
On the Top Mass Parameter in Monte-Carlo Event-Generators

This talk reports on old and new work Oliver Jin,
Simon Plätzer and Daniel Samitz

arXiv:1807.06617

arXiv:2004.12915

arXiv:2404.09856

arXiv:2504.xxxxx

André H. Hoang

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$\int dk \Pi$ Doktoratskolleg
Particles and Interactions



FWF
Der Wissenschaftsfonds.

Most Precise Top Mass Measurements Method

LHC+Tevatron: Direct top mass measurements

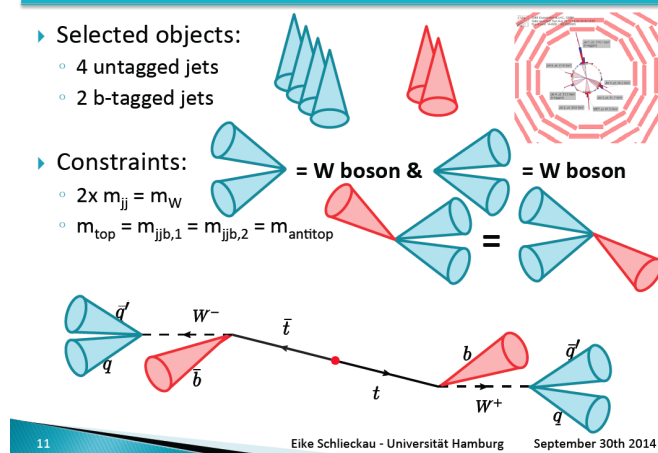
Kinematic Fit

Selected objects:

- 4 untagged jets
- 2 b-tagged jets

Constraints:

- $2 \times m_{jj} = m_W$
- $m_{top} = m_{jjb,1} = m_{jjb,2} = m_{antitop}$



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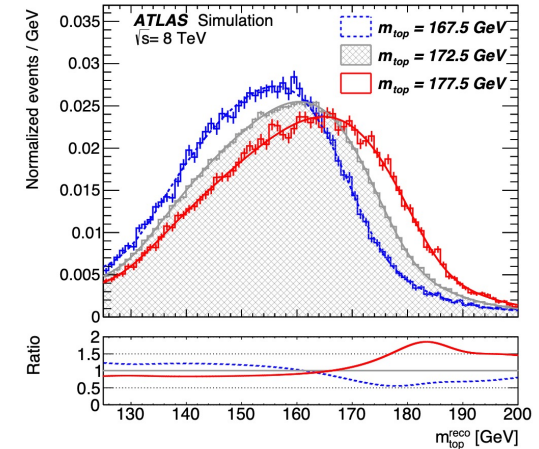
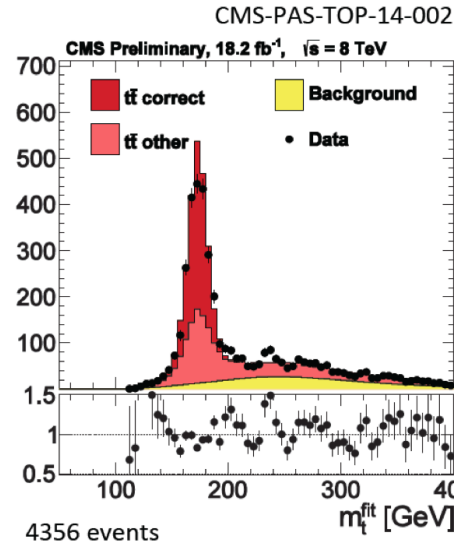
Eike Schlieckau - Universität Hamburg

September 30th 2014

kinematic mass determination

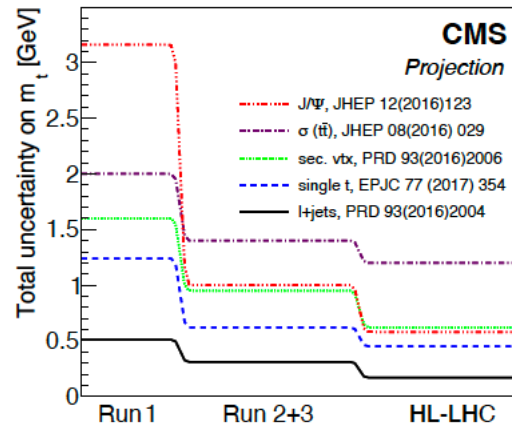
based on the picture of a top quark particle

Determination of the best-fit value of the Monte-Carlo top quark mass parameter



$$m_t^{\text{MC}} = 171.77 \pm 0.37 \text{ GeV}$$

CMS collaboration. arXiv: 2302.01967



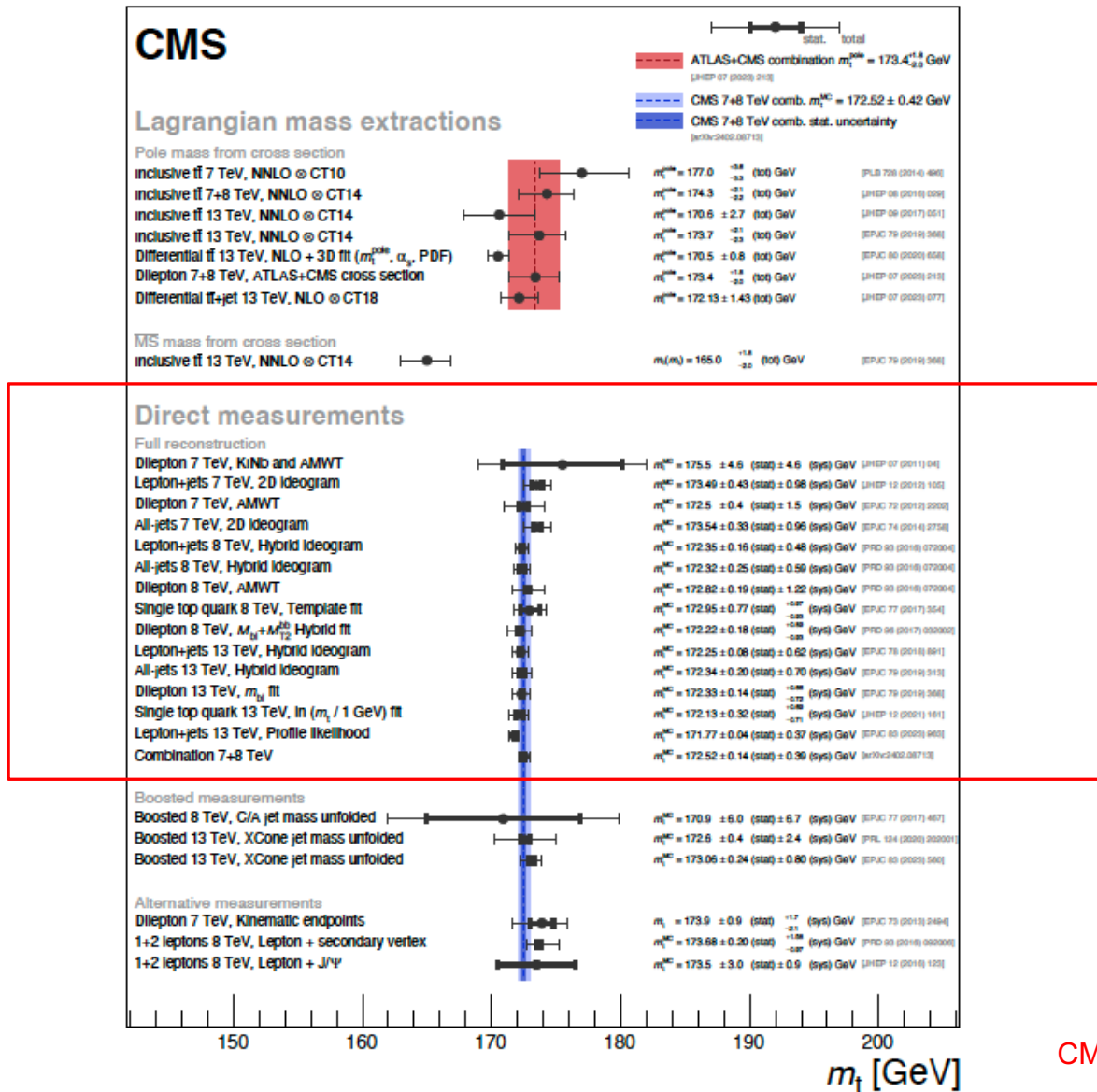
⊕ High top mass sensitivity

⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

← $\Delta m_t \sim 200 \text{ MeV}$ (projection)

Top Mass Measurement



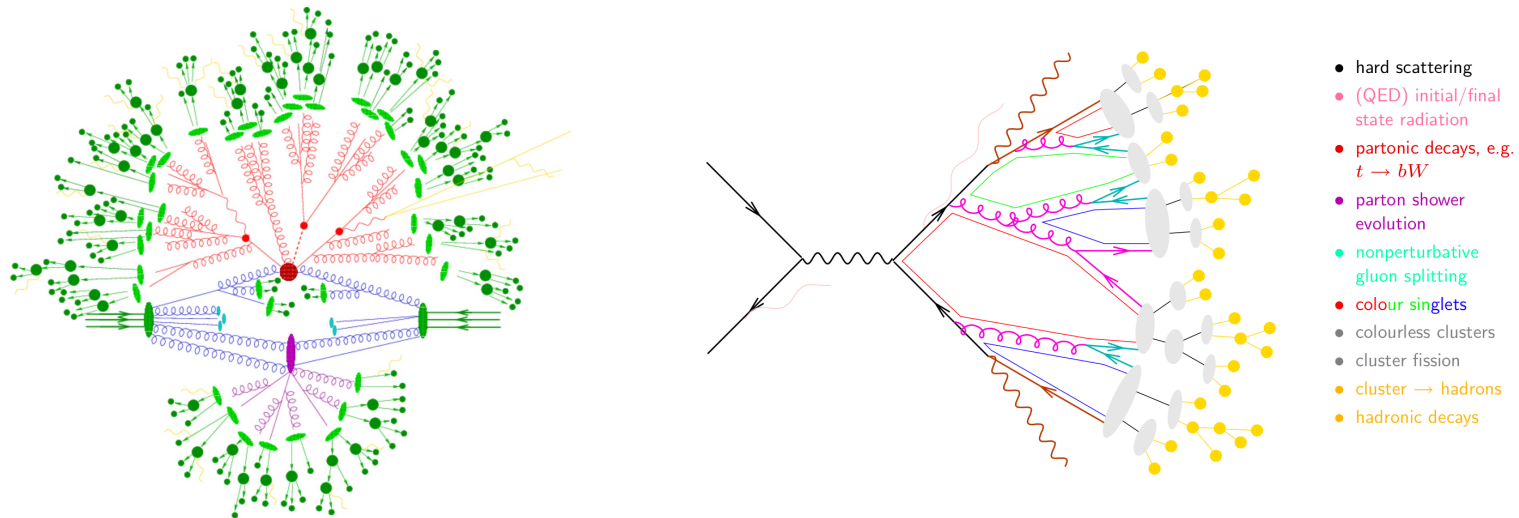
What is m_t^{MC} ?

What does the question mean in the first place?

→ It means that we can provide the relation
where δm^{scheme} can be **computed in pQCD**

$$m_t^{\text{MC}} = m_t^{\text{scheme}}(\mu) + \frac{\alpha_s(\mu)}{\pi} \delta m^{\text{scheme}} + \dots$$

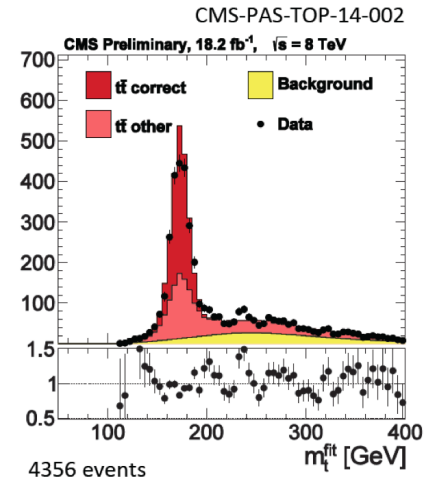
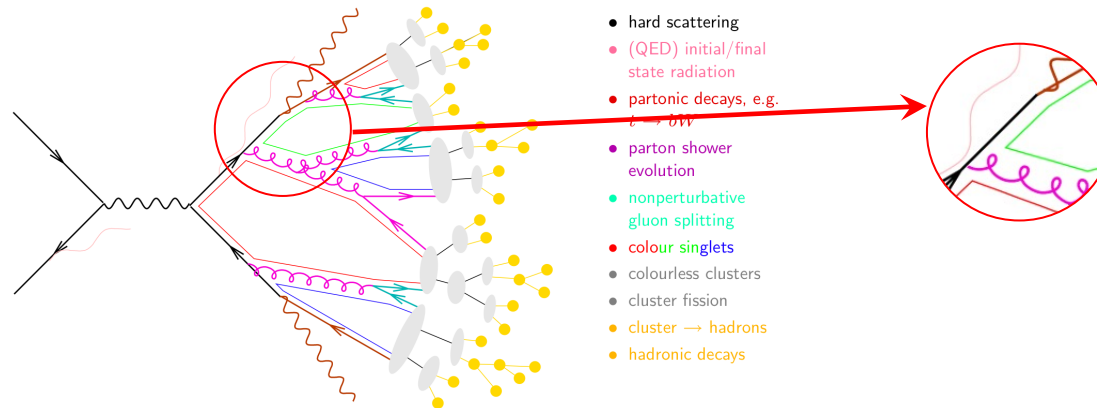
The issue is complicated as we must understand and control the interplay of the different components of MC event generators.



The nature of question is intrinsically theoretical.

What is m_t^{MC} ? (Answer 1.0)

We can give a rough answer already from a simple consideration:



Direct measurements are based on the picture of a top quark particle

- Direct measurements are based on reconstructed top quarks on the top quark resonance
- Employed MCs (Pythia, Herwig) are based on the narrow width limit for on-shell top quark
- MCs model QCD, hadronization and unstable particle effects + m_t^{MC} = mass in propagator

$\Rightarrow m_t^{\text{MC}}$ is close to the top quark pole mass m_t^{pole}

Conservative statement: $m_t^{\text{MC}} = m_t^{\text{pole}} + \mathcal{O}(\Gamma_t, \Lambda_{\text{QCD}})$

“Within a precision of about 1 GeV we can consider the top quark as a physical particle with a rest mass which is close to the pole mass.”

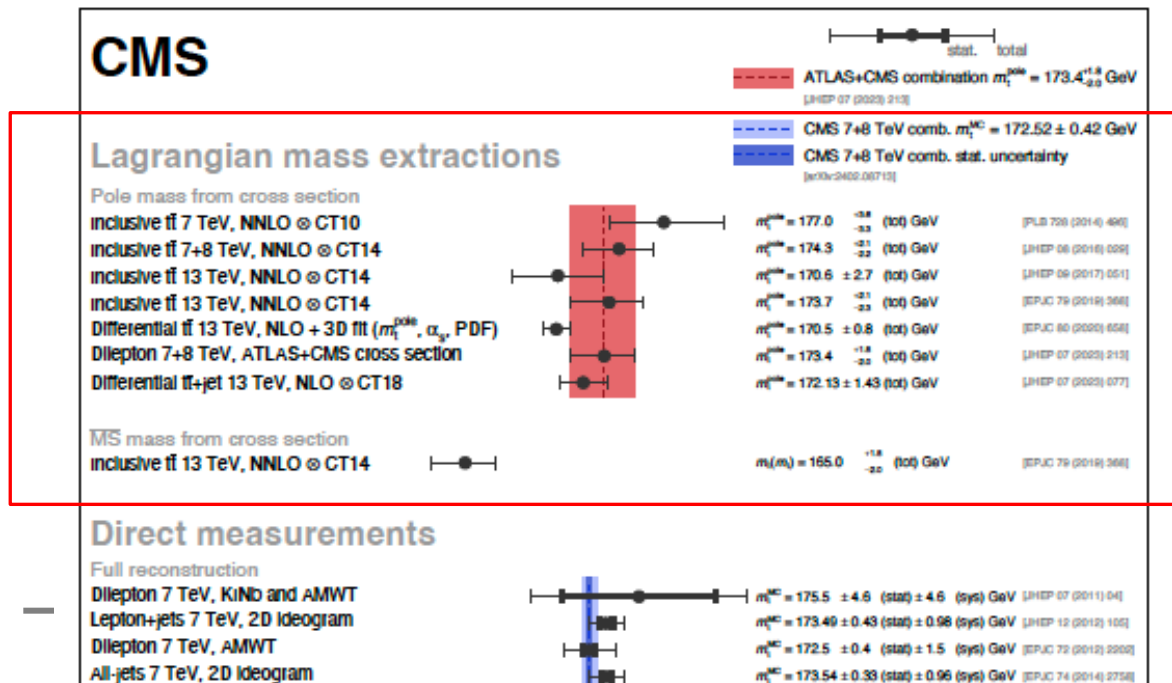
But we want to do better than that ! $\rightarrow$ “MC top mass interpretation problem”

Approaches to remedy the m_t^{MC} problem

- Indirect top quark mass measurements

→ ATLAS/CMS

- Unfold data to the parton level $t\bar{t}$ hard matrix element to be compared to N(N)LO fixed order calculations for on-shell top quarks → extraction of m_t^{pole} , $m_t(m_t)$, $m_t^{\text{MSR}}(80 \text{ GeV})$
Makela, Lipka, Moch, AHH, 2301.03546
- MC modelling aspects now contained in the hadron-to-parton unfolding carried out with the MC generator (different systematics)
- Uncertainties not as small as for direct determinations as observables are of more inclusive character



Approaches to remedy the m_t^{MC} problem

- 'Hadron' level analytic QCD predictions for top mass determinations

- Fat top jets for boosted tops with soft drop grooming
 - MPI currently provides a practical limitation for LHC
 - UE/MPI still sizeable
- Energy correlators for boosted top
 - top decay opening angle
 - theory predictions still missing

Mantry, Pathak, Stewart, AHH (2017)
Mantry, Pathak, Stewart, AHH (2019)
ATL-PHYS-PUB-2021-034

Holguin, Moul, Pathak, Procura (2022)
Holguin, Moul, Pathak, Procura,
Schöfbeck (2023)

→ Aim: Compare theory predictions with particle level data

- This talk is about work to truly understand and control the MC top quark mass parameter m_t^{MC} → Improve/understand MCs so that direct measurements can be interpreted reliably.

Mass Extraction and Renormalization Schemes

The Principle of Top Mass Determinations

- Top quark is not a physical particle (“parton picture”)
- Top mass defined from theoretical prescriptions (renormalization schemes)
- Different schemes are related by a perturbative series.

$$m_t^A - m_t^B = \sum_n c_n \alpha_s^n(\mu)$$

$$\hat{\sigma}(Q, m_t^A, \alpha_s(\mu), \mu; \delta m^A) = \hat{\sigma}(Q, m_t^B, \alpha_s(\mu), \mu; \delta m^B)$$

Parton level cross section are formally scheme-invariant, but practically there is scheme-dependence due to truncation of perturbation theory:
→ We have to pick “adequate” schemes so that higher order corrections are small.

- For comparison with exp. data one has to account for non-perturbative corrections

$$\sigma^{\text{exp}} = \hat{\sigma}(Q, m_t^X, \alpha_s(\mu), \mu; \delta m^X) + \sigma^{\text{NP}}(Q, \Lambda_{\text{QCD}})$$

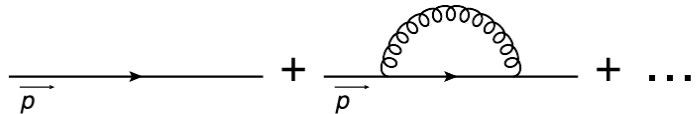
Typically at LHC: $\sigma^{\text{NP}} \sim \left(\frac{\Lambda_{\text{QCD}}}{Q} \right)^n, \quad n = 1$

Linear effects arise from color neutralization processes due to kinematic cuts.
High precision control over NP effects needed as well.

Mass Extraction and Renormalization Schemes

Top Mass Renormalization Schemes

- Related to different treatments of the top self energy.



$$\overline{p} \rightarrow \overline{p} + \overline{p} \text{ (with gluon loop) } + \dots \sim \frac{i}{\not{p} - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

Calculated in the limit of vanishing infrared cutoff

Large linearly IR-sensitive contributions

$O(\Lambda_{\text{QCD}})$ renormalon behavior at higher orders

$$\Sigma(p, m_t^0, \mu) \sim m_t^0 \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu) \right] + \dots$$

Pole Mass:

- Absorb ALL self-energy corrections into the mass

$$m_t^{\text{pole}} = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu) \right] \right\} + \dots$$

- RG-invariant:

$$\frac{d}{d \ln \mu} m_t^{\text{pole}} = 0$$

Realizes naive picture of a free top quark.

(Standard mass for most FO-NLO/NNLO calculations.)

- Large contributions in A^{fin} absorbed into m_t^{pole} that cannot cancel with other linearly sensitive corrections in the cross section: $O(\Lambda_{\text{QCD}})$ renormalon problem !
- Renormalon ambiguity: 110 – 250 MeV

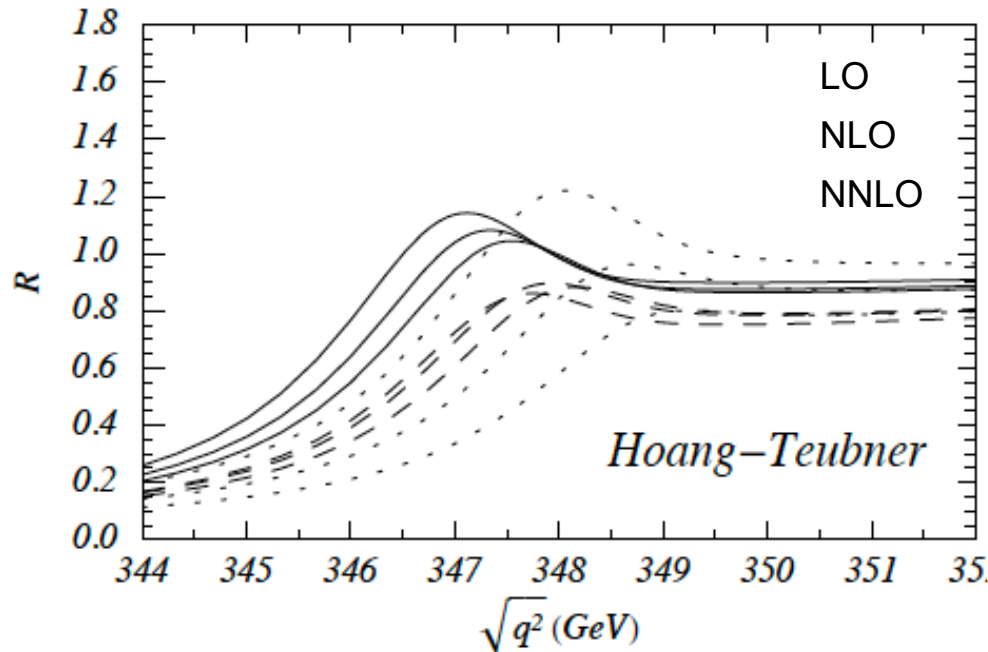
AAH, Jain, Lepenik et al; arXiv: 1704.01580

Beneke, Nason, Steinhauser, arXiv: 1605.03609

Mass Extraction and Renormalization Schemes

The renormalon problem of the pole mass

Top-antitop toponium cross section at the ILC : m_t^{pole}



Beneke, AHH, Melnikov, Nagano, et al, hep-ph/0001286

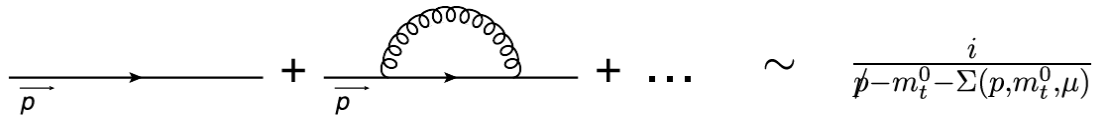
NNLO: Beneke, Kiyo, Marquard, et al; arXiv: 1506.06864

- Mass sensitive toponium resonance mass does not show any convergence.
- Pole mass has intrinsic ambiguity leads to an unphysical enhancement of sensitivity to low scales in perturbative predictions that is not related to any hadronization effects (due to top quark decay).

Mass Extraction and Renormalization Schemes

Top Mass Renormalization Schemes

- Related to different treatments of the top self energy.



$$\overline{p} \rightarrow \overline{p} + \overline{p} \text{ (with gluon loop) } + \dots \sim \frac{i}{\not{p} - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

Large linearly IR-sensitive contributions:

$O(\Lambda_{\text{QCD}})$ renormalon behavior at higher orders

$$\Sigma(p, m_t^0, \mu) \sim m_t^0 \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/\mu) \right] + \dots$$

MSbar mass:

- Absorb $1/\epsilon$ term into the mass (MSbar): $\overline{m}_t(\mu) = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(\mu)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) \right] \right\} + \dots$
- All self-energy corrections from scales above $\mu > m_t$ are absorbed into $m_t(\mu)$
→ IR-safe short-distance mass definition

- RG-evolution similar to α_s : $\frac{d}{d \ln \mu} \overline{m}_t(\mu) = -\overline{m}_t(\mu) \left(\frac{\alpha_s(\mu)}{\pi} \right) + \dots$

- Renormalon-free (“short-distance mass”)
- Only adequate for $\mu \gtrsim m_t$

- MSbar and pole mass differ by ~ 10 GeV: $m_t^{\text{pole}} - \overline{m}_t(\mu) = \frac{4}{3} \left(\frac{\alpha_s(\mu)}{\pi} \right) \overline{m}_t(\mu) + \dots$

Mass Extraction and Renormalization Schemes

MSR mass: → hybrid scheme

- Absorb also virtual top quark fluctuations also into the mass:

$$m_t^{\text{MSR}}(R) = m_t^0 \left\{ 1 + \left(\frac{\alpha_s(R)}{\pi} \right) \left[\frac{1}{\epsilon} + \ln(4\pi e^{-\gamma_E}) + A^{\text{fin}}(m_t^0/R) \right] \right\} - R \left(\frac{\alpha_s(R)}{\pi} \right) A^{\text{fin}}(1) + \dots$$

- All self-energy corrections from scales $> R$ are absorbed into $m_t(R)$
 - IR-safe short-distance mass definition
 - adequate for scales $R \lesssim m_t$

- RG-evolution similar is linear in R :
$$\frac{d}{d \ln R} m_t^{\text{MSR}}(R) = -\frac{4}{3} R \left(\frac{\alpha_s(R)}{\pi} \right) + \dots$$

- Renormalon-free
- MSR and pole mass are numerically close for small R :

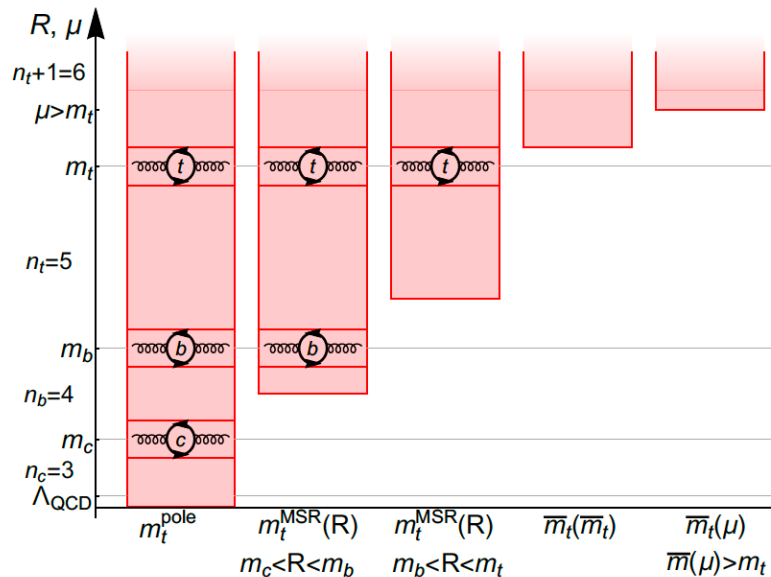
$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = \frac{4}{3} \left(\frac{\alpha_s(R)}{\pi} \right) R + \dots$$

→ $m_t^{\text{MSR}}(1\text{-}2 \text{ GeV})$ is a renormalon-free proxy for the pole mass

- Agrees with MSbar mass at $R=m_t$
$$m_t^{\text{MSR}}(m_t) = \overline{m}_t(m_t)[1 + \mathcal{O}(\alpha_s^2)]$$

Mass Extraction and Renormalization Schemes

Self-energy contribution absorbed into mass schemes:



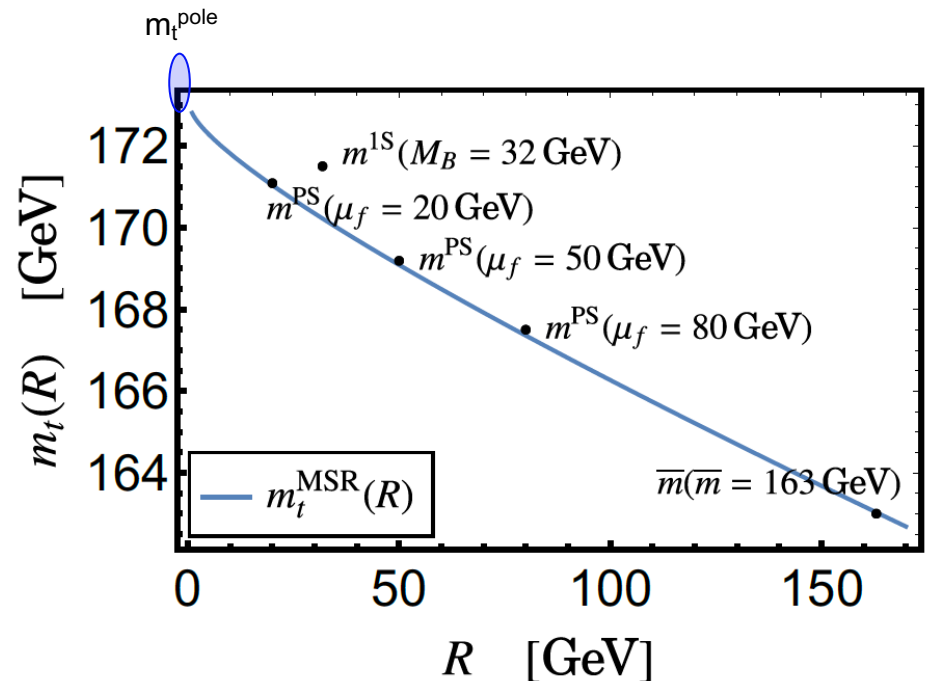
pole mass

MSR mass

MSbar mass

$$\overline{p} \rightarrow \overline{p} + \overline{p} \text{ (with self-energy loop)} + \dots \sim \frac{i}{\not{p} - m_t^0 - \Sigma(p, m_t^0, \mu)}$$

Numerical values of different mass schemes:
MSR mass encodes RG-flow for all short-distance masses



All short-distance masses (MSbar, MSR, PS, 1S) can be converted into each other with a precision of 10-20 MeV !

Available software libraries:

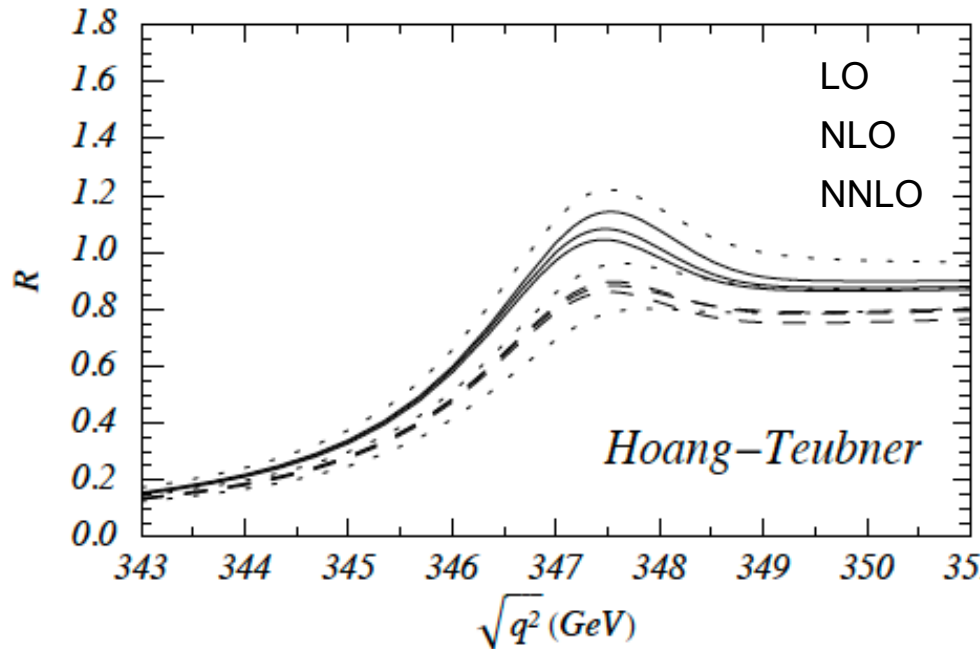
Rundec, Revolver

Mass Extraction and Renormalization Schemes

toponium average momentum



Top-antitop toponium cross section at the ILC: $m_t^{1S} \approx m_t^{MSR} (20 \text{ GeV})$

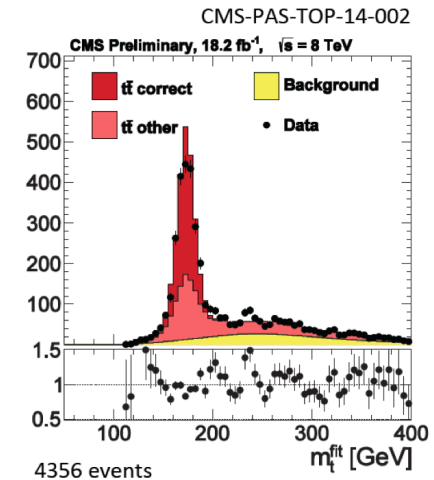
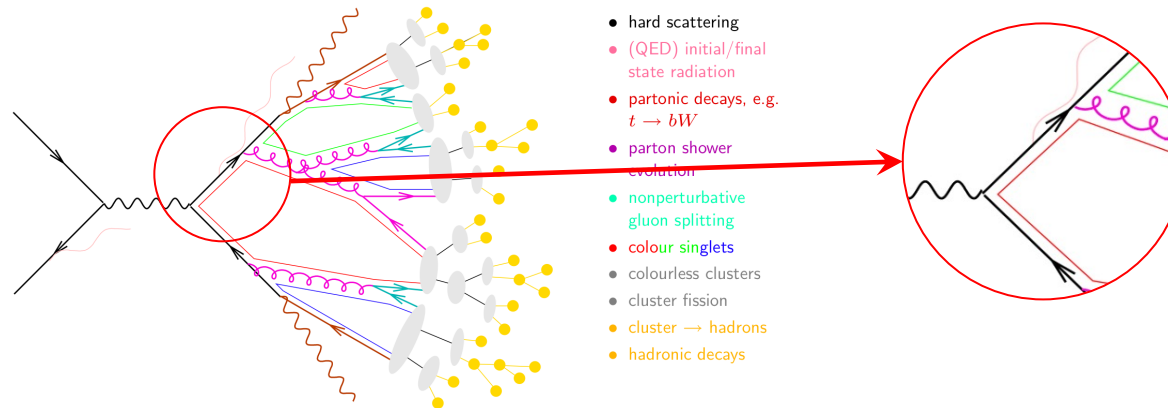


Beneke, AHH, Melnikov, Nagano, et al, hep-ph/0001286

- Mass sensitive toponium resonance mass shows stability under radiative corrections

What is m_t^{MC} ? (Answer 1.1)

We can give a more concrete answer:



- Parton shower resums perturbative corrections by an evolution from high to low scales
- Shower evolution terminates at shower cutoff $Q_0 \approx 1\text{-}2$ GeV $\leftarrow$ IR cutoff
- All linearly IR sensitive perturbative corrections are cut off
- MC provides hadronization corrections but not to the top quark: $\Gamma_t = 1.4$ GeV
- m_t^{MC} is renormalon-free
- m_t^{MC} depends on the cutoff definition (and thus on the MC)

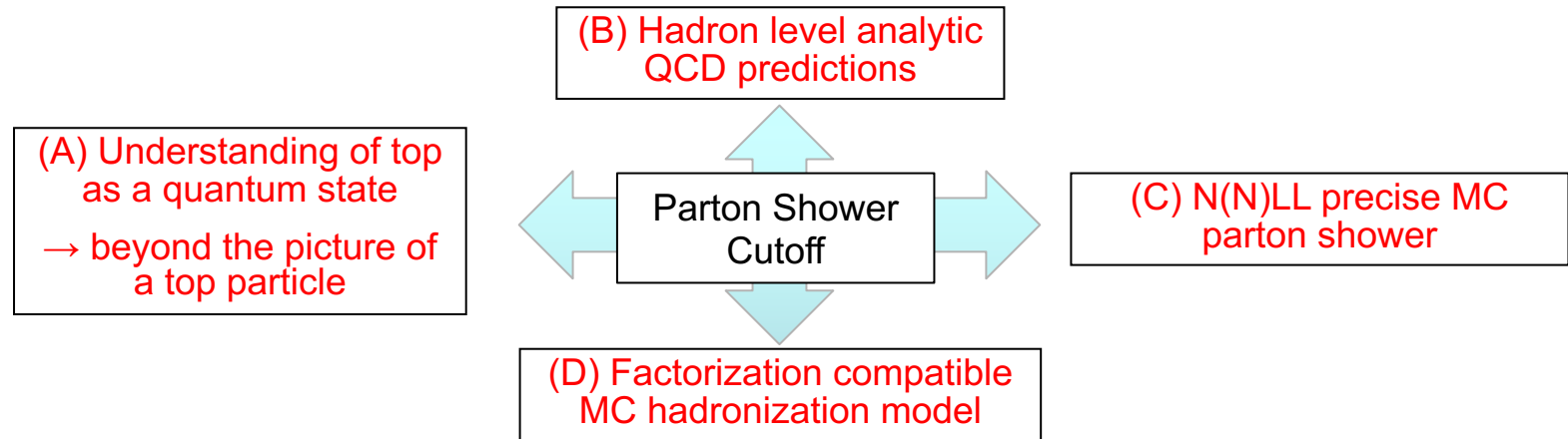
$\Rightarrow m_t^{\text{MC}}$ is more closely related to the MSR mass:

AHH, Stewart, 0808.0222

$$m_t^{\text{MC}} = m_t^{\text{MSR}}(Q_0) + Q_0 \alpha_s(Q_0) \times \text{const.}$$

What is m_t^{MC} ? (Answer 2.0)

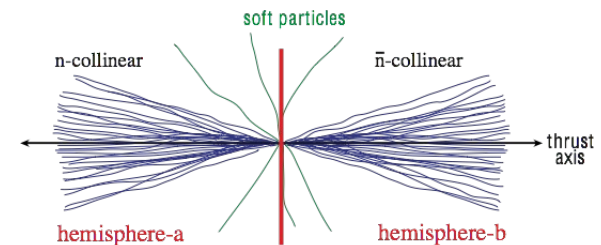
There are 4 essential ingredients to resolve the problem from first principles:



Currently there is only 1 observable class where all 4 ingredients are available.

Jet mass based event-shape observables
in e^+e^- collisions for boosted top pair
production in dijet region:

→ 2-jettiness, thrust, ... (decay insensitive, NNNLL)



Rest of this talk:

- Discuss interplay of (A) – (D) provide conceptual and practical basis to determine and control m_t^{MC} for MC event generators
- Review (A) – (C) from previous work. New development for (D)
- Explicit realization for e^+e^- event-shape top resonance distribution (e.g. 2-jettiness) for Herwig 7.2 as a proof of principle

(A) Beyond the picture of a top particle

The top quark does not hadronize due to its large width $\Gamma_t \gg \Lambda_{\text{QCD}}$. It therefore has some characteristics of a physical particle (hadron).

BUT: If we stick to the picture of a physical top particle the only mass that is ever relevant is the pole mass = pole of the top propagator.

Due to the top quarks color charge, however, this picture is too restricted when we want to understand the MC top quark mass.

What we mean by a top quark is however related to

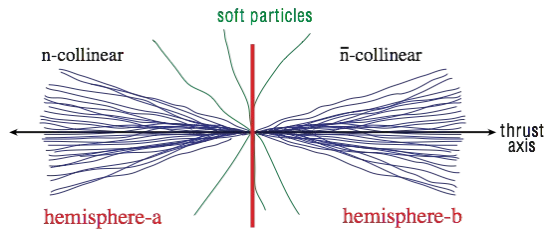
well
known
aspect

- a particular experimental measurement prescription (of a color singlet state)
 - calculations/simulations must properly account color neutralization effects
 - implies that we need accurate hadron level QCD predictions/simulations

novel
aspect

- the way how we treat soft gluons in the top rest frame
 - MC simulations impose an IR cut Q_0 of the parton shower gluon evolution
 - the shower cutoff Q_0 acts as a resolution scale
 - changes the physical meaning of the top quark mass in the simulation, but also the scheme of hadronization corrections for the entire process
 - impact of the shower cutoff needs to be quantified and controlled accurately

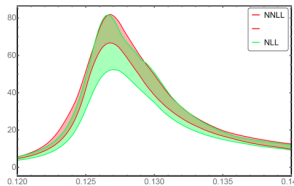
(B) Boosted top eventshapes



$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q} \quad \tau \xrightarrow{0} \frac{M_1^2 + M_2^2}{Q^2} \quad Q = E_{\text{c.m.}}$$

2-jettiness: insensitive to details of top decay

Hadron level SCET:



$$\frac{d\sigma}{d\tau}(\tau, Q, m, \delta m) = \int_0^{Q\tau} d\ell \underbrace{\frac{d\hat{\sigma}_s}{d\tau}\left(\tau - \frac{\ell}{Q}, Q, m, \delta m\right)}_{\text{parton cross section}} S_{\text{mod}}(\ell)$$

Non-perturbative shape function
(universal for massive and massless quarks)

Fleming, Mantry, Stewart, AHH (2007)

Bachu, Mateu, Pathak, Stewart, AHH (2022)

- S_{mod} leading nonperturbative corrections only from large-angle soft radiation: linear sensitive to Λ_{QCD}
- Any top mass renormalization scheme can be implemented $m_t^{\text{pole}} = m + \delta m$
- Can be calculated with a finite IR cutoff Q_0 for the parton cross section
- IR cutoff $Q_0 = \text{factorization scale}$ for parton-level vs. hadronization corrections
 - ▶ Defines scheme for S_{mod} (large-angle soft radiation): $S_{\text{mod}}(\ell) \rightarrow S_{\text{mod}}(\ell, Q_0)$
 - ▶ Defines scheme for parton distribution: $\frac{d\hat{\sigma}}{d\tau}(\tau, Q, m) \rightarrow \frac{d\hat{\sigma}}{d\tau}(\tau, Q, m, Q_0)$

(C) Angular ordered parton shower (Herwig)

→ Coherent Branching algorithm (default Herwig shower):

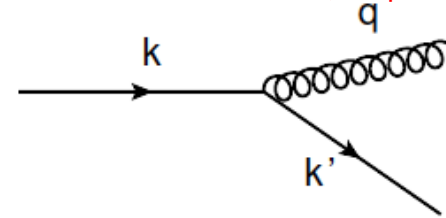
Dokshitzer, Fadin, Khoze (1982)
 Bassetto, Ciafaloni, Marchesini (1983)
 Catani, Marchesini, Webber (1991)
 Gieseke, Stephens, Webber (2003)

$$k'^{\mu} = zk^{-} \frac{n^{\mu}}{2} + \frac{k'^2 - q_{\perp}^2}{zk^{-}} \frac{\bar{n}^{\mu}}{2} - q_{\perp}^{\mu}$$

$$q^{\mu} = (1-z)k^{-} \frac{n^{\mu}}{2} + \frac{q^2 - q_{\perp}^2}{(1-z)k^{-}} \frac{\bar{n}^{\mu}}{2} + q_{\perp}^{\mu}$$

momentum conservation:

$$k^2 = \frac{k'^2}{z} + \frac{q^2}{1-z} + \frac{q_{\perp}^2}{z(1-z)}$$



evolution variables: $z, \tilde{q} = \frac{q_{\perp}^2}{z^2(1-z)^2}$

color coherence of soft gluon emissions → angular ordering: $z_i^2 \tilde{q}_i^2 > \tilde{q}_{i+1}^2$

probabilities from
splitting functions and
Sudakov form factors

→ NLL precise analytic jet mass distribution (mass generated from one boosted quark)

$k^2 \approx$ hemisphere mass (does not account for out of cone radiation)



$$J(Q^2, k^2 - m^2, m^2) = \delta(k^2 - m^2)$$

$$+ \int_0^{Q^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_0^1 dz P_{QQ}[\alpha_s(z(1-z)\tilde{q}), z, m] \theta\left(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\right)$$

$$\times \left[zJ(z^2\tilde{q}^2, z(k^2 - m^2) - z^2(1-z)\tilde{q}^2) - J(\tilde{q}^2, k^2 - m^2) \right]$$

(C) Angular ordered parton shower (Herwig)

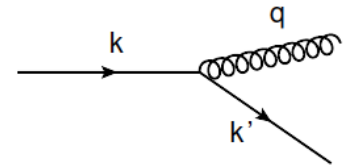
Parton level cross section

Catani, Trentadue, Turnock Webber (1993)

$$\frac{d\hat{\sigma}}{d\tau} = \int dk^2 dk'^2 \delta\left(\tau - \frac{k^2 + k'^2}{Q^2}\right) J(Q^2, k^2) J(Q^2, k'^2)$$

AHH, Plätzer, Samitz (2018)

- Agrees exactly with partonic cross section obtained from analytic factorized SCET calculations at NLL!
- CB is NLL precise for inclusive event shapes.
- For massless quarks and massive quarks



- Analytic calculation: for vanishing shower cutoff $Q_0=0$: $m_t^{\text{MC}} = m_t^{\text{pole}}$ (one-shell self energy contribution does not arise in CB!)

BUT: Parton showers in MC generators have an finite shower cutoff Q_0 to prevent infinite multiplicities → acts as finite resolution scale that is physical for the MC

- We track the dominant linear dependence on Q_0 from large-angle soft and ultra-collinear (=soft in top rest frame) radiation
- Matches to analogous results from analytic factorization theorem
- Realized accurately by Herwig's shower

$$q_{\perp} > Q_0$$

Linear Shower Cutoff Dependence (Herwig)

Massless quarks: (effects on large-angle soft radiation)

AHH, Plätzer, Samitz (2018)

$$\begin{aligned} \frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau}, Q, Q_0) &= \frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau}, Q) + \frac{1}{Q} \Delta_{\text{soft}}(Q_0) \frac{d^2\hat{\sigma}}{d\hat{\tau}^2}(\hat{\tau}, Q) \\ &= \frac{d\hat{\sigma}}{d\hat{\tau}}\left(\hat{\tau} + \frac{1}{Q} \Delta_{\text{soft}}(Q_0), Q\right) \end{aligned} \quad \Delta_{\text{soft}}(Q_0) = 16 Q_0 \frac{\alpha_s(Q_0) C_F}{4\pi} + \mathcal{O}(\alpha_s^2(Q_0))$$

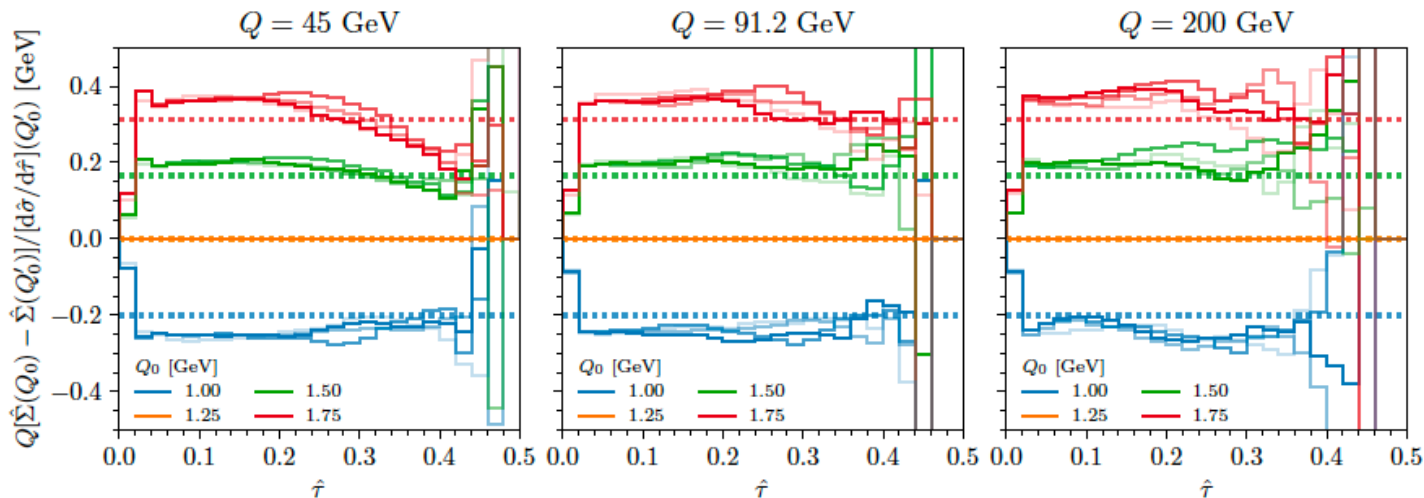
$$Q \frac{\hat{\Sigma}(\hat{\tau}, Q, Q_0) - \hat{\Sigma}(\hat{\tau}, Q, Q'_0)}{\frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau}, Q, Q'_0)} = \Delta_{\text{soft}}(Q_0, Q'_0)$$

→ 2-jettiness cumulant distribution:

$$\Delta_{\text{soft}}(Q_0, Q'_0) = 16 \int_{Q'_0}^{Q_0} dR \left[\frac{\alpha_s(R) C_F}{4\pi} \right]$$

Herwig CB shower versus pQCD:

(Herwig 'true' parton level had to be added)



Different lines: different matrix element and matching schemes

Linear Shower Cutoff Dependence (Herwig)

AHH, Plätzer, Samitz (2018)

Massive quarks: (effects on large-angle-soft + ultra-collinear radiation)

Modifies pole of top propagator away from m_t^{pole} :

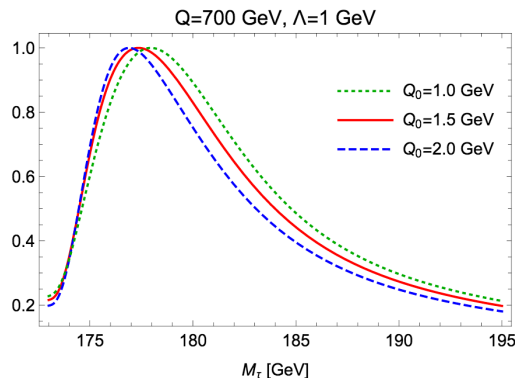
$$m_t^{\text{pole}} \rightarrow m_t(Q_0) = m_t^{\text{pole}} - \delta m(Q_0), \quad \delta m(Q_0) = 2/3 \alpha_s(Q_0) Q_0 + \dots$$

→ 2-jettiness resonance position:

$$\frac{d}{d \ln Q_0} \tau_{\text{peak}}^{\text{parton}}(Q_0) = \frac{C_F \alpha_s(Q_0)}{4\pi} \frac{Q_0}{Q} \left[16 - 8\pi \frac{m_t}{Q} \right]$$

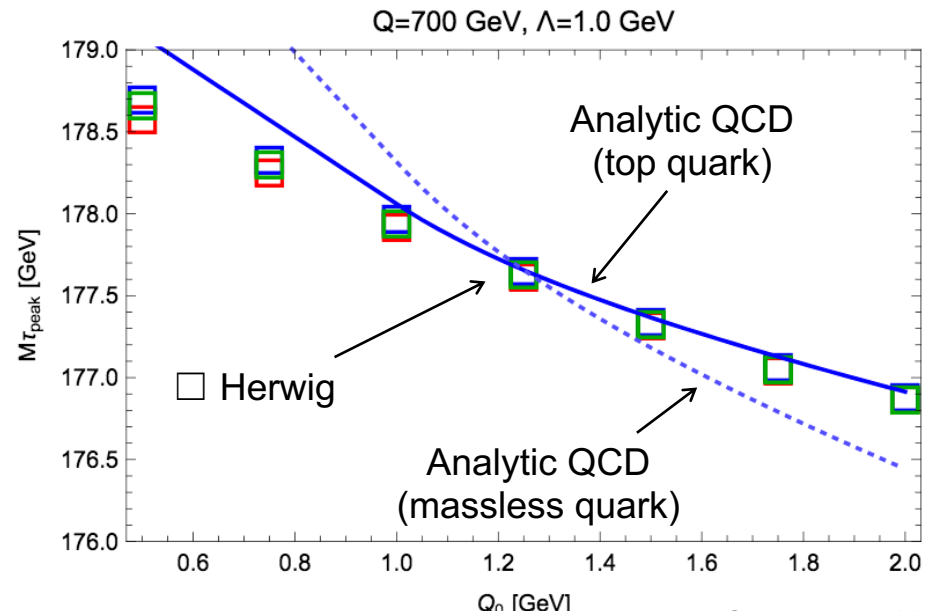
large-angle soft

Must be compensated
by hadronization
corrections in S_{mod}



both linear Q_0 -
contributions cancel

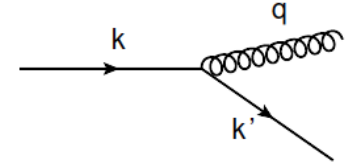
ultra-collinear (from all radiation except self-energy)



(C) Angular ordered parton shower (Herwig)

AHH, Plätzer, Samitz (2018)

For inclusive jet-mass-related event shapes the Herwig top mass parameter represents a Q_0 -dependent mass scheme that can be related to other mass schemes at NLO:



$$q_{\perp} > Q_0$$

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{MSR}}(Q_0) - \frac{2}{3} \left(1 - \frac{2}{\pi}\right) Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2(Q_0))$$

(1) Does this survive the hadronization model?

- Hadron level simulations should be Q_0 -independent
- Shower cut has to be considered as a factorization scale and its proper control in QCD is essential to control parton level and hadronization separately.

(2) How universal is the result? → Needs careful additional work and similar analyses for other types of observables.

(D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

Standard shower cut treatment for all current MC generators:

- Shower-cutoff scale Q_0 = one of many hadronization model parameters

BUT: To gain control over the shower's top mass parameter:

Plätzer arXiv:2204.06956

- The shower-cutoff scale Q_0 must be promoted to a factorization scale,
→ hadron level descriptions should be shower-cut independent.
- We must scrutinize the hadronization models to satisfy the constraints from pQCD concerning its behavior and shower-cut dependence
- For 2-jettiness **in Herwig**: tuning for different Q_0 values (including top mass sensitive data) must yield a Q_0 -dependent MC top mass parameter consistent with $m_t^{\text{CB}}(Q_0)$

(D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

This implies non-trivial QCD constraints on the properties of the hadronization migration matrix:

$$\frac{d\sigma}{d\tau}(\tau, Q) = \int d\hat{\tau} \underbrace{\frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau}, Q)}_{\text{parton shower}} \underbrace{T(\tau, \hat{\tau}, \{Q, Q_0\})}_{\text{hadronization model}}$$

$$T\left(\frac{k}{Q} = \tau - \hat{\tau}, \frac{k'}{Q} = \hat{\tau}, \{Q, Q_0\}\right)$$

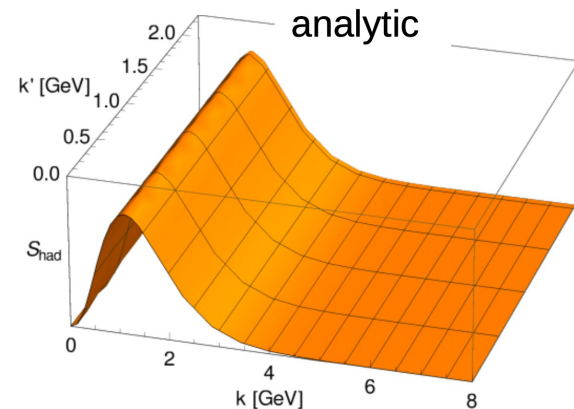
(1) Migration matrix should have this form:

$$T(\tau, \hat{\tau}, Q, Q_0) = T(\tau - \hat{\tau}, Q_0) = Q S_{\text{mod}}\left(\frac{\tau - \hat{\tau}}{Q}\right)$$

(2) Q_0 -dependence of the first moment constrained at NLO QCD:

$$\Omega_1(Q_0) \equiv \frac{1}{2} \int d\ell \ell S_{\text{had}}(\ell, Q_0)$$

$$\Omega_1(Q_0') = \frac{1}{2} \Delta_{\text{soft}}(Q_0', Q_0) + \Omega_1(Q_0)$$



Q_0 -dependent tuning analyses

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856

Tuning software: APPRENTICE

Reference tune = standard Herwig e^+e^- tune (Z-pole LEP data [3180 observable bins])

Reference data = high precision simulated data for $Q_0 = 1.25$ GeV for

- Z-pole LEP data [3180]
- Z-pole 2-jettiness [peak region]
- $t\bar{t}$ 2-jettiness at $E_{\text{cm}} = 700$ and 1000 GeV [peak region]

Q_0 -dependent tunes: tunes to reference data for different shower cut Q_0 values

Tuned parameters: 6 tuning parameters + m_t^{Herwig}

Default model

- m_g (force gluon splitting)
- PSplit (cluster fission, mass distr.)
- Cl_{max} (cluster fission, condition)
- Cl_{pow} (cluster fission, condition)
- PwtSquark (cluster hadronization)
- PwtDIquark (cluster hadronization)

Dynamic model

- Qgtilde (forced gluon splitting)
- Qqtilde (cluster fission splitting)
- Cl_{max} (cluster fission, condition)
- Cl_{pow} (cluster fission, condition)
- PwtSquark (cluster hadronization)
- PwtDIquark (cluster hadronization)

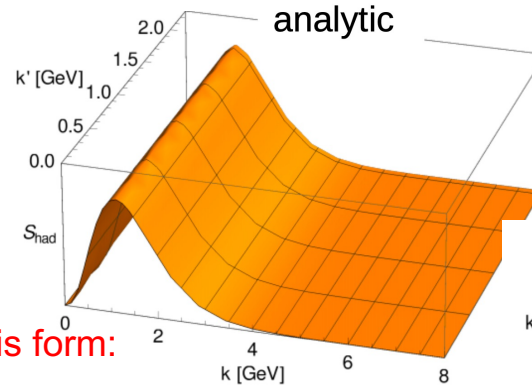
Interpolation grids: cubic and quartic polynomials

(D) Factorization compatible hadronization model

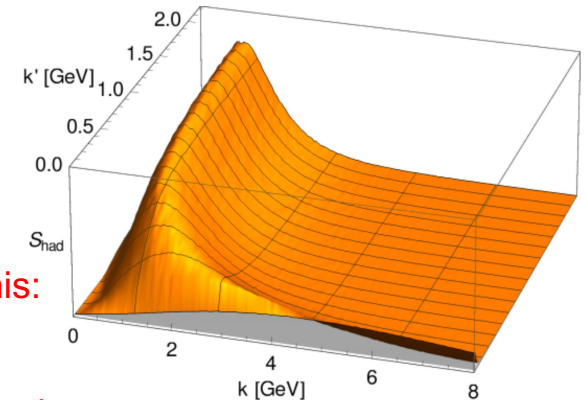
Results from Q_0 -tuned MC simulations: **default model**

$$T\left(\frac{k}{Q} = \tau - \hat{\tau}, \frac{k'}{Q} = \hat{\tau}, \{Q, Q_0\}\right)$$

migration matrix should have this form:



AHH, Jin, Plätzer, Samitz
arXiv:2404.09856



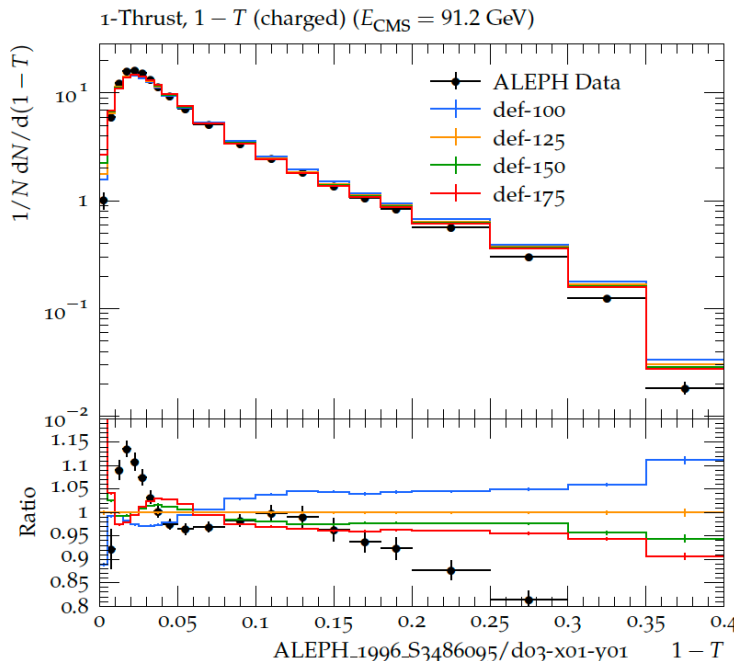
But it actually looks like this:

Peak region hadronization
inconsistent with QCD
factorization!

Description of observables at hadron level not
quite shower-cutoff independent (Thrust at $Q=M_Z$)

$Q_0 = (1.00, 1.25, 1.50, 1.75) \text{ GeV}$

(Reference data for tune: Simulation for $Q_0=1.25 \text{ GeV}$)

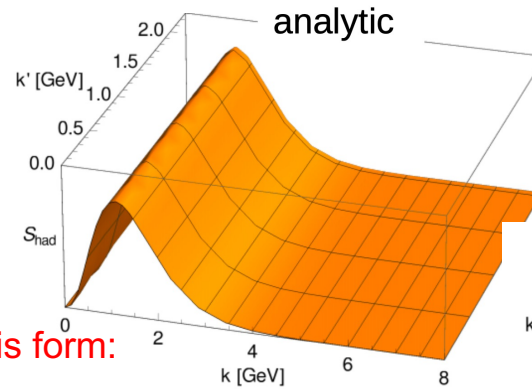


(D) Factorization compatible hadronization model

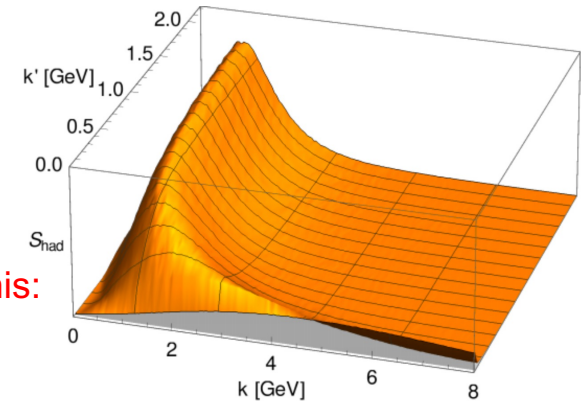
Results from Q_0 -tuned MC simulations: Default model

$$T\left(\frac{k}{Q} = \tau - \hat{\tau}, \frac{k'}{Q} = \hat{\tau}, \{Q, Q_0\}\right)$$

migration matrix should have this form:

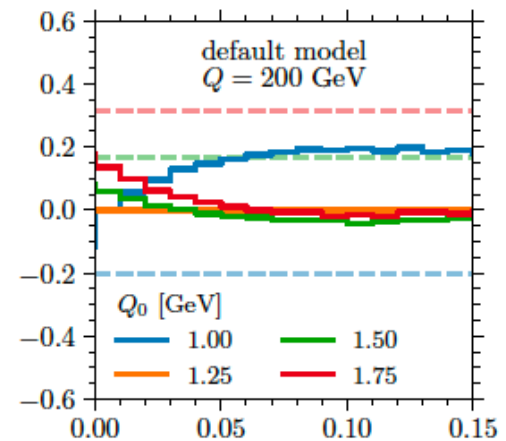
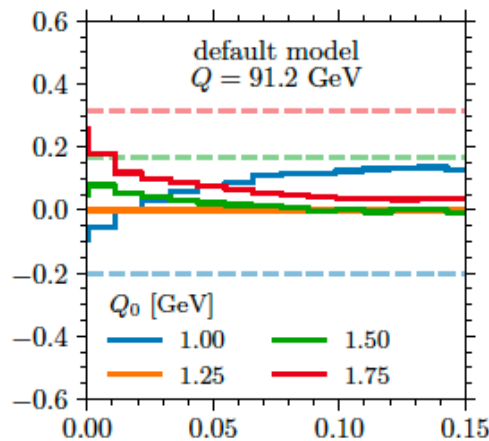
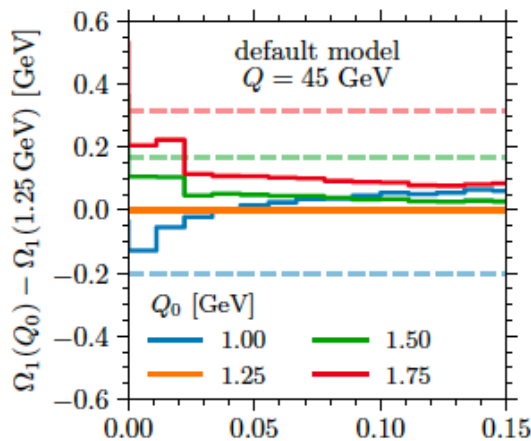


AHH, Jin, Plätzer, Samitz
arXiv:2404.09856



But it actually looks like this:

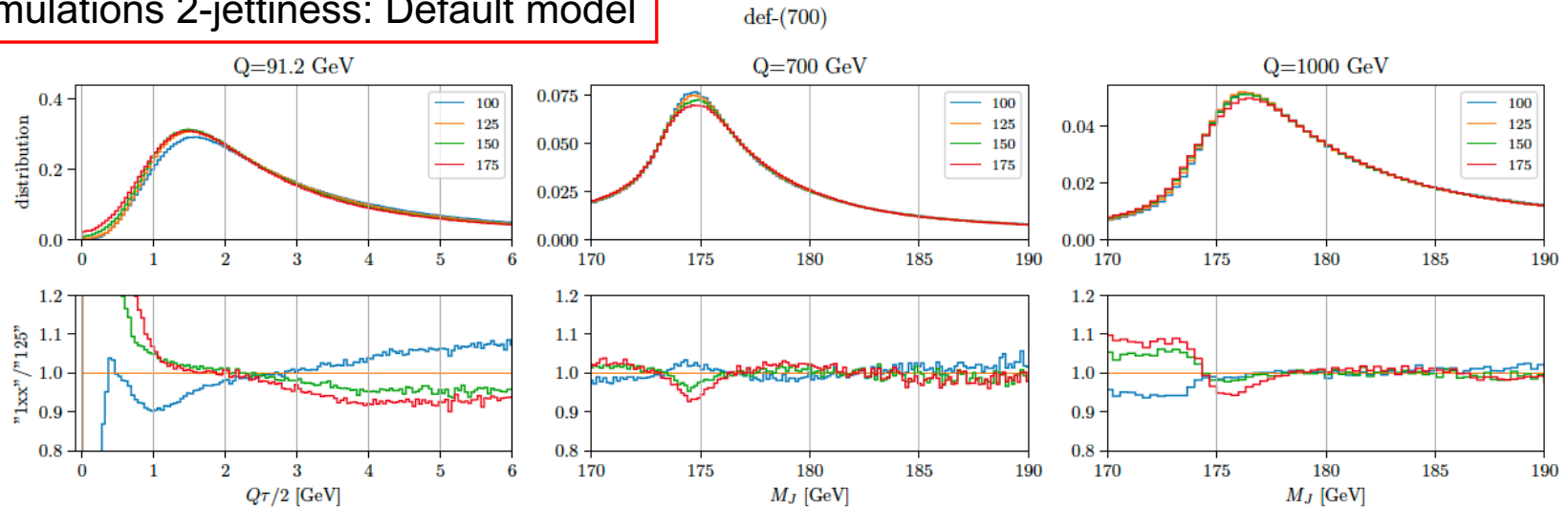
First moment does not satisfy the NLO QCD Q_0 - evolution well



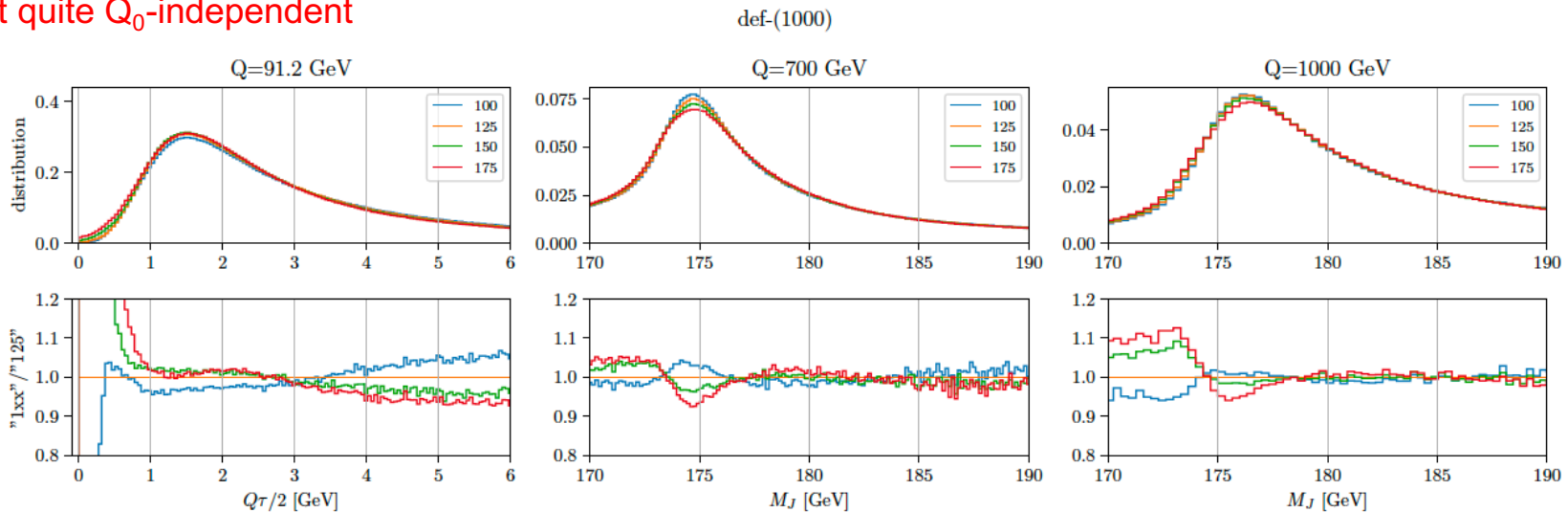
(D) Factorization compatible hadronization model

Results from Q_0 -tuned MC
simulations 2-jettiness: Default model

AHH, Jin, Plätzer, Samitz
arXiv:2404.09856



Not quite Q_0 -independent

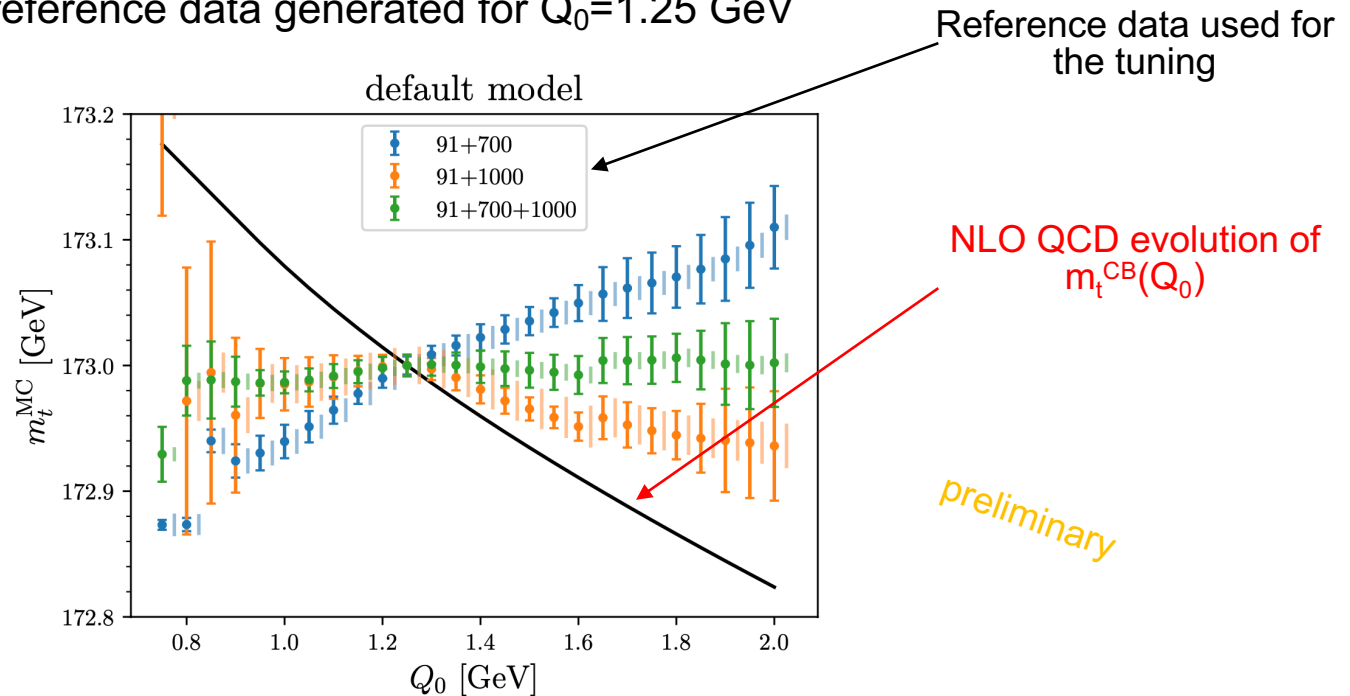


(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz to appear

Q_0 -dependent tunes m_t^{MC} :

- Also tune the top mass parameter m_t^{Herwig} for different Q_0 values (to reference data generated for $Q_0=1.25$ GeV)



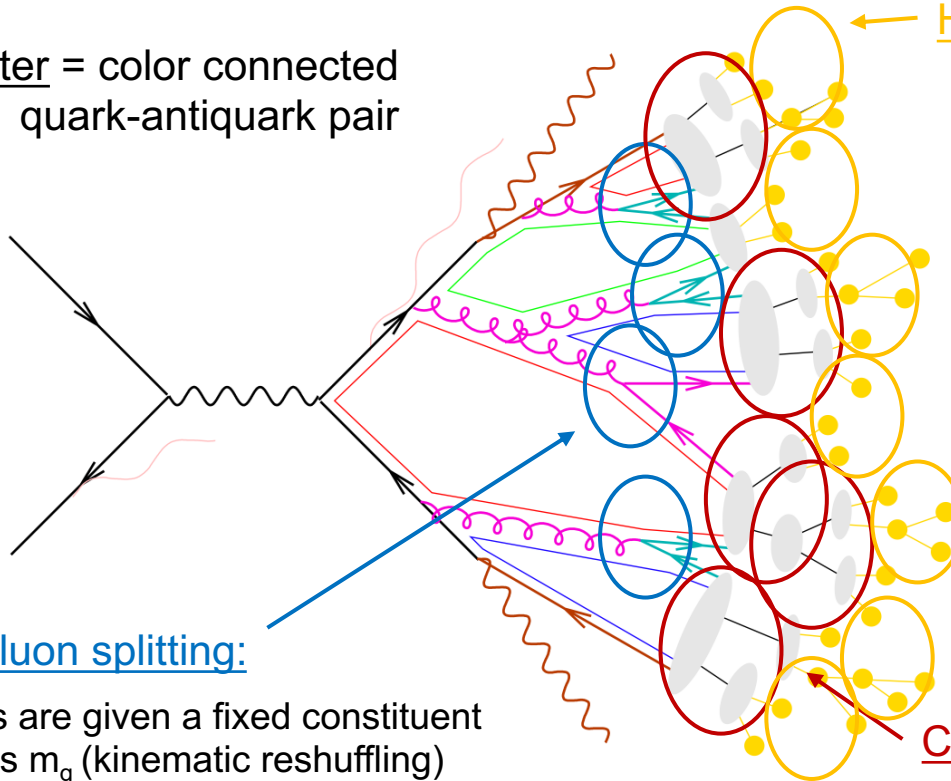
Default Herwig hadronization model modifies m_t^{MC} in an unphysical way incompatible with QCD factorization: uncertainty ~ 0.5 GeV

→ $m_t^{\text{Herwig}}(Q_0) \neq m_t^{\text{CB}}(Q_0)$ for the default hadronization model

(D) Factorization compatible hadronization model

Default cluster hadronization model:

Cluster = color connected quark-antiquark pair



Hadron formation and decays

- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \rightarrow bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster $\rightarrow$ hadrons
- hadronic decays

Forced gluon splitting:

- Gluons are given a fixed constituent masses m_g (kinematic reshuffling)
- Isotropic decay into light qqbar pair in gluon rest frame

Cluster fission:

- Cluster fission as a 1-dim process along the qqbar axis
- Adhoc functional ansatz for cluster mass distribution

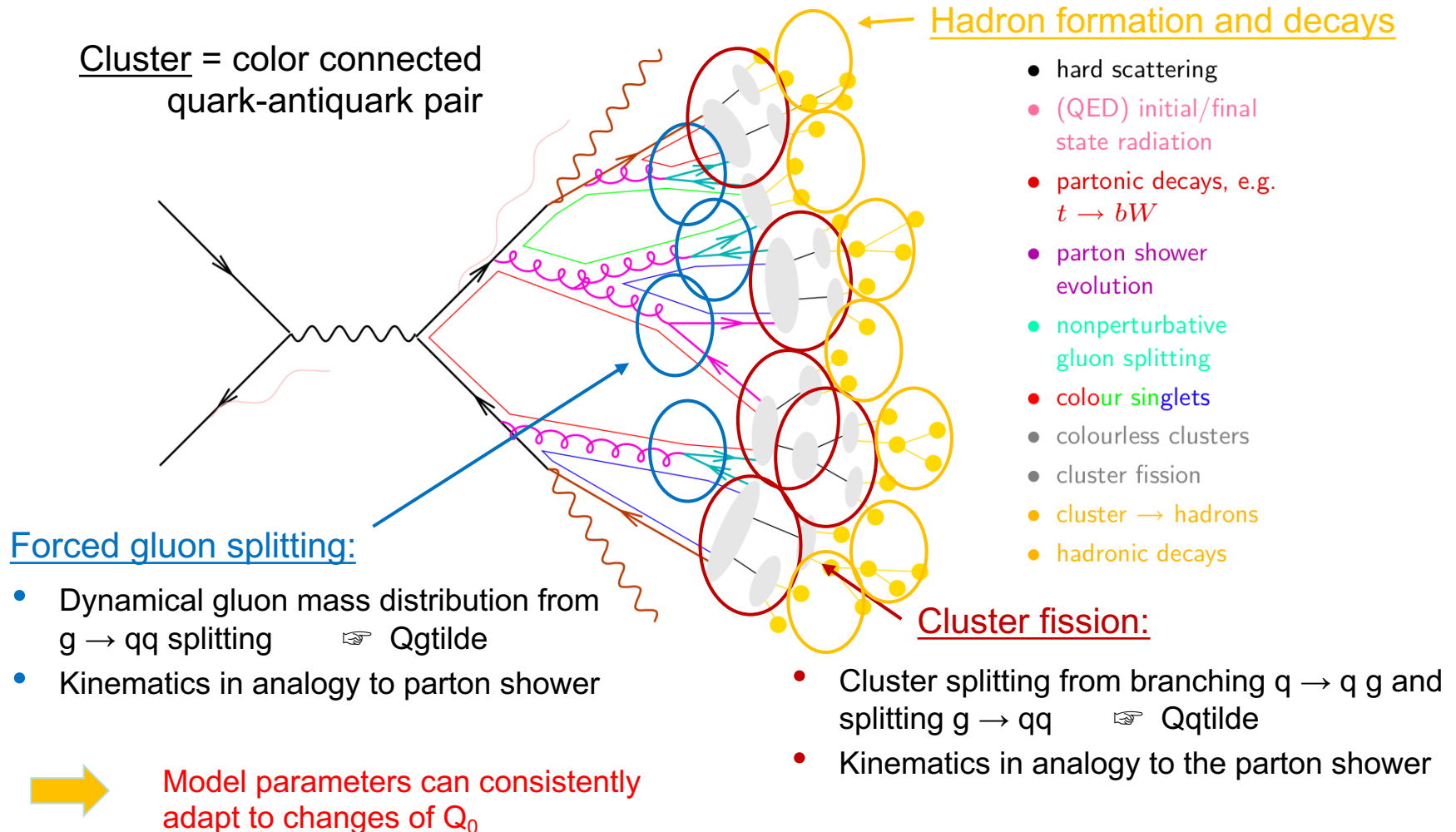


Ad hoc modelling: not designed to adapt Q_0

(D) Factorization compatible hadronization model

AHH, Jin. Plätzer, Samitz 2024.09856

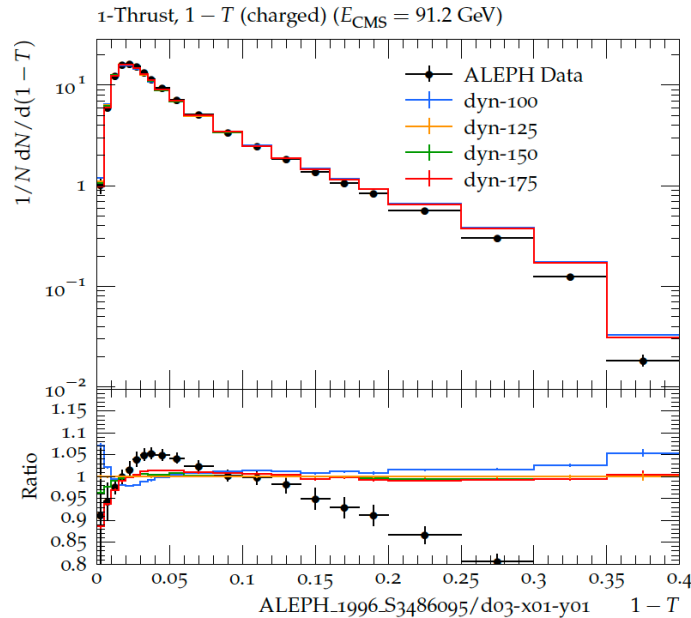
Dynamical cluster hadronization that mimics aspects of parton shower dynamics:



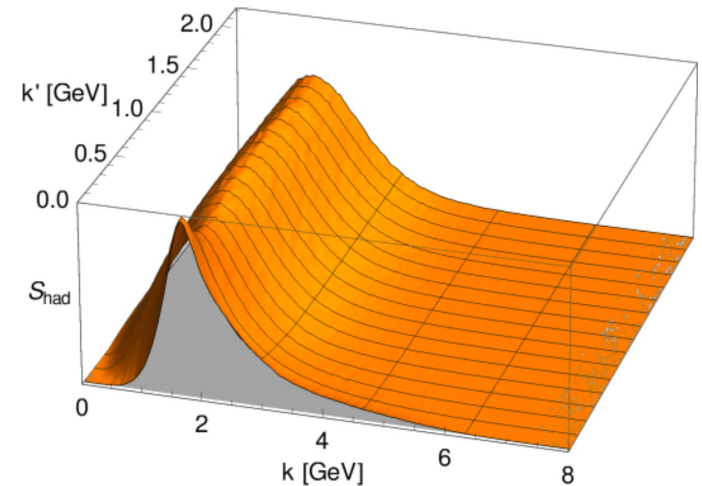
(D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz 2024.09856

Migration function much better consistent with QCD factorization



Observables much less dependent on Q_0

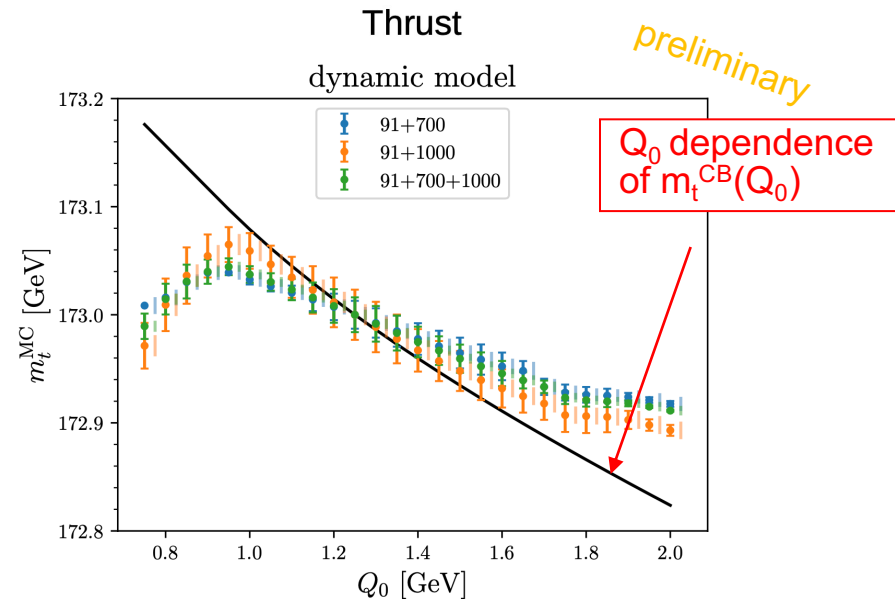


Tunes m_t^{MC} fully consistent with expectations from analytic QCD calculation

(“pseudo data” generated for $Q_0 = 1.25 \text{ GeV}$)

$$\Rightarrow m_t^{\text{Herwig}}(Q_0) = m_t^{\text{CB}}(Q_0)$$

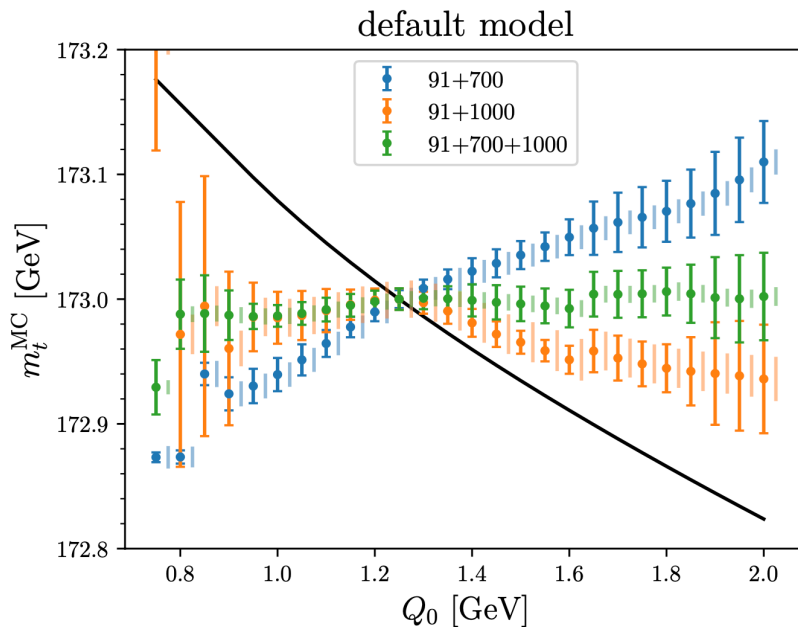
within a precision of better than 50 MeV



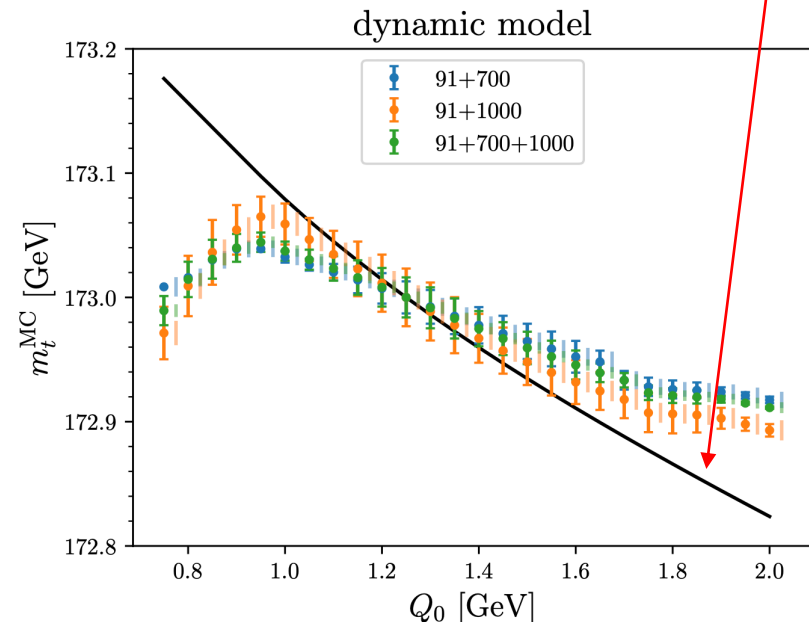
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Shower cutoff dependence of tuned MC top quark mass to reference data including top quark 2-jettiness distributions at 700 and/or 1000 GeV



m_t^{Herwig} ambiguous
at the level of 500 MeV



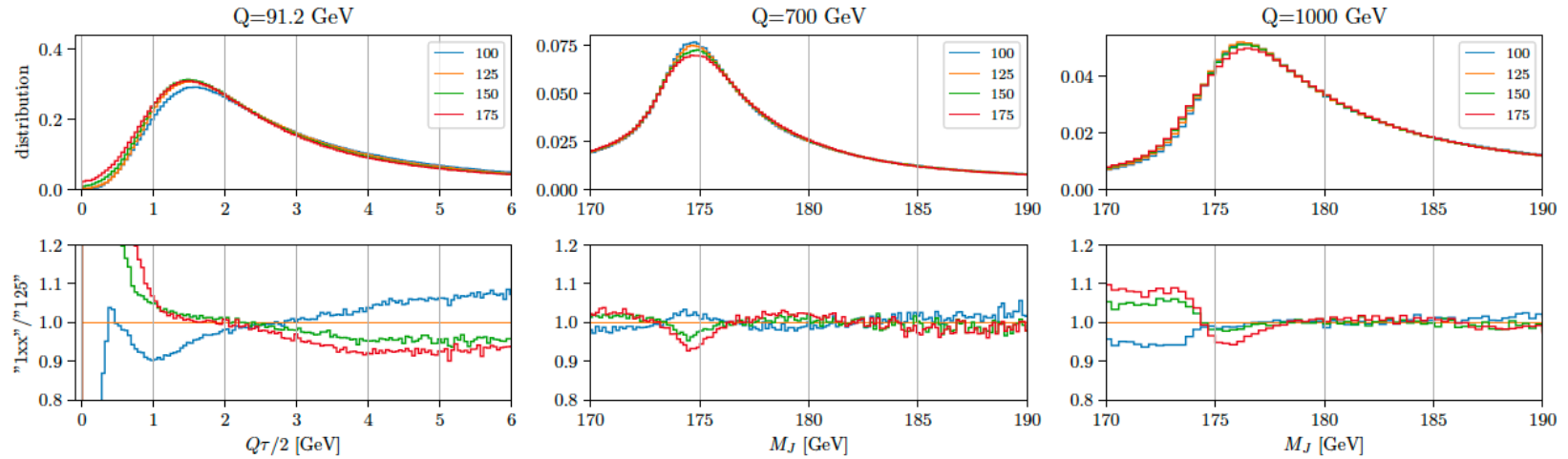
Q_0 dependence
expected from
 $m_t^{\text{CB}}(Q_0)$

Agreement of m_t^{Herwig} with $m_t^{\text{CB}}(Q_0)$
within 50 MeV

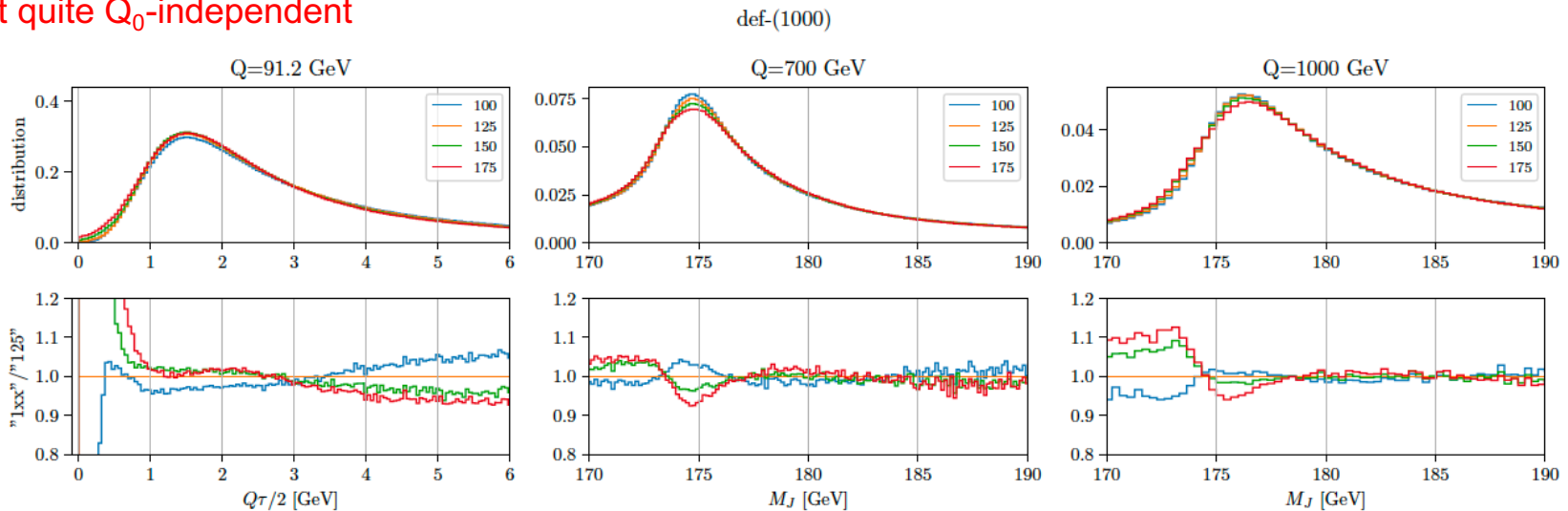
Old Default Model vs. New Dynamical Model

Results from Q_0 -tuned MC simulations: Default model

AHH, Jin, Plätzer, Samitz
to appear



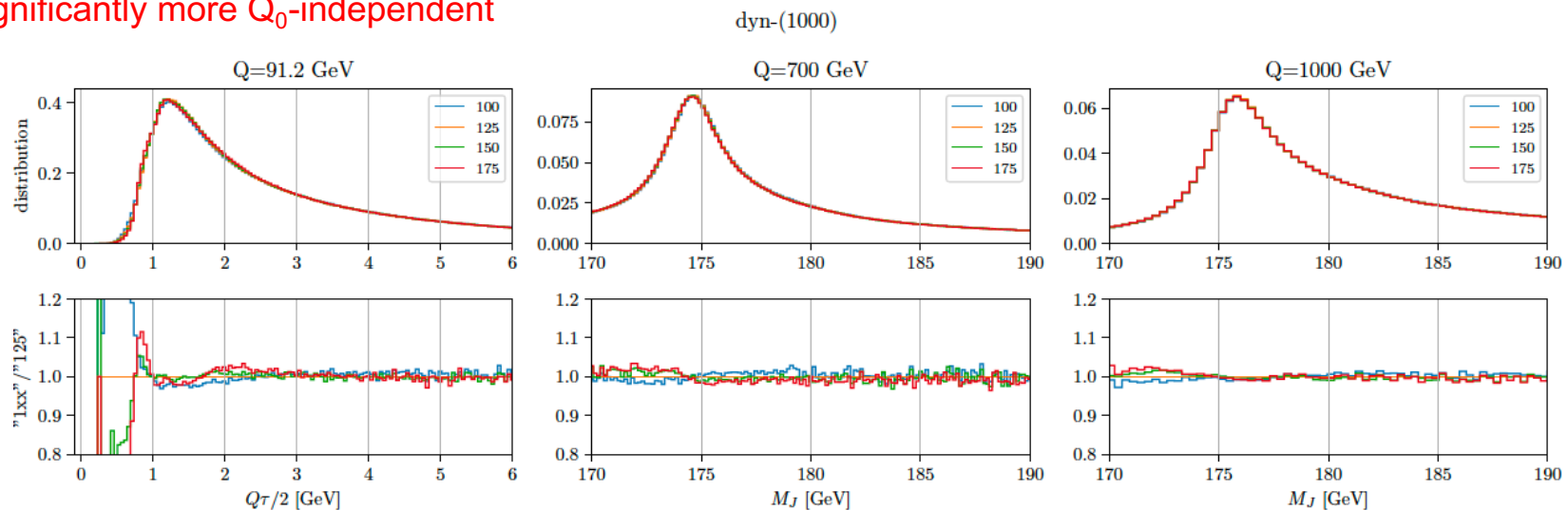
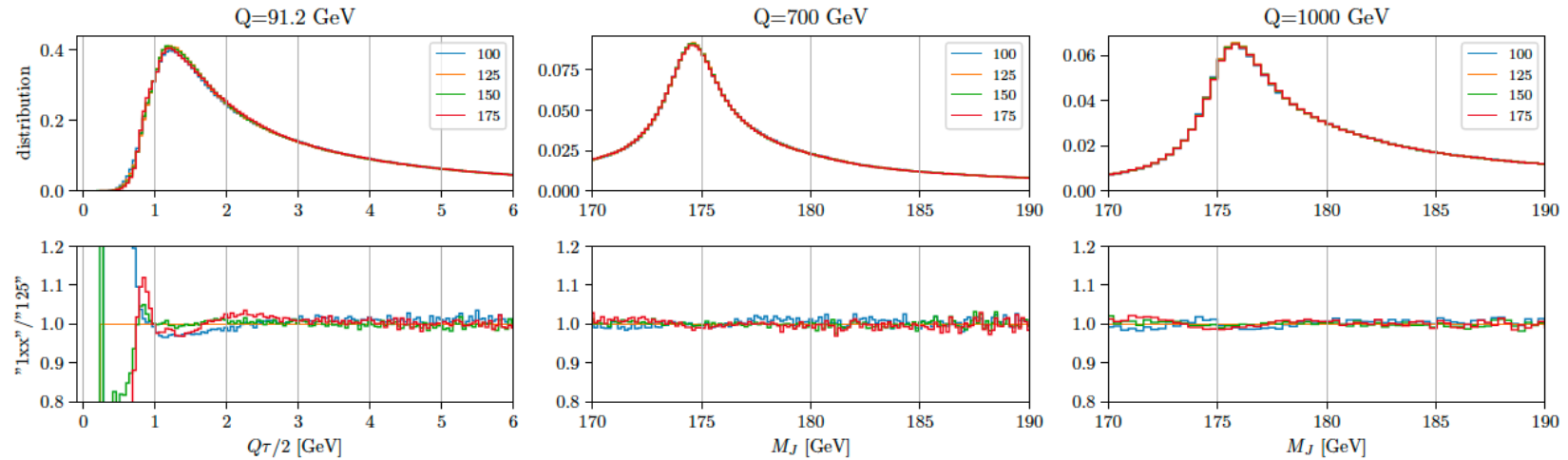
Not quite Q_0 -independent



Old Default Model vs. New Dynamical Model

Results from Q_0 -tuned MC simulations: Dynamical model

AHH, Jin, Plätzer, Samitz
to appear



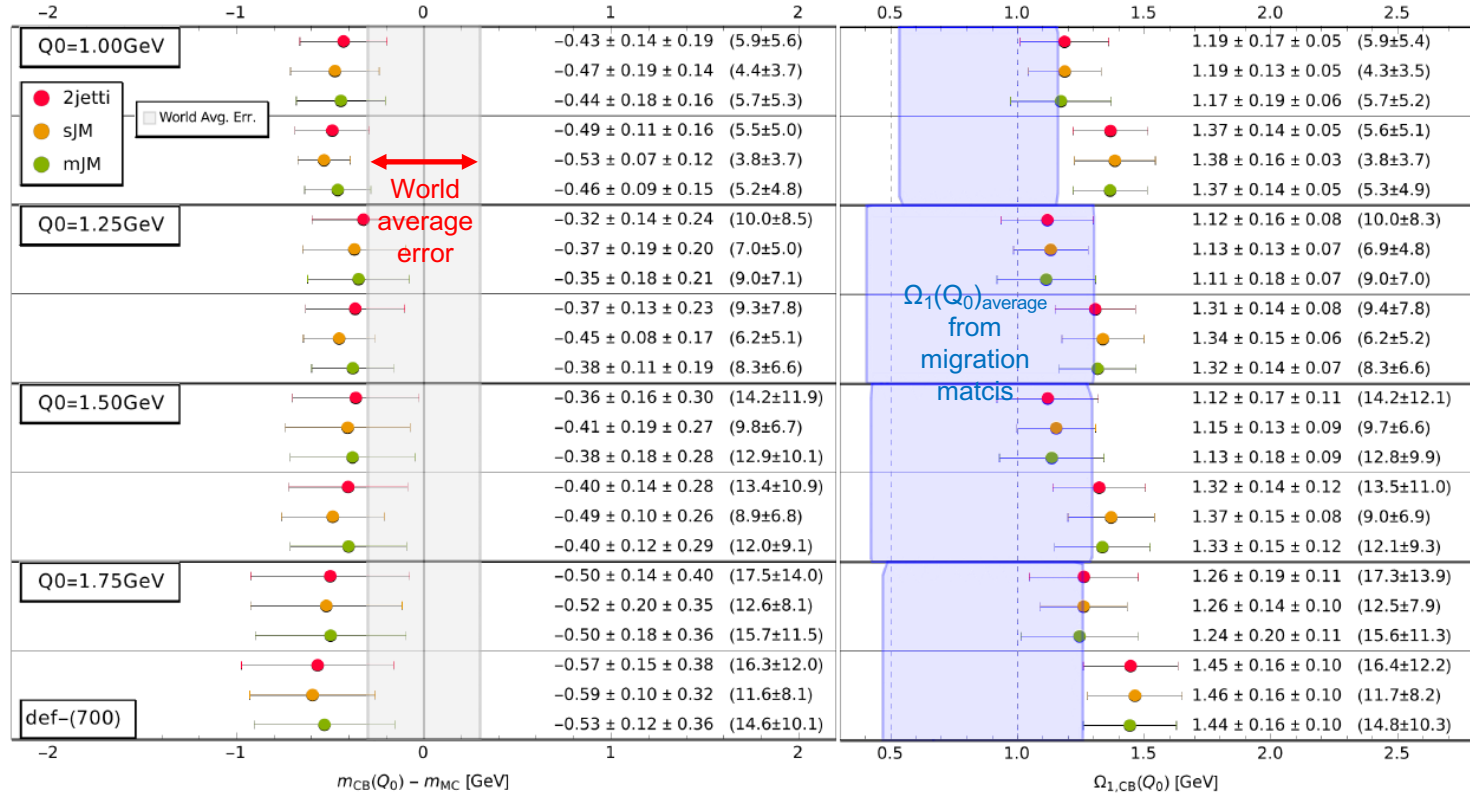
Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz to appear

Cross check: apply top mass calibration to determine $m_t^{\text{CB}}(Q_0)$

preliminary

default
model



Default: m_t^{MC} incompatible with $m_t^{\text{CB}}(Q_0)$

First moment of migration matrix with large variations, Q_0 -evolution not visible

Dynamical: m_t^{MC} compatible with $m_t^{\text{CB}}(Q_0)$

First moment of migration matrix with smaller variations, Q_0 -evolution clearly visible

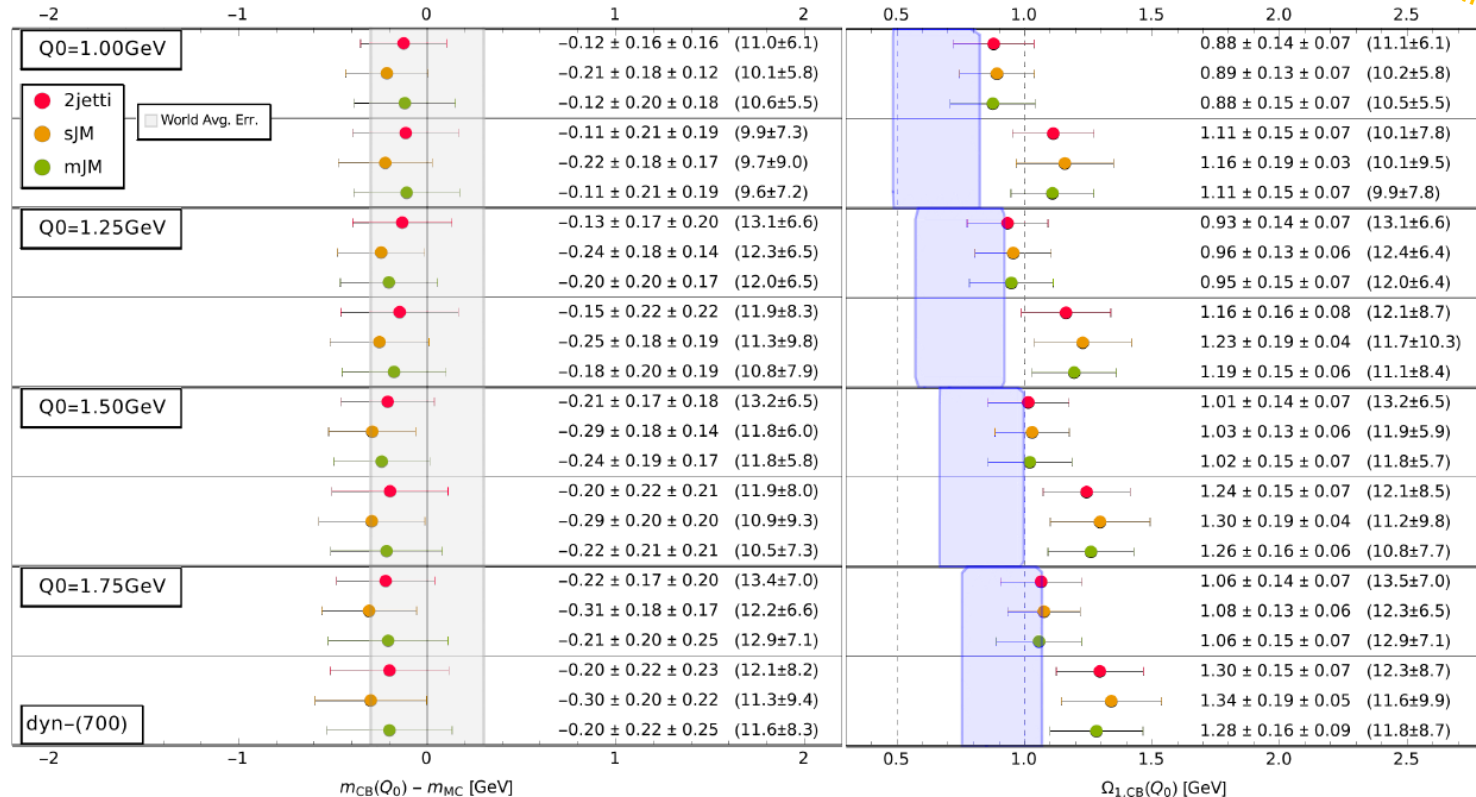
Old Default Model vs. New Dynamical Model

AHH, Jin, Plätzer, Samitz to appear

Cross check: apply top mass calibration to determine $m_t^{\text{CB}}(Q_0)$

preliminary

dynamical
model



Default: m_t^{MC} incompatible with $m_t^{\text{CB}}(Q_0)$

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Dynamical: m_t^{MC} compatible with $m_t^{\text{CB}}(Q_0)$

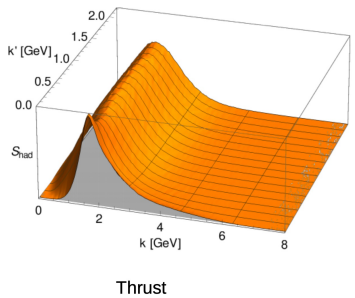
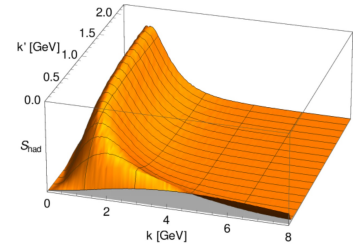
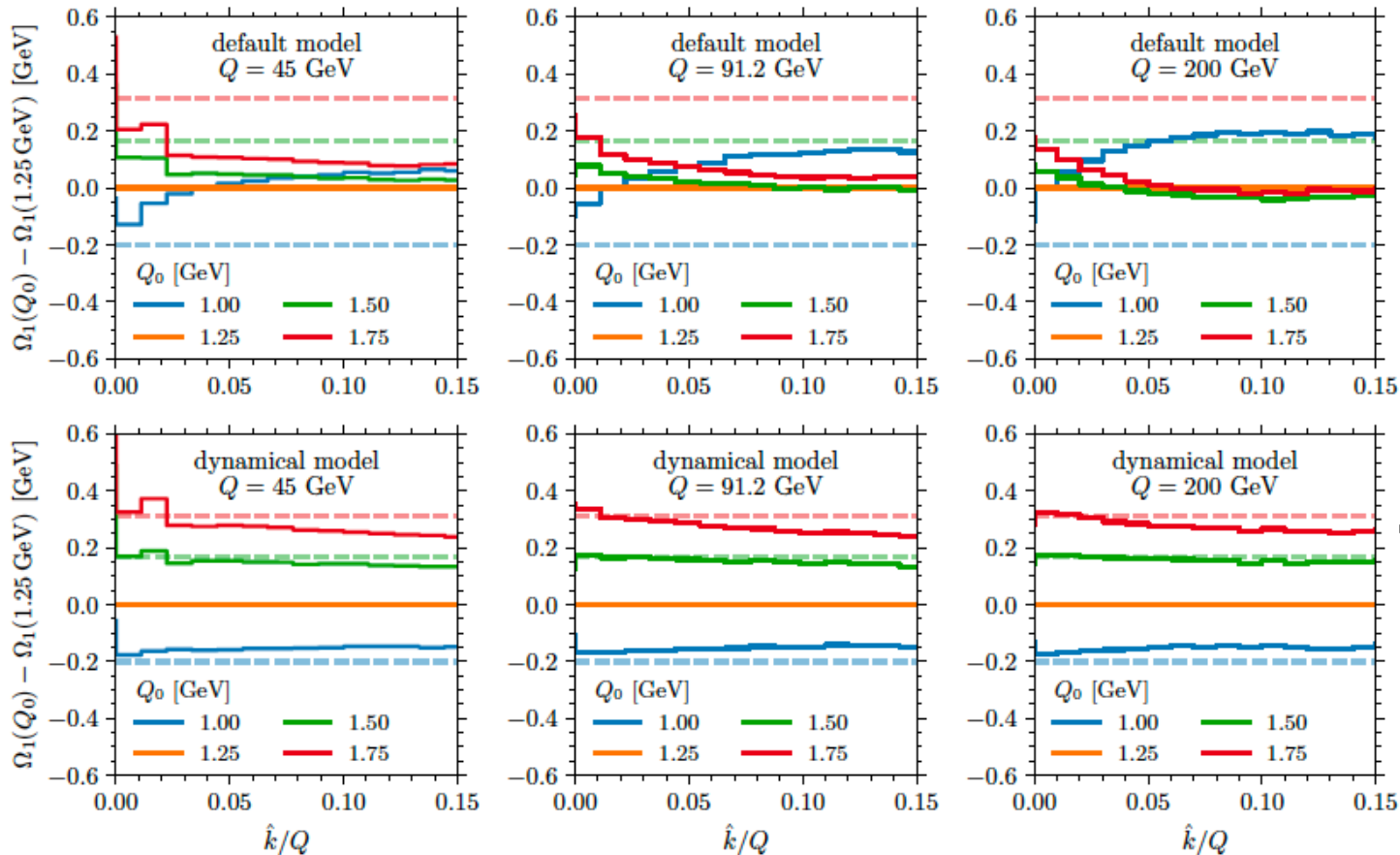
First moment of migration matrix with smaller uncertainties, Q_0 -evolution clearly visible

Old Default Model vs. New Dynamical Model

AHH, Jin. Plätzer, Samitz 2404.09856

Shower cutoff dependence of first moment Ω_1 of migration matrix from simulations for 2-jettiness $\rightarrow$ "MC scheme for hadronization correction"

$$\Omega_1^{\text{MC}}(\hat{k}, Q, Q_0) - \Omega_1^{\text{MC}}(\hat{k}, Q, Q_{0,\text{ref}} = 1.25 \text{ GeV})$$

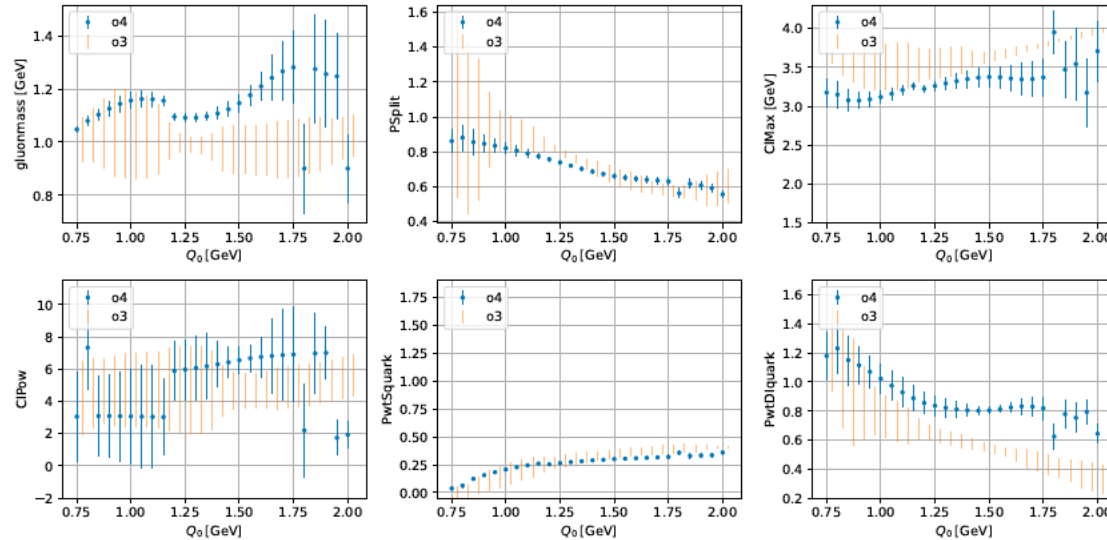


Old Default Model vs. New Dynamical Model

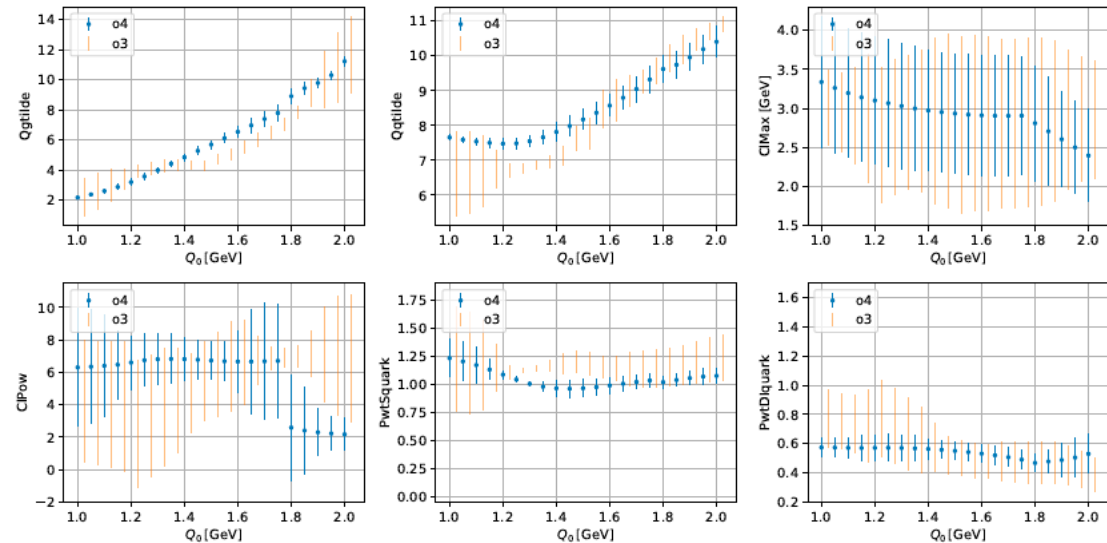
Tuned parameters for Q_0 -dependent tuning analyses (apart from m_t^{MC})

AHH, Jin, Plätzer,
Samitz
arXiv:2404.09856

Default
model

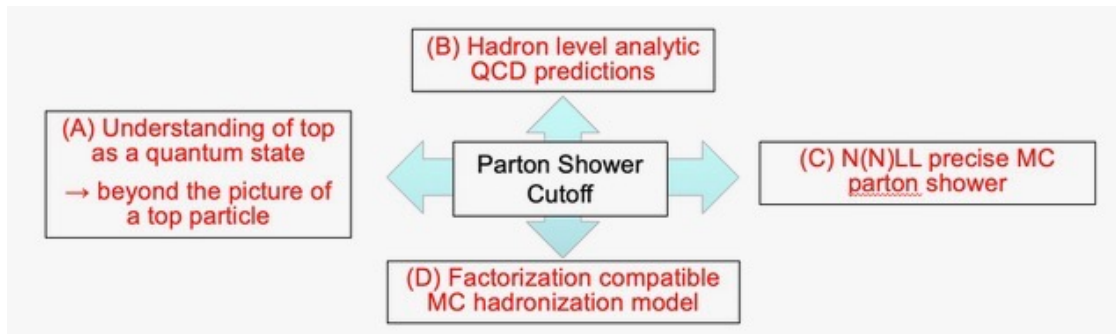


Dynamical
model



Final remarks and Outlook

- **Proof-of-principle**: It is possible to promote MC top mass parameter m_t^{MC} to a renormalization scheme so that its NLO relation to any other top mass renormalization scheme can be calculated.
- $\rightarrow m_t^{\text{Herwig}} = m_t^{\text{CB}}(Q_0)$
- Key aspect: Parton shower cutoff Q_0 = Factorization scale separating pQCD and npQCD
- Currently: Concretely working machinery available only for e^+e^- top jet masses via tuning analyses for different Q_0 values
- The realization of (A)-(D) in this work provides a concrete blueprint that can now be applied to other classes of observables more closely related to direct measurements.



Caveats/open questions:

- MC shower universality among MCs
- Observable dependence

$\rightarrow m_t^{\text{MC}}$ may not be universal

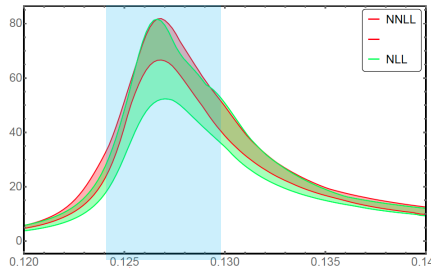
Final remarks and Outlook

- Main aim of future work: Generalization and study of universality
 - conceptual insights and applications for top mass and beyond
e.g. MC Hadronization corrections with controlled scheme dependence
 - complements general developments of MCs towards becoming QCD tools
- Future plans:
 - ▶ investigate dipole showers (N(N)LL), string hadronization (Pythia)
 - ▶ investigate other shower cutoff prescriptions
 - ▶ other observables, e.g. differential in top decay (→ e.g. $M_{\text{b-jet lepton}}$)
energy correlators
 - IR sensitivity & non-perturbative corrections
 - ▶ long-term aim: b-jets with small jet radius (non-global)
 - ▶ establish a m_t^{MC} verification tool box
 - final approach may be not as elaborate as shown in this talk
- Final remark: Understanding m_t^{MC} is a global endeavor where new developments on many aspects are needed (NLL-precise MC, analytic calculations, hadronization corrections)

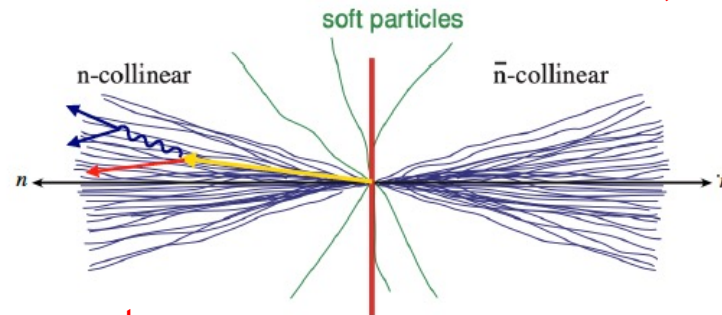
Outlook

Semileptonic top decay distributions with top state defined from 2-jettiness measurement

AHH, Regner, w.i.p.



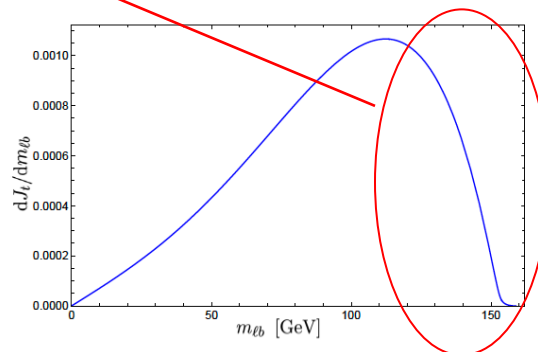
Top state defined by 2-jettiness measurement
(boosted top production)



b-jet = all hadrons in top
hemisphere

$$\frac{d\sigma}{d\tau dM_{b\ell}} \sim H_Q(Q) \times H_{m,t\bar{t}}(m_t) \times H_{m,t \rightarrow b}(m_t)$$

$$\times J_{\bar{t}-jet}(\text{incl.}) \otimes S_{\text{hemi}}(\text{non-pert.}) \otimes J_{b-jet}(\text{incl.}) \otimes S_{\text{ucs}}(\text{Fermi motion})$$

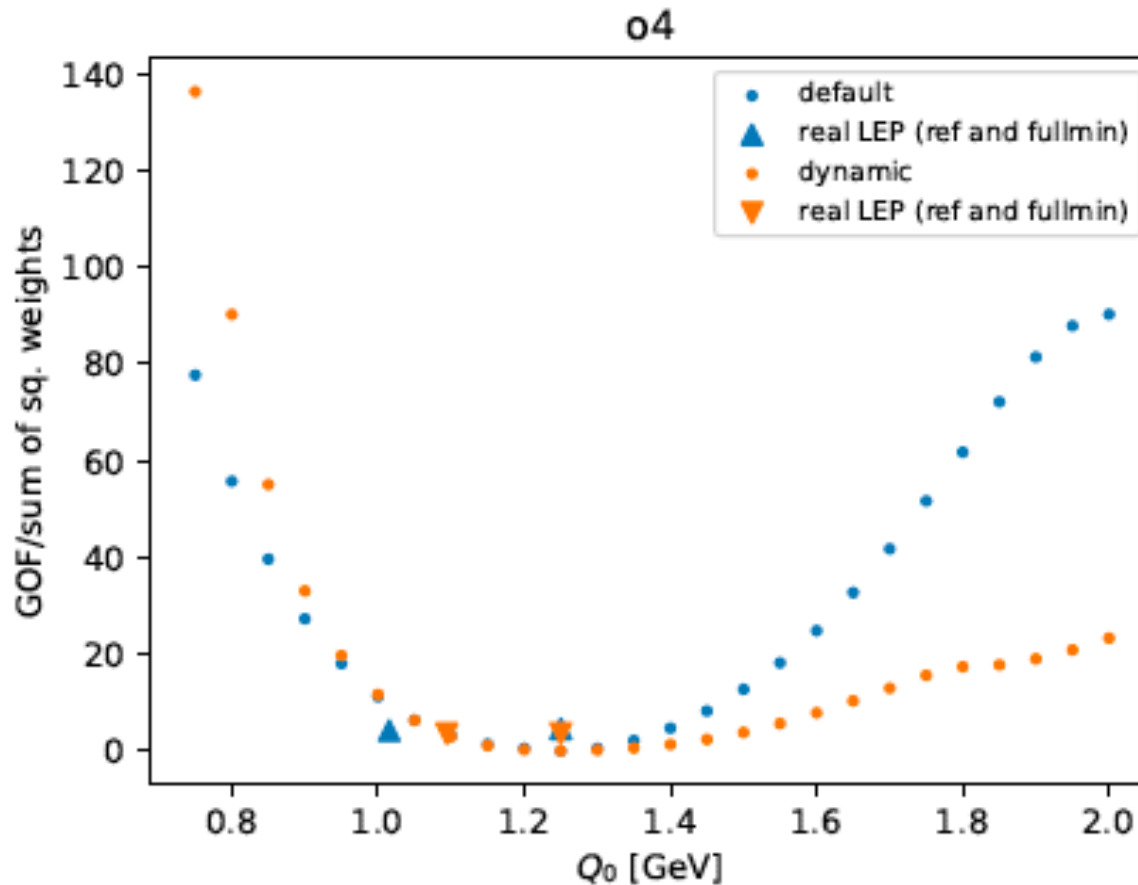


Top mass sensitive
endpoint region

Old Default Model vs. New Dynamical Model

AHH, Jin, Plätzer, Samitz arXiv:2404.09856

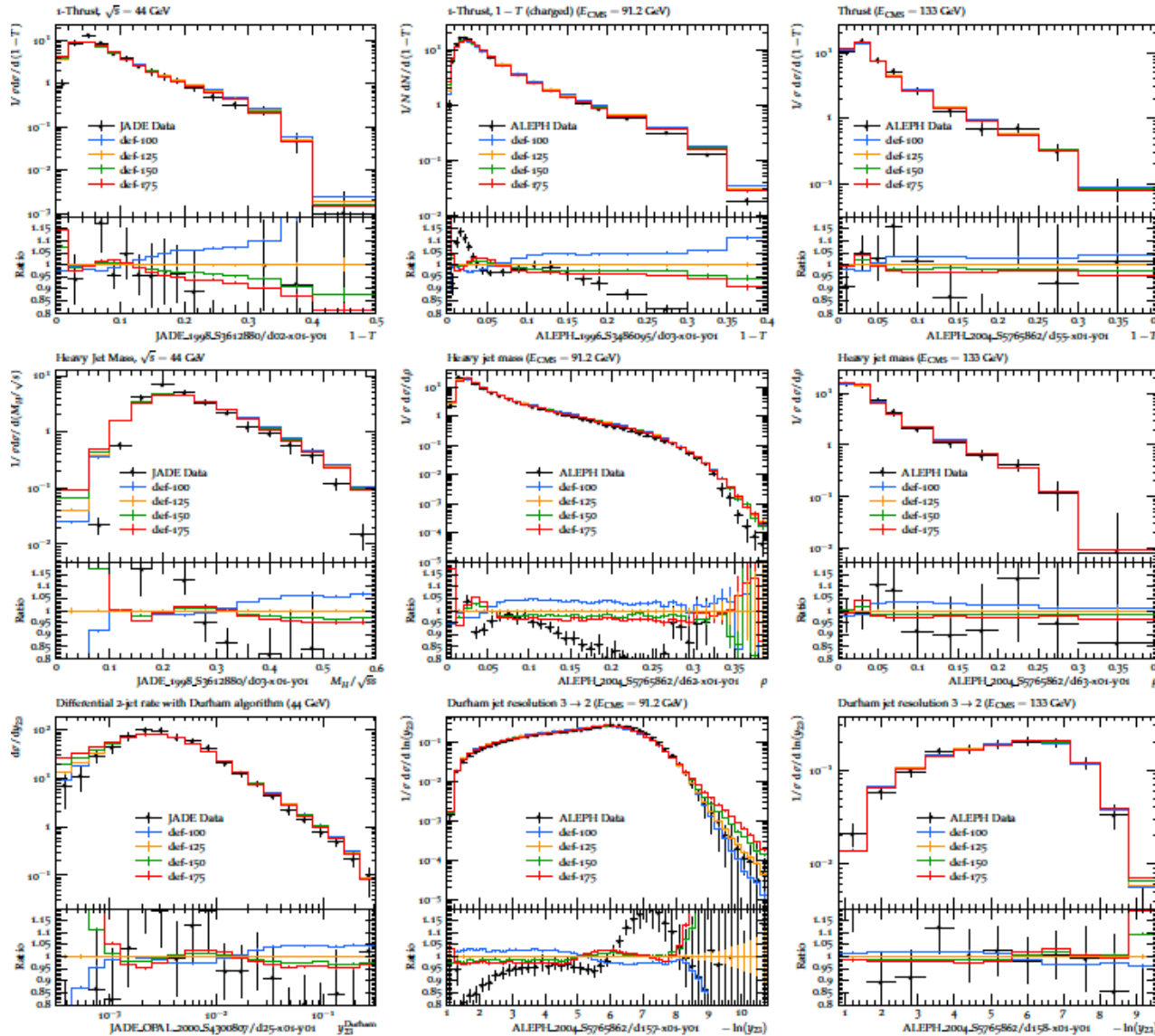
Shower cutoff Q_0 minimal χ^2 -values obtained in the tuning fits



Old Default Model vs. New Dynamical Model

AHH, Jin, Plätzer, Samitz arXiv:2404.09856

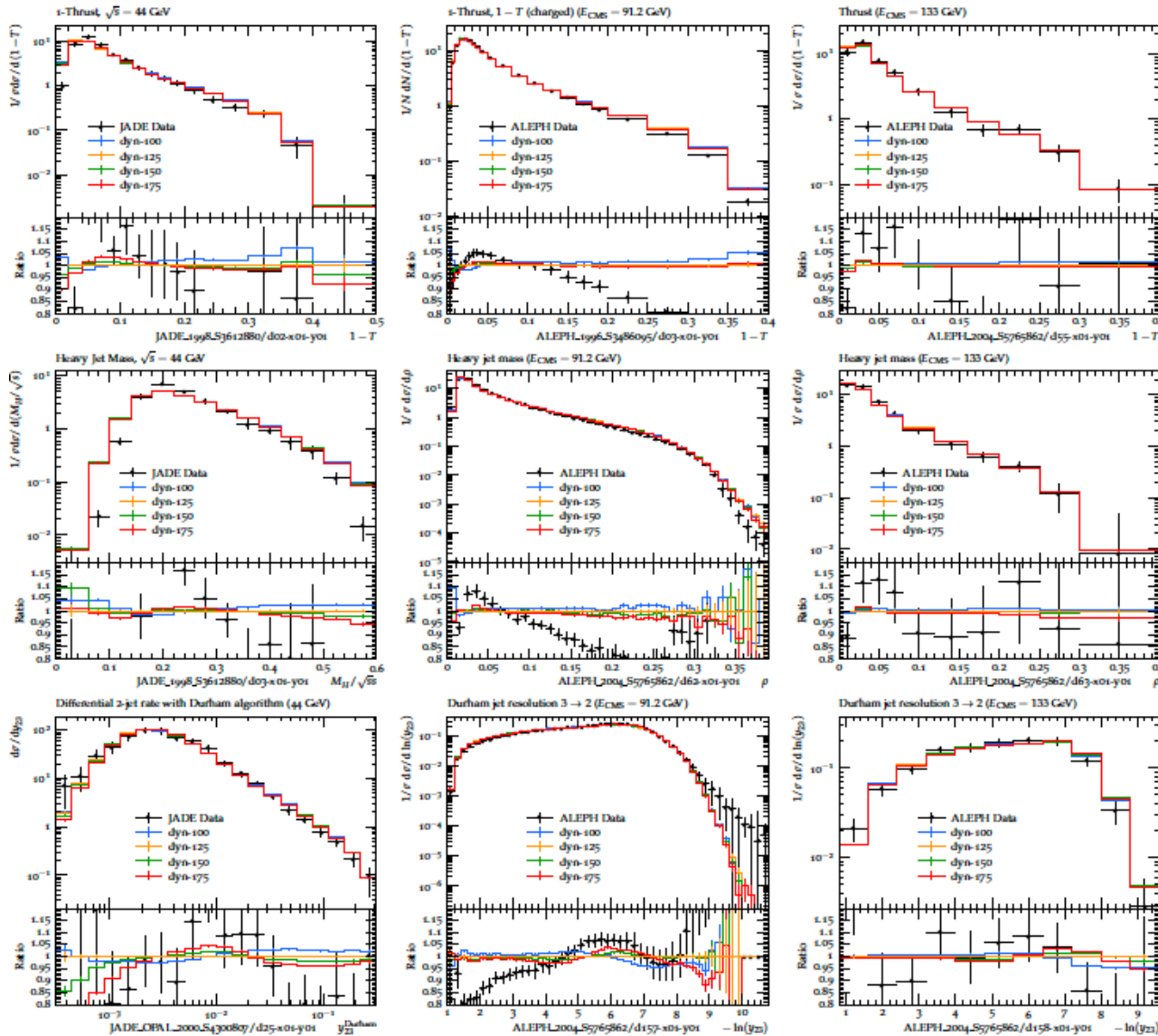
Default



Old Default Model vs. New Dynamical Model

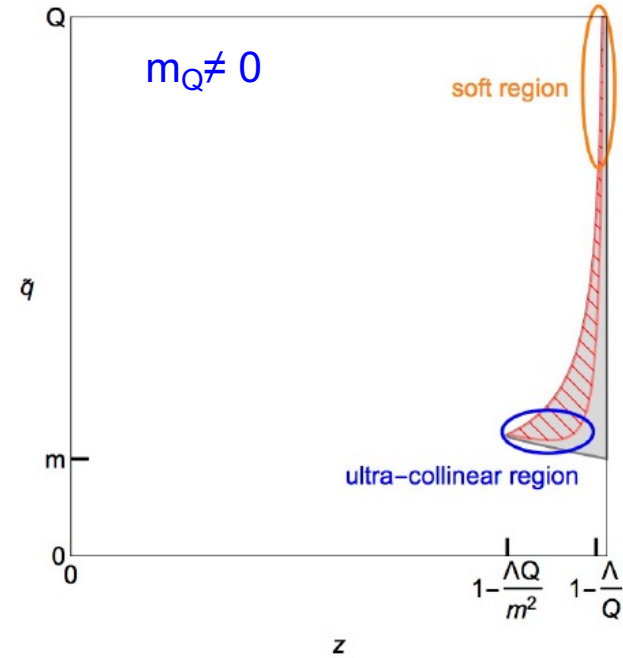
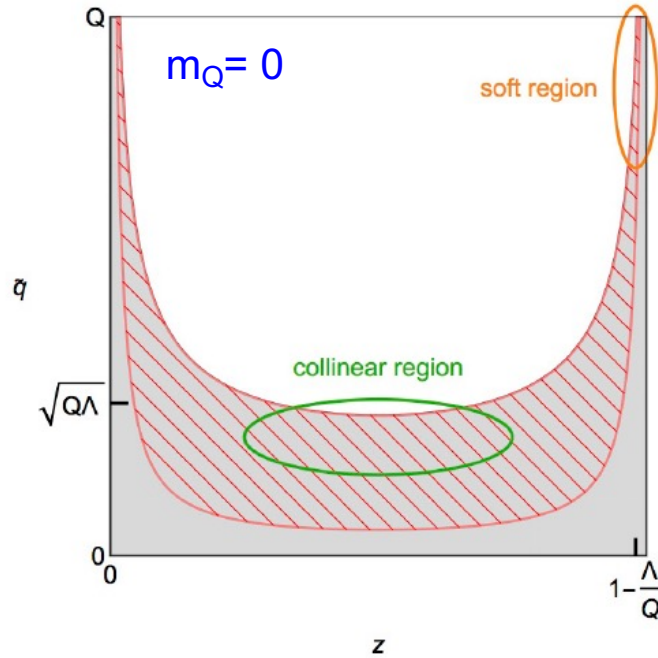
AHH, Jin, Plätzer, Samitz arXiv:2404.09856

Dynamical



Phase Space and Power Counting ($Q_0 \neq 0$)

AHH, Plätzer, Samitz arXiv:1807.06617



phase space regions for $\tau_{\text{peak}} \sim \frac{\Lambda}{Q} \ll 1$, $m = 0$		
	coherent branching	QCD factorization
n-coll.	$z \sim (1 - z) \sim 1$ $\tilde{q} \sim (Q\Lambda)^{\frac{1}{2}}$ $q_{\perp} \sim (Q\Lambda)^{\frac{1}{2}}$	$q^{\mu} \sim (\Lambda, Q, (Q\Lambda)^{\frac{1}{2}})$
soft	$1 - z \sim \frac{\Lambda}{Q}$, $z \sim 1$ $\tilde{q} \sim Q$ $q_{\perp} \sim \Lambda$	$q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$

phase space regions for $\tau_{\text{peak}} - \tau_{\text{min}} \sim \frac{\Lambda}{Q} \ll 1$, $m \neq 0$		
	coherent branching	QCD factorization
u. coll.	$1 - z \sim \frac{Q\Lambda}{m^2}$, $z \sim 1$ $\tilde{q} \sim m$ $q_{\perp} \sim \frac{Q}{m}\Lambda$	$q^{\mu} \sim (\Lambda, \frac{Q^2}{m^2}\Lambda, \frac{Q}{m}\Lambda)$
soft	$1 - z \sim \frac{\Lambda}{Q}$, $z \sim 1$ $\tilde{q} \sim Q$ $q_{\perp} \sim \Lambda$	$q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$