Mathematics inspired by string theory

Bernd Siebert

Fachbereich Mathematik
Universität Hamburg

LEXI Meeting Hamburg
Octobre 10, 2011

The Research Training Group 1670

"Mathematics inspired by string theory and QFT", 4/2011–

Size: 14 PhD positions, 3 postdocs

Leading Researchers:

Dept. of Mathematics:	Cortés, Kühn, Latschev, Richter, Runkel, Schweigert, Siebert, Wockel
Dept. of Physics:	Fredenhagen, Kniehl, Louis
DESY:	Schomerus, Teschner

Freeman Dyson: “Missed Opportunities”
Bull. AMS 78 (1972)

As a working physicist, I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce.

Mathematics $\longleftrightarrow$ Physics before 1950

- Mathematics as language for physical theories
- Many bilingual people, fruitful collaborations, e.g. Einstein/Weyl; but also tensions, e.g. von Neumann/Dirac, Schrödinger/Wigner ("Gruppenpest").
- Branches of physics and branches of mathematics corresponded, e.g.

Newtonian mechanics	$\longleftrightarrow$	calculus
electrodynamics	$\longleftrightarrow$	vector analysis
statistical physics	$\longleftrightarrow$	probability theory
general relativity	$\longleftrightarrow$	pseudo-Riemannian geometry
quantum mechanics	$\longleftrightarrow$	functional analysis

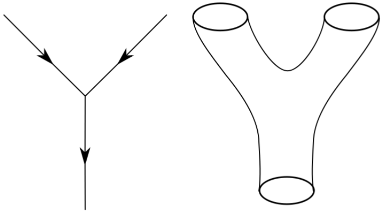
Fields medals since 1980 related to physics

1982	Connes, Yau
1986	Donaldson
1990	Drinfeld, Jones, Witten
1994	–
1998	Borcherds, Kontsevich
2002	–
2006	Okounkov, Werner
2010	Ngô, Smirnov

This makes 12 out of 28 rather directly linked to physics.

The stringy point of view in mathematics

Probe "space" by loops rather than by points:



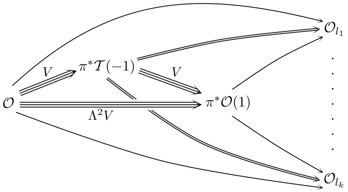
Supersymmetry brings in additional mathematical structure, depending on the physical context.

For example, BPS D-branes ("Dirichlet-type boundary conditions" for open strings) could be holomorphic objects in the IIA string, or Lagrangian in the IIB string.

The ubiquity of "space" in mathematics

Mathematically, "**space**" can be many kinds of things:

- A topological space ($\longrightarrow$ topology)
- A differentiable (Riemannian) manifold ($\longrightarrow$ differential geometry)
- A complex (algebraic) manifold ($\longrightarrow$ complex geometry/algebraic geometry)
- A (linear) representation of a group ($\longrightarrow$ representation theory of groups)
- A representation of a quiver ($\longrightarrow$ representation theory of algebras)



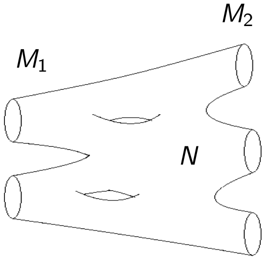
- A C^*-algebra ($\longrightarrow$ Connes' non-commutative geometry)
- ...

The stringy point of view. . .

. . . inspires new areas of research directly, for example:

I. Topological field theory (Atiyah, Segal, Witten):

Idea (dim $= n$**):** A closed manifold M of dimension n provides a *vector space* V_M of states (supported on M).
A compact manifold N of dimension $n+1$ with boundary $M_1 \cup M_2$ defines a linear map $\Phi_N : V_{M_1} \to V_{M_2}$.



Generalizations axiomatizing conformal field theory or treating all dimensions at once (extended TFT) have been very successful.

RTG: "String topology and holomorphic curves" (Latschev, Richter)
"Higher categorical geometry and homological methods for string backgrounds" (Richter, Schweigert, Wockel)

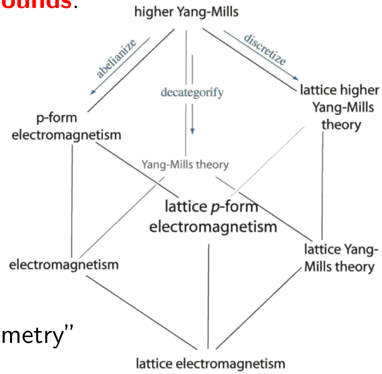
The stringy point of view. . .

. . . inspires new areas of research directly, for example:

II. Generalized geometries for string backgrounds:

Various string theories can be enriched, e.g.

Kähler manifold	$\longrightarrow$	generalized Kähler manifold (Hitchin)
	or even	
manifold	$\longrightarrow$	gerbe (higher categorical space)



RTG: "Differential geometry and supersymmetry" (Cortés, Louis))
"Higher categorical geometry and homological methods for string backgrounds" (Richter, Schweigert, Wockel)

The stringy point of view...

...inspires new areas of research directly, for example:

III. Mathematical conformal field theory:

Much of the structure of conformal field theories can be treated mathematically:

RTG: "Three-dimensional topological quantum field theories"
(Runkel, Schomerus, Teschner), [Schweigert]

The stringy point of view...

...and sometimes links areas of mathematics previously considered largely unrelated, mostly by *string dualities*. Mathematically most prominent:

Mirror symmetry:

IIA/IIB T-duality $\longrightarrow$ relates symplectic geometry of M and complex geometry on a topologically different mirror manifold $\check{M}$

Most useful: D-branes, both for explaning the mathematical nature of mirror symmetry and for manifestations of the phenomenon.

Several other dualities that are also mathematically interesting.

RTG: "Mirror symmetry" (Siebert)

QFT?

RTG: "Algebraic and arithmetic structures in perturbative quantum field theories" (Fredenhagen, Kniehl, Kühn)

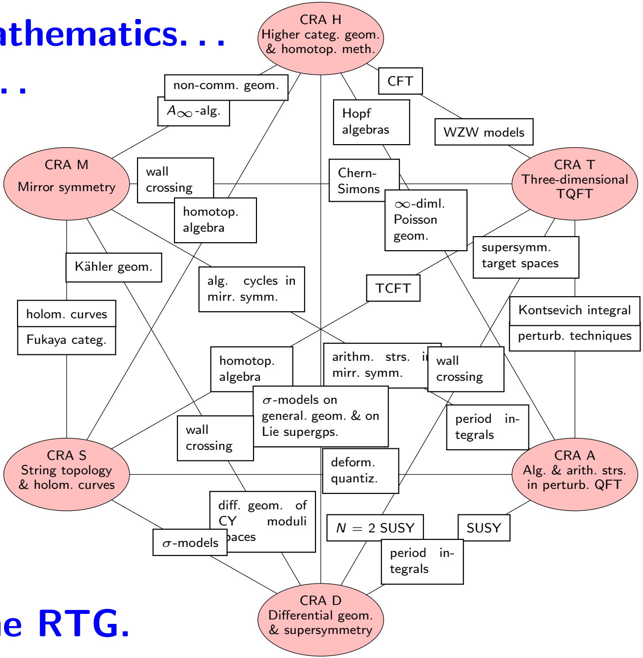
Investigates implications of the recently discovered algebraic structure of Feynman diagrams (Connes/Kreimer) to the evaluation of Feynman integrals (multi-zeta values, motivic nature).

Other topic: Use perturbative QFT for deformation quantization in infinite dimensions.

The technique of Feynman diagrams is becoming more popular also in mathematics, and hence is linked to several other areas within the RTG.

The unity of mathematics. . .

. . . and physics. . .



. . . as seen in the RTG.