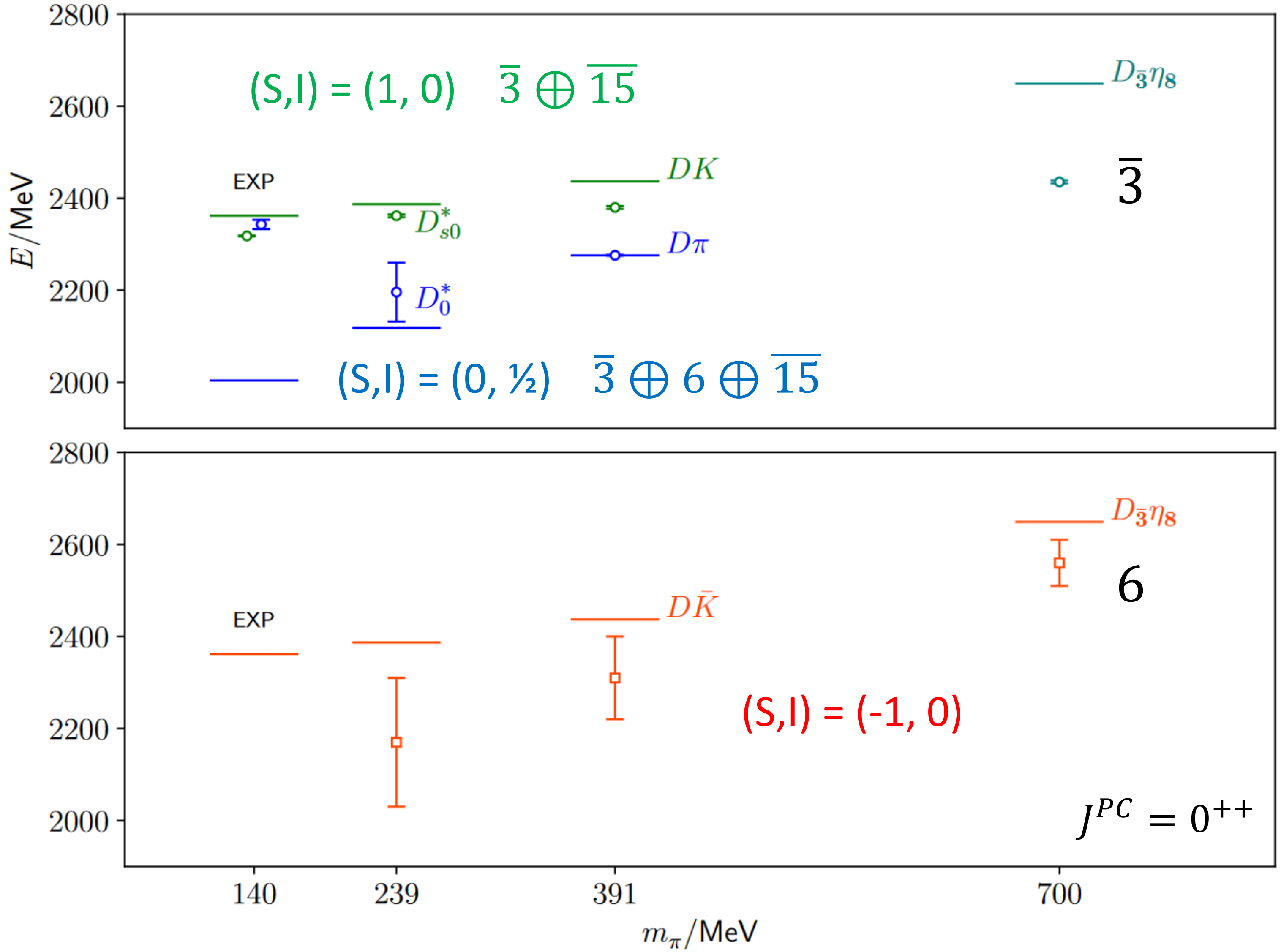
[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



Two-pole structure of D_0^*?
(contrib. from $\bar{3}$ and 6)

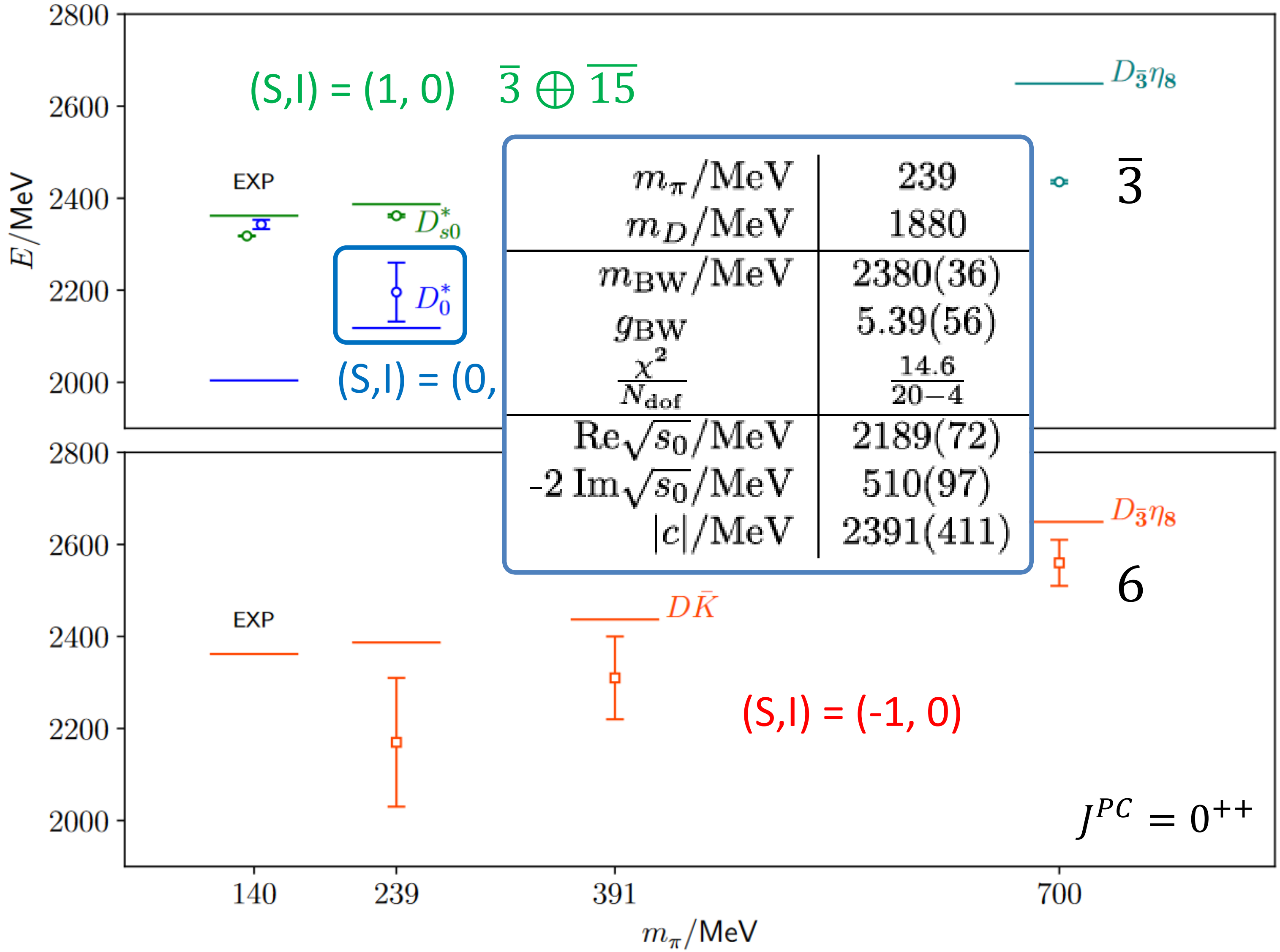
DK/π – dependence on m_π

[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



DK/π – dependence on m_π

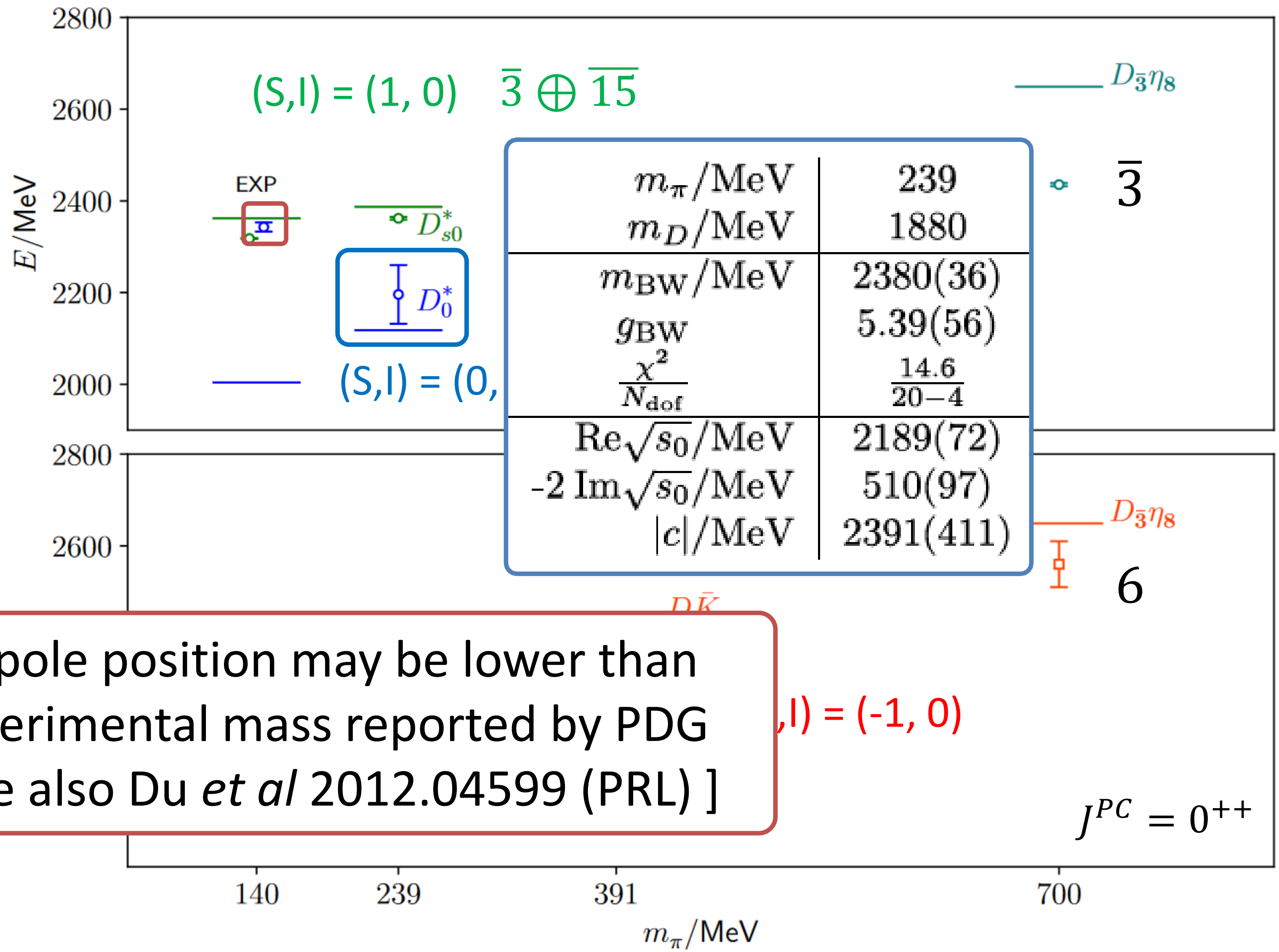
[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]



m_π/MeV	239
m_D/MeV	1880
m_{BW}/MeV	2380(36)
g_{BW}	5.39(56)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$
$\mathrm{Re}\sqrt{s_0}$/MeV	2189(72)
-2 $\mathrm{Im}\sqrt{s_0}$/MeV	510(97)
$\lvert c\rvert$/MeV	2391(411)

DK/π – dependence on m_π

[HadSpec, 1607.07093, 2008.06432, 2102.04974, 2403.10498]

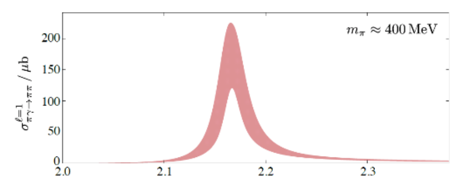


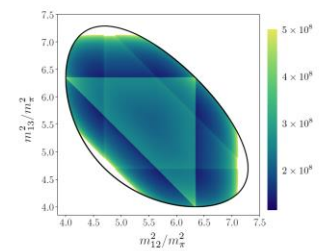
m_π/MeV	239
m_D/MeV	1880
m_{BW}/MeV	2380(36)
g_{BW}	5.39(56)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$
$\mathrm{Re}\sqrt{s_0}$/MeV	2189(72)
$-2\,\mathrm{Im}\sqrt{s_0}$/MeV	510(97)
$\lvert c\rvert$/MeV	2391(411)

D_0^* pole position may be lower than experimental mass reported by PDG [See also Du *et al* 2012.04599 (PRL)]

Summary

- A few examples of recent lattice QCD calculations of charm/charmonium(-like) mesons.
  - T_{cc} and T'_{cc} in coupled DD^*, D^*D^* scattering.
  - Scalar (0^{++}) and tensor (2^{++}) charmonium resonances (only one of each in energy region investigated).
  - [DK/π at SU(3)$_F$ sym. point and dependence on m_π]
- Many other calcs, e.g. π_1 (exotic $J^{PC} = 1^{-+}$), light scalars.
- Study evolution as vary light-quark masses.
- Effect of left hand cut?
- Three (or more!?) hadron scattering.
- Probe structure, e.g. transitions and form factors.





UNIVERSITY OF CAMBRIDGE

UKRI Science and Technology Facilities Council

DiRAC

www.hadspec.org

had spec