Automating dipole subtraction

K. Hasegawa

DESY Zeuthen

1. Introduction
2. Calculation of dipole terms
3. Summary

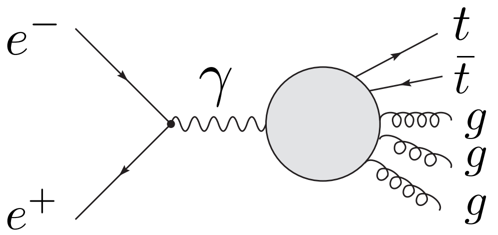
Collaboration with S. Moch and P. Uwer

Loop and Legs in Quantum Field Theory Sondershausen, 20 - 25 April 2008

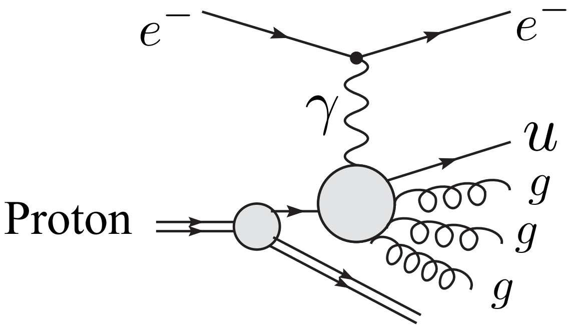
1. Introduction

Collider processes with several parton legs

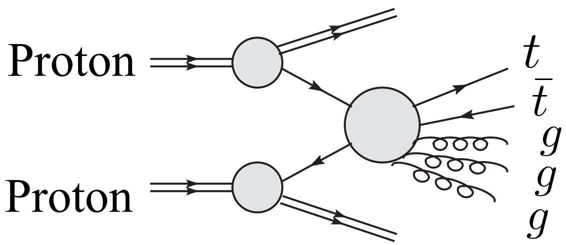
① e^+e^- annihilation into hadrons (LEP, ILC)



② Deep inelastic scattering (SLAC, HERA)



③ Hadron-hadron scattering (Tevatron, LHC)



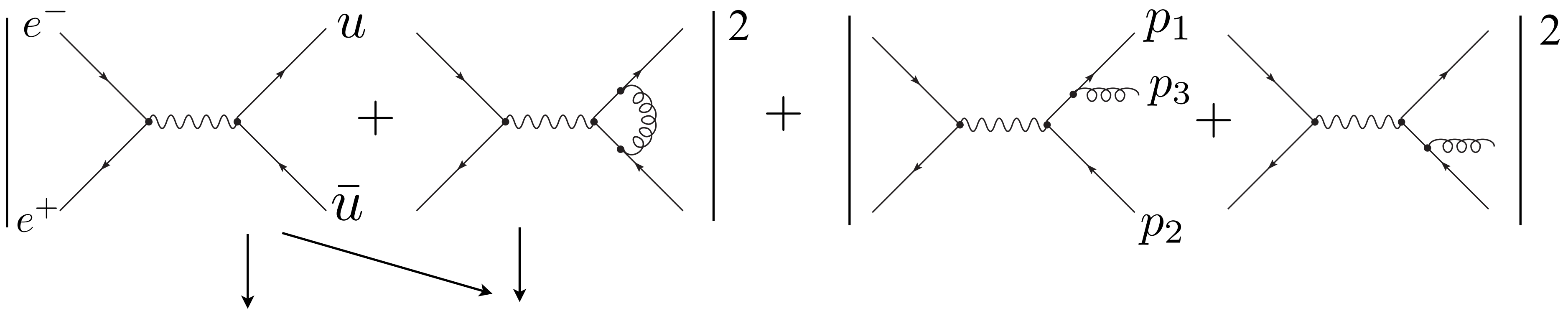
Leading order (LO) can be well calculated by some published softwares in automatic way

Typical ones : MadGraph, CompHep, FeynArts, HELAC/PHEGAS, Alpgen

Hope :

Software to calculate QCD Next-to-Leading order (NLO) in automatic way

□ QCD NLO calculation : a simplest example $e^+e^- \longrightarrow u\bar{u}$



$$\sigma(e^+e^- \to u\bar{u}) = \sigma_{\rm LO} + \underset{\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8+\pi^2+O(\epsilon))\right]}{\sigma_{\tt virtual}} + \underset{\left[+\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{19}{2}-\pi^2+O(\epsilon)\right]}{\sigma_{\tt real}}$$

$$= \sigma_0 N_c Q_u^2 \left(1 + \boxed{\frac{\alpha_s}{\pi}} + O(\alpha_s^2)\right)$$

Finite NLO: Infrared safety

In the case of several parton legs, it is difficult to obtain $|\mathrm{M}_{\rm real}|^2$ and to integrate phase spaces in D dimension analytically because of their too complicated expressions.

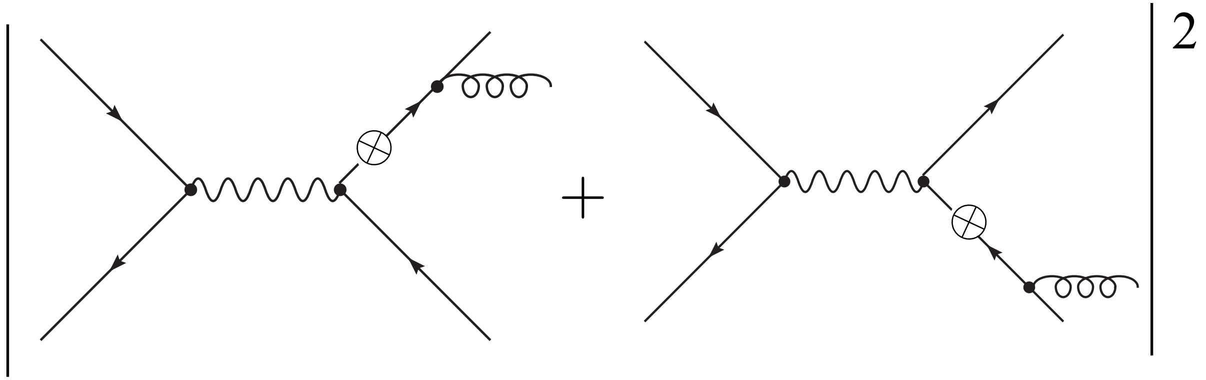
⇨ A practical and general procedure is found and formulated :

Dipole subtraction

Massless : S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
Massive : S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

☐ Dipole subtraction

Based on the general property that soft and collinear divergences can be factorized from their Born amplitude in universal ways



LO : m particles in final state

NLO : m particles with 1-loop diagram
(m+1) particles with real emssion

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virtual}}$$

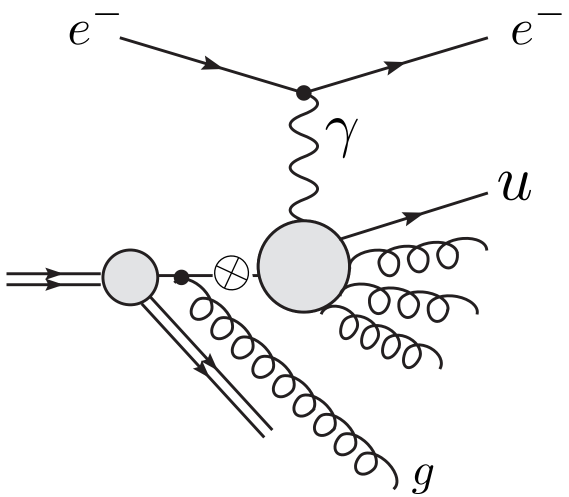
$$= (\sigma_{\text{real}} - \sigma_{\text{a}}) + (\sigma_{\text{virtual}} + \sigma_{\text{a}})$$

$$= \underbrace{\boxed{\int d\Phi_{m+1}\left[|\mathrm{M}_{\text{real}}|^2 - \sum_i \mathrm{D}_i\right]\Bigg|_{\mathrm{D}=4}}}_{\text{Finite}} + \underbrace{\int d\Phi_m\left[\underbrace{|\mathrm{M}_{\text{1-loop}}|^2 + \int d\Phi_1 \sum_i \mathrm{D}_i}_{\text{(Cancelation of } 1/\epsilon \text{ poles)}}\right]\Bigg|_{\mathrm{D}=4}}_{\text{Finite}}$$

Big advantage:

We need $|\mathrm{M}_{\text{real}}|^2$ and phase space integral in only D=4 dimension.
Phase space integral is finite

☐ Initial partons



Gluon emission from initial partons produces collinear singularity which is not cancelled by virtual correction $\sigma_{\mathrm{virtual}}$

⇩

Those singularities should be factorized into parton distribution function

In dipole subtraction:

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}} + \underline{\sigma_{\mathrm{C}}}$$

(Collinear-subtraction counter-term)

$$= \underbrace{\boxed{\int \mathrm{d}\Phi_{m+1}\left[|\mathrm{M}_{\mathrm{real}}|^2 - \sum_i \mathrm{D}_i\right]\Big|_{\mathrm{D}=4}}}_{\text{Finite}} + \int \mathrm{d}\Phi_m\left[|\mathrm{M}_{\text{1-loop}}|^2 + \mathrm{Y}\right]\Big|_{\mathrm{D}=4} + \underline{\int_0^1 \mathrm{d}z \int \mathrm{d}\Phi_m \Big[|\mathrm{M}_{\mathrm{Born}}|^2 \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\epsilon} \mathrm{P}^{ab}(z)} + \mathrm{X}\Big]\Big|_{\mathrm{D}=4}$$

$$\int \mathrm{d}\Phi_1 \sum_i \mathrm{D}_i = \mathrm{X} + \mathrm{Y}$$

X : Collinear divergence from incoming parton

Y : Others

□ Automatization

- Algorithm of dipole subtraction is combinatorics (We will see later)

⇨ Suitable for automatization

We construct a Mathematica code to prdoduce dipole terms

- Large volume calculation

Example : $gg \longrightarrow t\bar{t}ggg$

100 dipoles : $\sum_{i=1}^{100} \mathrm{D}_i$

Each dipole : $\mathrm{D}_i \simeq \mathrm{V} \cdot |\mathrm{M}_{\mathrm{Born}}(gg \to t\bar{t}gg)|^2$

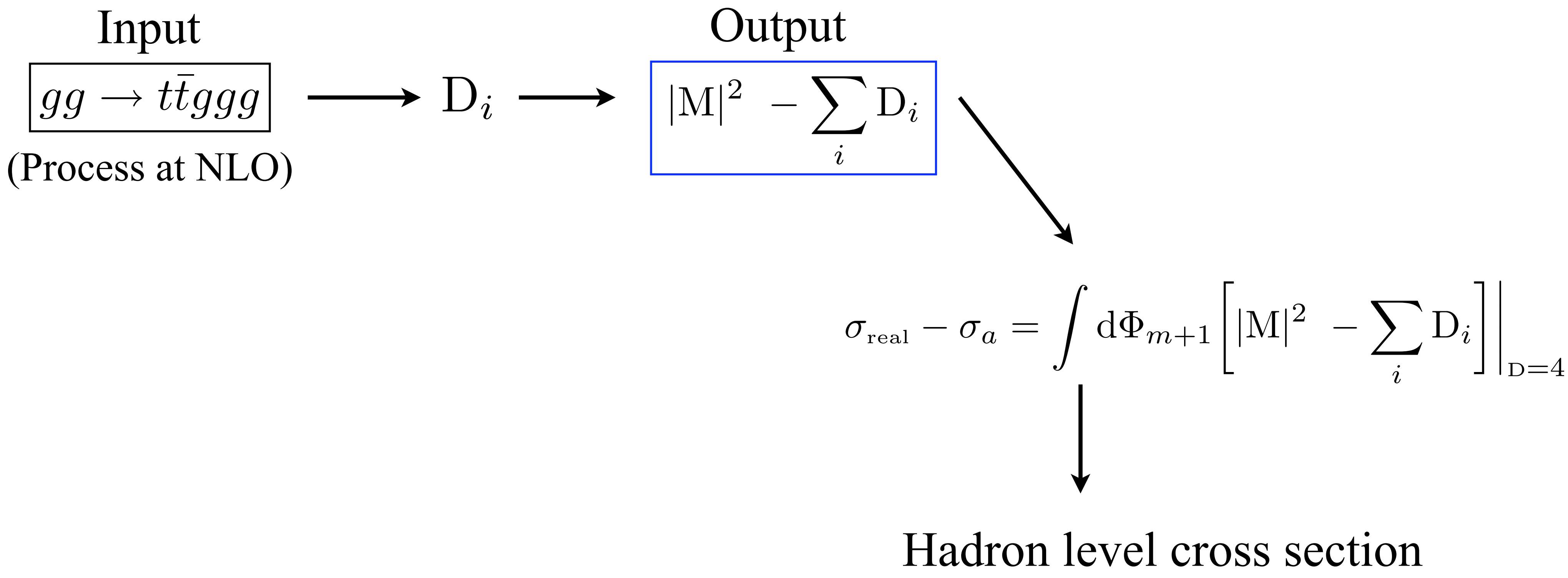
Singular part

Monte Carlo integral : Evaluate $\left(|\mathrm{M}_{\mathrm{real}}|^2 - \sum_i \mathrm{D}_i\right)$ at $\mathrm{O}(10^6)$ times

⇨ Fastest and automatic code to calculate $|\mathrm{M}|^2$ numeriacally is essential

We take MadGraph as one solution

Our aim is to construct a code to calculate QCD NLO corrections by dipole subtraction in automatic way



Input

Output

$$\boxed{gg \to t\bar{t}ggg} \longrightarrow \mathrm{D}_i \longrightarrow \boxed{|\mathrm{M}|^2 - \sum_i \mathrm{D}_i}$$

(Process at NLO)

$$\sigma_{\mathrm{real}} - \sigma_a = \int \mathrm{d}\Phi_{m+1} \left[|\mathrm{M}|^2 - \sum_i \mathrm{D}_i \right] \Big|_{\mathrm{D}=4}$$

Hadron level cross section

2. Calculation of dipole terms

☐ Algorithm

Example: LO $gg \to u\bar{u}$

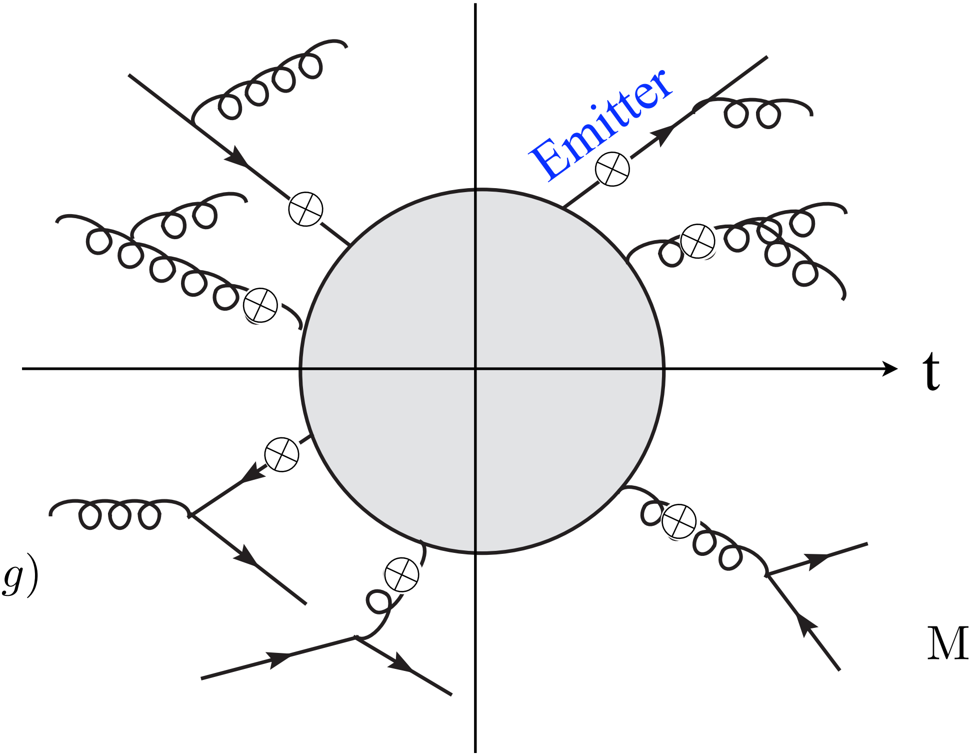
NLO $gg \to u\bar{u}g$

Initial parton=a,b

Final parton=i,j,k

Initial (a,i)

Final (i,j)



$\mathrm{M}_0(ug \to ug)$

$\mathrm{M}_0(gg \to gg)$

Diagonal

$\mathrm{M}_0(gg \to u\bar{u})$ = LO

(Reduced Born)

Non-Diagonal

Algorithm 1

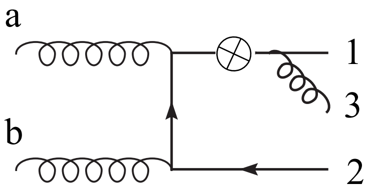
Choose all possible leg-pair which matches one of the seven patterns

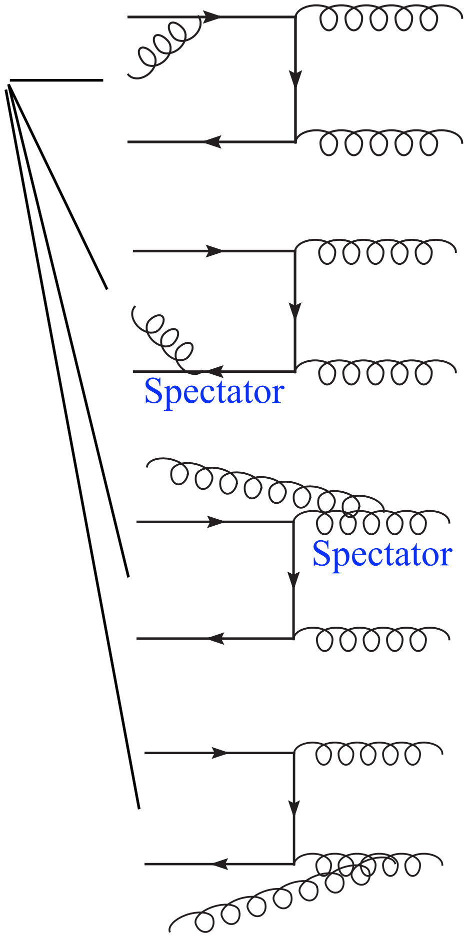
(a,i) or (i,j)

Example: NLO $gg \to u\bar{u}g$ Emitter: $(i,j) = (1,3)$

M × M*

Color factor





$= C_{13,1}A_{13,1}$

$+$

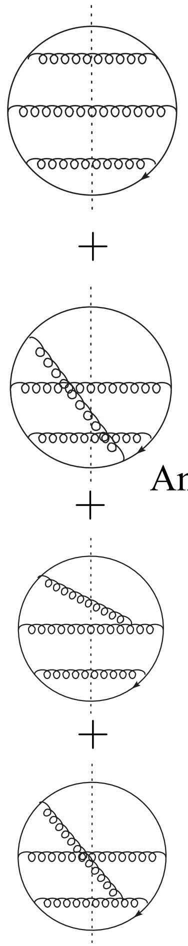
$= C_{13,2}A_{13,2}$

$+$

$= C_{13,a}A_{13,a}$

$+$

$= C_{13,b}A_{13,b}$



$\mathrm{Tr}[t^a t^b t^c t^c t^b t^a]$

$\mathrm{Tr}[t^a t^b t^c t^a t^c t^b]$ $\times(-1)$

Anti-quark

$= C_{13,2}(A_{13,2} - A_{13,1}) \equiv \underline{\mathrm{D}_{13,2}}$

$+C_{13,a}(A_{13,a} - A_{13,1}) \equiv \mathrm{D}_{13,a}$

$+C_{13,b}(A_{13,b} - A_{13,1}) \equiv \mathrm{D}_{13,b}$

||

0

(Color conservation)

Spectator : $\underline{k \neq i, j}$

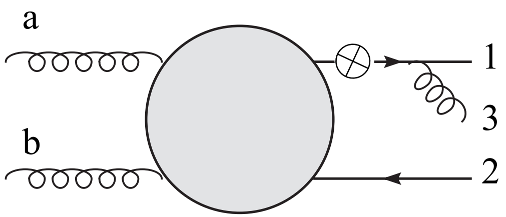
Algorithm 2

2. spectator 1. emitter pair	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\;b}$
(a, i)	$\mathrm{D}^{ai}_{\;k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

□ Dipole formulae and Color linked Born squared (CLBS)

$$\mathrm{D}_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{\mathrm{T}_k\cdot\mathrm{T}_{ij}}{\mathrm{T}_{ij}^2}\mathrm{V}_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1>_m$$

$$\mathrm{D}_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\left\langle gg\to\tilde{u}\tilde{\bar{u}}\right|\frac{\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}}{\mathrm{T}_{ug}^2}\mathrm{V}_{13,2}\left|gg\to\tilde{u}\tilde{\bar{u}}\right\rangle_2$$

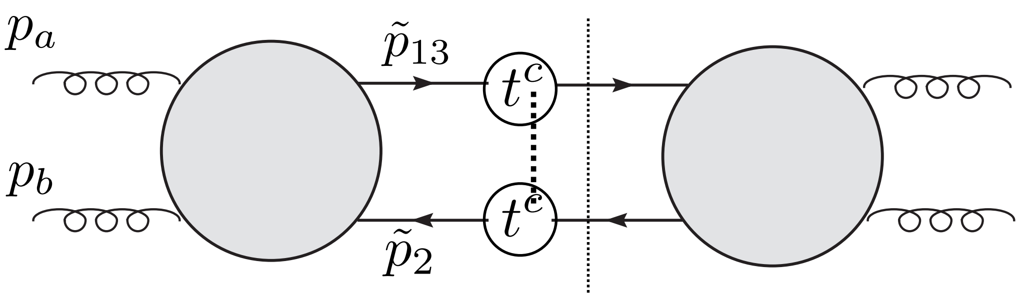


Dipole splitting function:

$$\mathrm{V}_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})}-(1+z_i)\right]$$

$$z_i = \frac{p_i\cdot p_k}{p_j\cdot p_k+p_i\cdot p_k} \qquad y_{ij,k} = \frac{p_i\cdot p_j}{p_i\cdot p_j+p_j\cdot p_k+p_k\cdot p_i}$$

Color linked Born squared : $\left\langle gg\to\tilde{u}\tilde{\bar{u}}\right|\mathrm{T}_{\bar{u}}\cdot\mathrm{T}_{ug}\left|gg\to\tilde{u}\tilde{\bar{u}}\right\rangle_2$



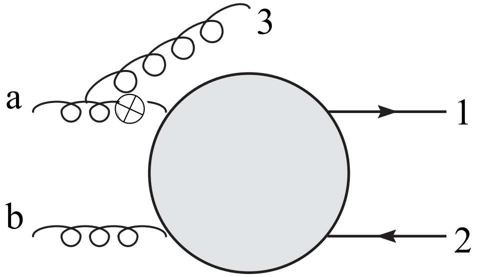
$$\mathrm{T}_{\mathrm{X}}^a = \begin{cases} t^a & (\mathrm{X} = \text{quark}) \\ f^a & (\mathrm{X} = \text{gluon}) \end{cases}$$

$$\tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}p_k^\mu$$

$$\tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}p_k^\mu$$

● Emitter = gluon case

$$\mathrm{D}^{a3}_{1}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_a\cdot p_3}\frac{1}{x_{31,a}}\Big\langle \underset{(\mu)}{\tilde{g}}g \to \tilde{u}\bar{u}\Big|\frac{\mathrm{T}_u\cdot\mathrm{T}_{gg}}{\mathrm{T}^2_{gg}}\mathrm{V}^{a3}\underline{(\mu,\nu)}\Big|\underset{(\nu)}{\tilde{g}}g \to \tilde{u}\bar{u}\Big\rangle_2$$

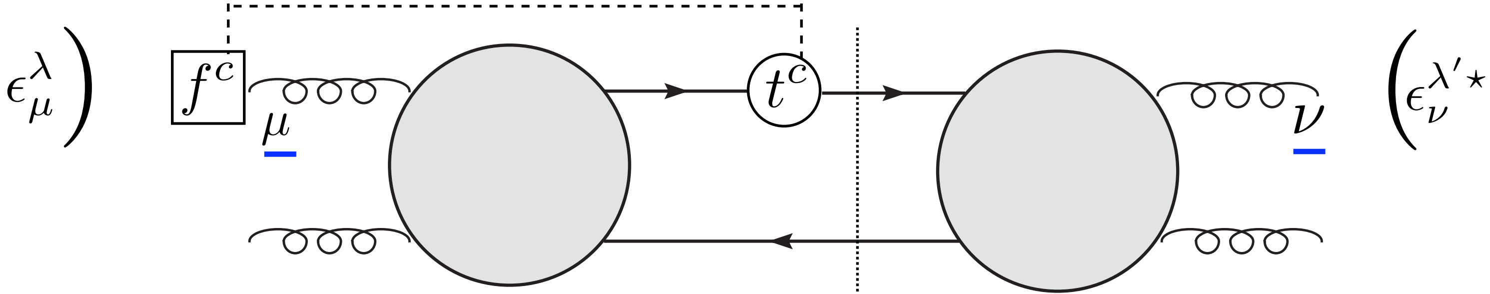


splitting function:

$$\mathrm{V}^{ai}_{k}(x,u)^{\underline{\mu\nu}} = 16\pi\alpha_s C_A\Big[-g^{\mu\nu}\Big(\frac{1}{1-x_{ik,a}+u_i}-1+x_{ik,a}(1-x_{ik,a})\Big)+\frac{1-x_{ik,a}}{x_{ik,a}}\frac{u_i(1-u_i)}{p_i\cdot p_k}\Big(\frac{p_i^\mu}{u_i}-\frac{p_k^\mu}{1-u_i}\Big)\Big(\frac{p_i^\nu}{u_i}-\frac{p_k^\nu}{1-u_i}\Big)\Big]$$

×

CLBS (Color linked Born squared)



(Amputate polarization vector)

□ Mathematica code

Input[$gg \to u\bar{u}g$] (Process at NLO)

↓

Creation of dipole terms

Write down all necessary D_i
Calculate D_i except for CLBS

↓

Fortran expression

Dipole 1

↓

Dipole 2

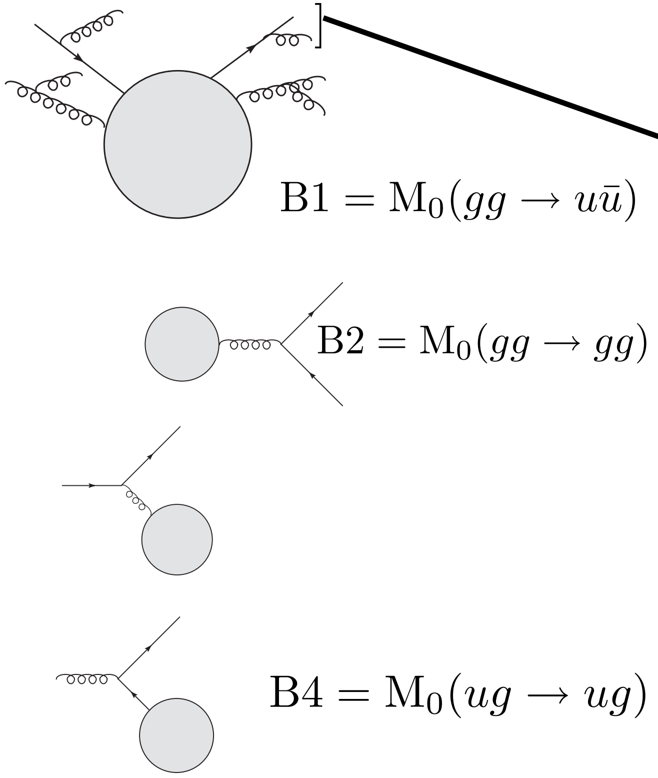
↓

Dipole 3

↓

Dipole 4

↓



$\mathrm{B1} = \mathrm{M}_0(gg \to u\bar{u})$

$\mathrm{B2} = \mathrm{M}_0(gg \to gg)$

$\mathrm{B4} = \mathrm{M}_0(ug \to ug)$

→ time

Number counting

Code : Search for all allowed combination by 3 do-loops with conditions

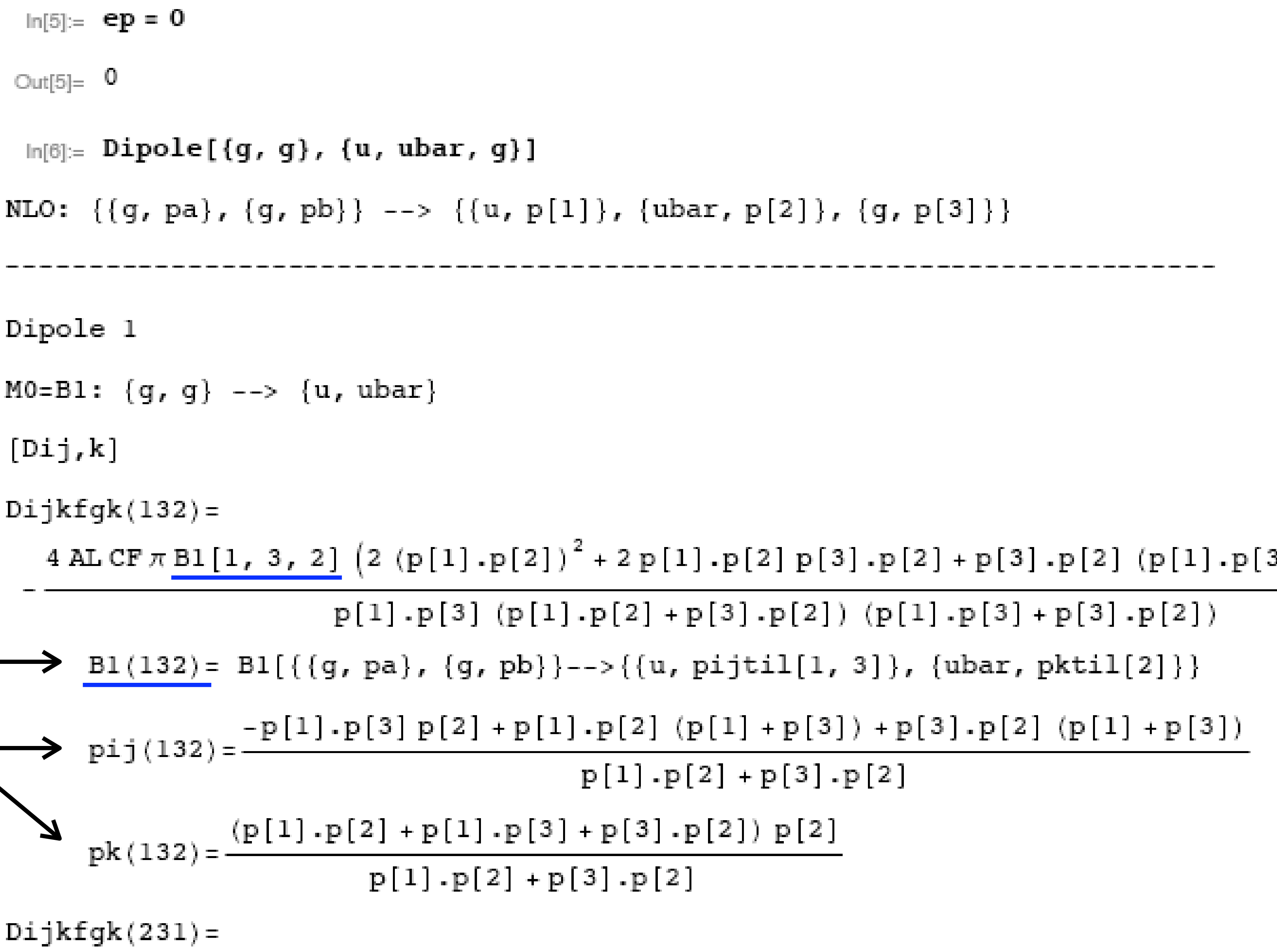
$$\mathrm{D}_{ij,k} = (\text{quark}, \text{gluon}, k)$$

1. Do-loop → gluon : $j = 1 \to n_g$
2. Do-loop → quark : $i = 1 \to n_{\text{final}}$
3. Do-loop → quark or gluon : $k = 1 \to n_{\text{final}} \neq i, j$

⇒ $\mathrm{D}_{13,2}$ → Calculate → Output

(Allowed)

Execution



In[5]:= ep = 0

Out[5]= 0

Input → In[6]:= Dipole[{g, g}, {u, ubar, g}]

Momentum assign → NLO: {{g, pa}, {g, pb}} --> {{u, p[1]}, {ubar, p[2]}, {g, p[3]}}

Diagonal → Dipole 1

Born → M0=B1: {g, g} --> {u, ubar}

$D_{ij,k}$ → [Dij,k]

$(ij, k) = (\text{quark}, \text{gluon}, k)$ → Dijkfgk(132)=

$$-\frac{4\,\text{AL}\,\text{CF}\,\pi\,\underline{\text{B1}[1,3,2]}\left(2\,(p[1].p[2])^2+2\,p[1].p[2]\,p[3].p[2]+p[3].p[2]\,(p[1].p[3\right.}{p[1].p[3]\,(p[1].p[2]+p[3].p[2])\,(p[1].p[3]+p[3].p[2])}$$

CLBS → $\underline{\text{B1}(132)=}$ B1[{{g, pa}, {g, pb}}-->{{u, pijtil[1, 3]}, {ubar, pktil[2]}}

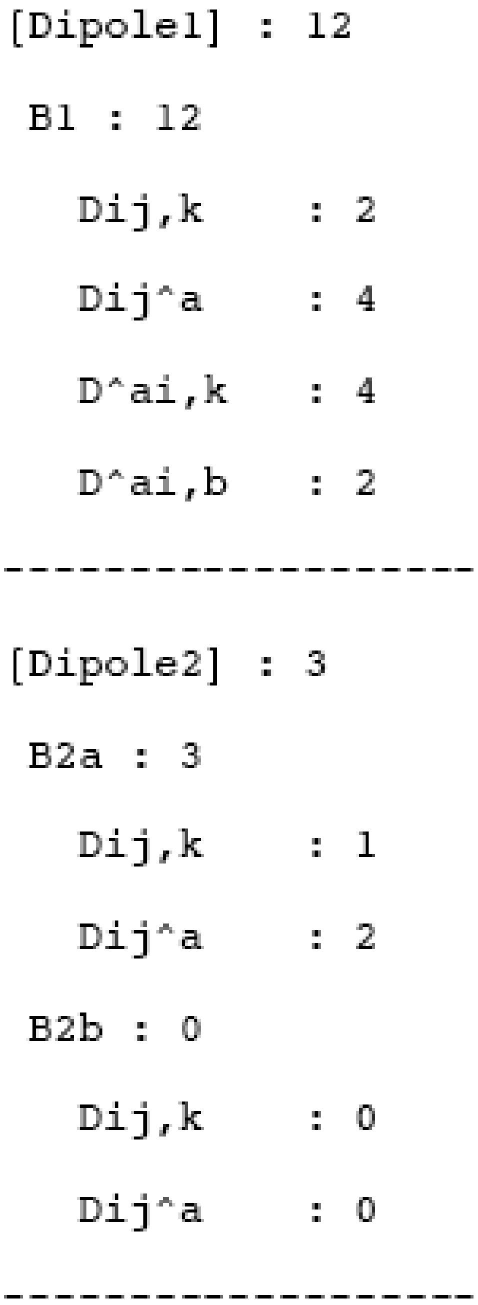
Reduced momenta →

$$\text{pij}(132)=\frac{-p[1].p[3]\,p[2]+p[1].p[2]\,(p[1]+p[3])+p[3].p[2]\,(p[1]+p[3])}{p[1].p[2]+p[3].p[2]}$$

$$\text{pk}(132)=\frac{(p[1].p[2]+p[1].p[3]+p[3].p[2])\,p[2]}{p[1].p[2]+p[3].p[2]}$$

Dijkfgk(231)=

Number of created Dipoles is shown
at the end :



```
[Dipole1] : 12

 B1 : 12

   Dij,k     : 2

   Dij^a     : 4

   D^ai,k    : 4

   D^ai,b    : 2

-------------------

[Dipole2] : 3

 B2a : 3

   Dij,k     : 1

   Dij^a     : 2

 B2b : 0

   Dij,k     : 0

   Dij^a     : 0

-------------------
```

$u - \bar{u}$ splitting → B2a

$d - \bar{d}$ splitting → B2b

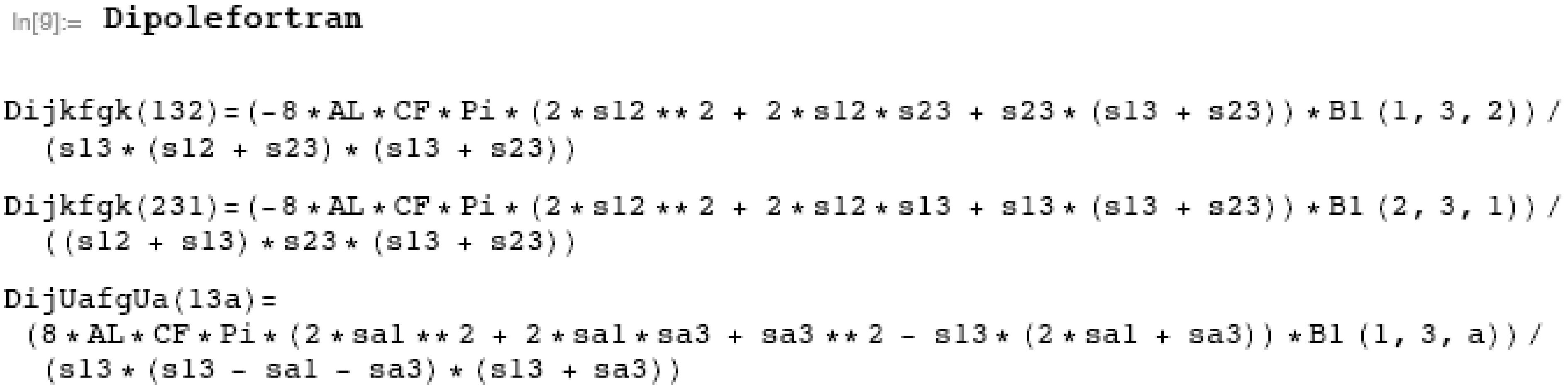
We can use these outputs for checking at this level before full calculation

Fortran expression

The outputs are converted into FORTRAN expression for the further calculations

$$\left[\begin{array}{ll}\text{Inner products:} & \mathrm{p}[i]\cdot\mathrm{p}[j] \;\to\; \dfrac{\mathrm{s}_{ij}}{2} \\ \text{Use command:} & \text{FortranForm[expression]}\end{array}\right.$$

Outputs



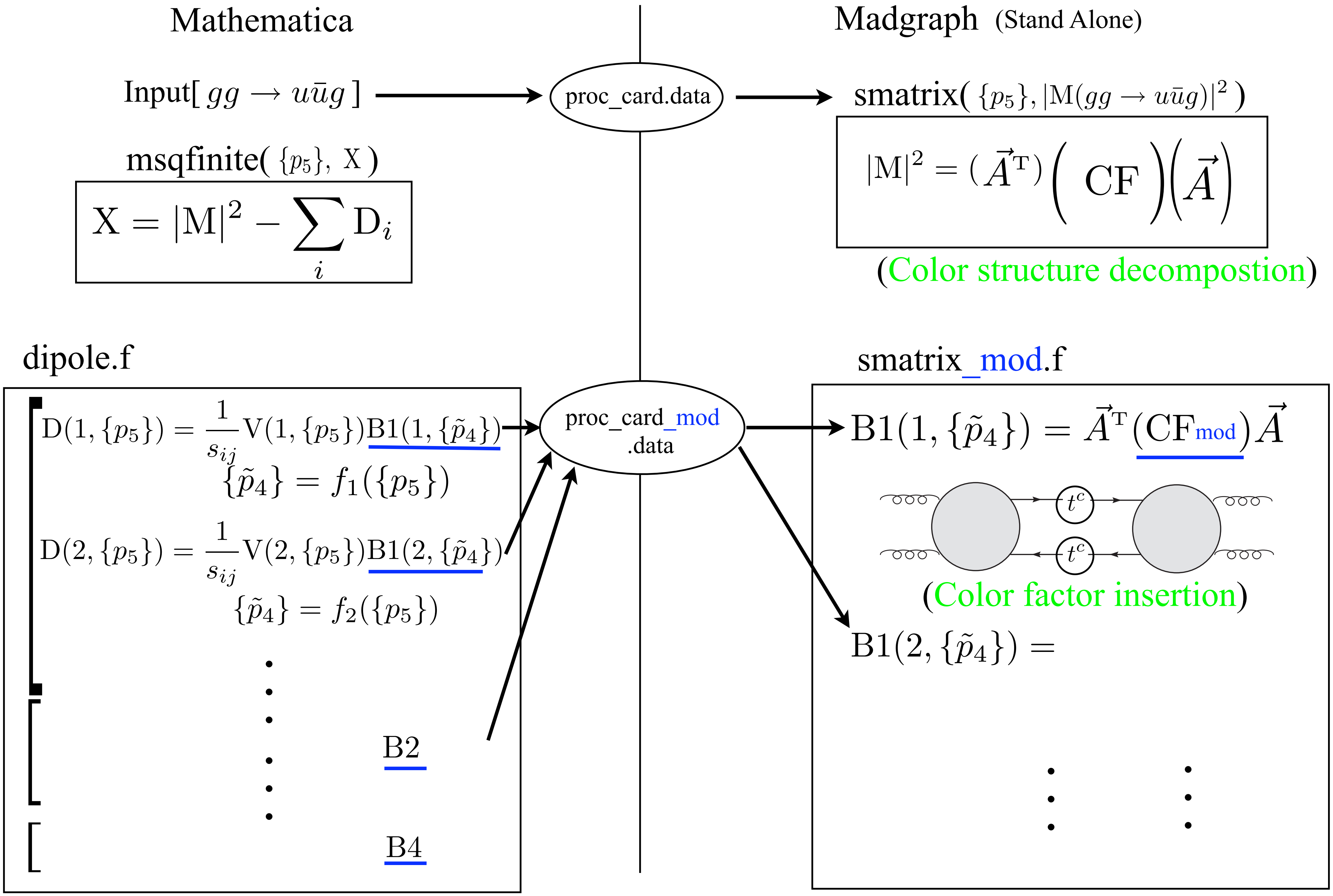
```
In[9]:= Dipolefortran

Dijkfgk(132)=(-8*AL*CF*Pi*(2*s12**2 + 2*s12*s23 + s23*(s13 + s23))*B1(1, 3, 2))/
   (s13*(s12 + s23)*(s13 + s23))

Dijkfgk(231)=(-8*AL*CF*Pi*(2*s12**2 + 2*s12*s13 + s13*(s13 + s23))*B1(2, 3, 1))/
   ((s12 + s13)*s23*(s13 + s23))

DijUafgUa(13a)=
  (8*AL*CF*Pi*(2*sa1**2 + 2*sa1*sa3 + sa3**2 - s13*(2*sa1 + sa3))*B1(1, 3, a))/
   (s13*(s13 - sa1 - sa3)*(s13 + sa3))
```

☐ Whole structure to calculate $|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$



Mathematica

Madgraph (Stand Alone)

Input[$gg \to u\bar{u}g$]

proc_card.data

smatrix($\{p_5\}, |\mathrm{M}(gg \to u\bar{u}g)|^2$)

$$|\mathrm{M}|^2 = (\vec{A}^{\mathrm{T}})\Big(\ \mathrm{CF}\ \Big)\Big(\vec{A}\Big)$$

(Color structure decompostion)

msqfinite($\{p_5\}$, X)

$$\mathrm{X} = |\mathrm{M}|^2 - \sum_i \mathrm{D}_i$$

dipole.f

$$\mathrm{D}(1, \{p_5\}) = \frac{1}{s_{ij}} \mathrm{V}(1, \{p_5\}) \underline{\mathrm{B1}(1, \{\tilde{p}_4\})}$$

$$\{\tilde{p}_4\} = f_1(\{p_5\})$$

$$\mathrm{D}(2, \{p_5\}) = \frac{1}{s_{ij}} \mathrm{V}(2, \{p_5\}) \underline{\mathrm{B1}(2, \{\tilde{p}_4\})}$$

$$\{\tilde{p}_4\} = f_2(\{p_5\})$$

⋮

⋮ B2

⋮ B4

proc_card_mod.data

smatrix_mod.f

$$\mathrm{B1}(1, \{\tilde{p}_4\}) = \vec{A}^{\mathrm{T}} \underline{(\mathrm{CF}_{\mathrm{mod}})} \vec{A}$$

(Color factor insertion)

$$\mathrm{B1}(2, \{\tilde{p}_4\}) =$$

⋮ ⋮

- Emitter = gluon case

$$\mathrm{V}_{\mu\nu}(\mathrm{CLBS})^{\underline{\mu\nu}} = \mathrm{V}_{\lambda\lambda'}(\mathrm{CLBS})_{\underline{\lambda\lambda'}}$$

Lorentz indices

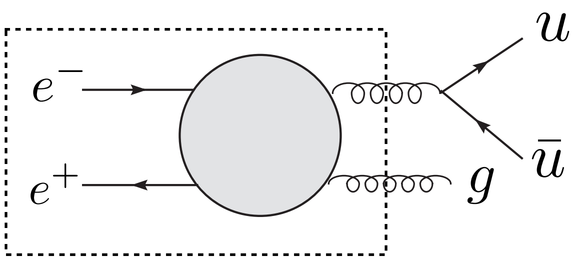
circular polarozation indices

Helicity amplitude

It can be calculated by Madgraph routine

- Discard of non-existing Born process

NLO: $e^+e^- \to u\bar{u}g$



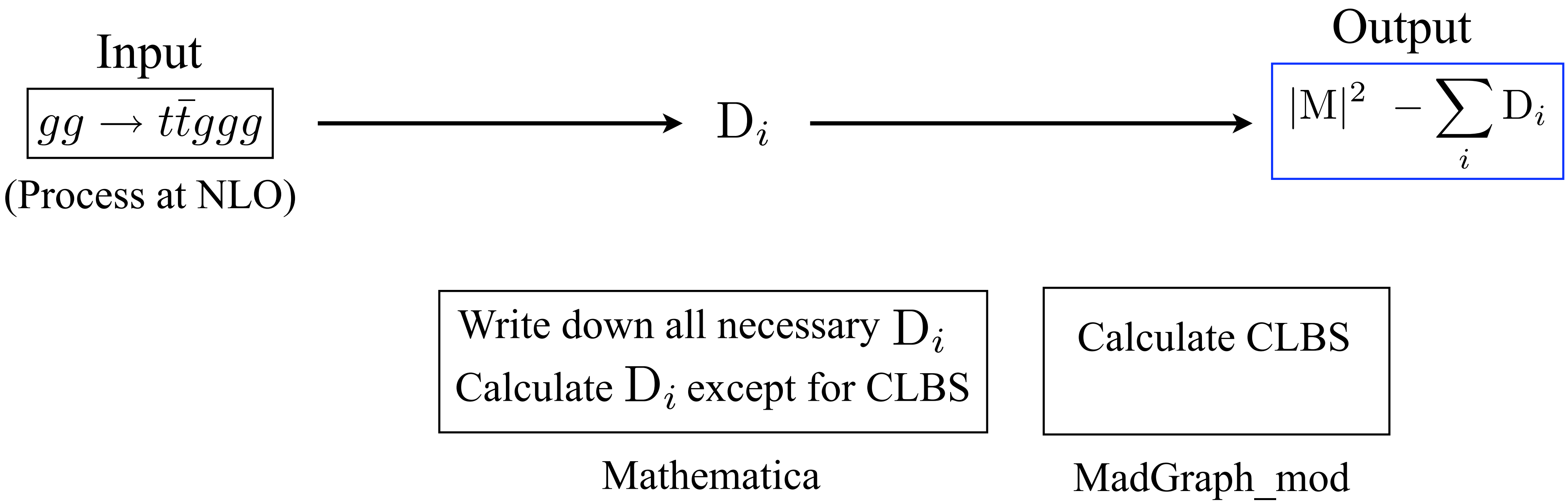
We can ask Madgraph to check whether each reduced Born exist or not

This reduced Born does not exist

Discard it

3. Summary

Construction of a code to calculate QCD NLO corrections by dipole subtraction in automatic way



Input

$gg \to t\bar{t}ggg$

(Process at NLO)

D_i

Output

$|\mathrm{M}|^2 - \sum_i \mathrm{D}_i$

Write down all necessary D_i
Calculate D_i except for CLBS

Mathematica

Calculate CLBS

MadGraph_mod

□ Plan

- Compeleted code will be published
- Automatization to parton and hadron level cross section