QUANTUM ALGORITHMS FOR CHARGED PARTICLE TRACK RECONSTRUCTION IN THE LUXE EXPERIMENT

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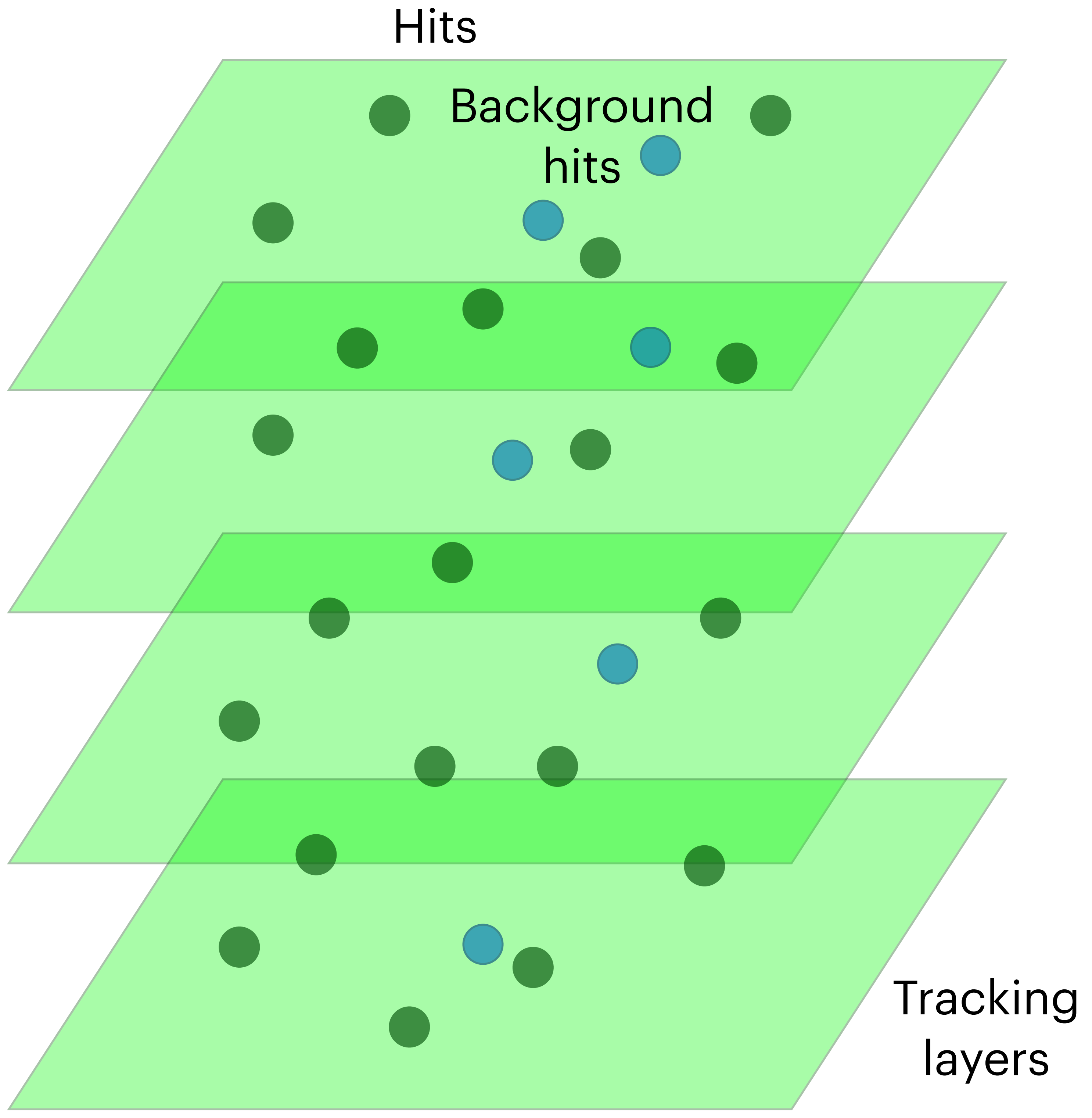
FH Scientific Computing Seminar, 31st January 2025

TRACKING BASICS



- Input: hits left in detector layers by particles.
- **Pattern recognition/track finding:** connect hits to form candidate tracks. Paths expected to be straight (in absence of magnetic field) or curved.
  - **Global** (considering all hits at once) vs **local** (seeded) approaches.
- **Track fitting**: mathematically optimise the track to determine properties, e.g. momentum, charge and origin.

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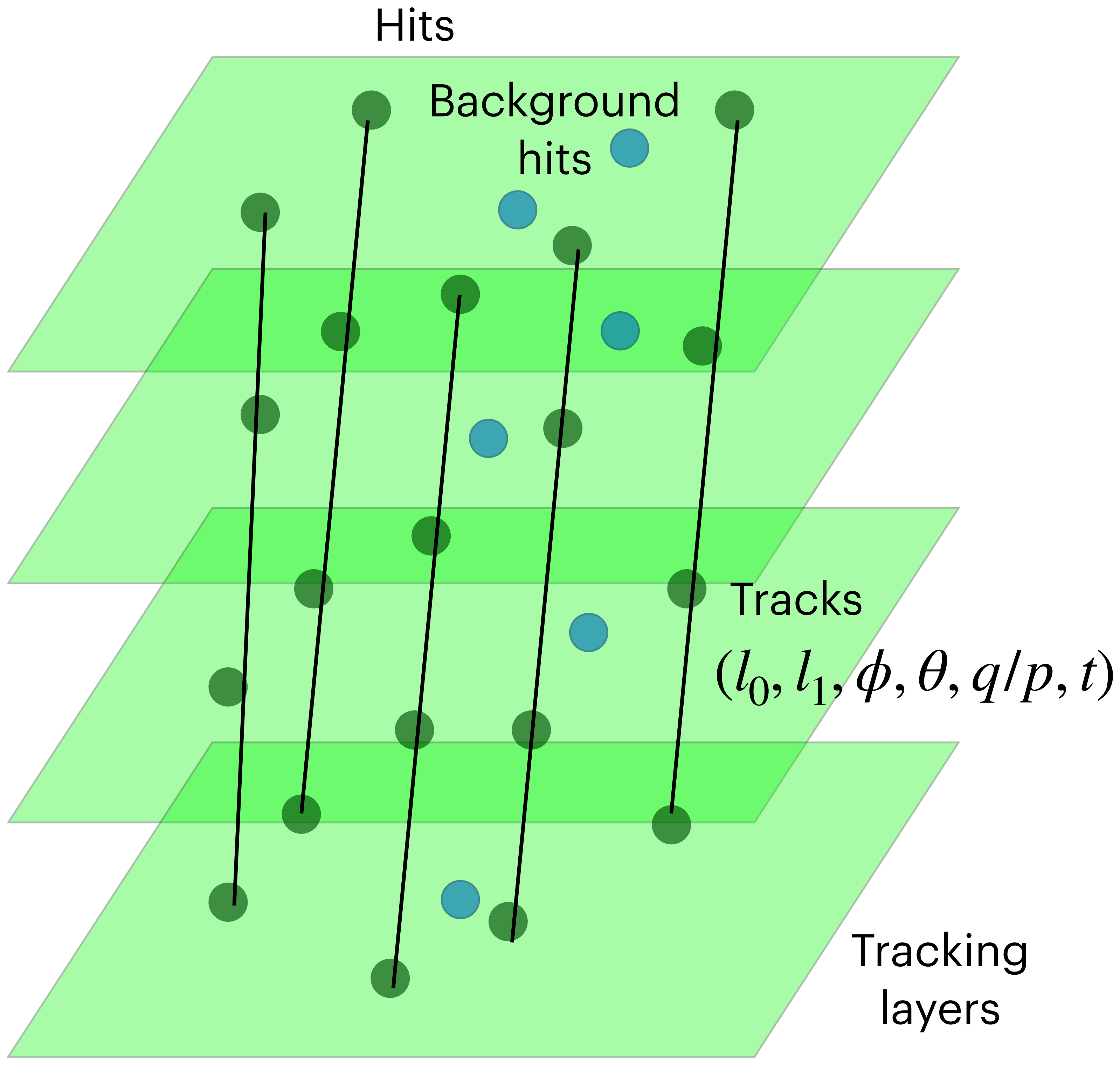
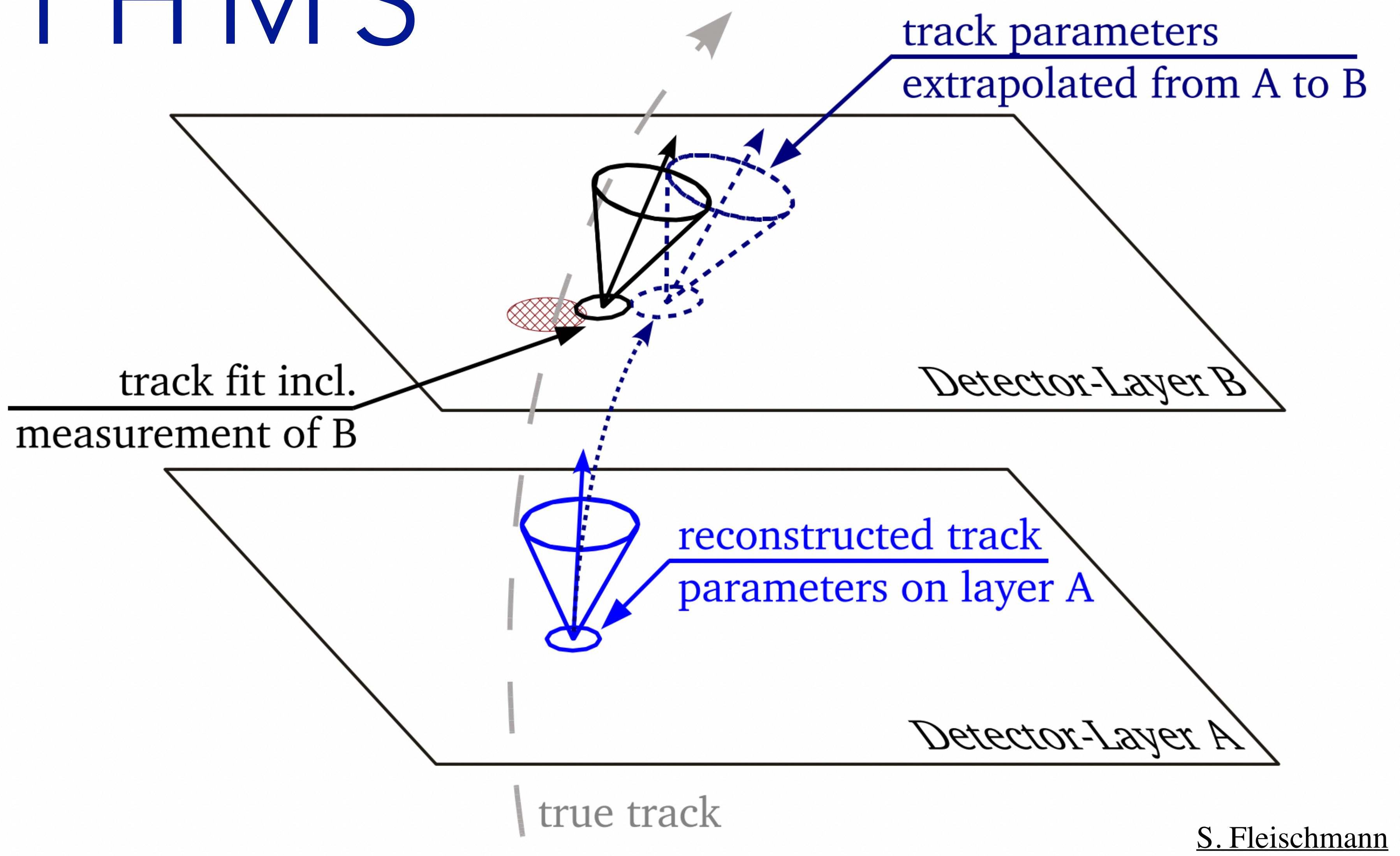
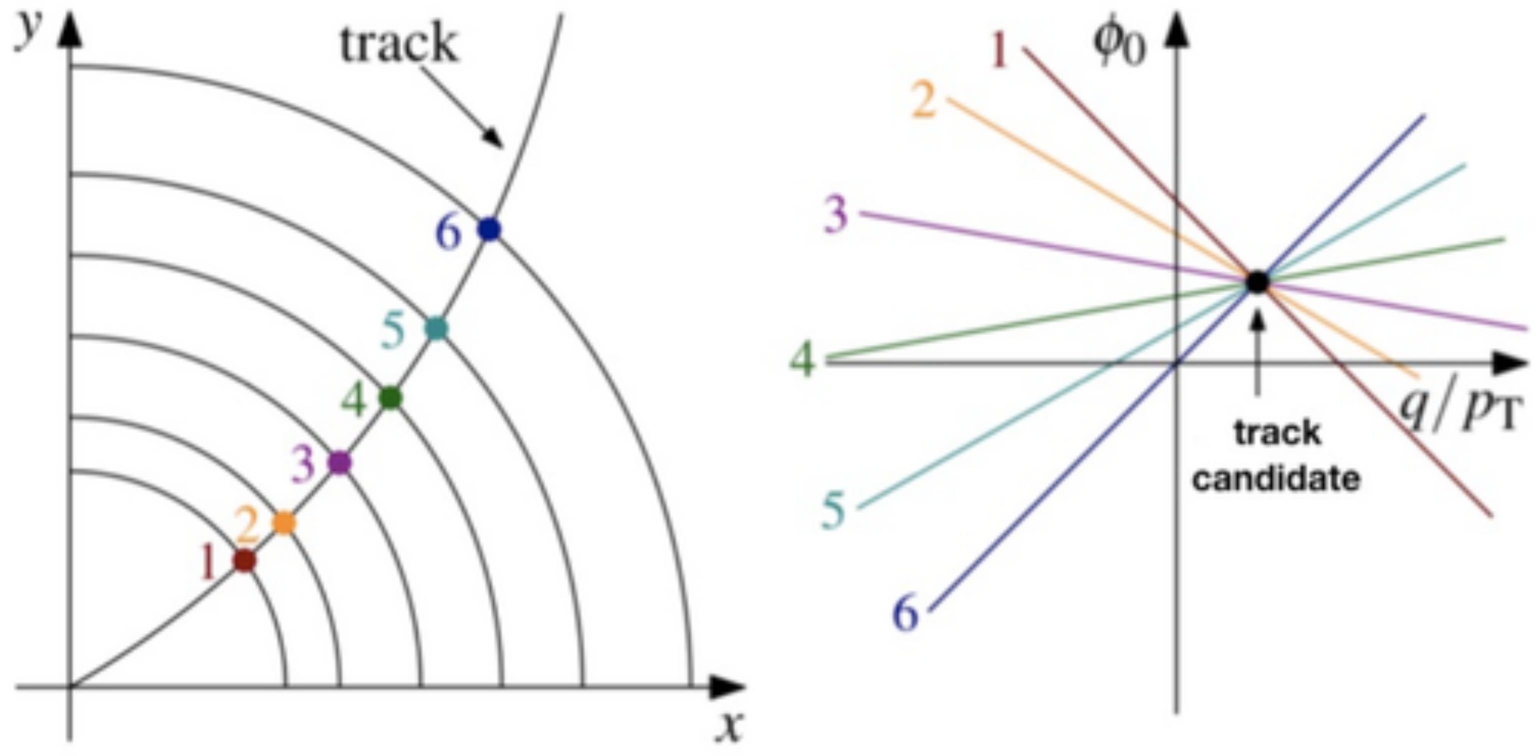


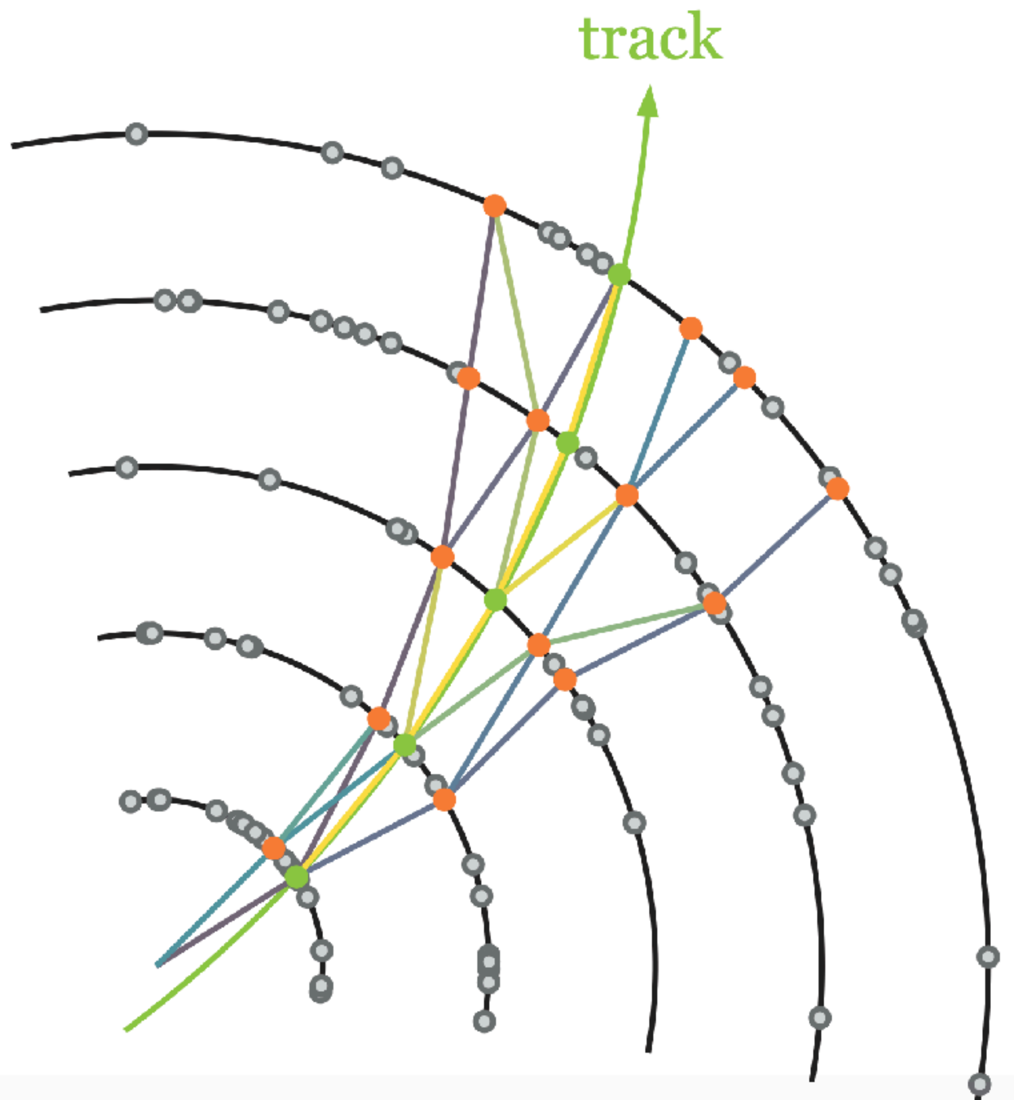
EXAMPLE CLASSICAL ALGORITHMS



- **Kalman Filter** (fitting to estimate track parameters, accounting for uncertainties and material effects, can also be used in track finding)

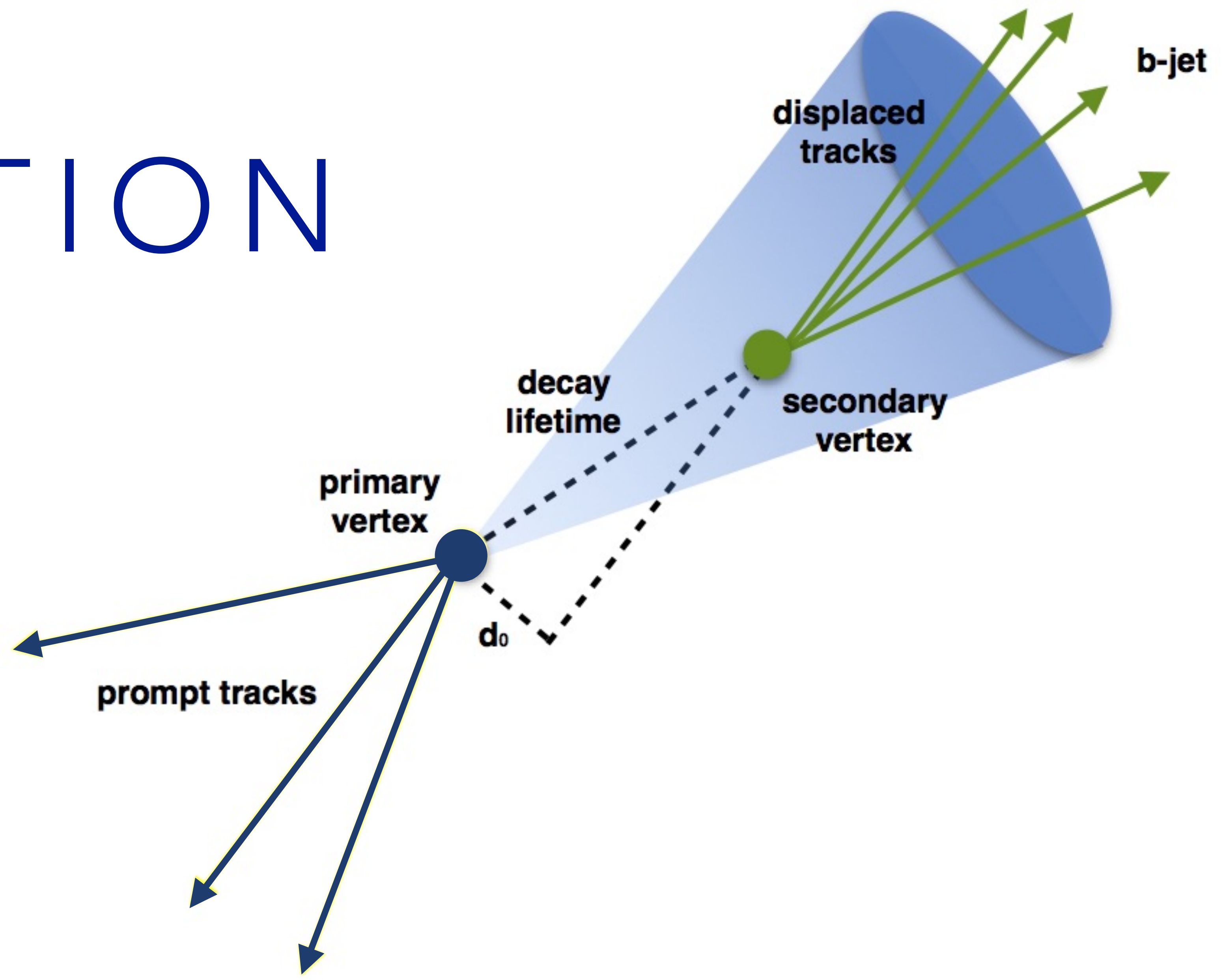


- **Hough transform** (global, maps detector hits into a parameter space, where tracks manifest as clusters)

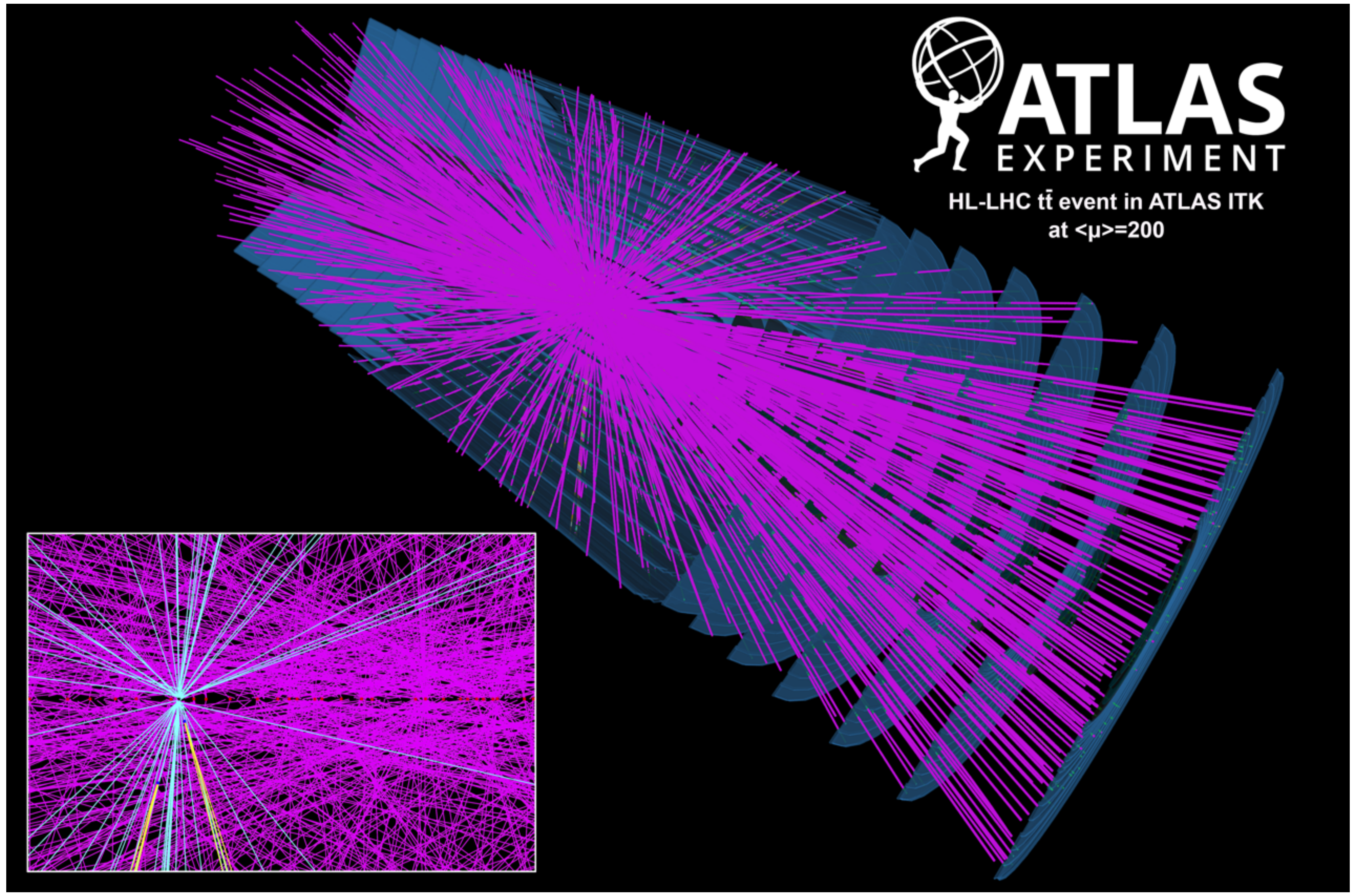


- **Combinatorial Kalman Filter** (local, KF combined with track finding, able to reconstruct multiple tracks from a seed, suited for high occupancy environment)

PARTICLE TRACK RECONSTRUCTION

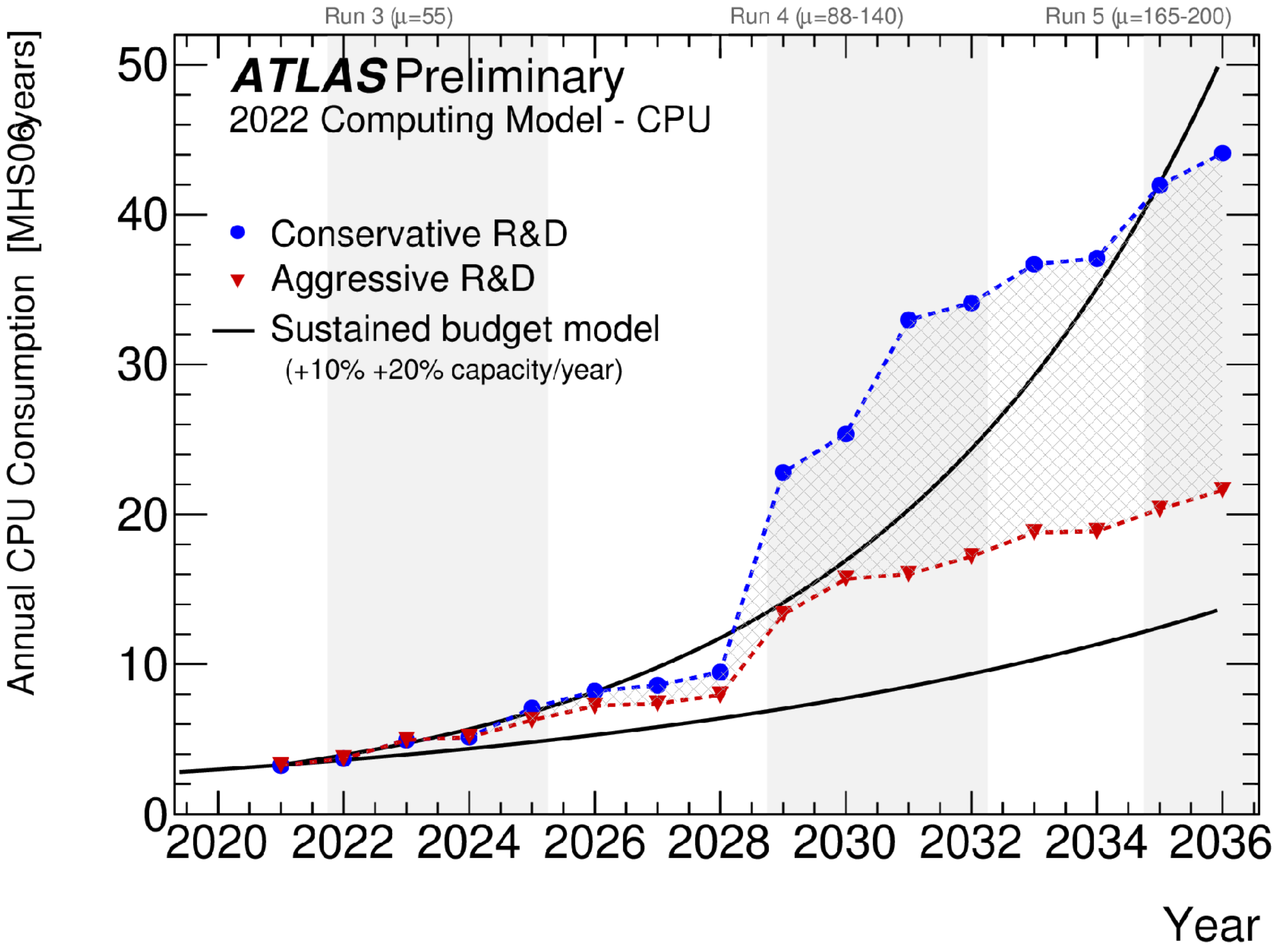


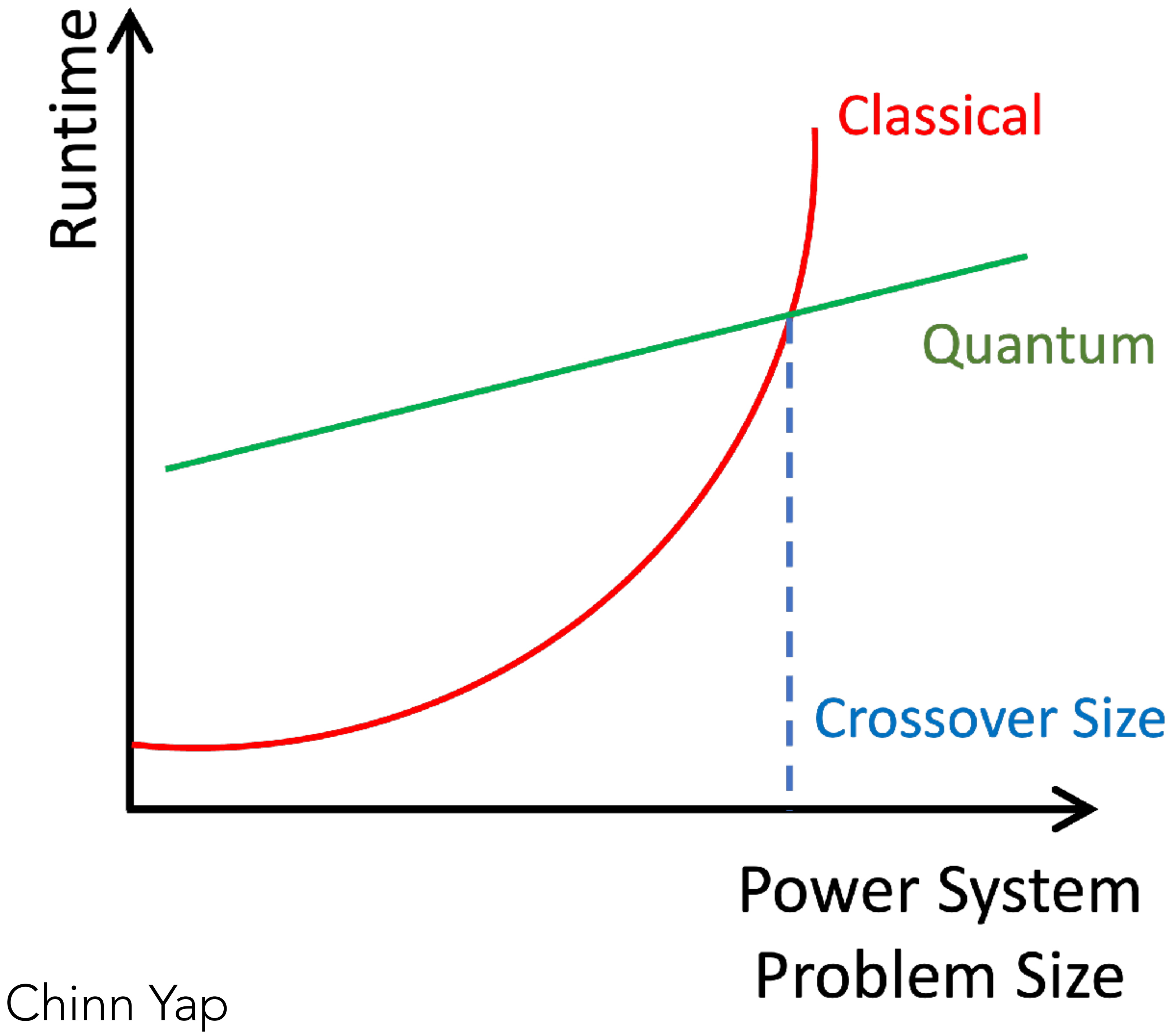
- Central in event reconstruction:
  - Identify particles produced
  - Determine its momentum (with magnetic field)
  - Reconstruct primary/secondary vertices.
- Distinguish genuine signals from background.
- Higher track multiplicities → increasing complexities.



WHY QUANTUM?

- CPU needs growing.
- Tracking is particularly CPU intensive due to its combinatorial complexity.

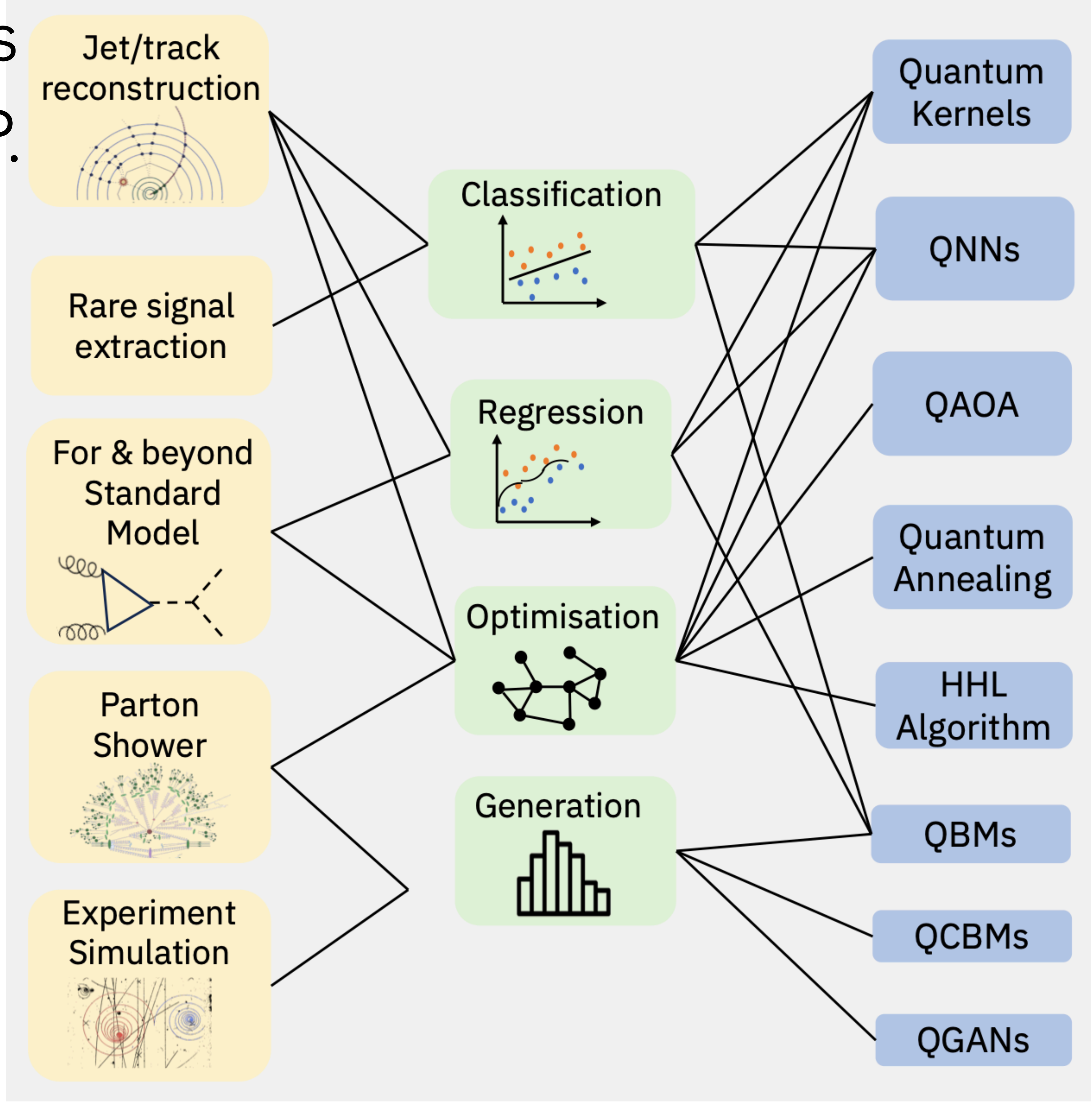




- Quantum computers have the potential to substantially speed up certain problems.

QC FOR HEP

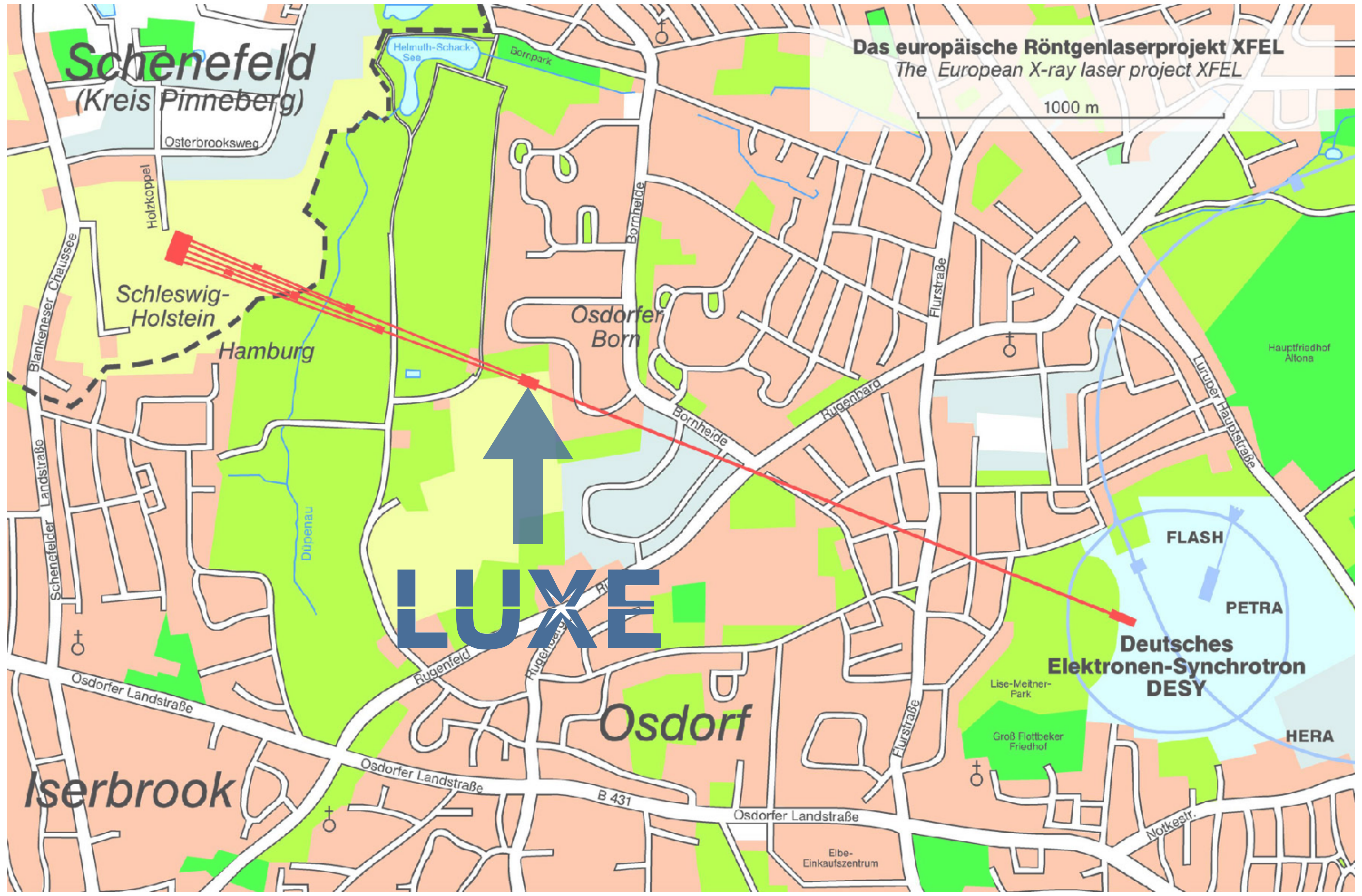
- Quantum computing has been studied in various applications in theoretical and experimental HEP.
- Summary of the QC4HEP WG: arXiv:2307.03236
- DESY. QUANTUM
  https://quantum.desy.de/
  https://quantum-zeuthen.desy.de/

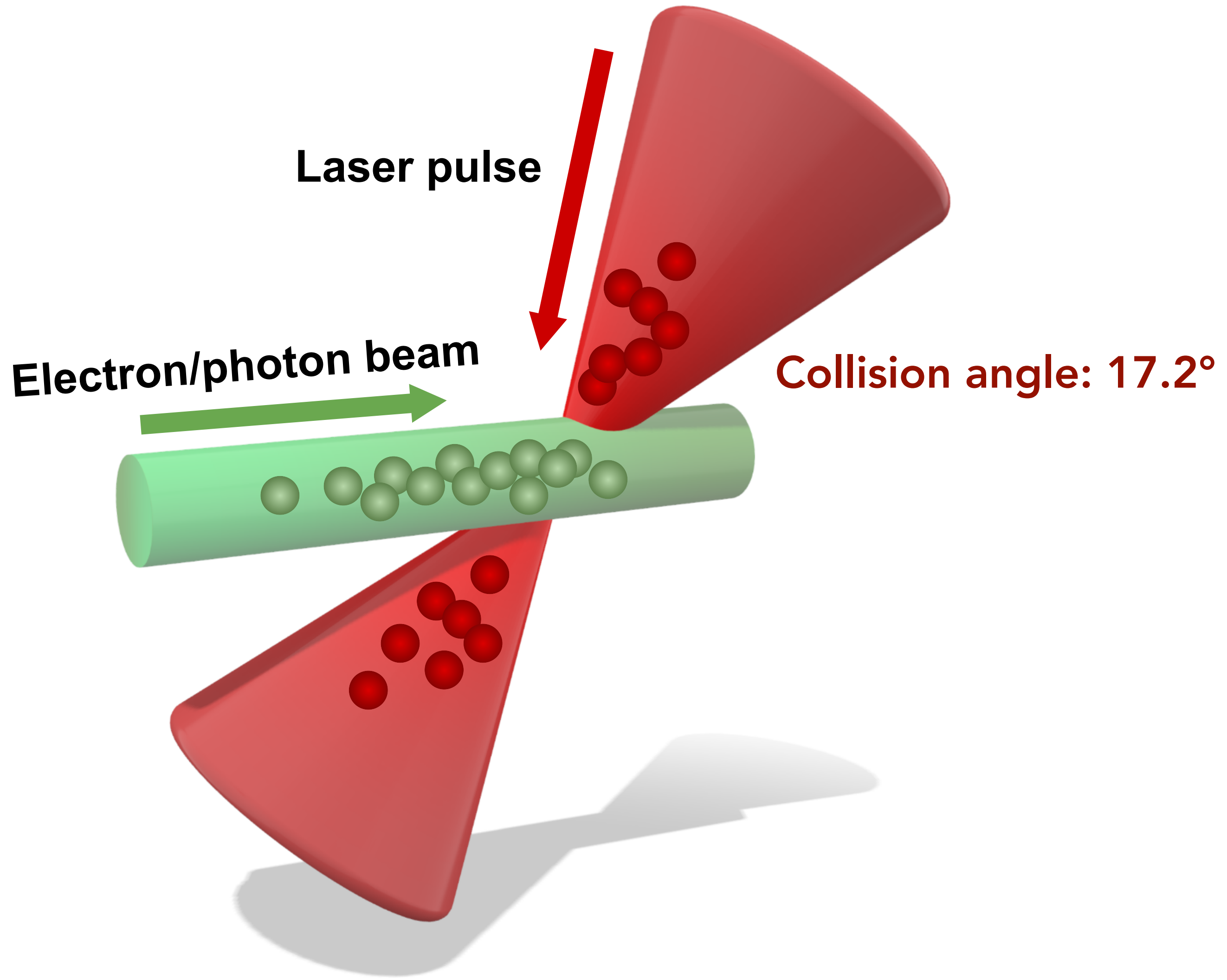


LUXE

LUXE: LASER UND XFEL EXPERIMENT

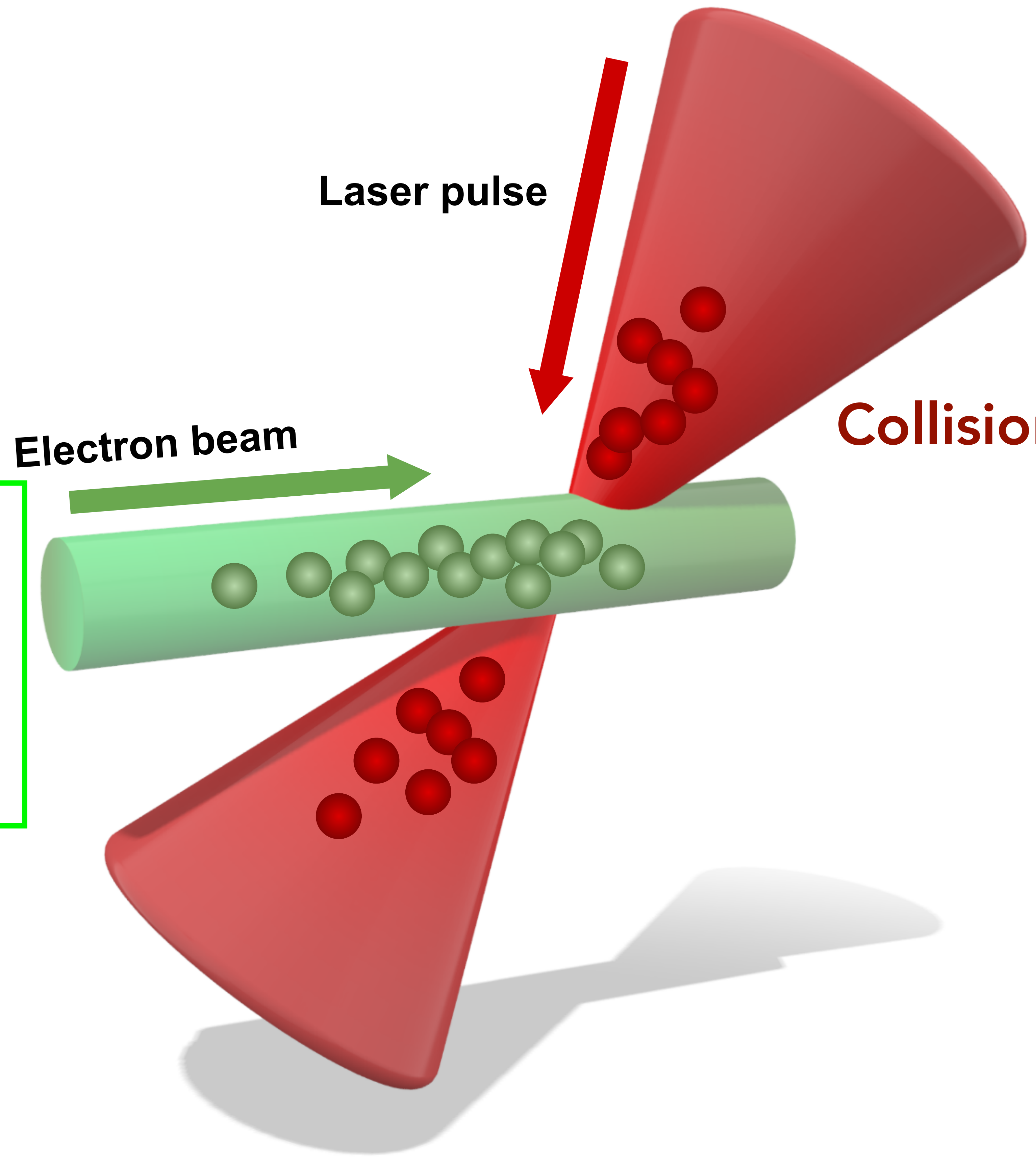
- LUXE is a proposed new experiment at DESY and European XFEL.
- Studies strong-field QED in collisions of XFEL electron beam and high-power laser.
- Two modes: directly collide electron beam or first convert electron beam into γ beam.
- 40 TW laser in phase-0, 350TW in phase-1.
- Website: https://luxe.desy.de, CDR, TDR





ELECTRON-LASER COLLISIONS

High intensity → **large E-field**



Collision angle: 17.2°

Lorentz boost: electrons "see" larger E-field of the laser in their rest frame: $E^* = \gamma_e E_L(1+\cos\theta)$

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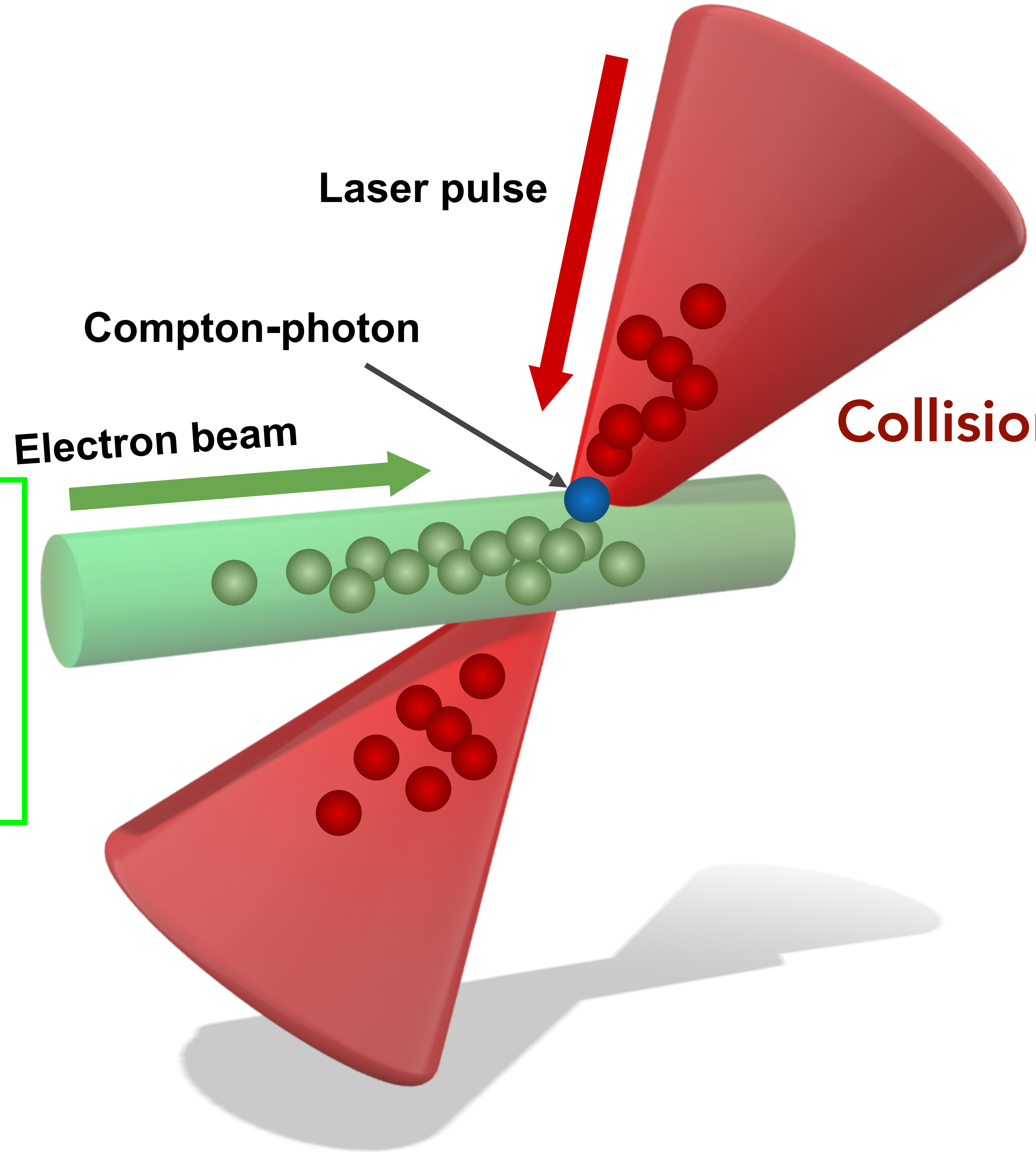

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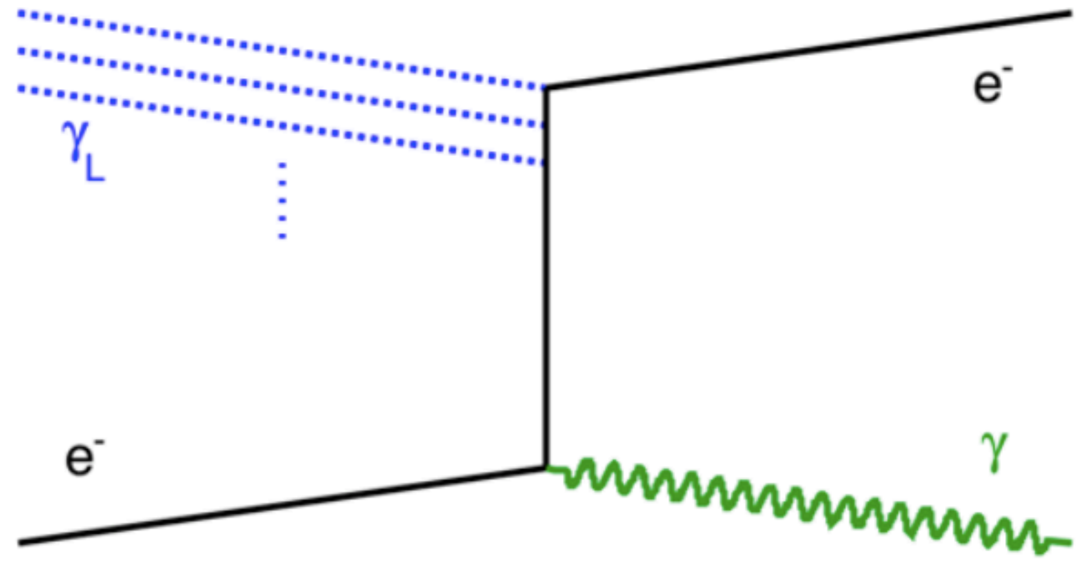
$E^* = \gamma_e E_L(1 + \cos\theta)$

Non-linear Compton scattering:

$$e^- + n\gamma_L \rightarrow e^- + \gamma_C$$



GeV photon

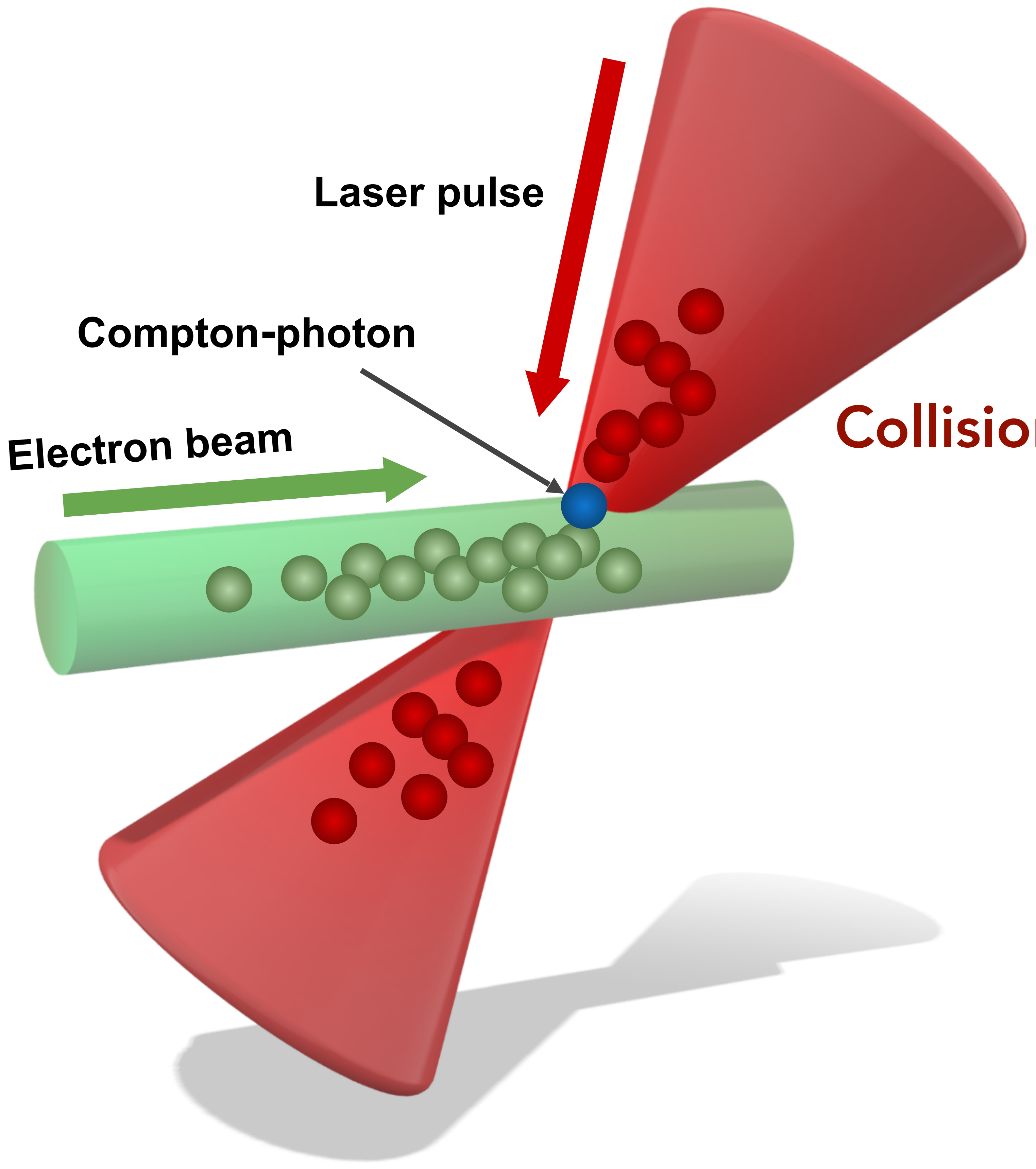
ELECTRON-LASER COLLISIONS

Field intensity parameter

$$\xi = \frac{m_e}{\omega_L}\frac{E_L}{E_{cr}}$$

Schwinger limit

$$E_{cr} = \frac{m_e^2 c^3}{e\hbar} = 1.32 \times 10^{18}\ \text{V/m}$$



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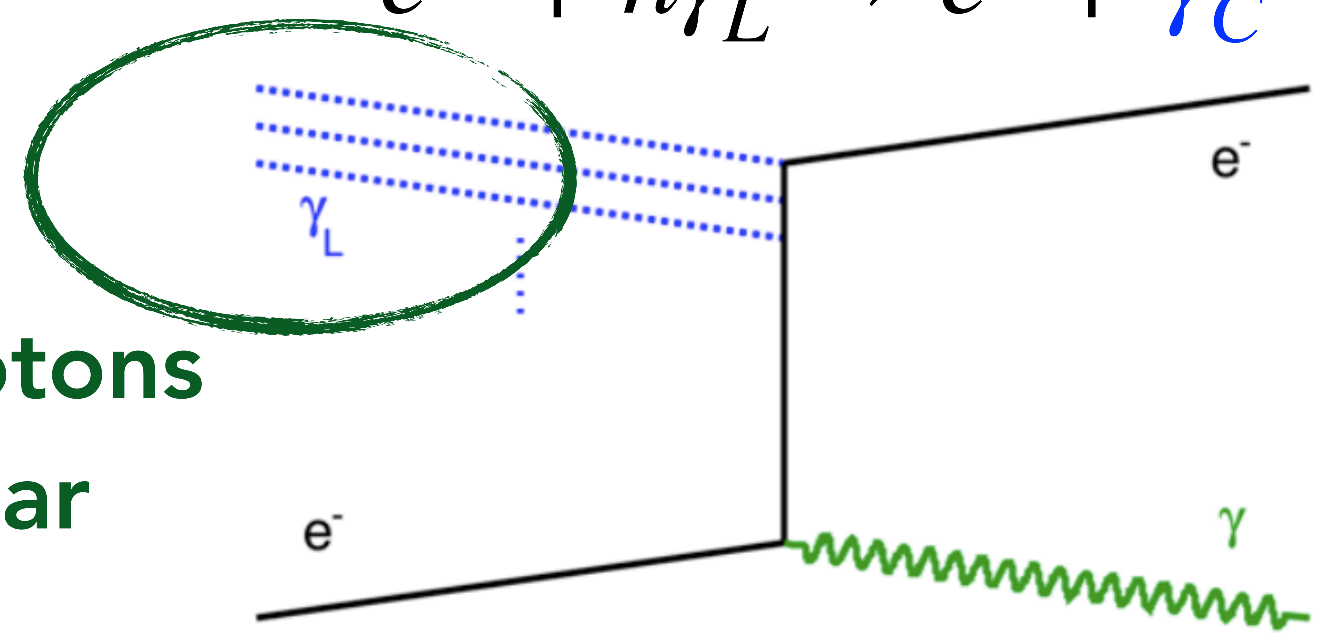

Multiple photons
= non-linear

GeV photon

All-order process, i.e. non-perturbative for $\xi \sim O(1)$

$$P(n\gamma_L \rightarrow \gamma) \propto \alpha\xi^{2n}$$

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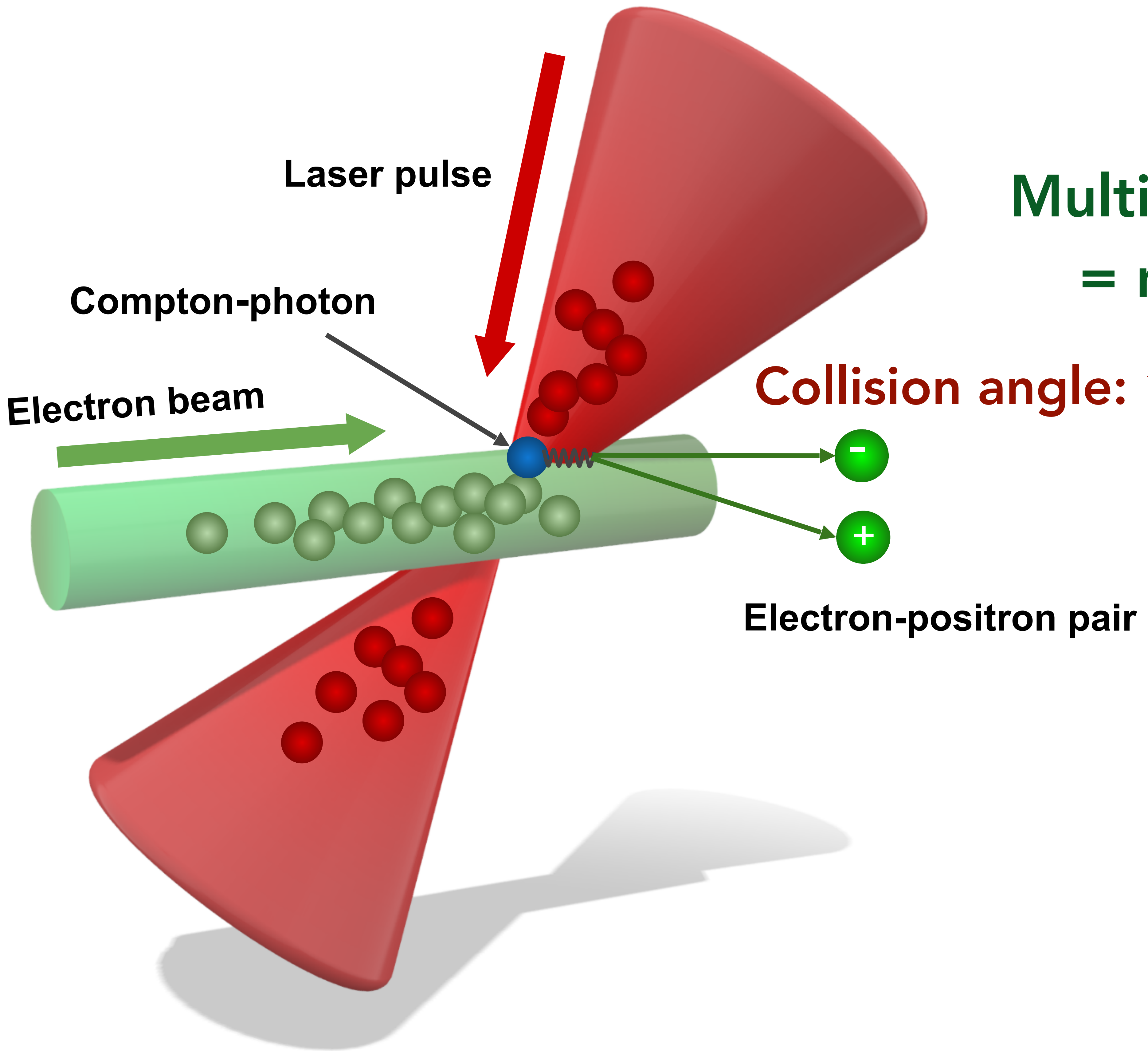

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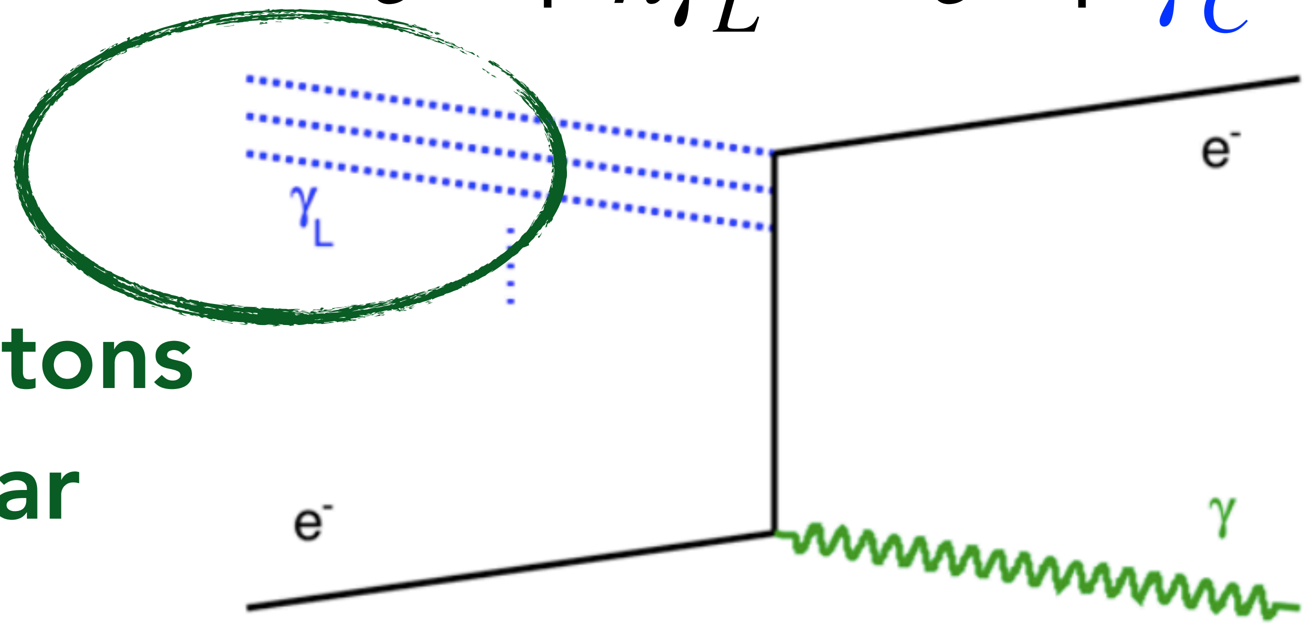
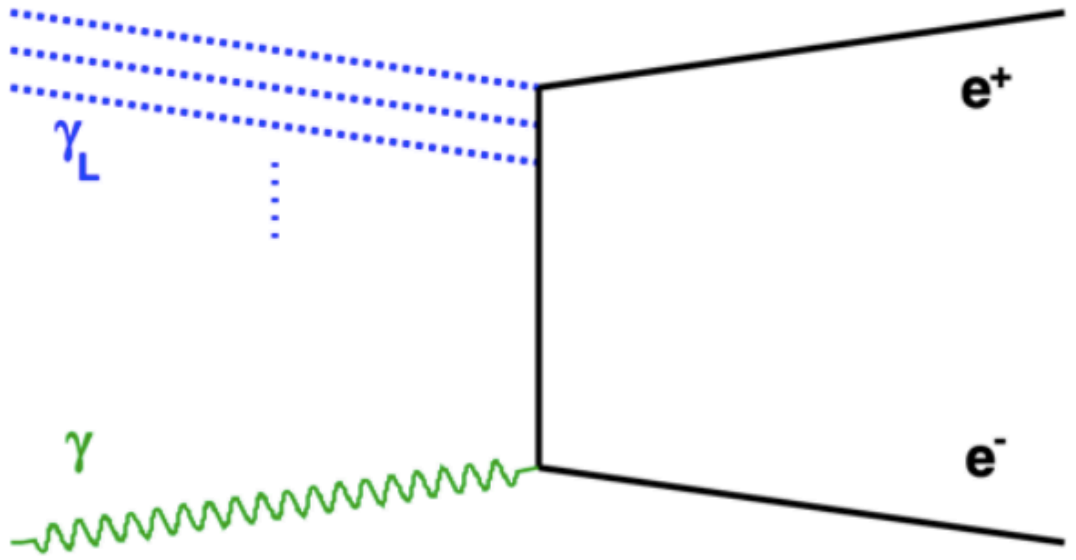


Multiple photons = non-linear

GeV photon

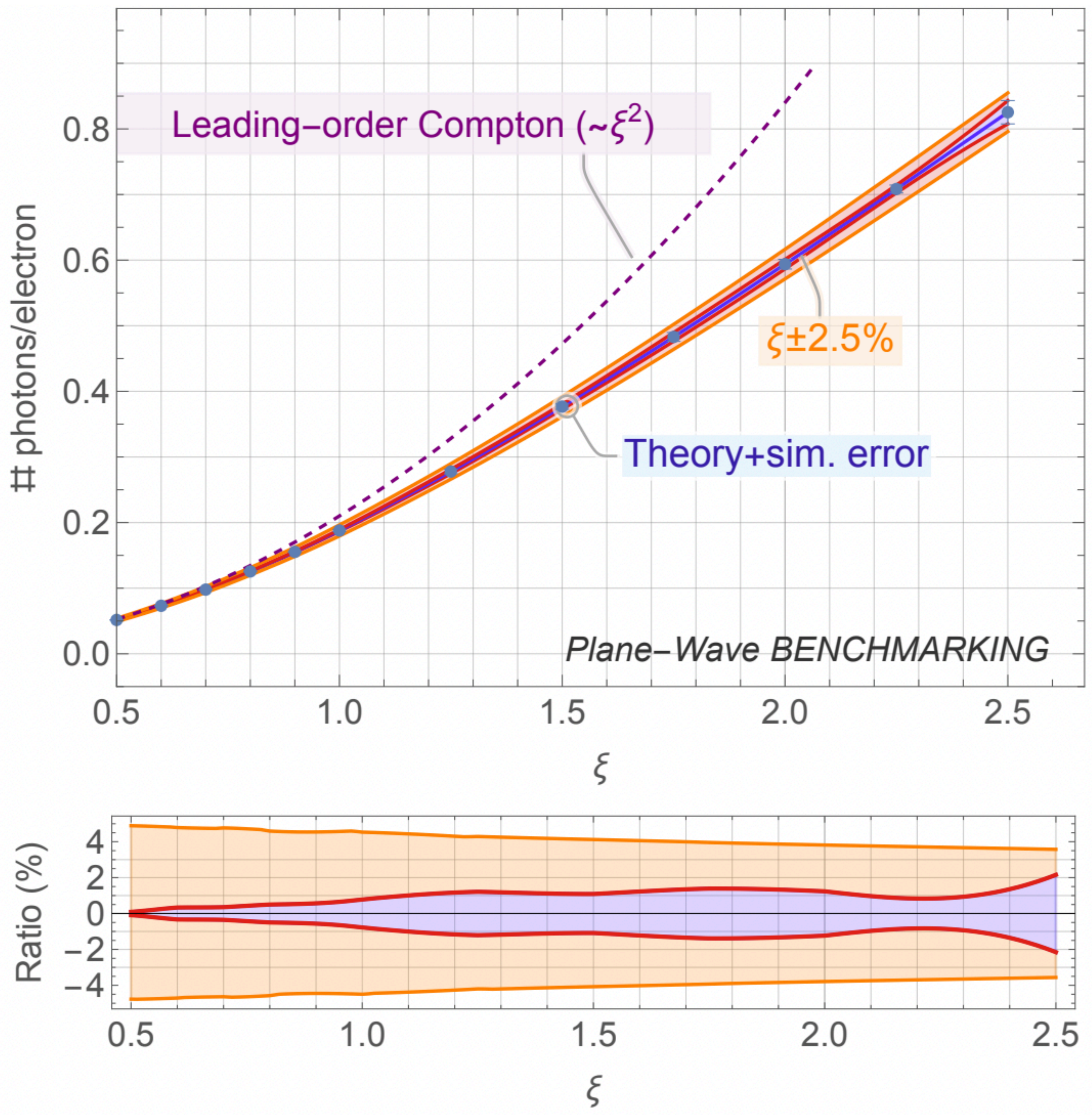
Non-linear Breit Wheeler:

$$\gamma_C + n'\gamma_L \rightarrow e^+ + e^-$$

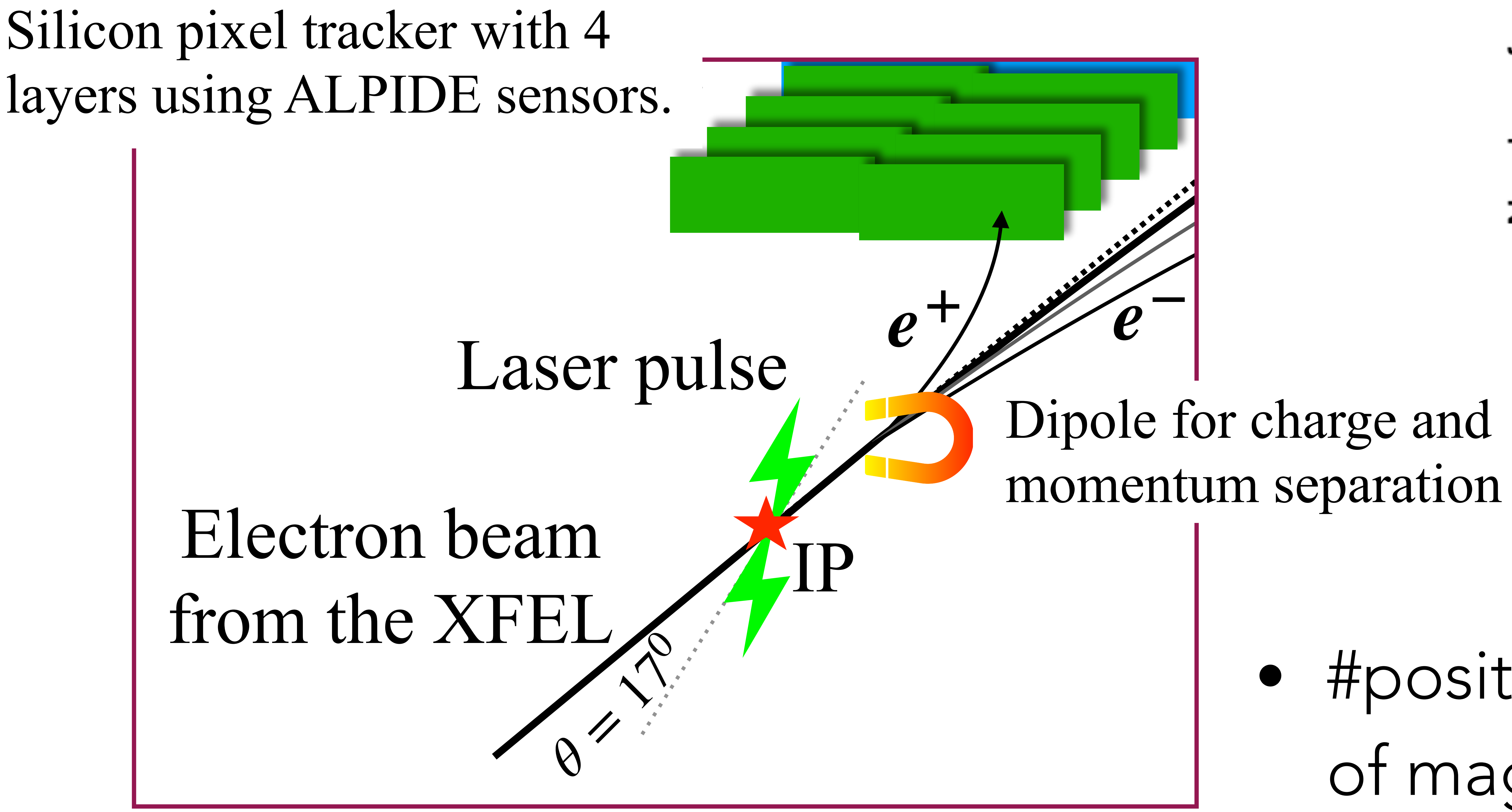


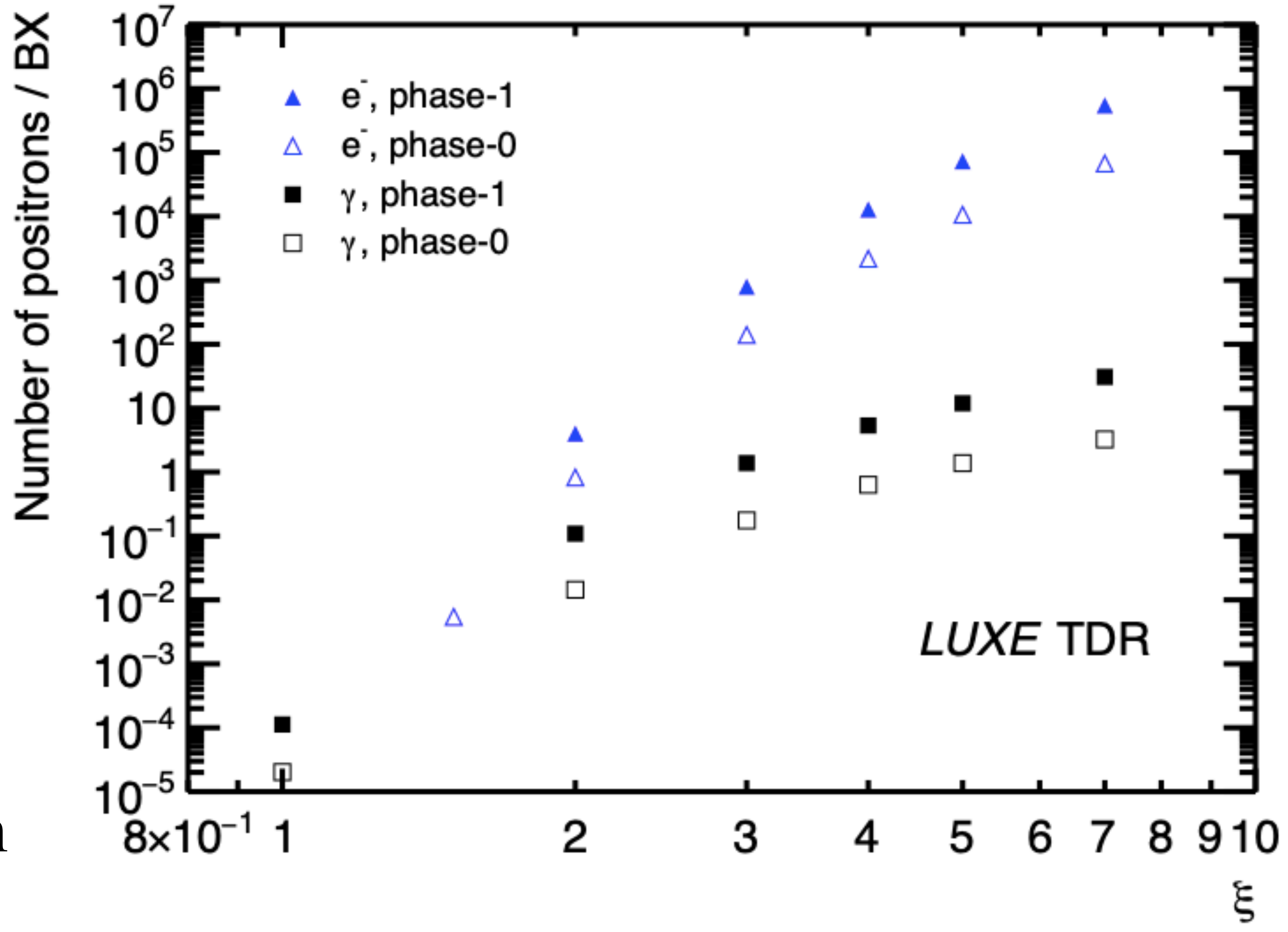
LUXE GOALS

- Main aim: **precision measurements** in a **transition** from perturbative to non-perturbative QED.
- Vary ξ between ~ 0.1 – 20 and conduct measurements:
  - e.g. # photons radiated, # positrons from non-linear Breit-Wheeler, etc.



POSITRON MEASUREMENT



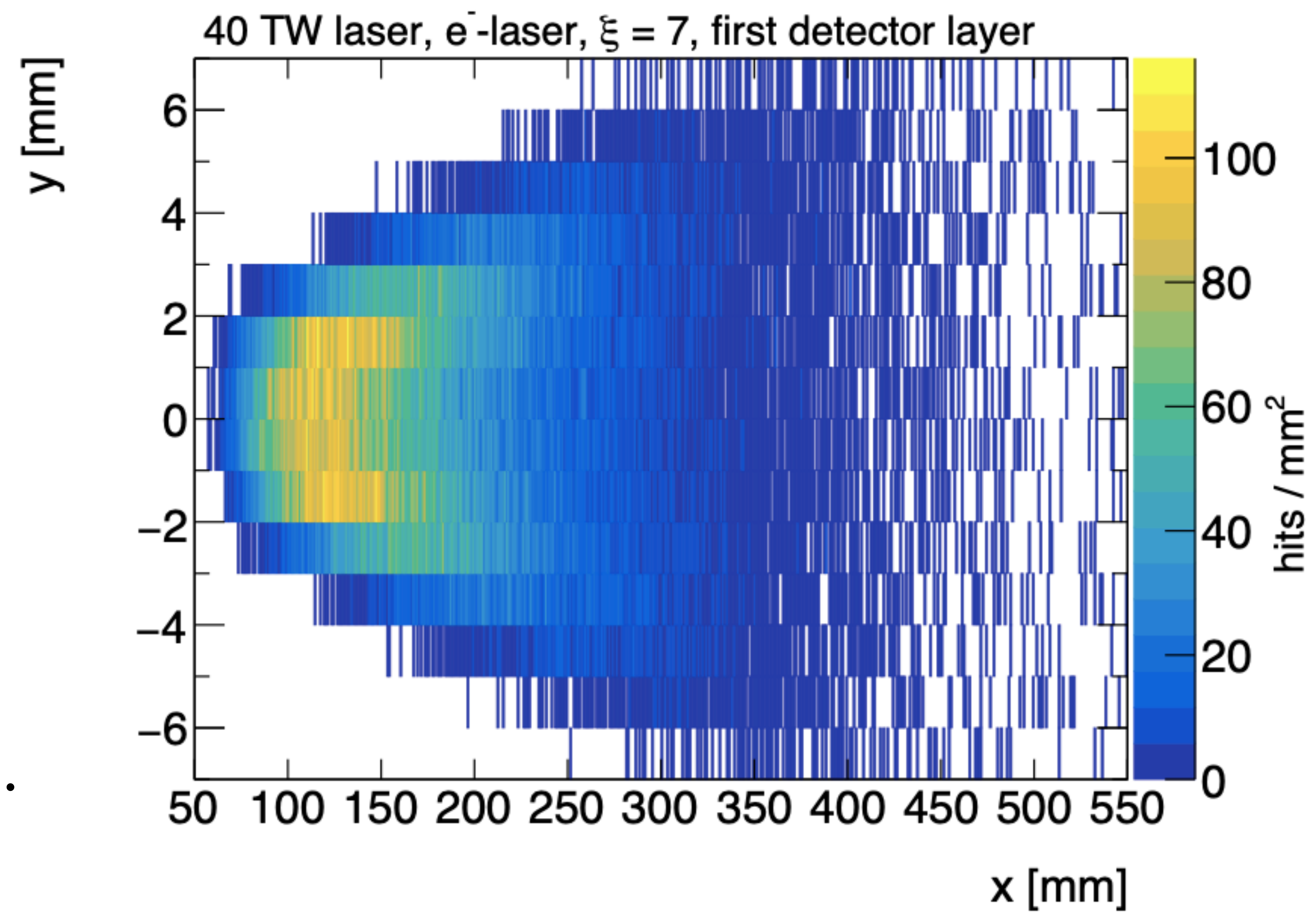


- #positron rate vs ξ spans over 10 orders of magnitudes.
- For precise positron rate measurement, reconstruct particle path with tracking.

LUXE TRACKING CHALLENGE

- Tracking at LUXE becomes challenging due to combinatorics at high track multiplicities (up to 10^6 at phase-0).

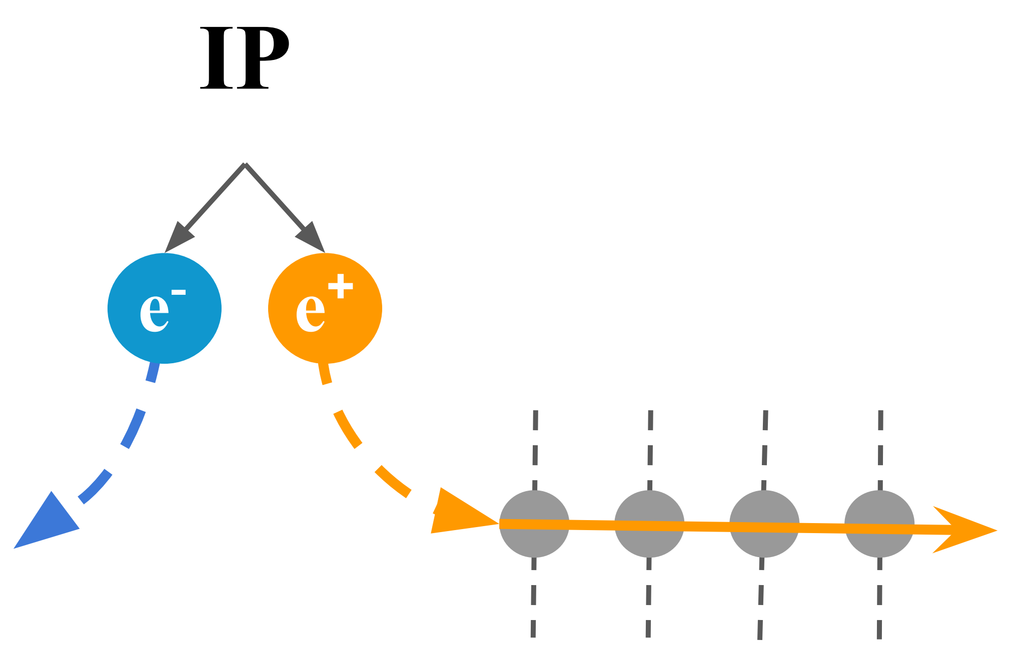
- At phase-0 (40 TW laser), **occupancies** at the pixel detector reach 100 particles/mm^2.
  - Orders of magnitudes higher than other planned HEP experiments, e.g. HL-LHC.



- Quantum computing may offer an advantage. In our paper ([arXiv:2304.01690]()), we study various tracking methods using quantum and classical algorithms.

TRACKING AT LUXE

OVERVIEW OF THE WORKFLOW



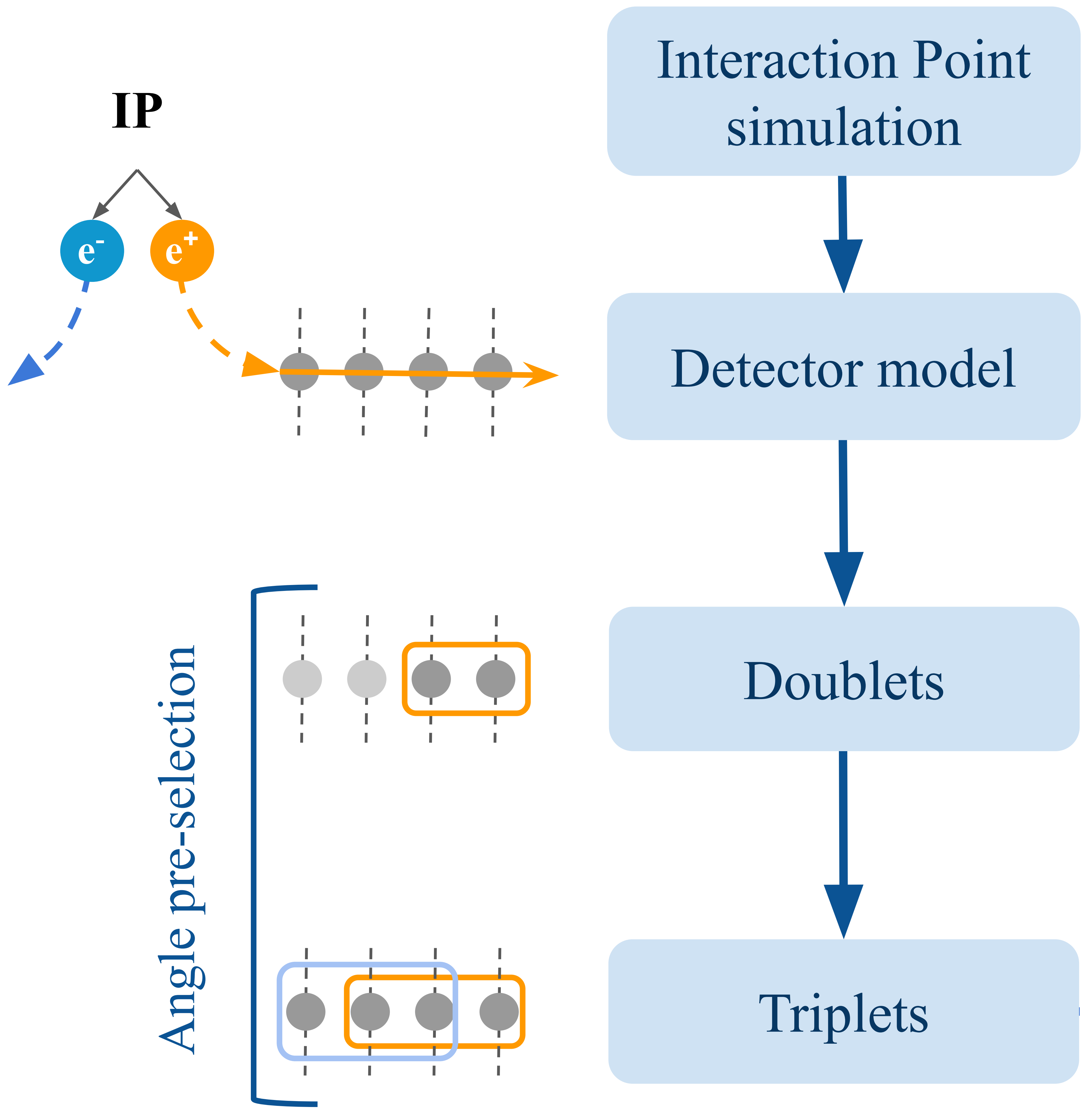
Interaction Point simulation

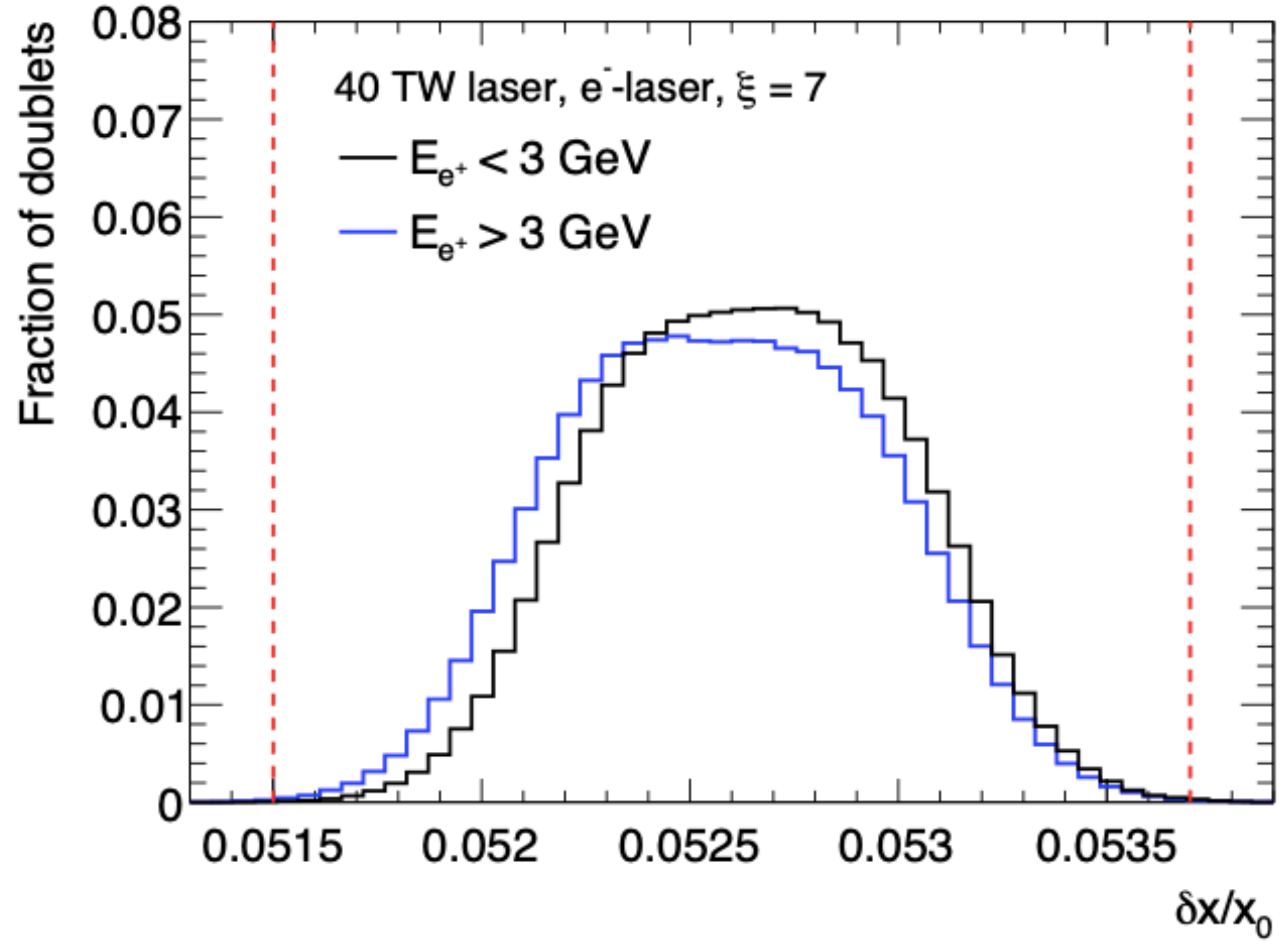
Detector model

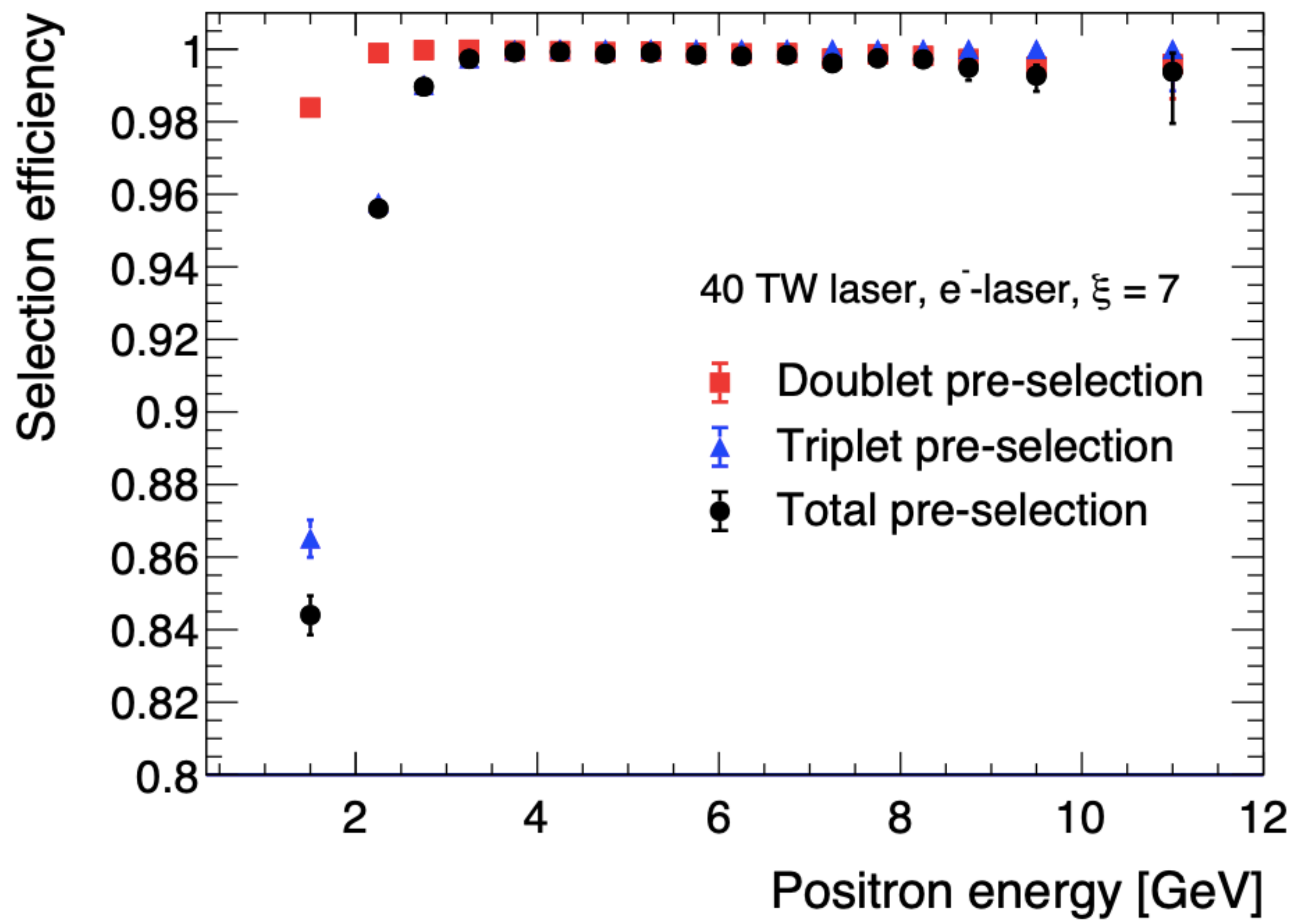
PTARMIGAN arXiv:2108.10883

Custom fast tracker simulation with simplified detector setup

OVERVIEW OF THE WORKFLOW







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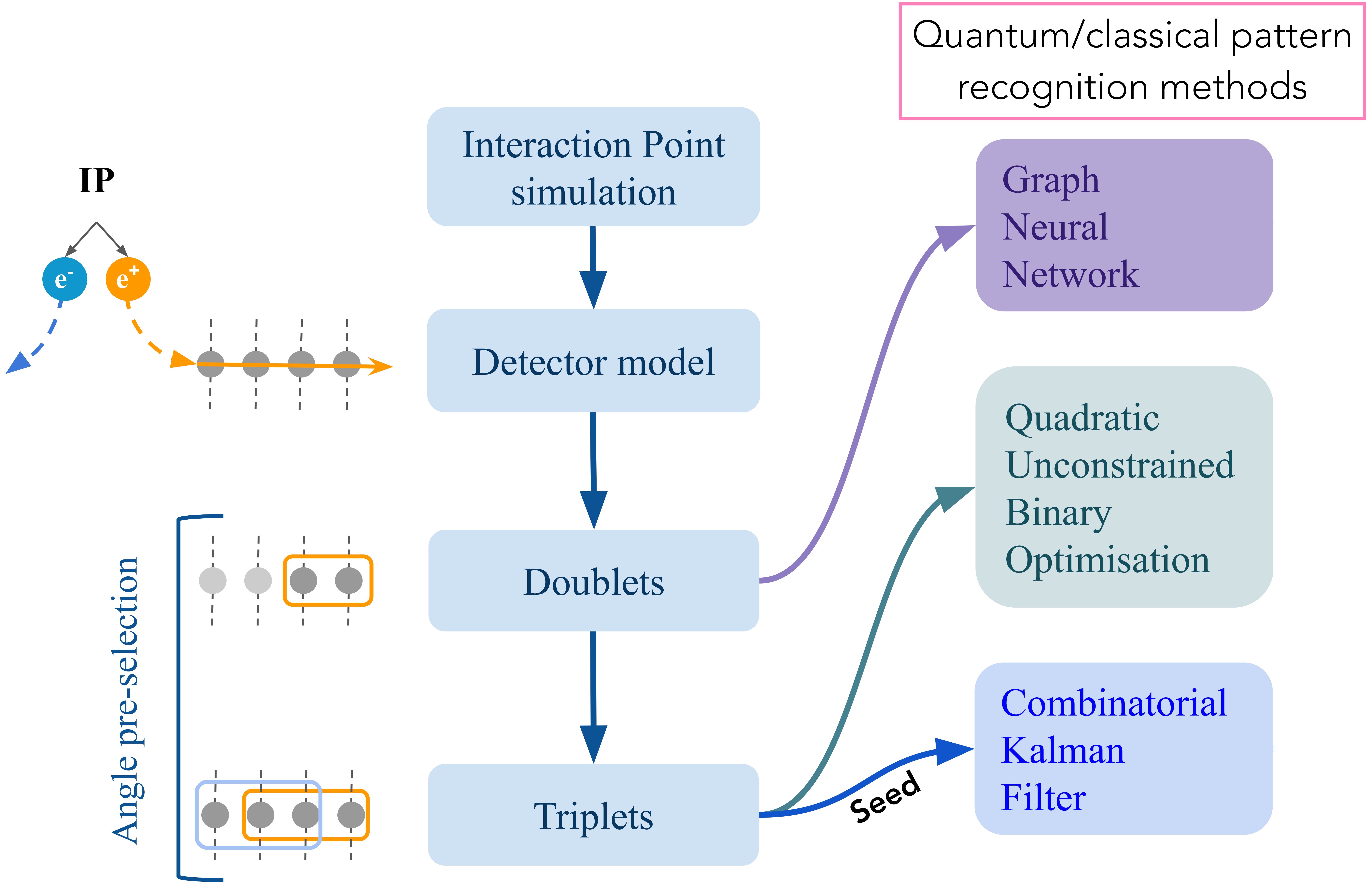

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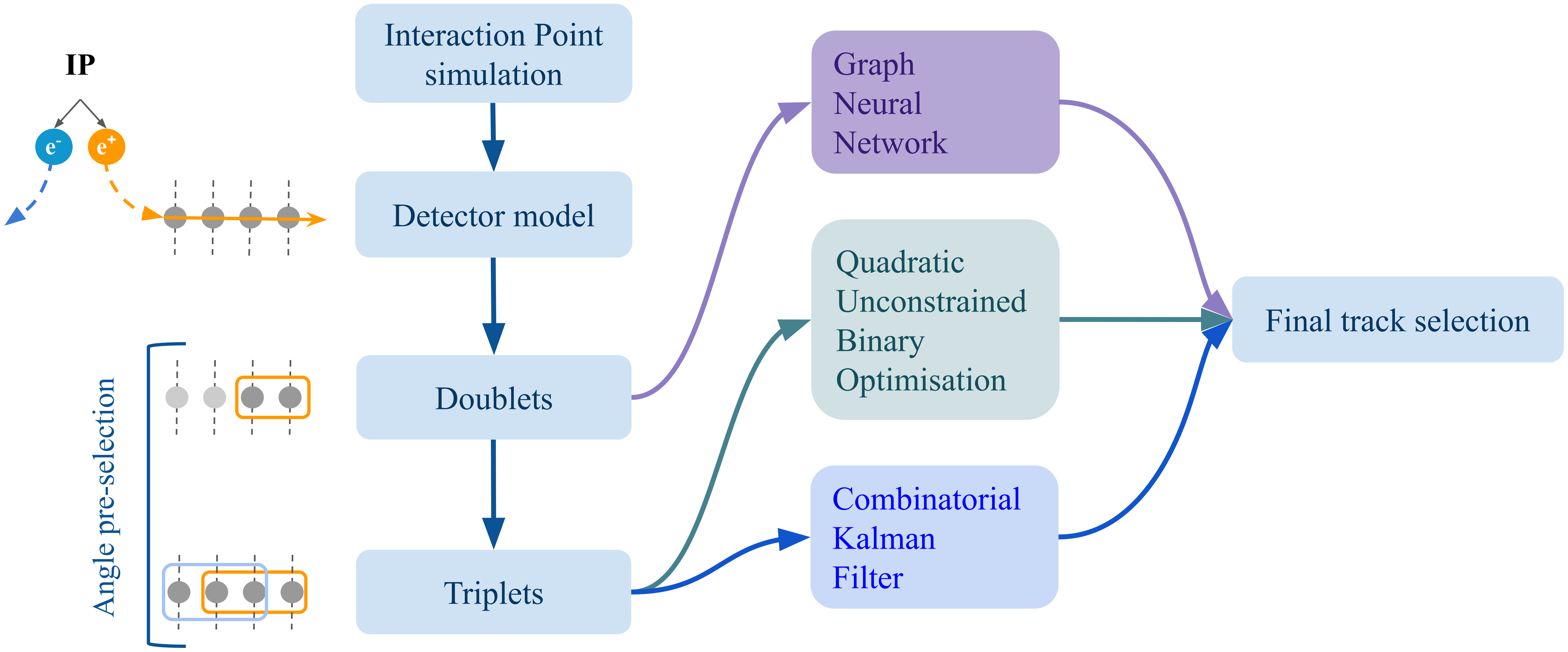
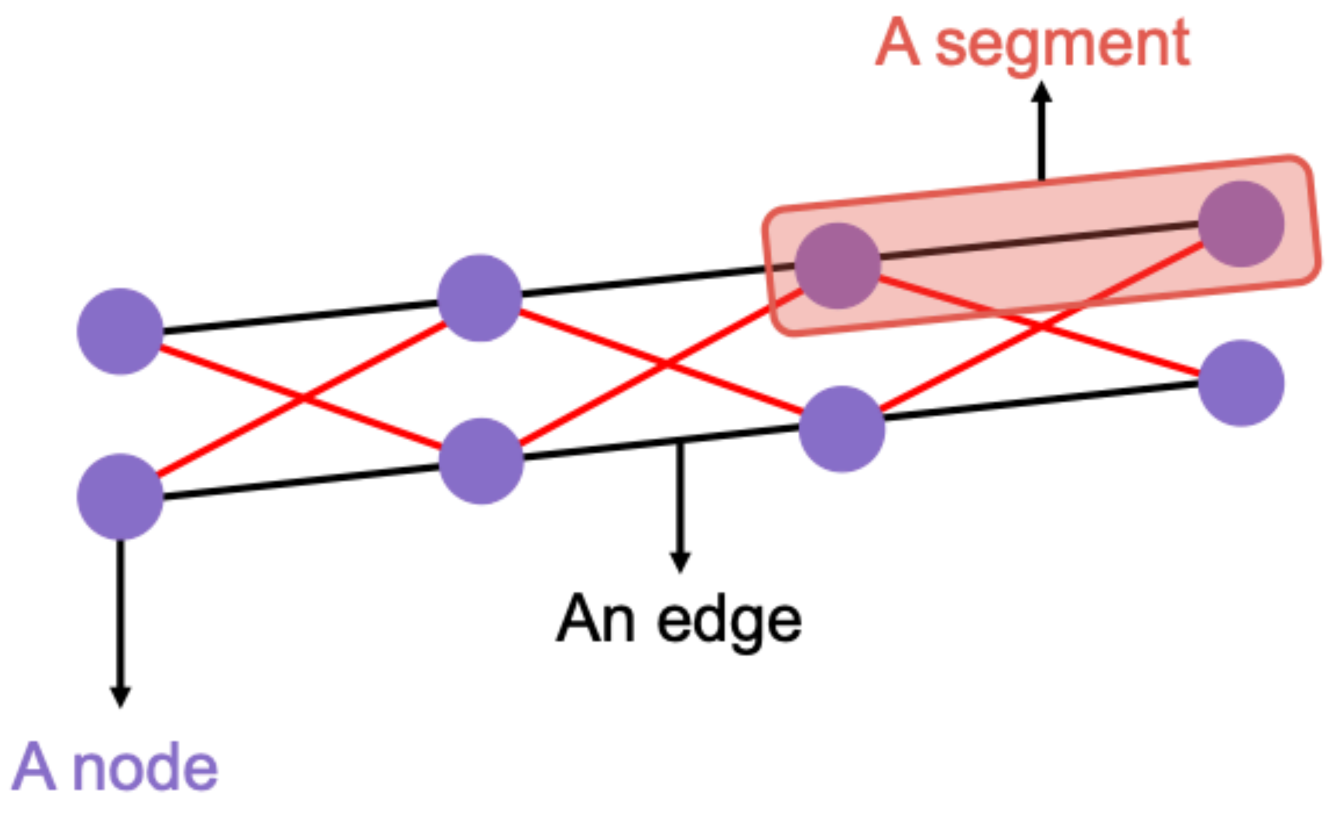
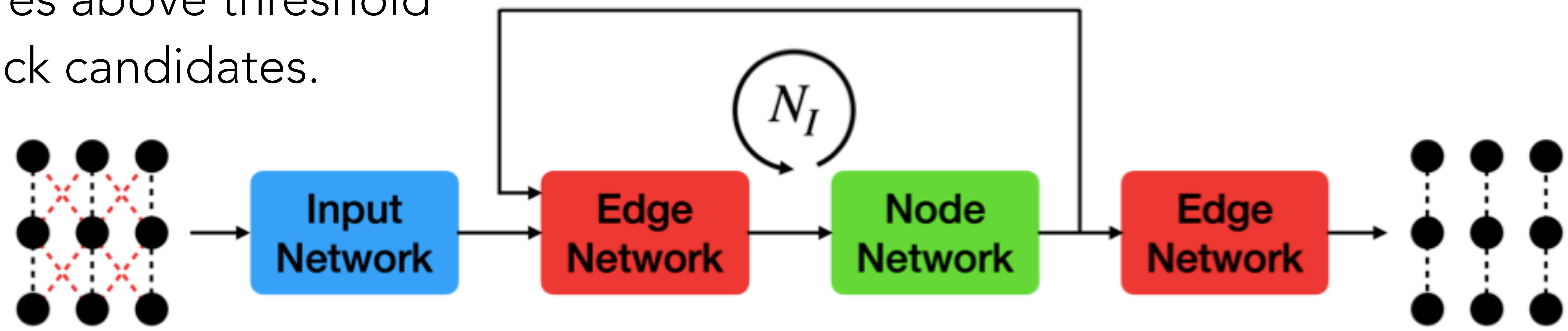


QUANTUM GRAPH NEURAL NETWORK (GNN)

- Doublet classification.
- Graph constructed from doublets, where the hits are nodes and the connections between the hits are edges.
- Hybrid quantum-classical model with 10 hidden features (qubits).
- N_I (4=#tracker layers) iterations of alternating edge and node networks applied.
- Edge/doublet with scores above threshold are retained to form track candidates.





HEP.TrkX: arXiv:1810.06111, Exa.TrkX: arXiv:2103.06995, Q.TrkX: arXiv:2109.12636

QUBO

Quadratic
Unconstrained
Binary
Optimisation

- Triplet classification.
  - Find the best set of triplets which can form tracks by minimising the QUBO, given by the states of T_i, T_j.

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j, \quad T_i, T_j \in \{0,1\}$$

Weighting triplet T_i with quality a_i

Compatibility b_{ij} between two triplets

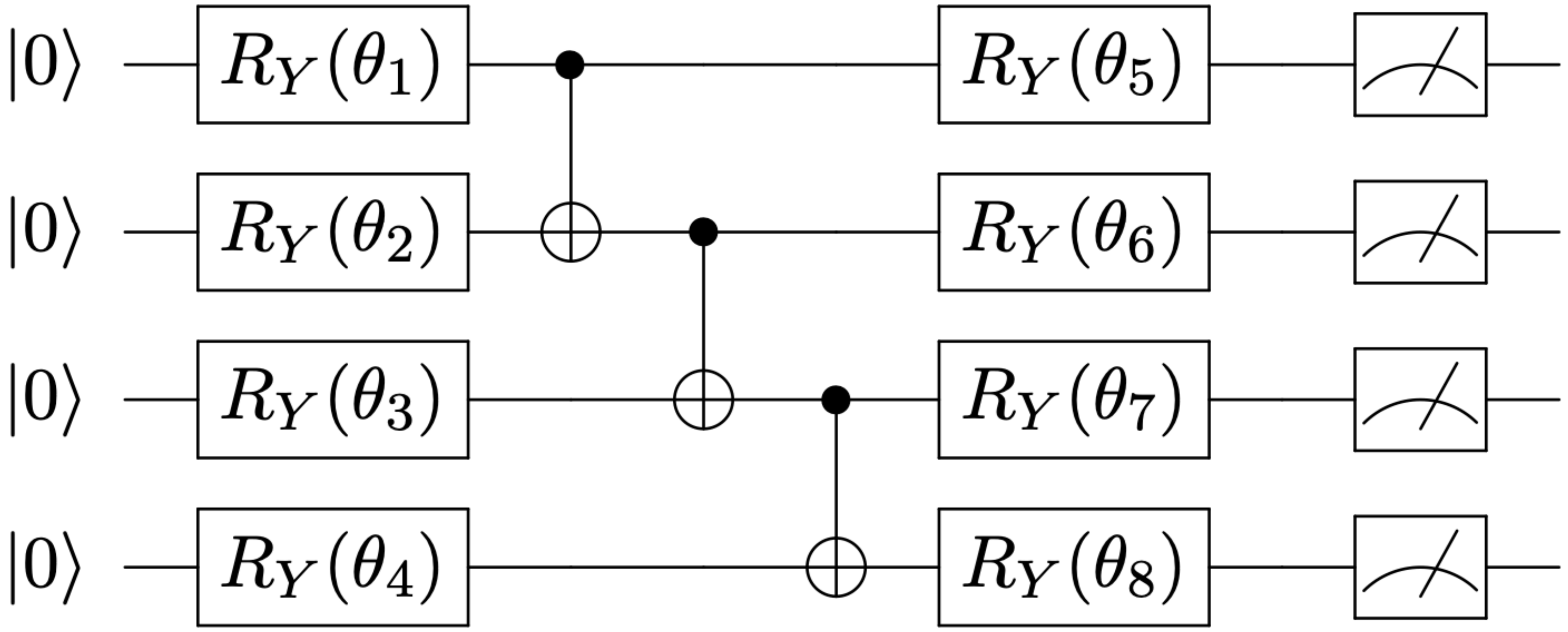
$$b_{ij} = \begin{cases} -S(Ti,Tj), & \text{if } (T_i,T_j) \text{ form a quadruplet}, \\ \zeta & \text{if } (T_i,T_j) \text{ are in conflict}, \\ 0 & \text{otherwise.} \end{cases}$$

- The QUBO can be mapped to an Ising Hamiltonian.
- Minimising the QUBO is equivalent to finding the ground state of the Hamiltonian.

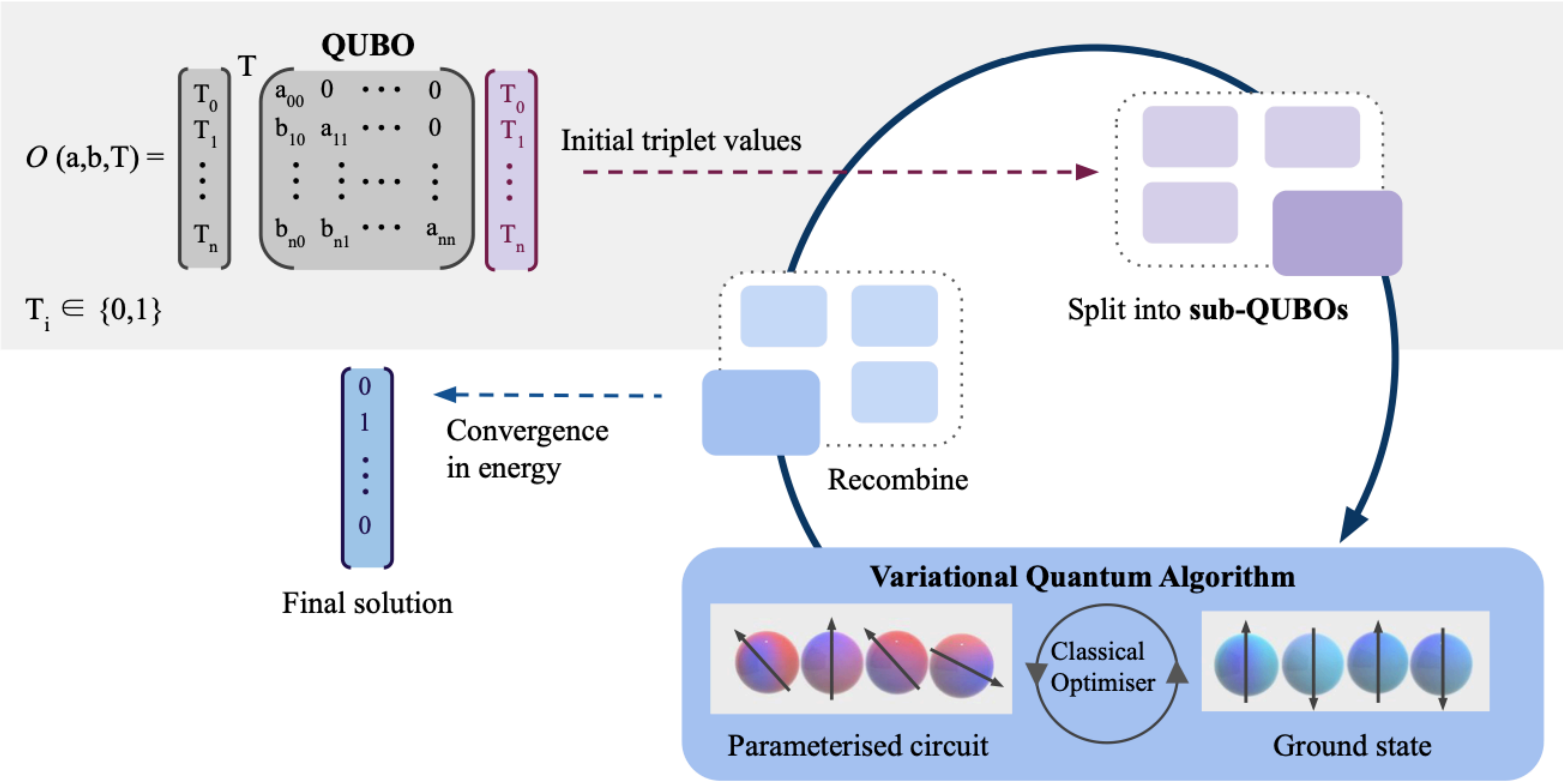
Find T_i, T_j that minimises QUBO!

QUBO MINIMISATION USING VQE

- The ground state is found using **Variational Quantum Eigensolver (VQE)**, a hybrid quantum-classical algorithm.
    - Nakanishi-Fujii-Todo (NFT) optimiser.
    - QUBO is partitioned into sub-QUBOs of the size of the quantum device (7 qubits assumed) to be solved iteratively.



- Exact solution using **matrix diagonalisation** used as benchmark.


QUBO
O (a,b,T) =
$\begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{bmatrix}^T \begin{bmatrix} a_{00} & 0 & \cdots & 0 \\ b_{10} & a_{11} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ b_{n0} & b_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ T_n \end{bmatrix}$
$T_i \in \{0,1\}$
Initial triplet values
Split into **sub-QUBOs**
Recombine
Convergence
in energy
$\begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$
Final solution
Variational Quantum Algorithm
Classical
Optimiser
Parameterised circuit
Ground state

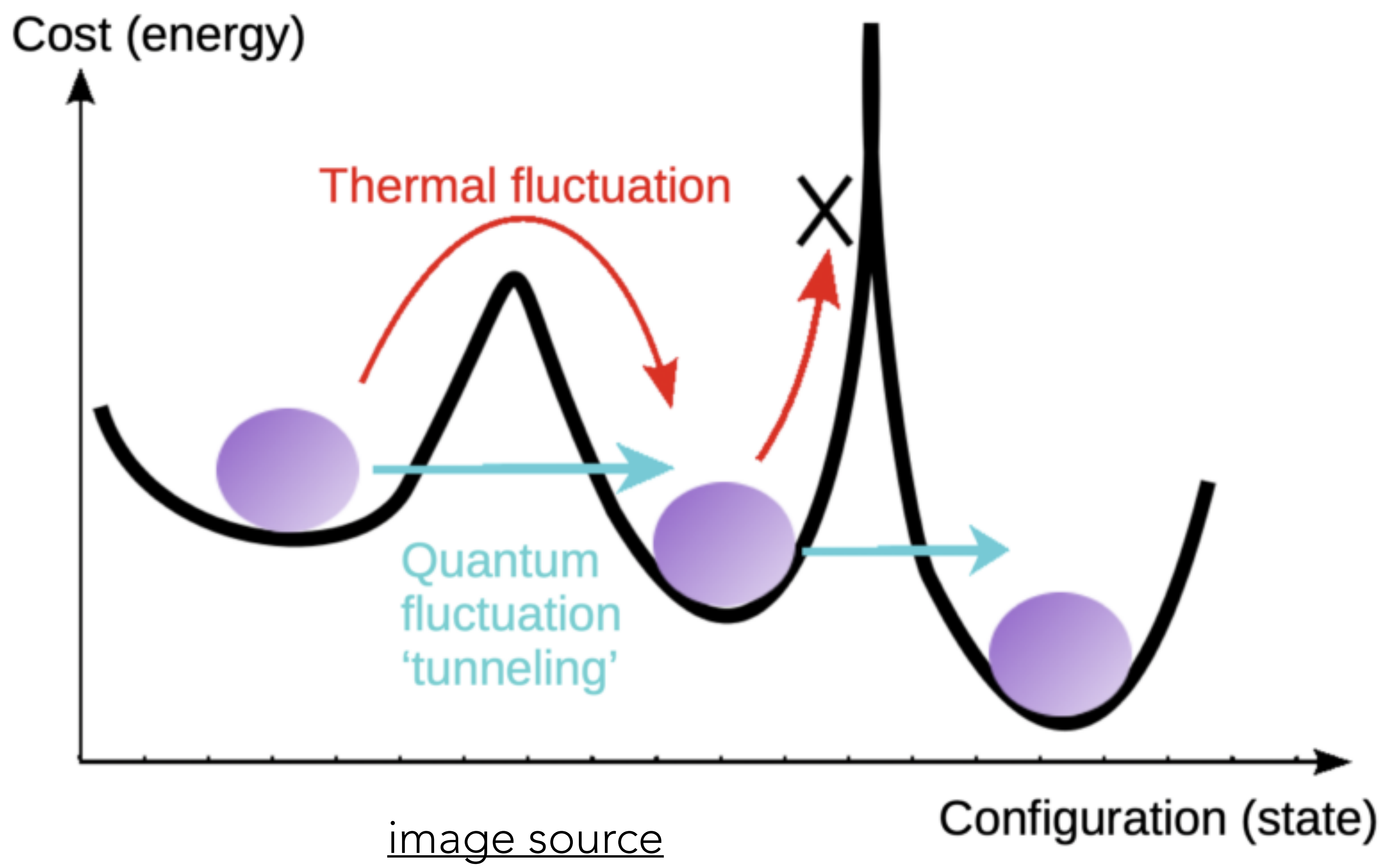
QUANTUM ANNEALING

- Besides VQE, quantum annealing can also be used.

- Quantum annealers specialise in solving combinatorial optimisation problems.

- Advantage: > 5000 qubits in D-Wave.
  - Study impact of QUBO partitioning.

- Simulated (thermal) annealing as benchmark.



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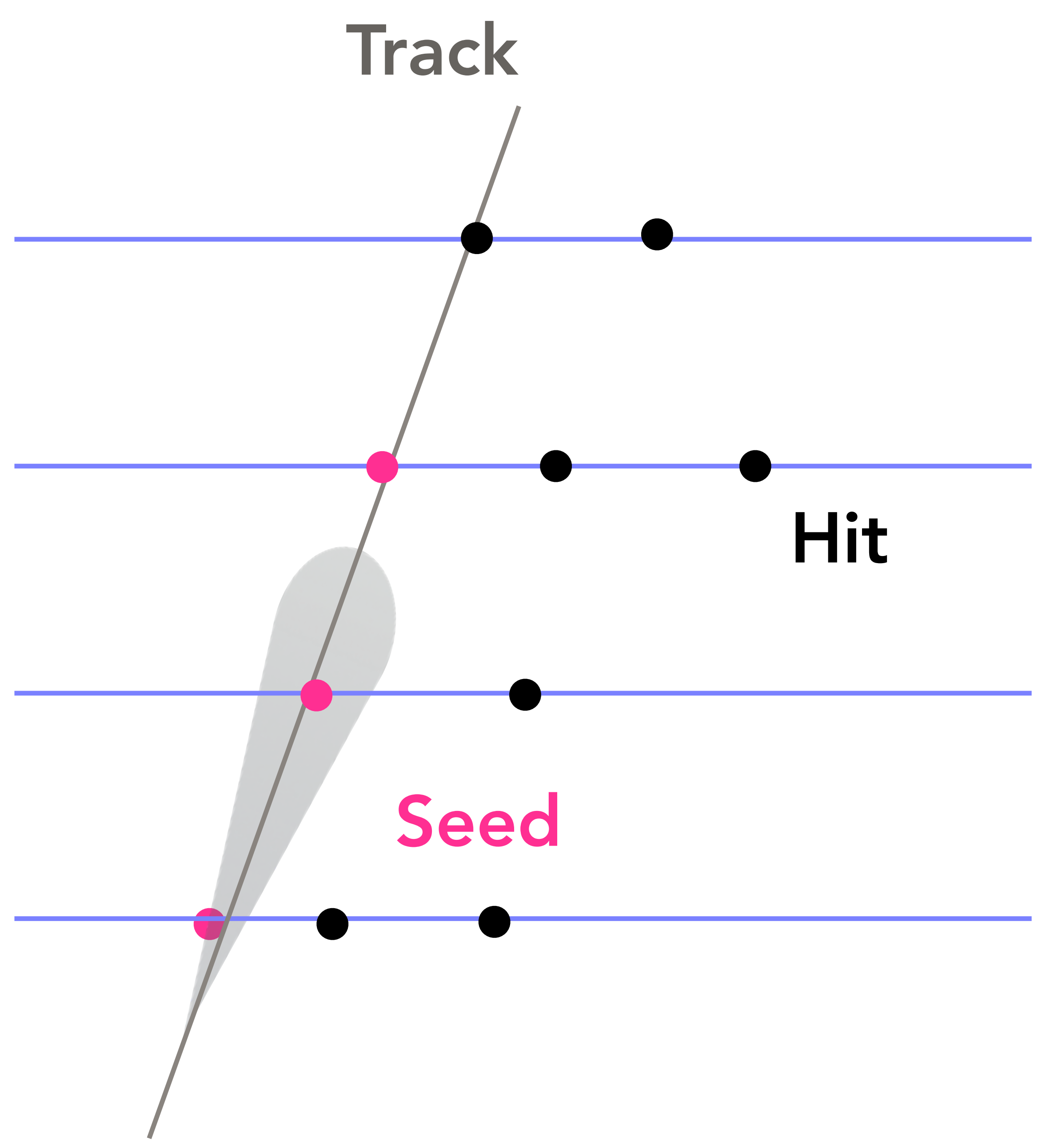
COMBINATORIAL KALMAN FILTER (CKF)

- As classical benchmark, CKF in a common tracking software (ACTS) used.
  - State-of-the-art experiment-independent toolkit for tracking, providing modular and efficient algorithms.

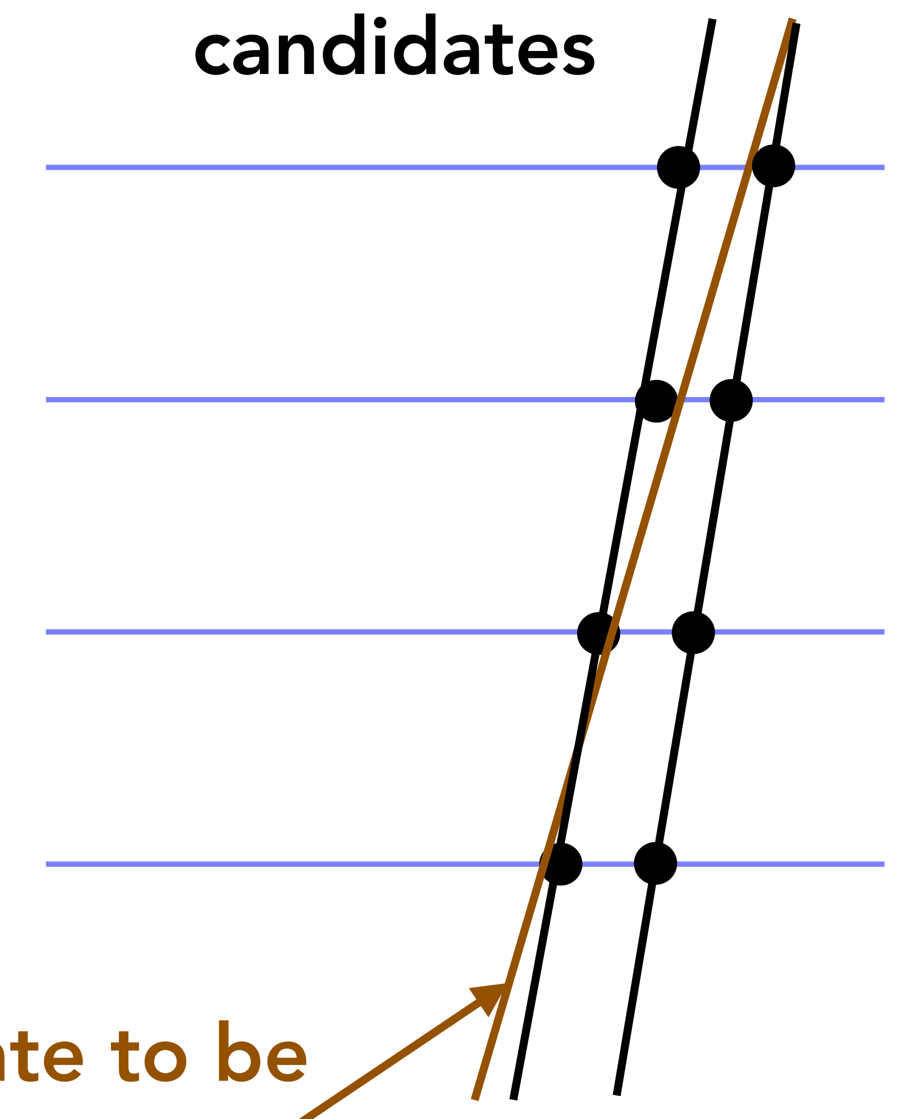
- Triplets from first three layers are used as **seeds**.

- Starting from each seed, the CKF extrapolates the particle's trajectory to the next detector layers.
  - Track parameters predicted and updated layer by layer.



FINAL TRACK SELECTION

- Tracks are required to have **4 hits**, found either directly with classical CKF tracking or by combining selected doublets/triplets into quadruplets in the GNN/QUBO approaches.
- Tracks are fitted and **ambiguity solving** applied to remove worse quality tracks with shared hits from the track collection.
  - No track is allowed to have more than 1 shared hit.
  - Local and combinatorial nature of CKF mean duplicates and hit sharing could be more common than global tracking approaches.



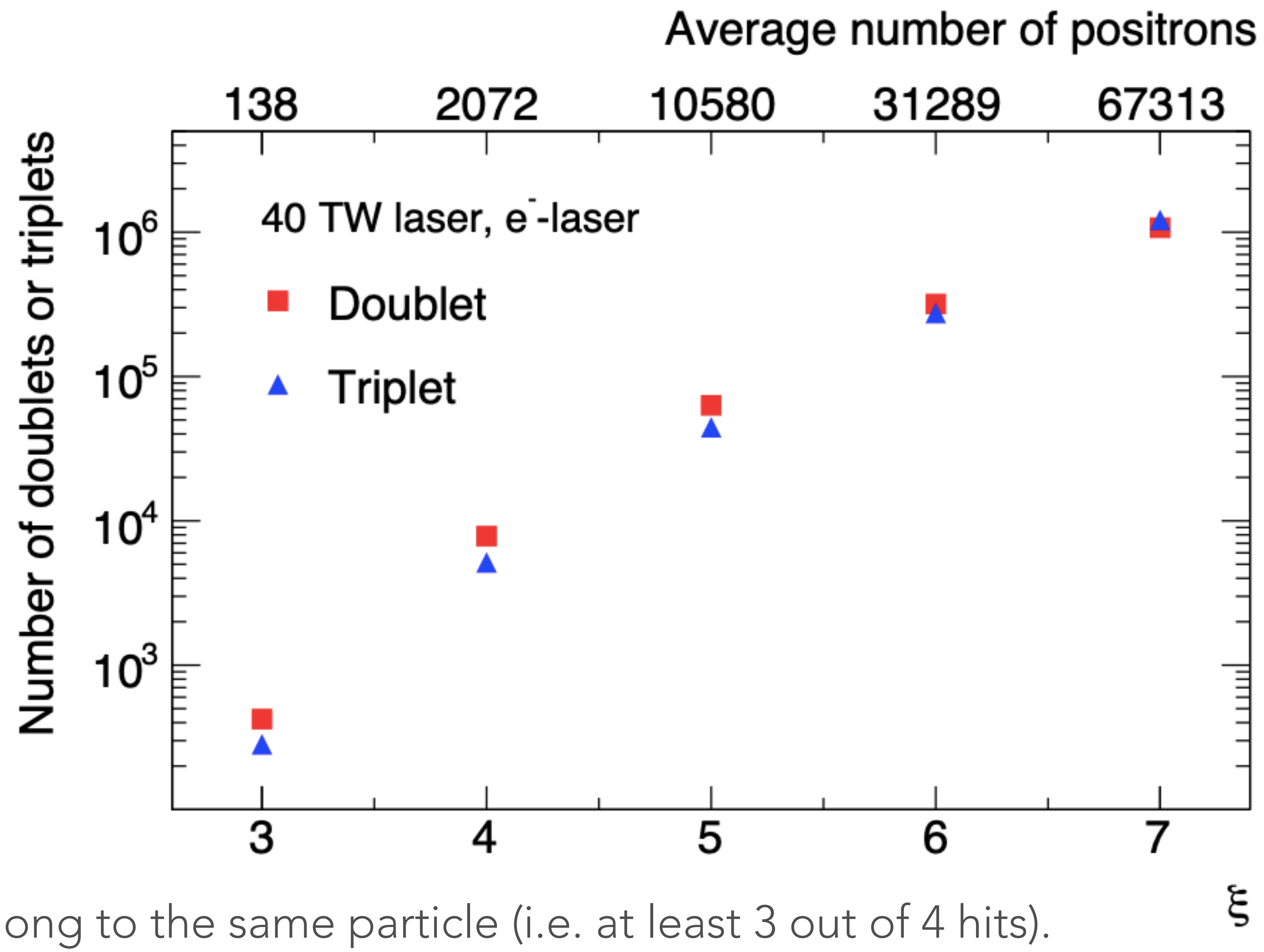
COMPARISON

- Compare performance of these tracking methods for ξ = 3 – 7 in LUXE phase-0 e-laser interactions, where the number of positrons are between 140 and 67,000.

- Two metrics:

$$\text{Efficiency} = \frac{N_{\text{tracks}}^{\text{matched*}}}{N_{\text{tracks}}^{\text{generated}}}$$

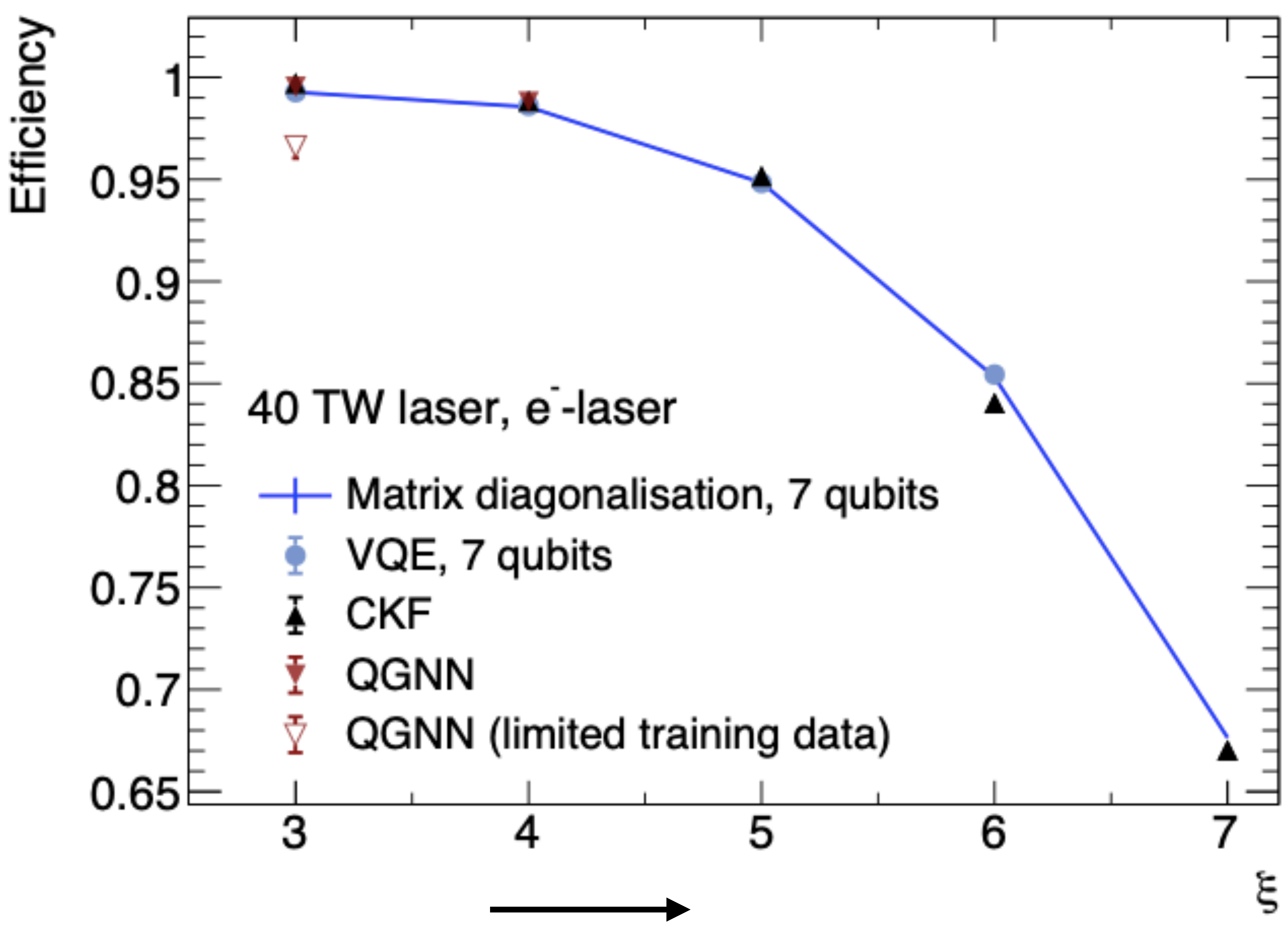
$$\text{Fake rate} = \frac{N_{\text{tracks}}^{\text{fake}}}{N_{\text{tracks}}^{\text{reconstructed}}}$$

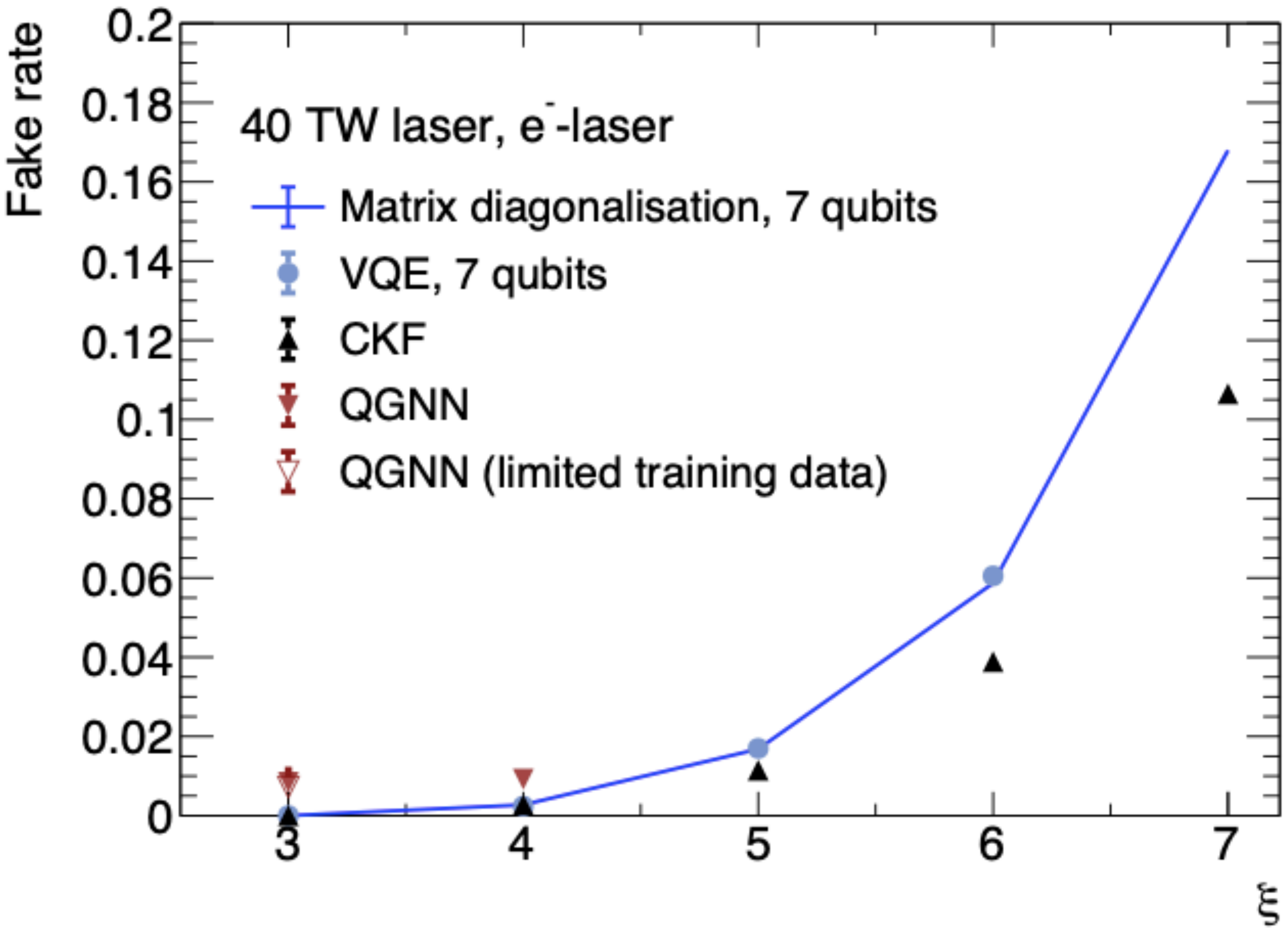


*A track is considered matched if the majority of its hits belong to the same particle (i.e. at least 3 out of 4 hits).

RESULTS (SIMULATED)

RESULTS

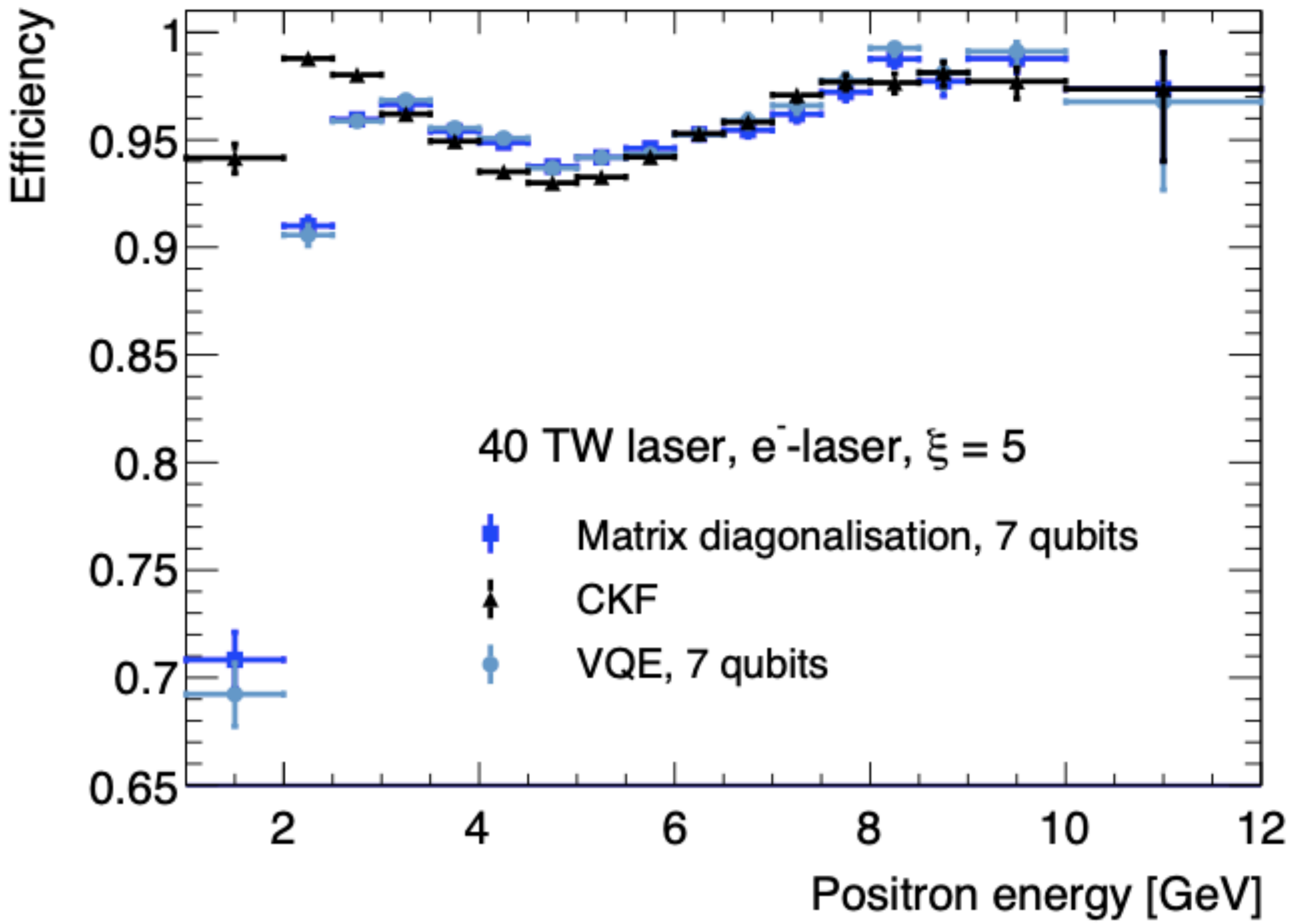


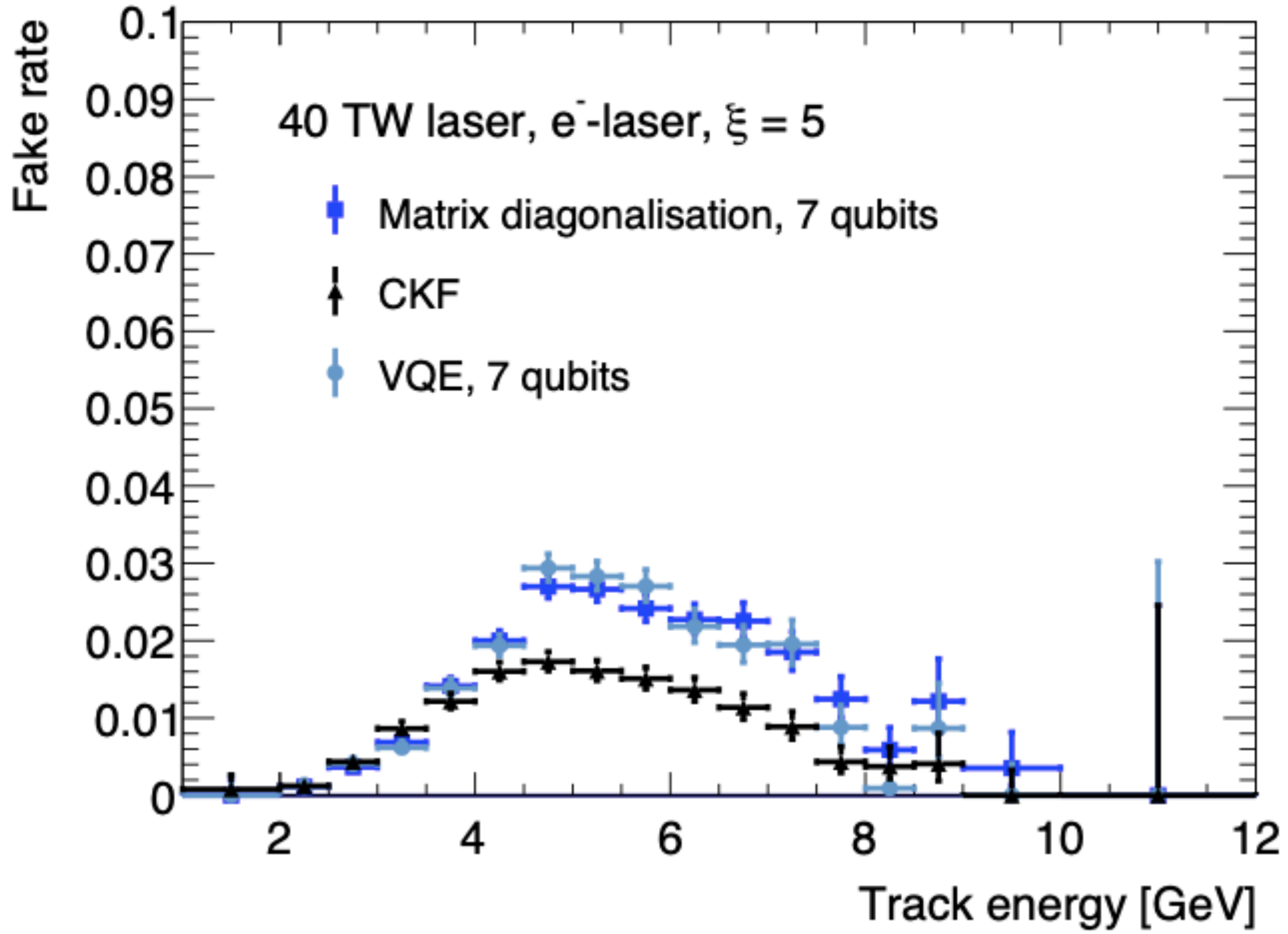


Increasing complexities/#particles

PERFORMANCE VS ENERGY

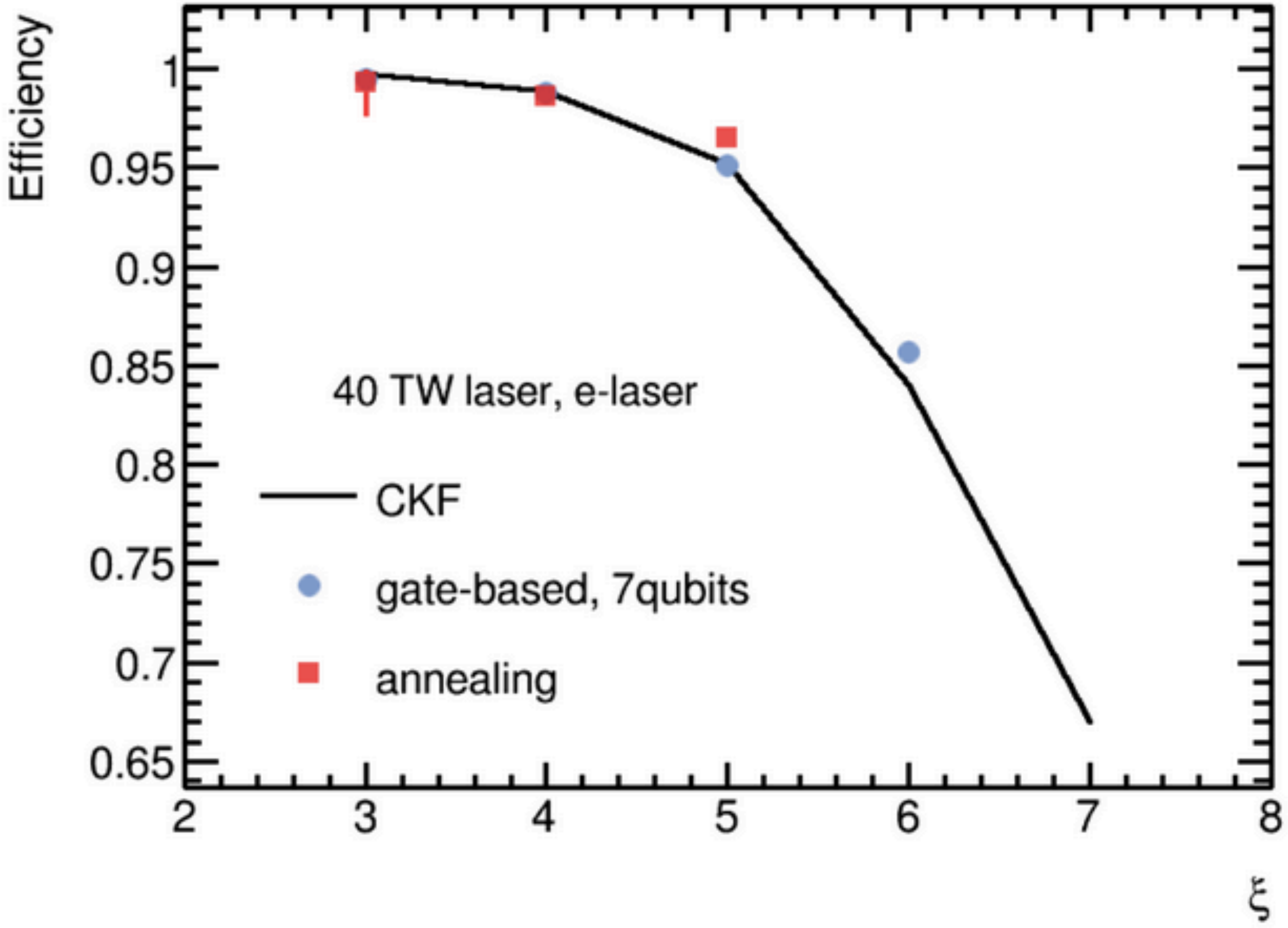
GNN results not available

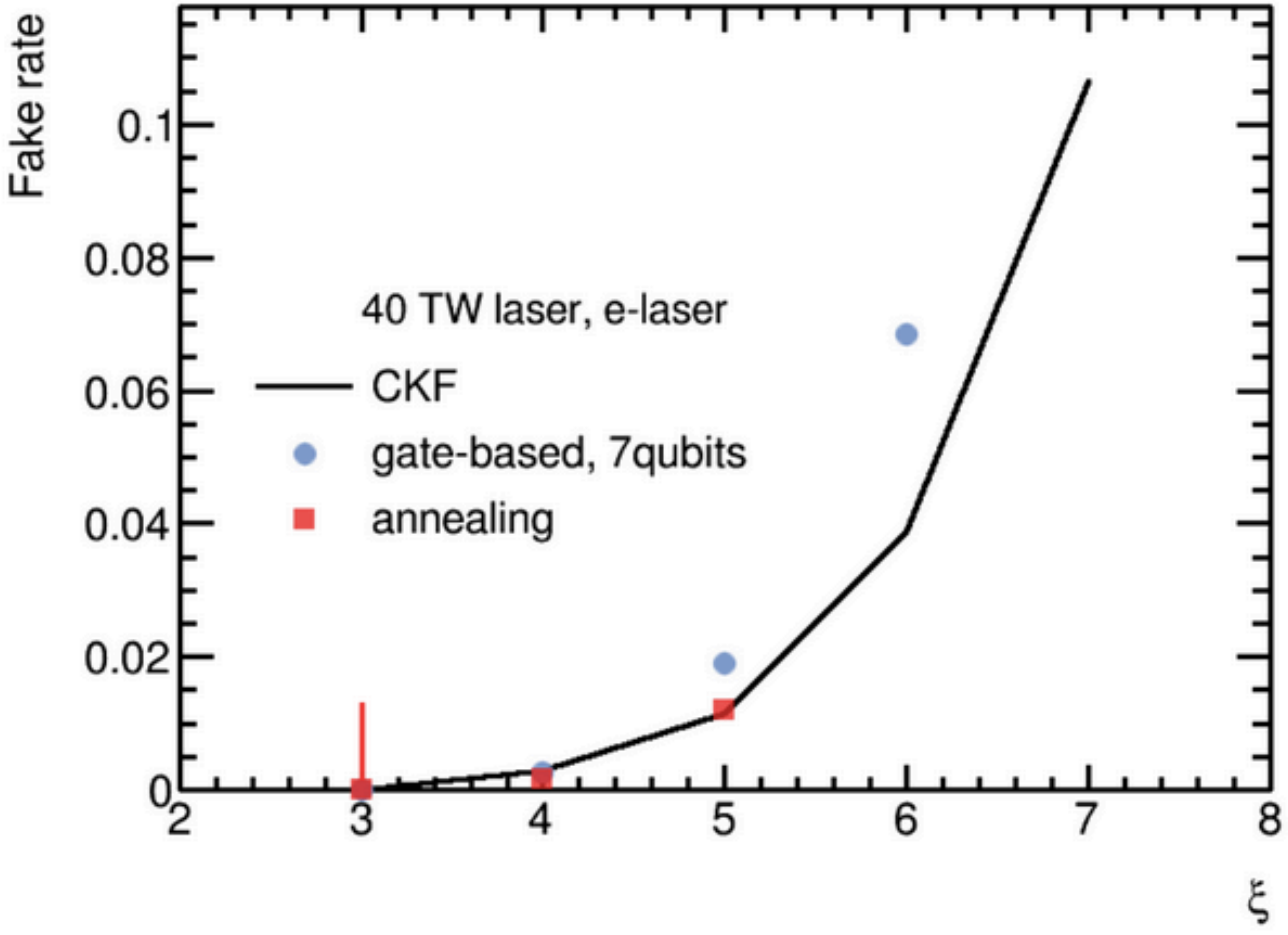




QUBO SIZE STUDY WITH SIMULATED ANNEALING

- No partitioning of QUBO.





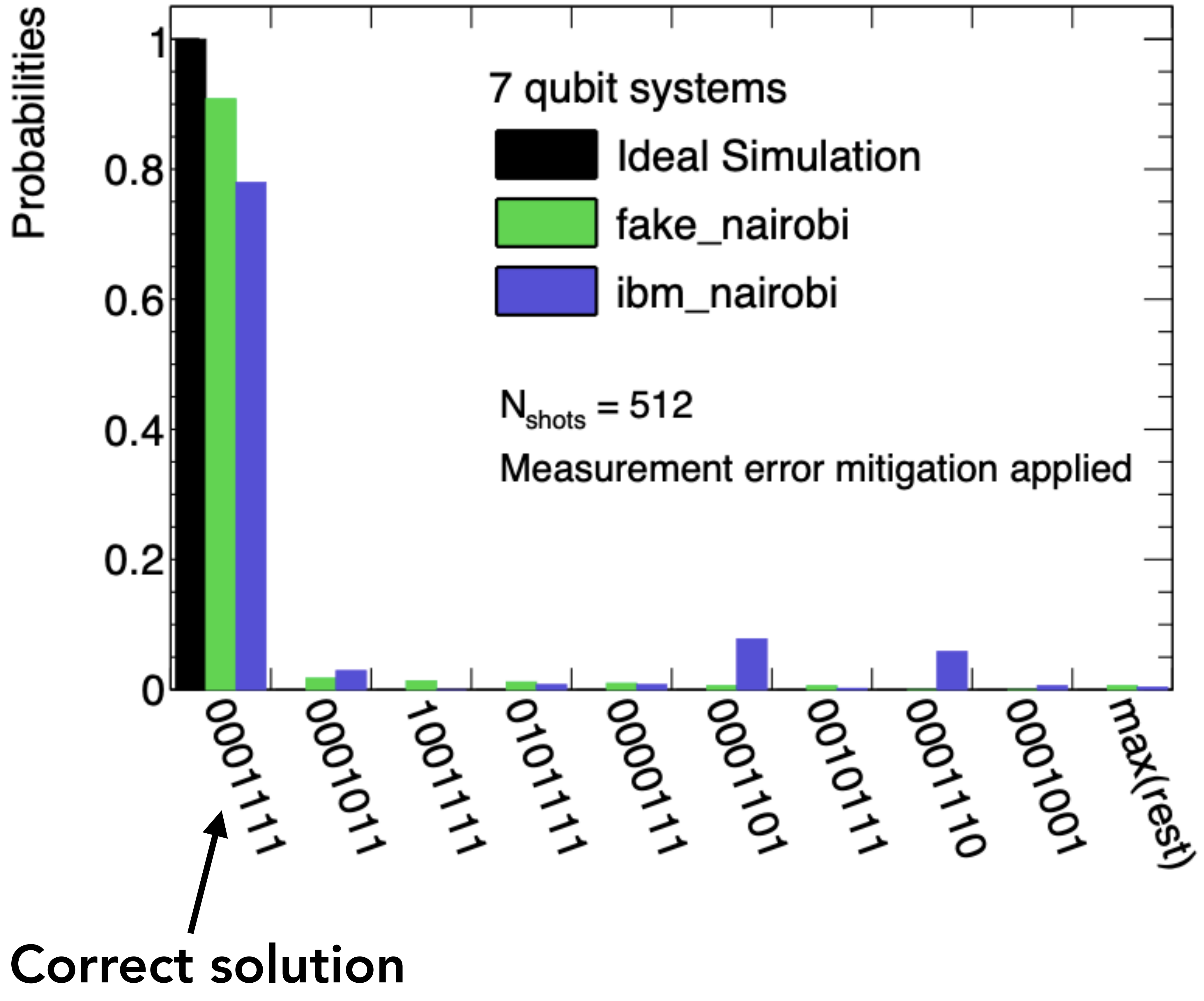
RESULTS (QUANTUM HARDWARE)

TEST ON IBM QUANTUM COMPUTER

- Results shown so far obtained using classical simulations of quantum hardware without noise.

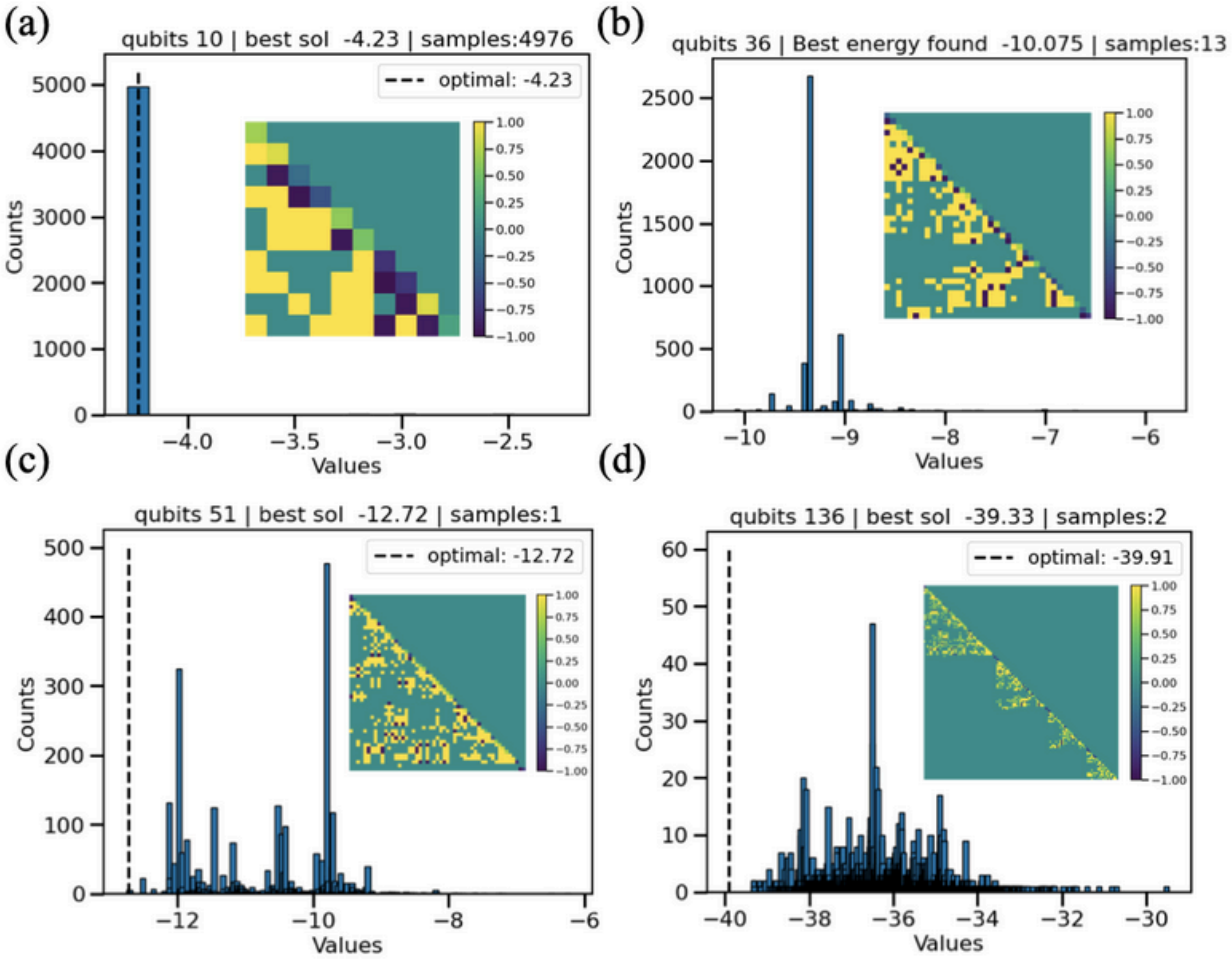
- To study how well VQE works, we study an example with 7 triplets (matching the #qubits of the device tested).

- Compare results from running on **quantum hardware (IBM Nairobi)** to **ideal simulation** as well as a **simulated device with noise**.



TEST ON QUANTUM ANNEALER

- Real D-wave annealer at Forschungszentrum Jülich
  - QUBO examples with sizes 10 — 136.
  - Ground state found for smaller QUBO sizes.

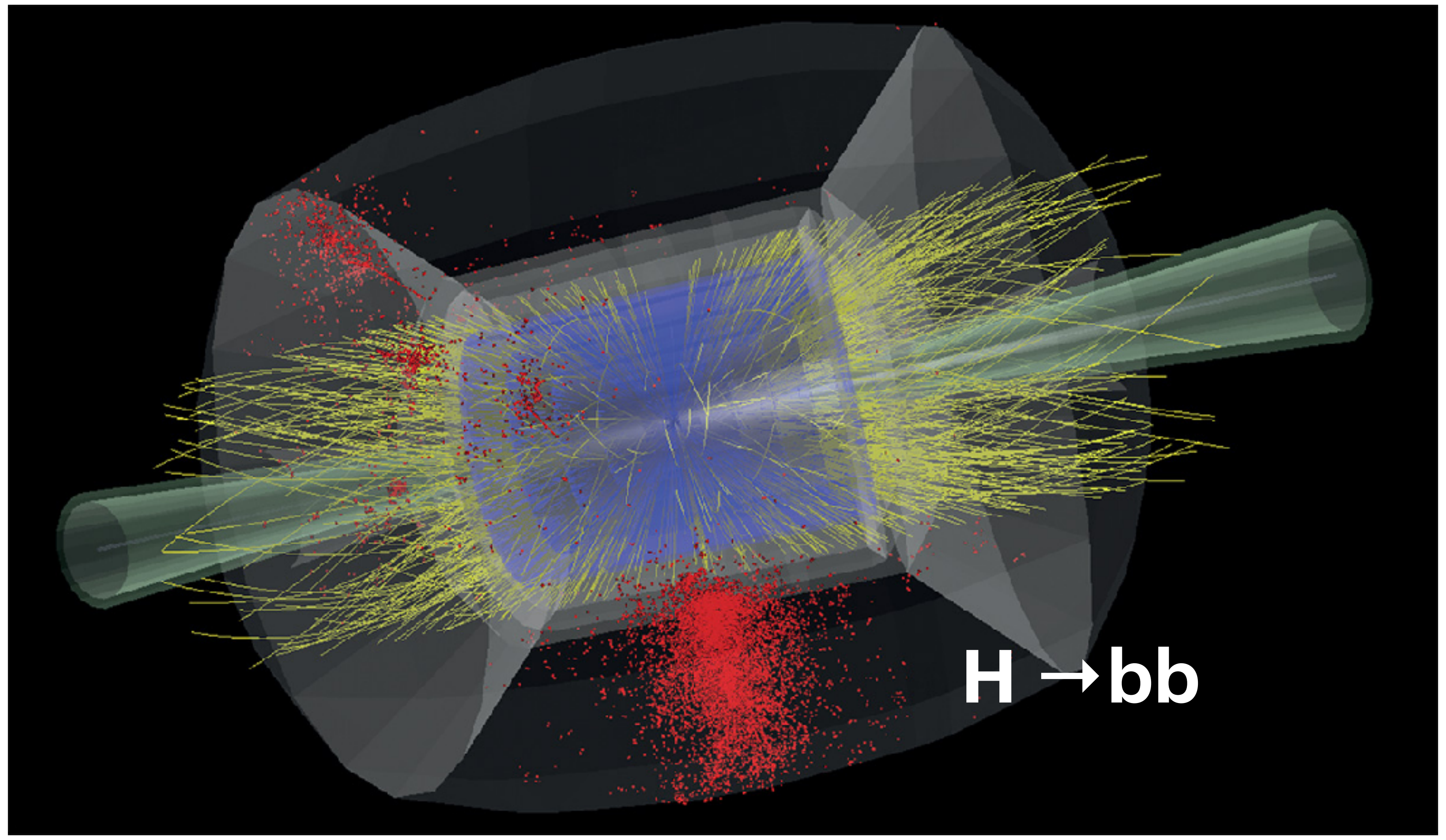


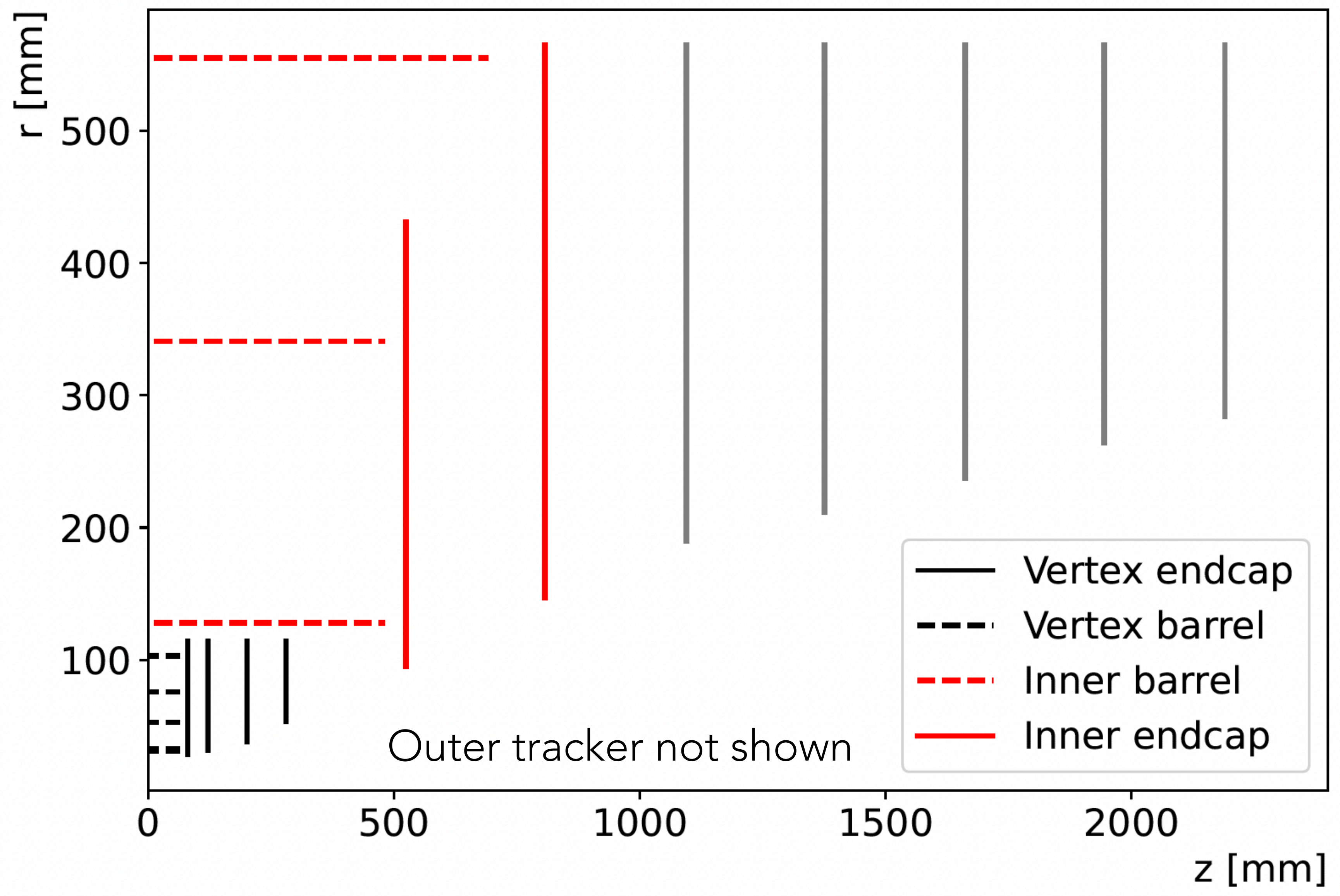
SUMMARY AND OUTLOOK

- Demonstrated the feasibility of tracking at LUXE using a quantum approach in arXiv:2304.01690.
    - Quantum approach with a GNN or QUBO (VQE or annealing).
    - Achieved similar performance as classical tracking (CKF).
    - Tests on real hardware.
- Outlook: how to extend our tracking approach to 4D?
    - O(10ps) timing resolution achievable in future tracking detector.
        - Constrain time-of-flight compatibility and reduce hit combinations.

4D TRACKING DETECTORS

- Many experimental upgrades/proposals include such technology in all tracking layers, e.g. LHC Run 5.
- Huge beam-induced background at muon colliders.
  - Timing information essential to filter out these background.
- Timing resolution of 30ps (vertex detector) – 60ps.





MUON COLLIDERS

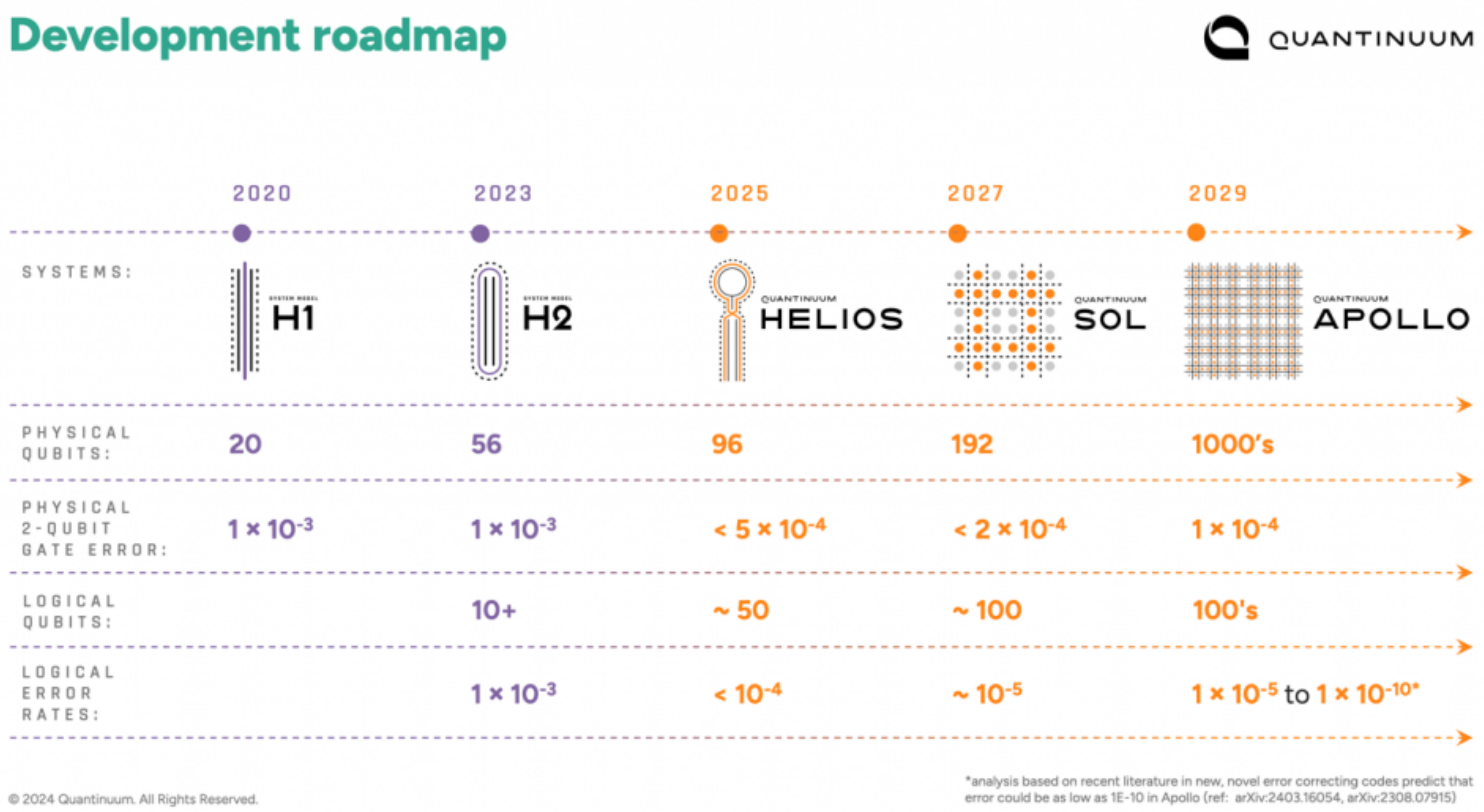
- With muon collider as a test case, develop 4D tracking by adding timing information to the QUBO.
  - QUBO parameters: 1D (timing only), 3D (spatial) and 4D (spatial+timing).
- Massive long-lived charged particle signal overlaid with the beam-induced background at a 10 TeV muon collider.

1.1 TeV LLCP					
Efficiency [%]			**Fake rate [%]**		
1D	**3D**	**4D**	**1D**	**3D**	**4D**
59.9	70.0	81.5	0.0	0.0	0.0

CONCLUSIONS

- Feasibility of tracking using quantum approach demonstrated for LUXE, and extended to 4-dimension for muon colliders.

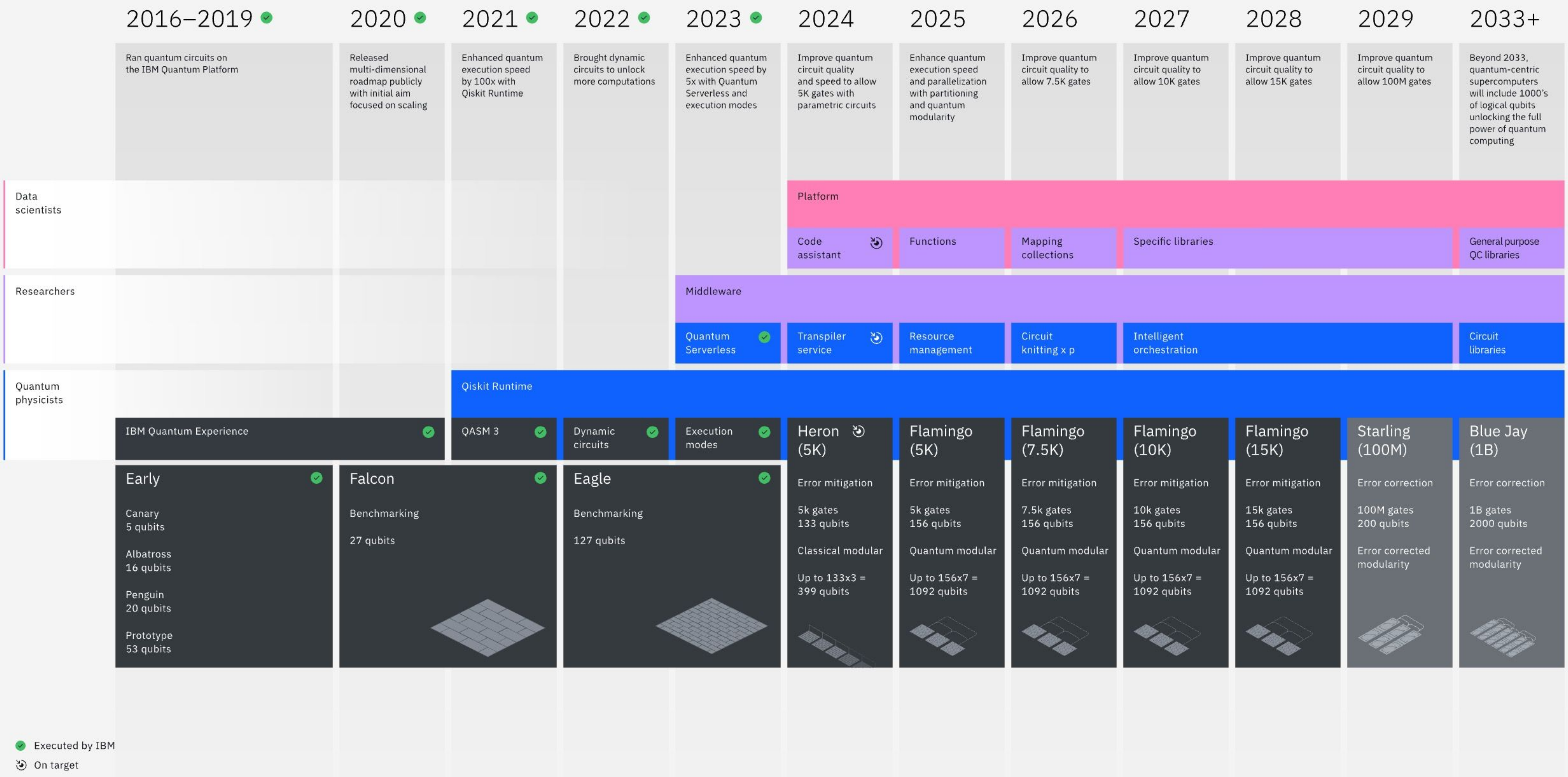
- Quantum hardware and algorithms are advancing rapidly, bringing practical HEP applications closer to reality.



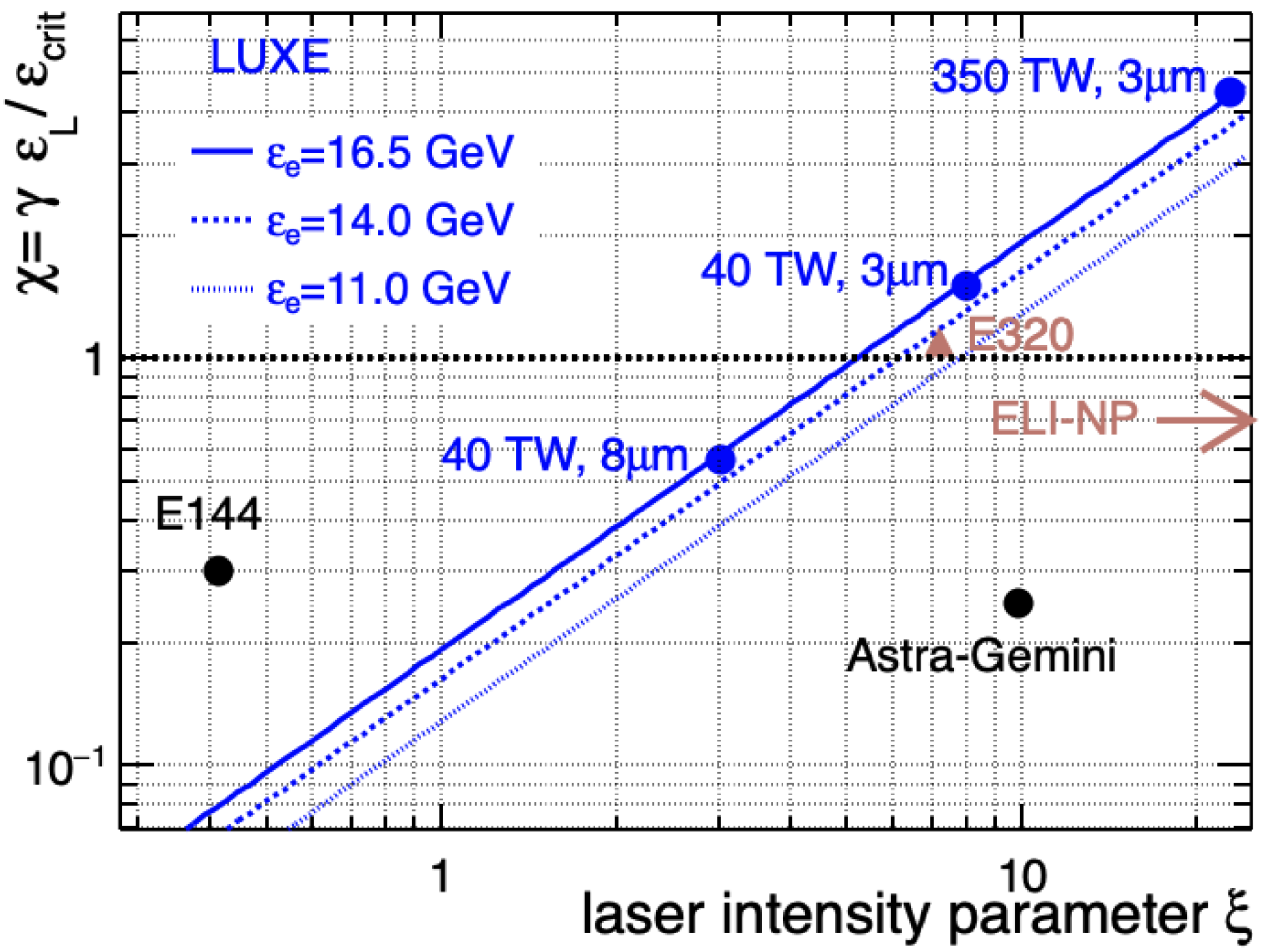
THANK YOU

Development Roadmap

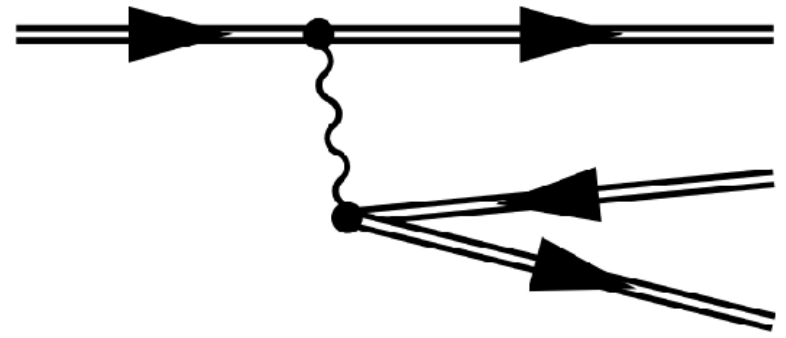
IBM **Quantum**



LUXE PARAMETER SPACE



- SLAC Experiment 144 in the 90s
  - reached $\chi_e \leq 0.25$, $\xi<0.4$, within perturbative regime, but with observable non-linear effects.
  - observed $e^- + n\gamma_L \rightarrow e^-e^+e^-$ trident process.



- Other ongoing/proposed experiments: SLAC-E320 (US), Astra-Gemini (UK), ELI-NP (RO)

LASER

- Titanium:Sapphire laser based on chirped pulse amplification (CPA) technology. Laser photon wavelength 800 nm (or 1.55 eV).

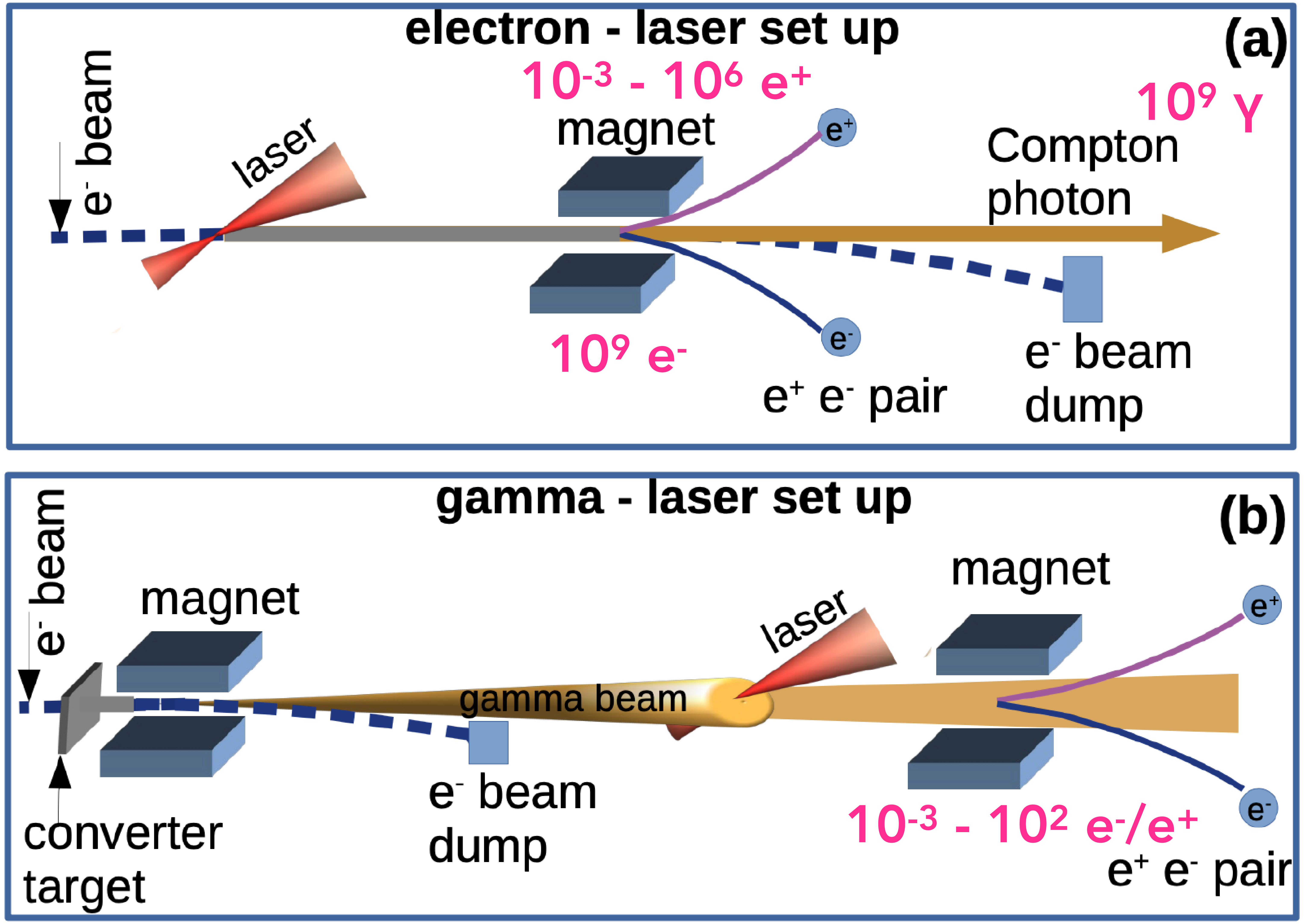
- Different ξ values can be reached by focussing/defocussing the laser.

- Need exceptional shot-to-shot stability (1%).

	Phase-0	Phase-1
Laser power (TW)	40	350
Peak intensity in focus ($\times 10^{20}$W/cm^2)	<1.33	<12
Dimensionless peak intensity ξ	<7.9	<23.6
Quantum parameter χ_e for E_e=16.5 GeV	<1.5	<4.45
Laser focal spot waist (μm)	≥3	
Laser pulse duration (fs)	30	

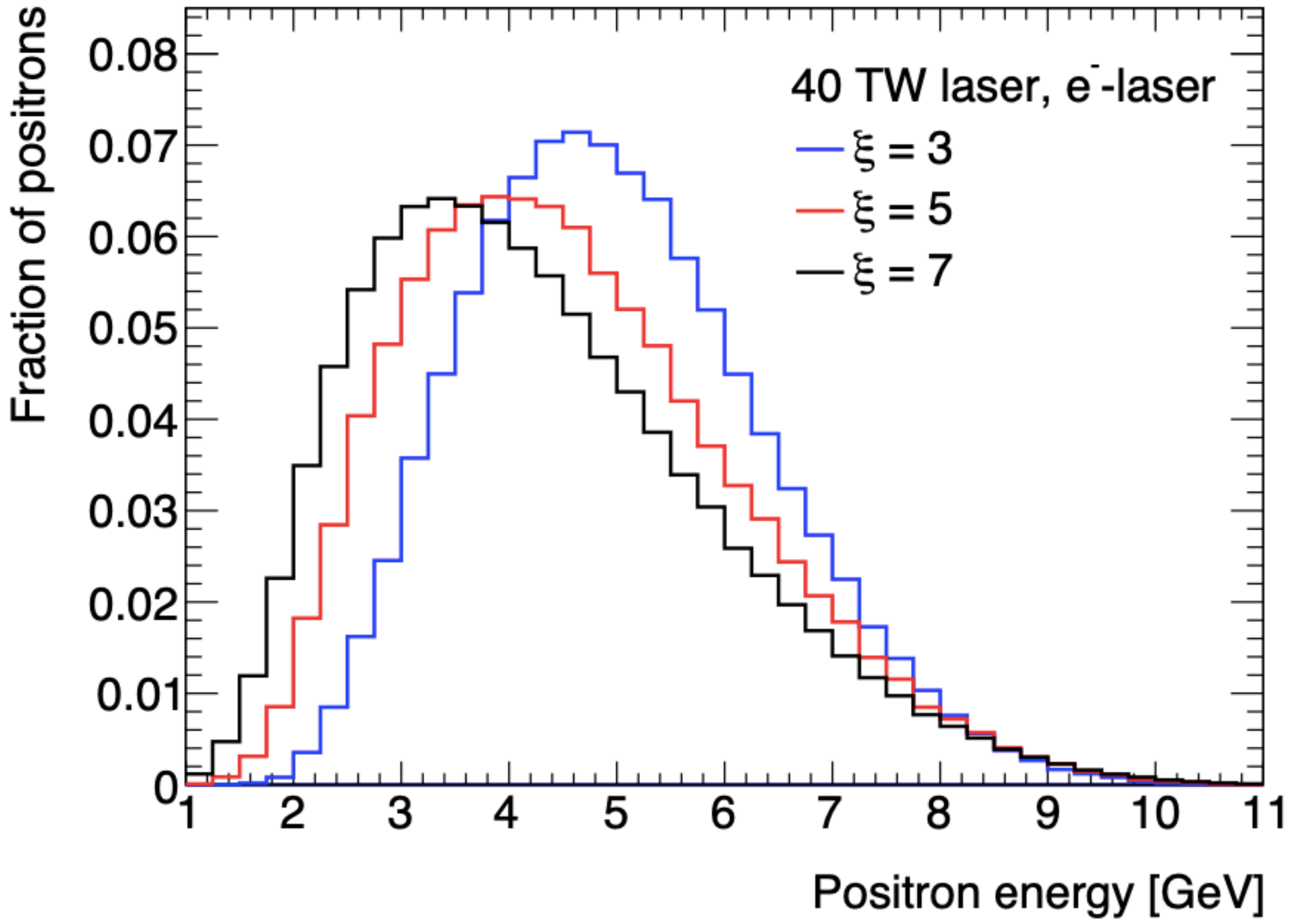
EXPERIMENTAL SETUP

- Use of dipole magnet to separate particles.
- Large variation in particle fluxes.

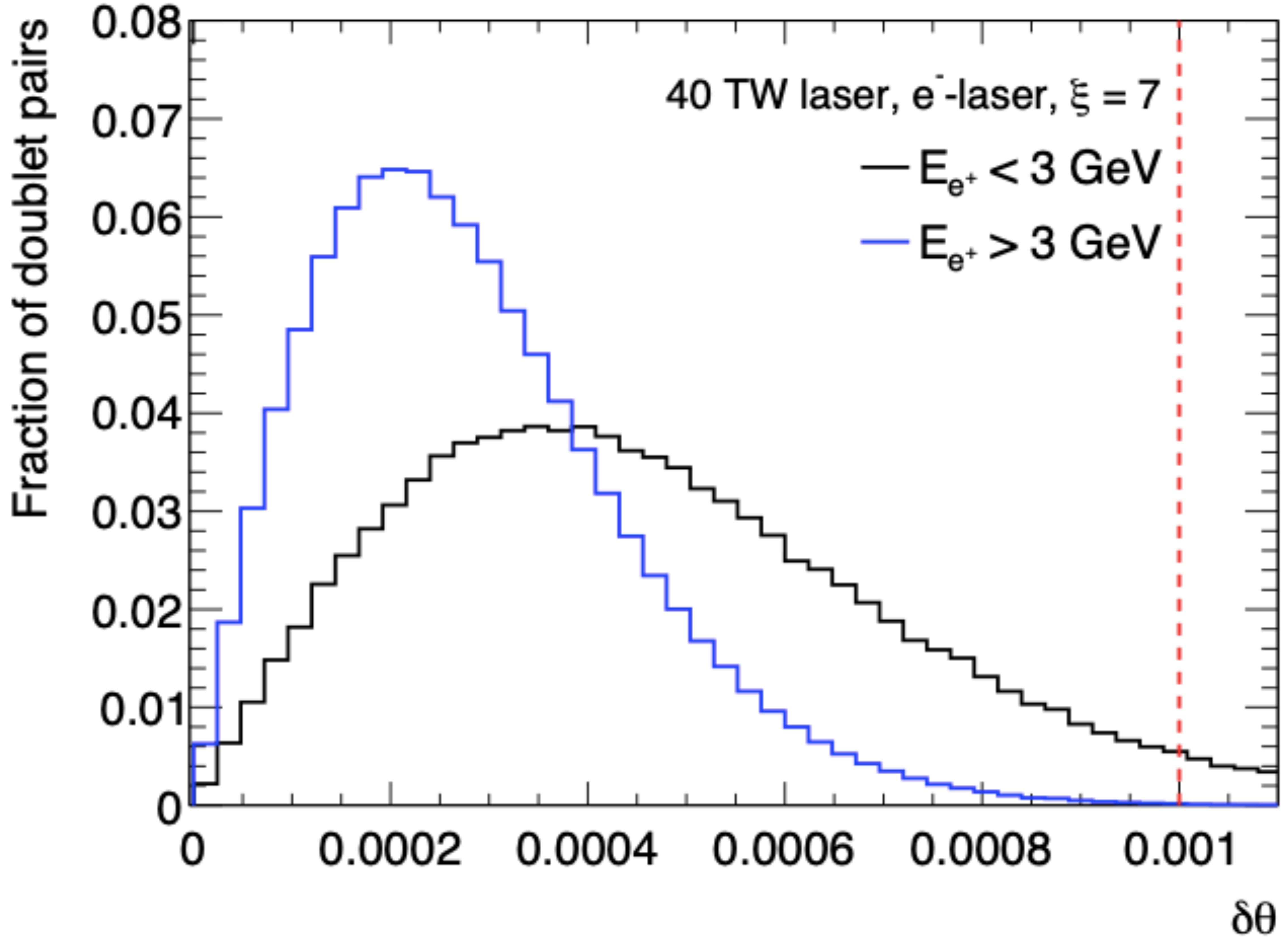


DETECTORS

- Different detectors to measure electron, positron and photon with suitable technologies for the expected range of flux, resolution and other requirements.

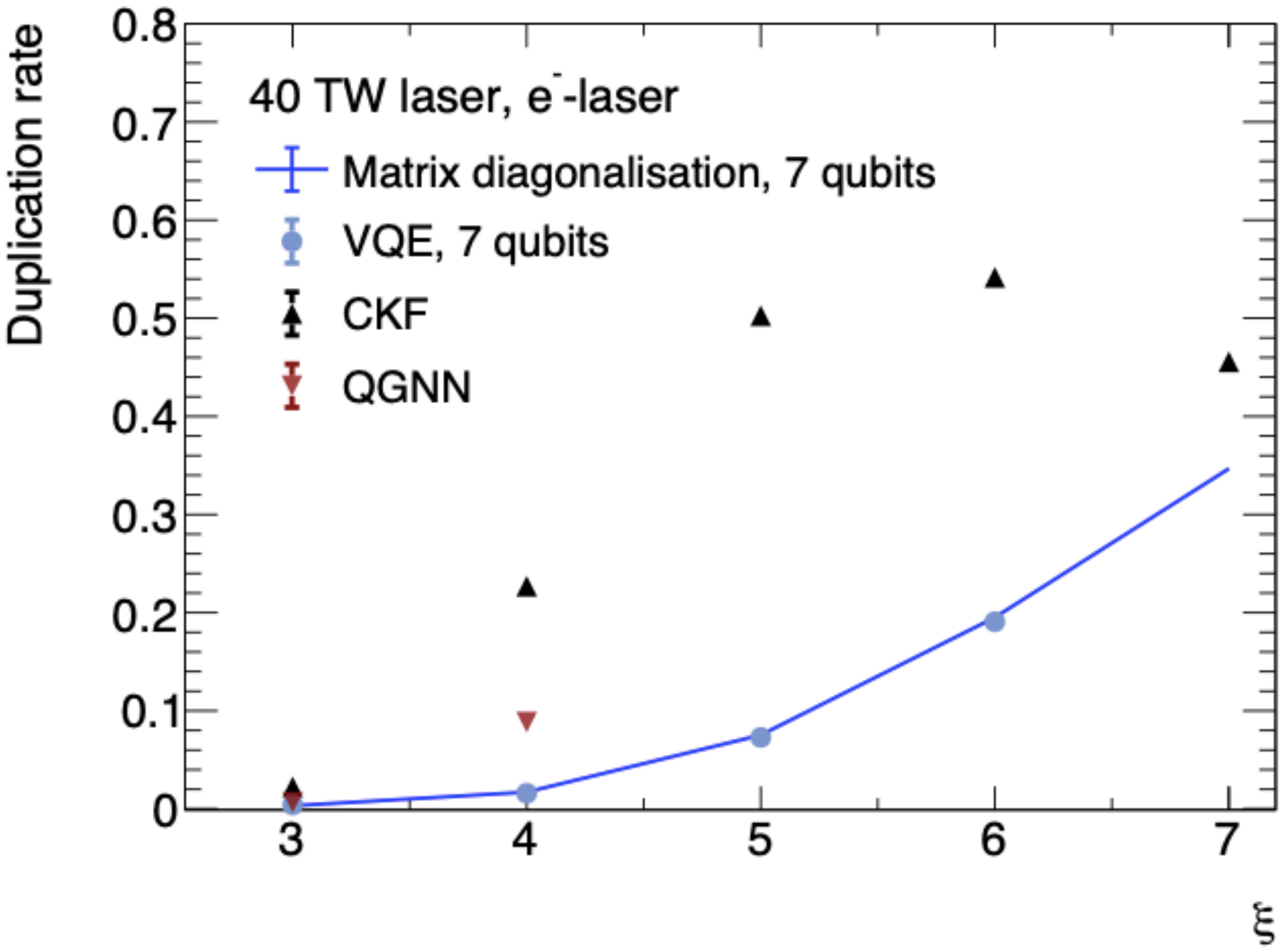


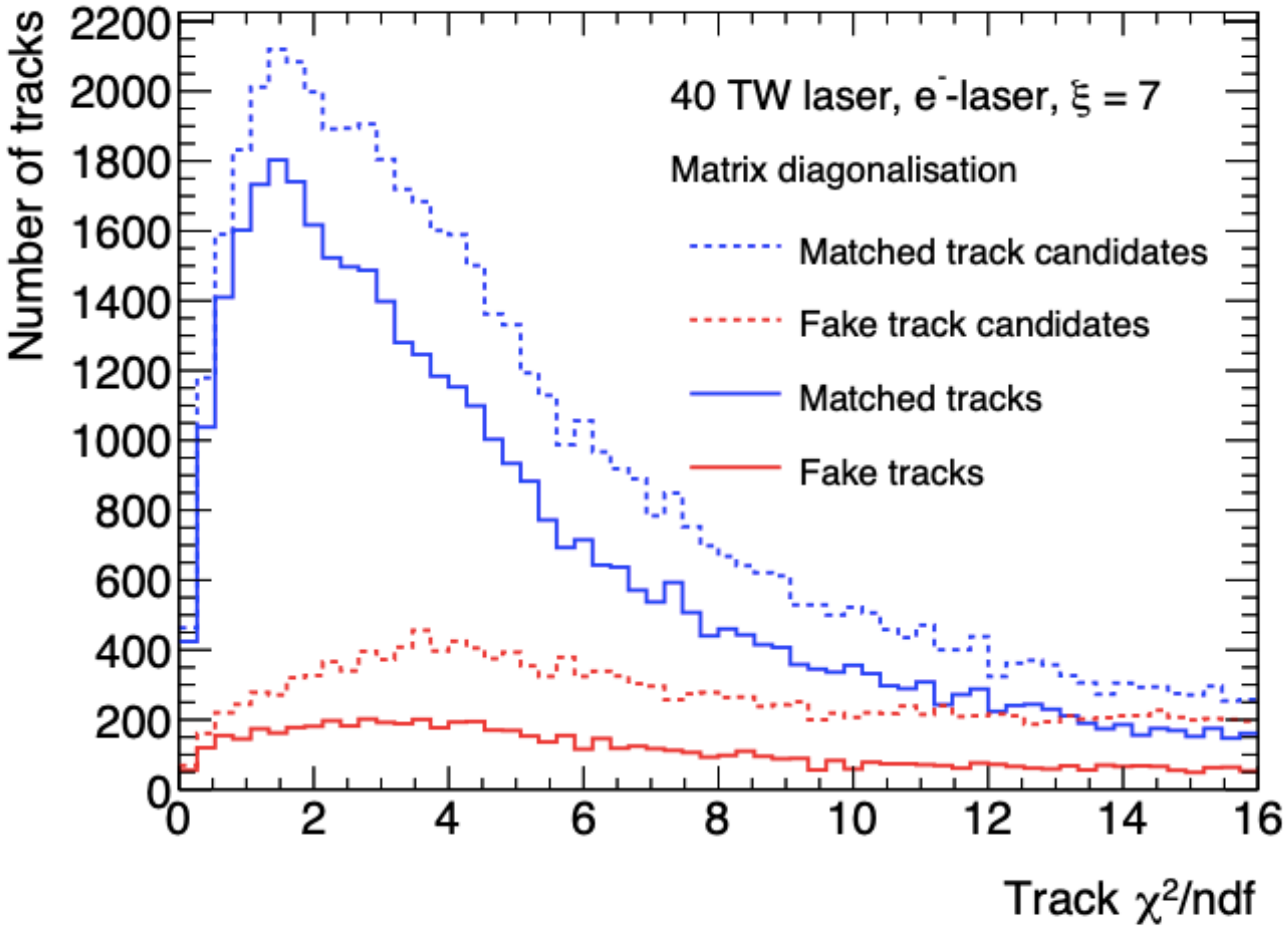
Fraction of positrons
40 TW laser, e$^-$-laser
ξ = 3
ξ = 5
ξ = 7
Positron energy [GeV]
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
9
10
11



Fraction of doublet pairs
40 TW laser, e$^-$-laser, ξ = 7
E_{e^+} < 3 GeV
E_{e^+} > 3 GeV
δθ
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.0002
0.0004
0.0006
0.0008
0.001

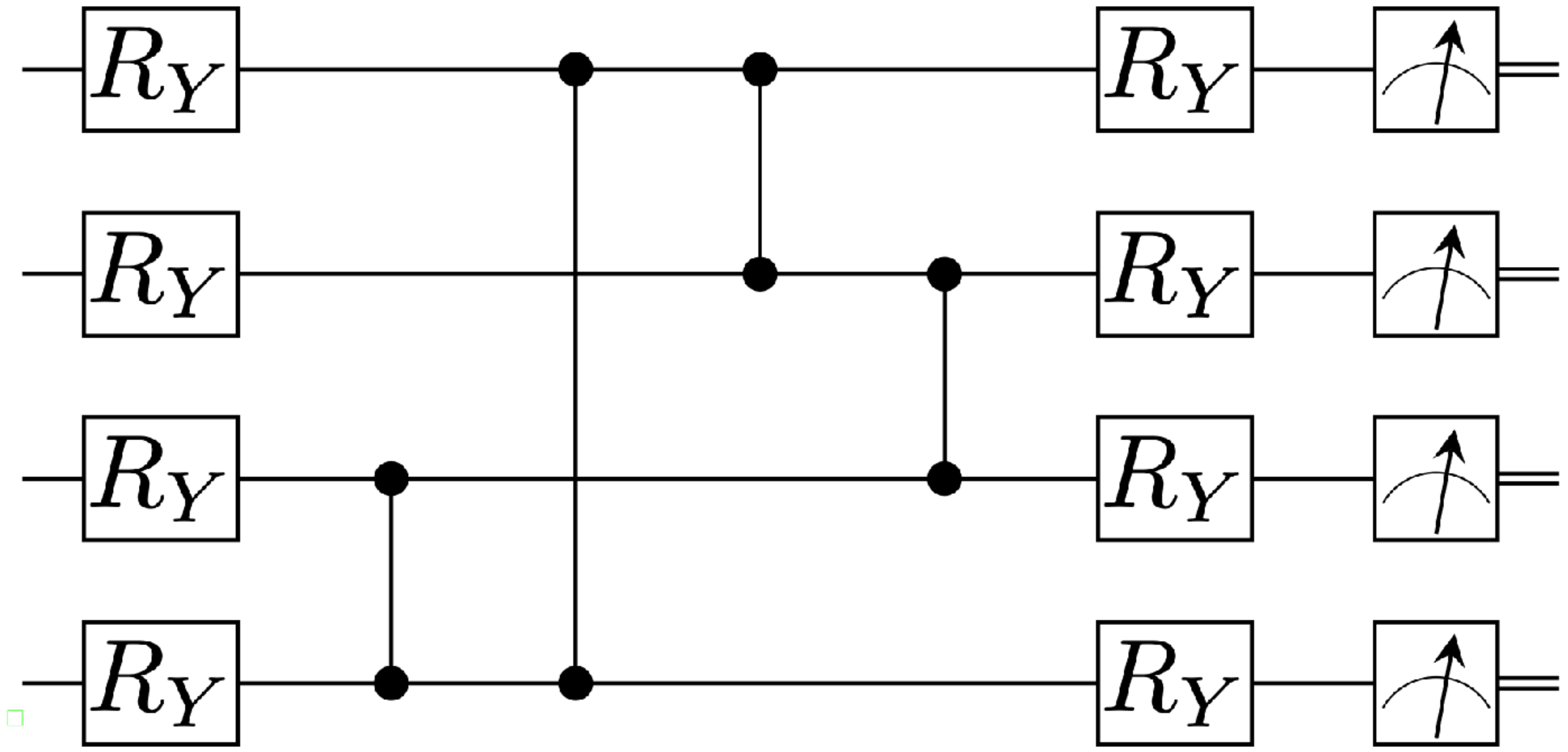
TRACK POST PROCESSING





GNN

- Circuit 10 with two layers and 10 qubits used.



(d) Circuit 10 in four qubits and single layer configuration. Adapted from Sim et al. (2019).

- EdgeNet and the NodeNet are applied alternately four times to allow the node features to be updated using farther nodes, as determined in a scan of the optimal model parameters.

GNN

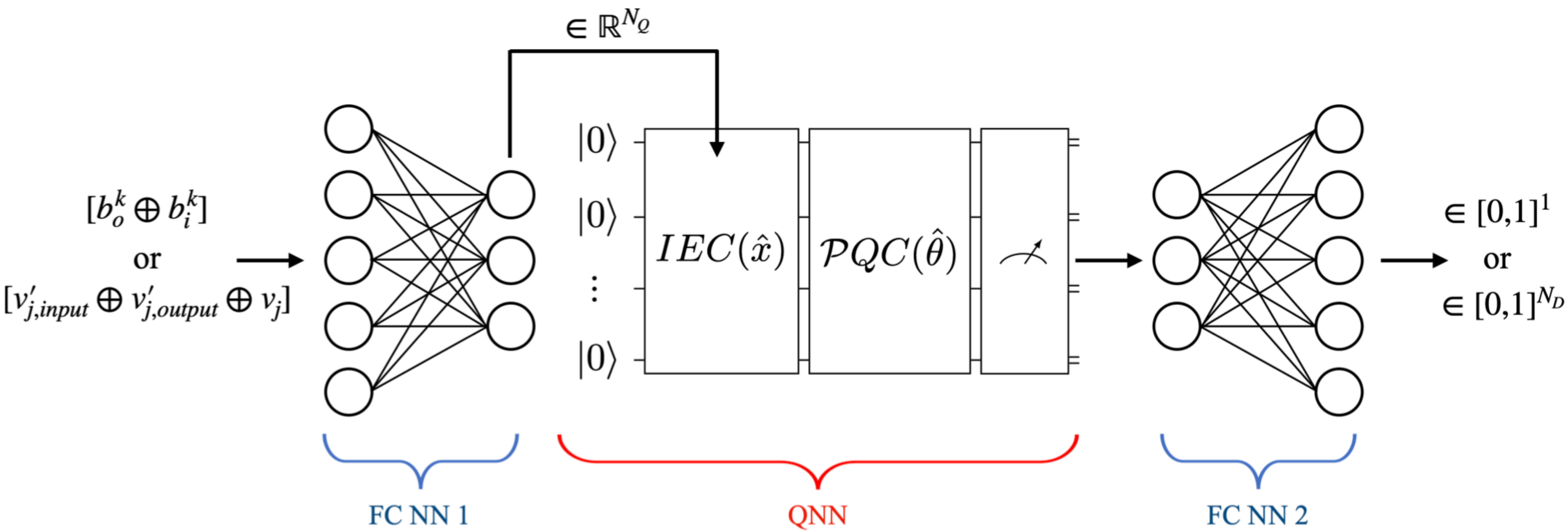
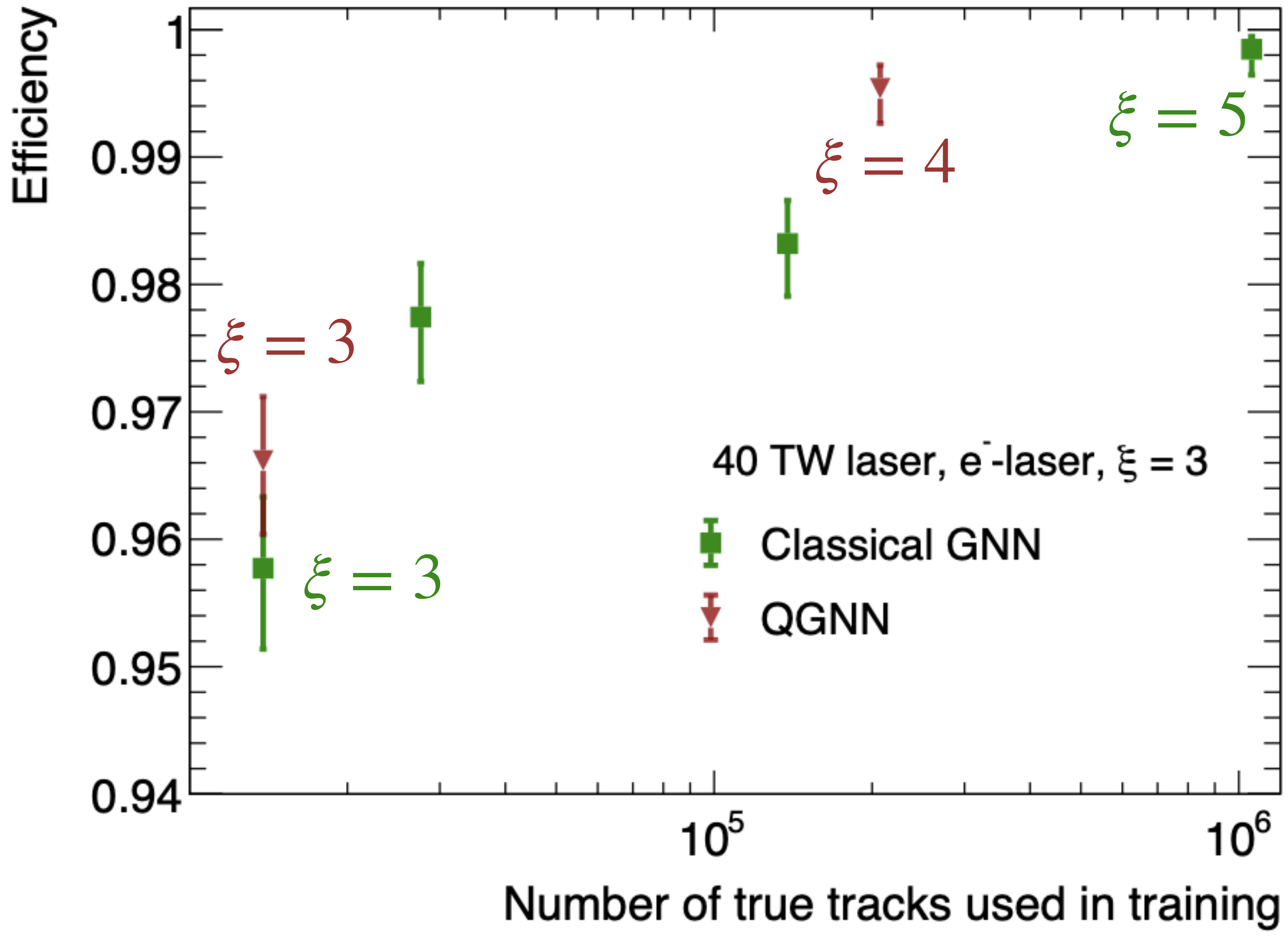


Fig. 6 The Hybrid Neural Network (HNN) architecture. The input is first fed into a classical fully connected Neural Network (FC NN) layer with sigmoid activation. Then, its output is encoded in the QNN with the Information Encoding Circuit (IEC). Next, the Parametrized Quantum Circuit (PQC) applies transformations on the encoded states. The output of QNN is obtained as expectation values for each qubit that is measured. A final FC NN layer with sigmoid activation is used to combine the results of different qubit measurements. The same HNN architecture is used in Edge (upper input and output dimension) and Node Networks (lower input and output dimension) with different parameters. The input and output dimension sizes change according to the network type. Details of the dimensions of each layer are given in Table 1.

GNN

- classical GNN with the same architecture, but with 128 node hidden features

- 90 BXs for training and 10 BXs for inference.



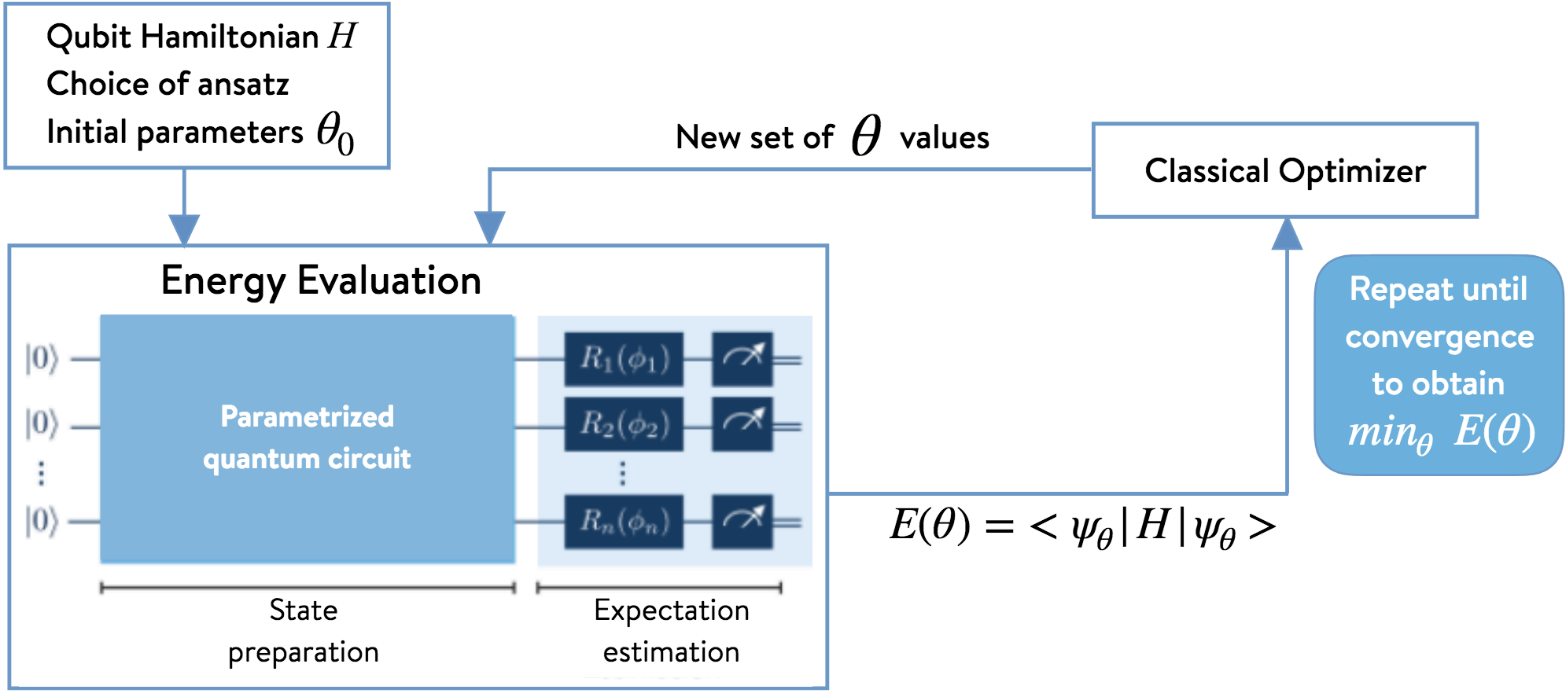


Image from http://openqemist.1qbit.com/docs/vqe_microsoft_qsharp.html

METHOD COMPARISON

Methods	GNN	QUBO	CKF
Starting point	Doublet	Triplet	Seed
Local/global	Global	Global	Local
Scope	Pattern recognition only	Pattern recognition only	Pattern recognition + track fitting
Classical benchmark	Classical GNN	Matrix diagonalisation	-