

Update of the ILD $e^+e^- \rightarrow ZHH$ analysis.

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34th Future Colliders @ DESY meeting
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HELMHOLTZ



Higgs potential and Higgs self-coupling

Higgs potential in SM after SSB

$$V(h) = \frac{1}{2}m_H^2 h^2 + \lambda_{HHH} \nu h^3 + \frac{1}{4}\lambda_{HHHH} h^4$$

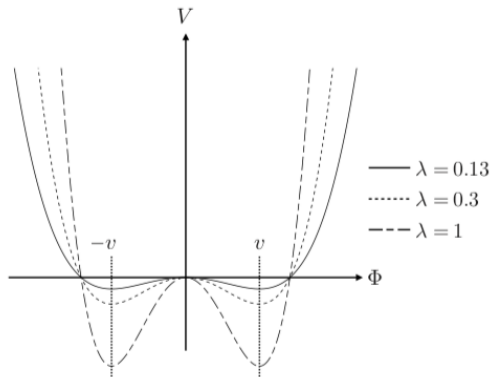
with $\lambda_3^{SM} = \lambda_4^{SM} = \frac{m_H^2}{2\nu^2}$

Measure λ

- → determine shape of **Higgs potential**
- → establish **Higgs mechanism** experimentally
- → determine how the Universe froze in the EW sector, giving mass to gauge bosons, fermions, and the Higgs itself

BSM: deviations in $\lambda \rightarrow$ new physics in Higgs sector

- deviation of λ can be large even if all other couplings are SM-like



Higgs potential and Higgs self-coupling

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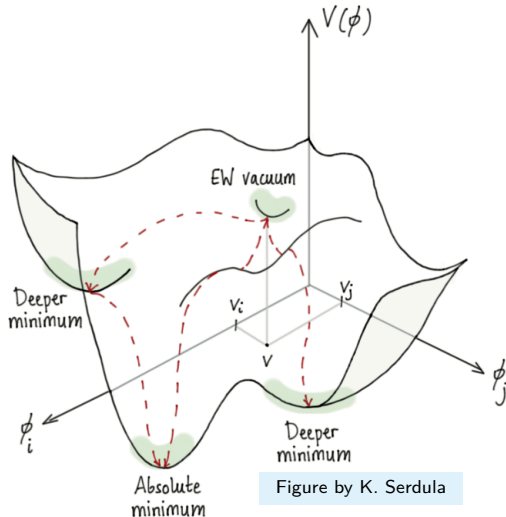
with $\lambda_3^{SM} = \lambda_4^{SM} = \frac{m_H^2}{2v^2}$

Measure λ

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BSM: deviations in λ → new physics in Higgs sector

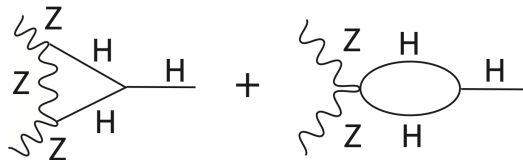
- deviation of λ can be large even if all other couplings are SM-like



Access to the Higgs self-coupling

"Indirect" access at loop-level:

- through loop-order-corrections found from EFT fits using single Higgs measurements and running at two different E_{cm}



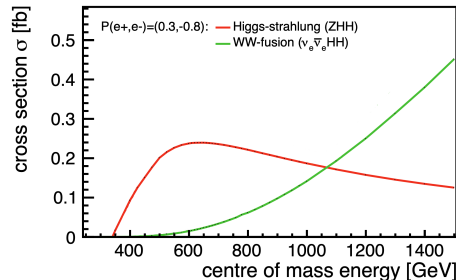
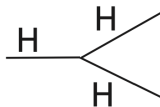
"Direct" access at tree-level:

- through double-Higgs production

$$\frac{\Delta\lambda_{HHH}}{\lambda_{HHH}} = c \cdot \frac{\Delta\sigma_{HHx}}{\sigma_{HHx}}$$

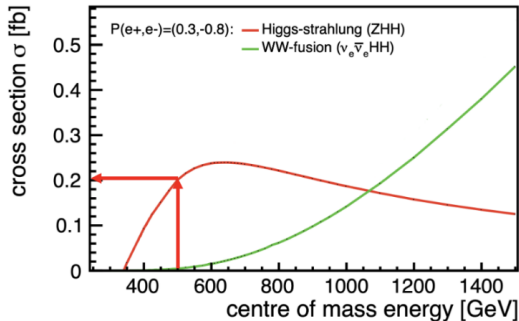
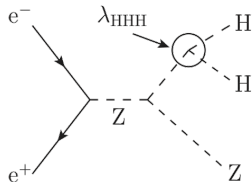
→ cross section measurement

- model-independent which is crucial for handling potentially large deviations in λ_{HHH} while all other couplings are SM-like

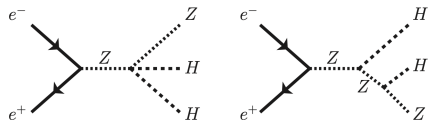
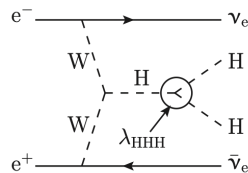


Higgs pair production in e^+e^- collisions

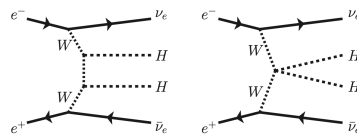
Di-Higgs strahlung:
dominant below 1 TeV



WW fusion:
dominant above 1 TeV



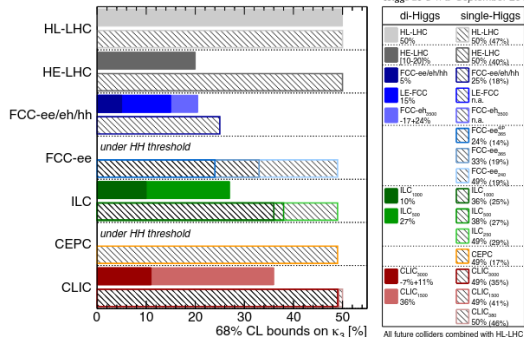
→ constructive interference



→ destructive interference

As of last EPPSU

Higgs@FC WG September 2019

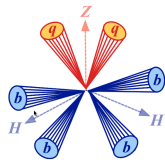
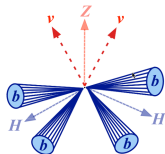
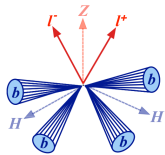


- 27% precision from DESY-THESIS-2016-027
- Performed at proposed E_{CM} of 500 GeV
- ILD detector concept (DBD '13, IDR '20)
- Precision reach after running 4 ab^{-1} at 500 GeV at ILC ($HH \rightarrow bbbb + HH \rightarrow bbWW$)
 - $\rightarrow \Delta\sigma_{ZHH}/\sigma_{ZHH} = 16.8\%$
 - $\rightarrow \Delta\lambda_{SM}/\lambda_{SM} = 26.6\%$
- and with additional running scenario at 1 TeV
 - $\rightarrow \Delta\lambda_{SM}/\lambda_{SM} = 10\%$

$\rightarrow 8\sigma$ observation of $e^+e^- \rightarrow ZHH$
 \rightarrow only $3.x\sigma$ observation of λ_{SM}

- Improvements in reconstruction tools have the potential to bring the sensitivity to **better than 20%** [ILC Snowmass white paper '22]

Analysis improvements



3 channels with $Z \rightarrow \ell\ell (ee, \mu\mu), \nu\nu, qq$ and $HH \rightarrow bbbb$

Flavor tagging

- Crucial for identifying the many, many b-jets
- Major progress seen in past decade for flavor tagging using ML tools

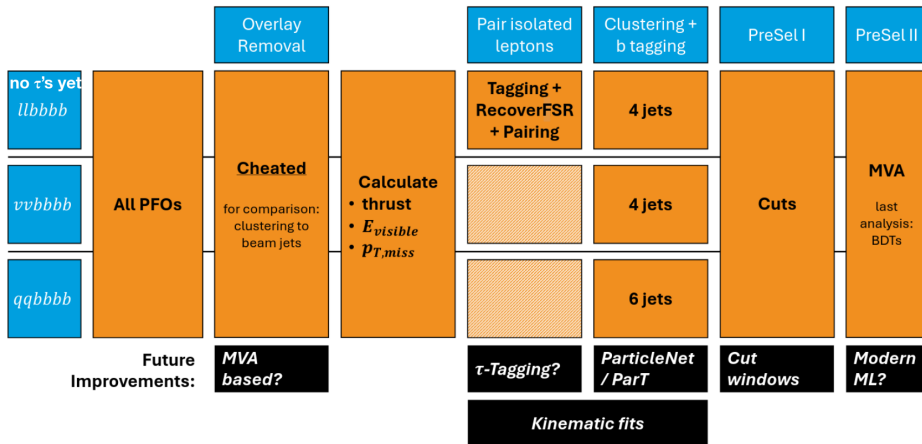
Kinematic reconstruction

- ν within jet from b -hadron decay \rightarrow new ν -correction
- kinematic fitting for better estimating event kinematics
- important input for the **event selection** with ML

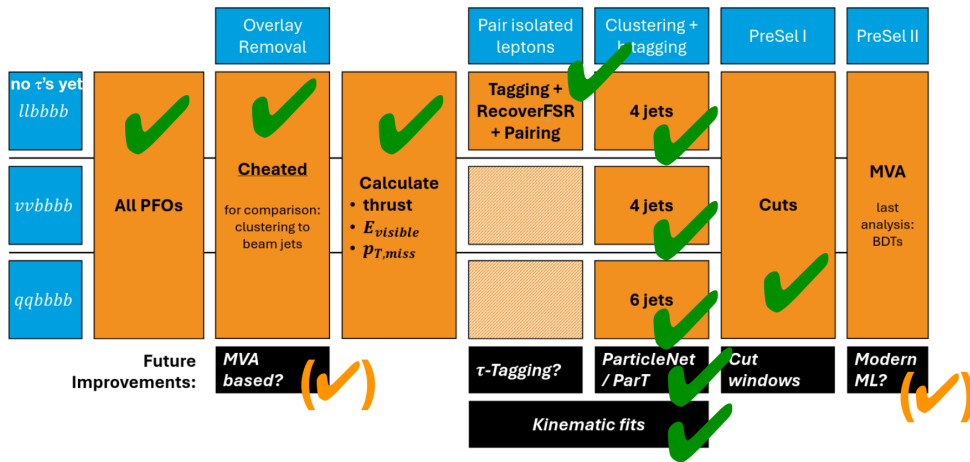
Increasing ECM by 10%

- increases the statistics by $\sim 15\%$
- gives slightly better sensitivity reach
- benefits another flagship measurement, the ttH coupling

Analysis Flow



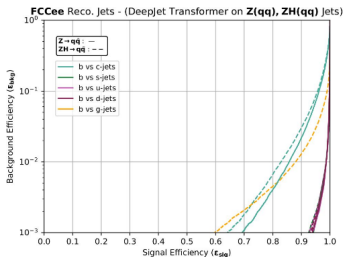
Analysis Flow



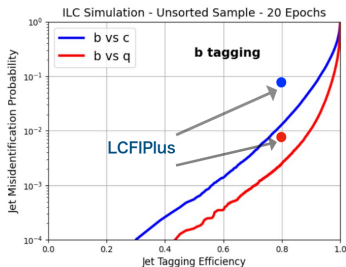
- Note that we are very close to running the full analysis chain!

Flavor tagging with ML

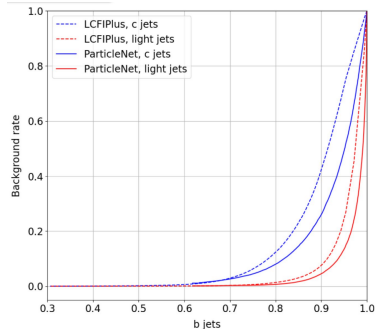
- LCFIPlus: Standard reconstruction tool for jet clustering and flavor tagging at the time of the last analysis
- ML models (DeepJet, ParticleNet, ParT) show highly improved rejection compared to LCFIPlus
 - ParticleNet and ParticleTransformer are ready to use in analysis
 - but still tweaking to improve performance



Flavor tagging performance of DeepJet using IDEA delphes simulation.
E. Ploerer et al. '24



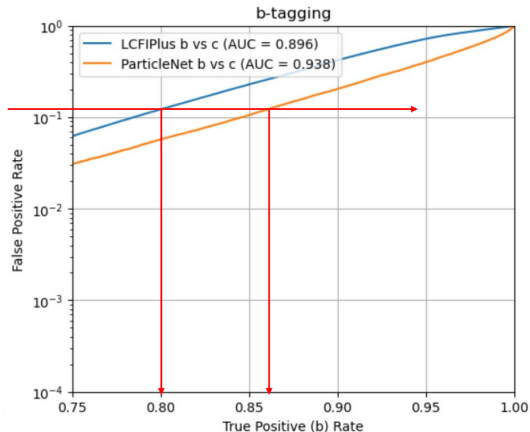
Flavor tagging performance of LCFIPlus vs. ParT using ILD full simulation.
R. Tagami et al. '24



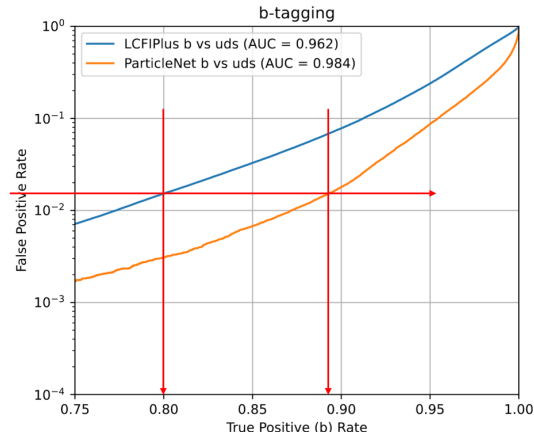
Flavor tagging performance of LCFIPlus vs. ParticleNet using ILD full simulation.
M. Meyer '23

Flavor tagging with ML

Courtesy of B. Bliewert



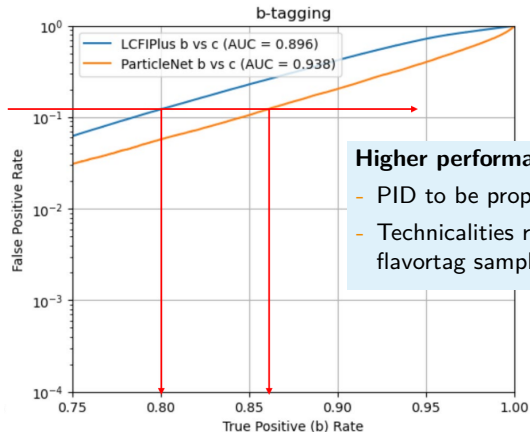
b vs. c @ same background rejection:
7% absolute / 9% relative improvement



b vs. uds @ same background rejection:
9% absolute / 11% relative improvement

Flavor tagging with ML

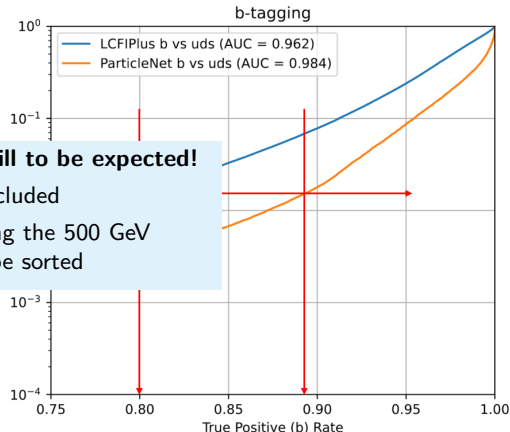
Courtesy of B. Bliewert



Higher performance still to be expected!

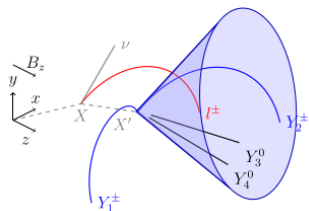
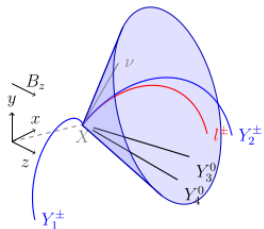
- PID to be properly included
- Technicalities regarding the 500 GeV flavortag samples to be sorted

b vs. c @ same background rejection:
7% absolute / 9% relative improvement



b vs. uds @ same background rejection:
9% absolute / 11% relative improvement

Kinematic reconstruction



ν -correction [arXiv:2105.08480](https://arxiv.org/abs/2105.08480)

Missing neutrinos from semi-leptonic-decays can be corrected if:

- charged lepton is associated to a secondary vertex
- charged lepton is a single track but another vertex exist in the jet

But only down to a sign ambiguity!

Solution: kinematic fit to test \pm correction

ErrorFlow [arXiv:2110.13731](https://arxiv.org/abs/2110.13731)

Parametrize sources of uncertainties for *individual* jets:

$$\sigma_{E_{jet}} = \sigma_{Det} \oplus \sigma_{Conf} \oplus \sigma_{\nu} \oplus \sigma_{Clus} \oplus \sigma_{Had} \oplus \sigma_{\gamma\gamma}$$

- σ_{Det} : Detector resolution
- σ_{Conf} : Particle confusion in Particle Flow Algorithm
- σ_{ν} : Neutrino correction
- both developed for the simpler ZH final state but shown to work well on ZHH events too

Kinematic reconstruction

Kinematic fitting

Exploit well-known initial state in e^+e^- colliders:

- Improve kinematics, e.g. mass resolution
- Hypothesis testing
- Jet-pairing

χ^2 -function to minimise:

$$L(y) = \Delta y^T \mathbf{V}(y)^{-1} \Delta y + 2 \sum_{k=1}^m \lambda_k f_k(a, y)$$

$f(a, y)$: fit model with a parameters and y measurements

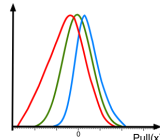
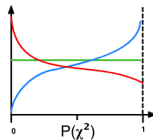
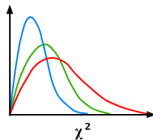
- usual fitting: given y , find a
- kinematic fit: given a , find y (Δy)

4C fit \rightarrow four constraints

- E conservation
- p_x conservation
- p_y conservation
- p_z conservation

\rightarrow

Chi2 and probability
+
Corrections (Δy) to the
jets and leptons



- well-estimated errors
- under-estimated errors
- over-estimated errors

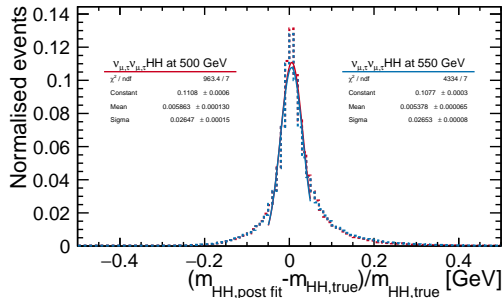
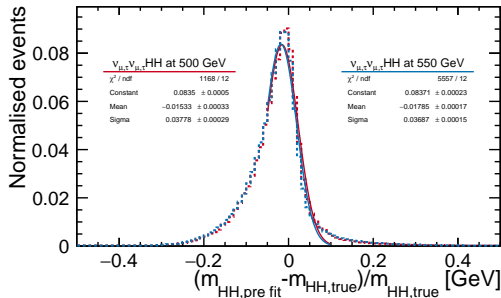
$$pull(y) = \frac{y_m - y_f}{\sqrt{\sigma_m^2 - \sigma_f^2}}$$

Kinematic reconstruction

Improving kinematic reconstruction: mass resolutions

ν -correction + ErrorFlow + Kinematic fit (4-momentum conservation)

→ improves $m(HH)$ distribution with better resolution



Input for event selection

Observables from kinematic fitting

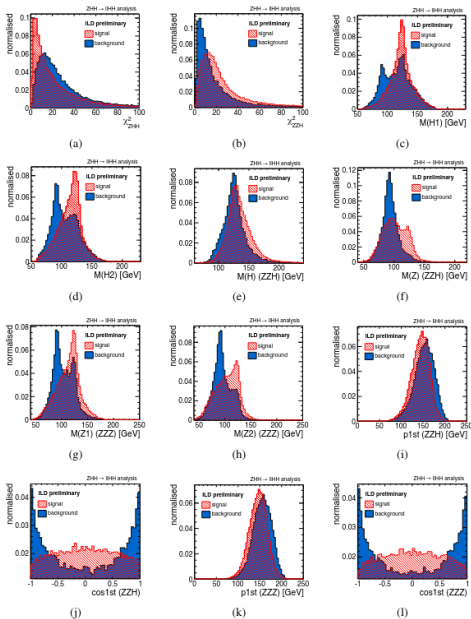
→ helps discriminating ZZH and ZZZ from ZHH

For lepton channel with 2 leptons and 4 jets signature:

- χ^2_{ZHH} and χ^2_{ZZH}
- $M_{j_1 j_2}(H)$ and $M_{j_3 j_4}(H)$ i.e. jet-pairing with ZHH hypo
- $M_{j_1 j_2}(Z)$ and $M_{j_3 j_4}(H)$ i.e. jet-pairing with ZZH hypo + momentum and angle for leading dijet
- $M_{j_1 j_2}(Z)$ and $M_{j_3 j_4}(Z)$ i.e. jet-pairing with ZZZ hypo + momentum and angle for leading dijet

Kinematic fitting works even better than in the past

→ expect improved event selection in interplay with new ML tools



Matrix elements method for ZZH/ZHH discrimination

Another powerful observable for event selection with matrix element method

- for each event, y , calculate event-likelihood for both ZHH and ZZH

$$p_i(\mathbf{y}) = \frac{1}{\sigma_i \cdot A_i} \int \left| \underbrace{\mathcal{M}_i(x)}_{\text{LO matrix element}} \right|^2 \underbrace{W_i(y|x)}_{\text{transfer function}} \epsilon_i(x) d\Phi_n(x)$$

- $e^+e^- + \text{ILD} \rightarrow$ well defined initial state + precisely measured final states
- but ISR+BS, detector transfer function (TF), missing neutrino... not directly measured
- so far only matrix elements \mathcal{M}_{ZHH} and \mathcal{M}_{ZZH}

[More details in PUBDB-2025-00387](#)

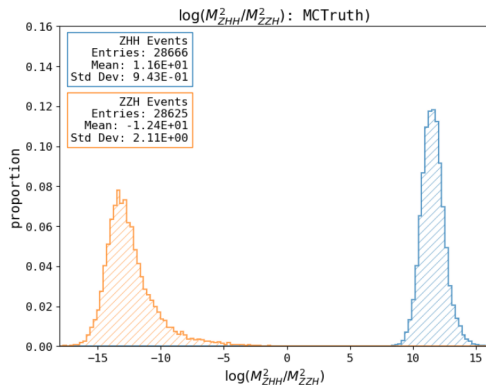
Matrixelements for ZZH/ZHH discrimination

B. Bliewert @ ECFA '24

generator level check

- excellent separation

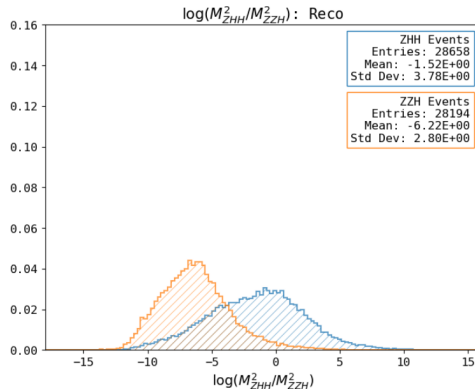
MEM type \ Event data	MC truth	Reco
	ME only	ME+DTF
	-	-



naive MEM

- separation power lost
➔ need to describe smearing with TFs

MEM type \ Event data	MC truth	Reco
	ME only	ME+DTF
	-	-



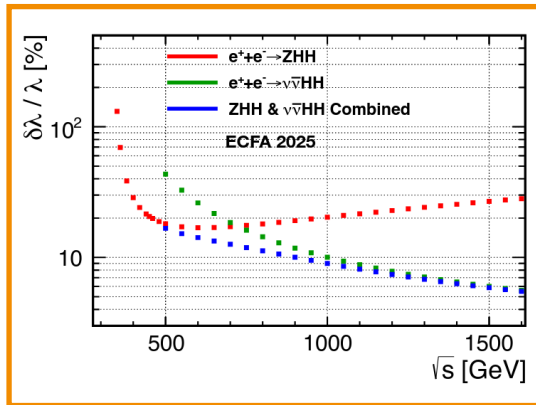
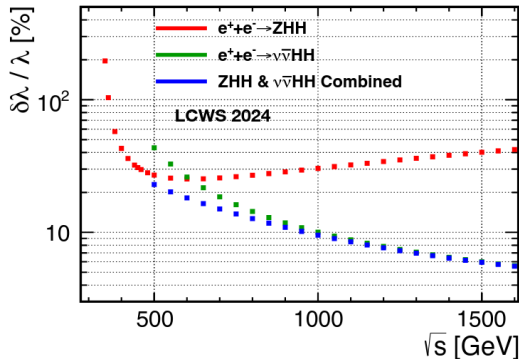
- Projections at ILC500 (DESY-THESIS-16-027) found 27% precision on λ_{HHH} @ 500 GeV with ZHH
 - S, B, and significances for each channel with $Z \rightarrow ee, \mu\mu, \nu\nu, bb, qq$ and $HH \rightarrow bbbb$ and both polarisations: 22.8%

	eebbbb		mumubbbb		nunubbbb		bbbbbb		qqbbbb		comb sig	comb. X-sec. uncert
Pol	-80,+30	+80,-30	-80,+30	+80,-30	-80,+30	+80,-30	-80,+30	+80,-30	-80,+30	+80,-30		
Significance (meas.) Claude	1.07	0.92	1.26	1.1	1.5	1.54	1.57	1.58	1.55	1.64	4.41	0.227
χ^2	1.14	0.85	1.59	1.21	2.25	2.37	2.46	2.50	2.40	2.69	19.46	
s Claude (Tab 9.1)	3.9	2.9	5.1	3.8	5.6	3.6	8.5	5.9	12.6	8.3		
b Claude (Tab 9.1)	7	4.2	8.9	5.3	6.9	1.1	21.9	7	55	16		
s/sqrt(s+b)	1.18	1.09	1.36	1.26	1.58	1.66	1.54	1.64	1.53	1.68	4.39	0.228
χ^2	1.40	1.18	1.86	1.59	2.51	2.76	2.38	2.70	2.35	2.83	19.29	

- apply changes to signal s per channel and polarisation, re-calculate combined cross-section significance
 - flavour tag improvement: 22.8% \rightarrow 17.2%
 - 10% rel. efficiency improvement per jet @ same bkg level
 - enters to 3rd power (i.e. 3 out of 4 jets tagged)
 - kin. sel. improvement: 17.2% \rightarrow 16%
 - 10% rel. efficiency improvement @ same bkg
- include additional channels (as in DESY-THESIS-16-027): 16% \rightarrow 11.2%
 - $Z \rightarrow \tau\tau$, $HH \rightarrow bbWW$, $HH \rightarrow bb\tau\tau$ and "other"
- convert to $d\lambda/\lambda(SM)$ with sensitivity factor incl. m_{HH} weighting (1.62): $d\lambda/\lambda(SM) = 18\%$

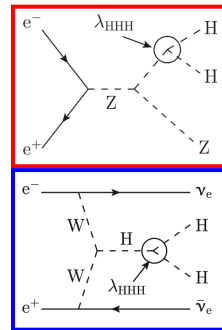
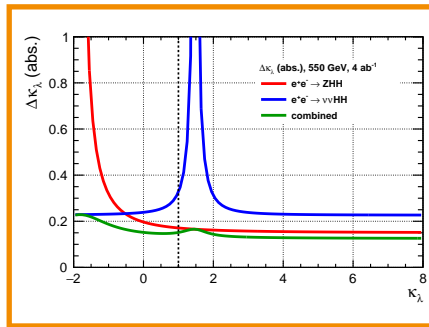
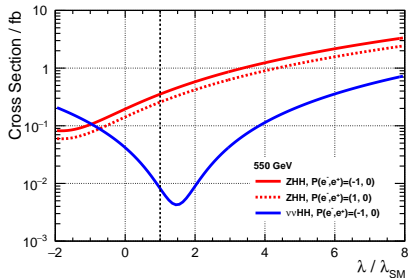
ECM Dependency

Courtesy of Jenny and J. Tian



- 18% precision on $\lambda_{HHH}(SM)$ @ 500 GeV with ZHH
- 15% precision on $\lambda_{HHH}(SM)$ @ 550 GeV with ZHH + $\nu\nu HH$

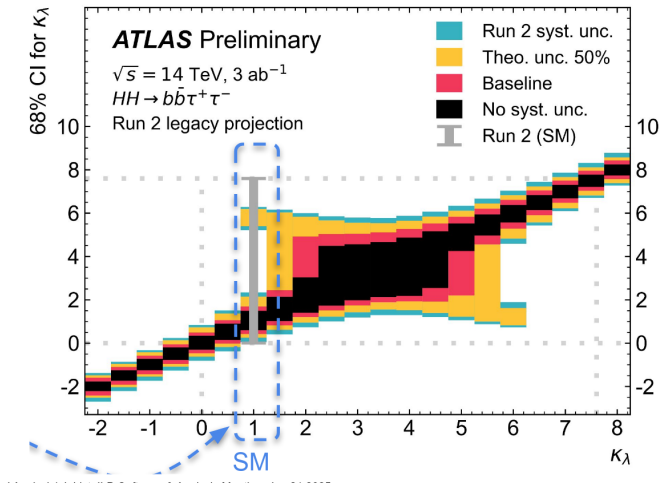
Beyond the SM



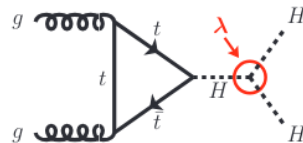
- The cross sections of the two channels have very different dependency of λ
- $\nu\nu HH$ contributes to ZHH already @ 550 GeV even though ZHH is still leading production
- **ILC550: Complementarity of ZHH and $\nu\nu HH$ gives at most an absolute error on κ_λ of 0.2 for any value of λ_{HHH} down to $\kappa_\lambda < -1$**

Comparison with HL-LHC

ATLAS BSM value projection - $bb\tau\tau$ only



Note only the $bb\tau\tau$ channel with Run 2; update for EPPSU with more channels and more data to be expected

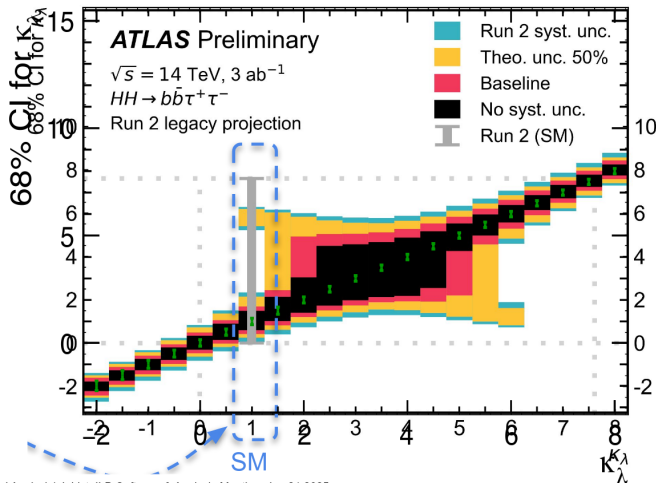


Gluon fusion: similar σ vs. λ to WW fusion

Comparison with HL-LHC

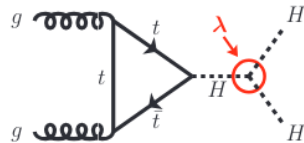
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Courtesy of Jenny



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ILC 550 GeV



Gluon fusion: similar σ vs. λ to WW fusion

Conclusion and next steps

Results submitted for the for ECFA report:

- 18% precision on $\lambda_{HHH}(SM)$ with 4 ab^{-1} @ 500 GeV with ZHH
- 15% precision on $\lambda_{HHH}(SM)$ with 4 ab^{-1} @ 550 GeV with $ZHH + \nu\nu HH$
 - reflects how well we can do *right now* with current tools
 - target even better with flavor tag improvements and setting up a new event selection with ML etc.
- strong precision reach across values of $\lambda \neq \lambda_{SM}$
 - due to the complementarity of ZHH and $\nu\nu HH$

Analysis is moving forward! Next steps:

- MC production @ 550 GeV
- Optimise flavor tag @ 550 GeV
- Re-do full event selection
- etc...

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Thank you.