

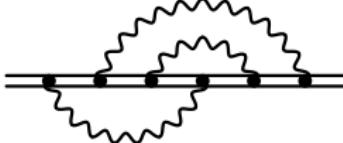
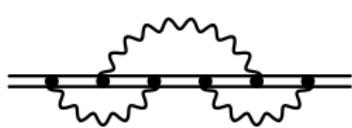
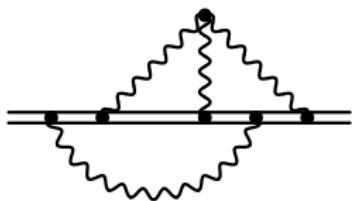
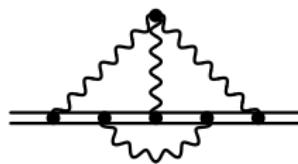
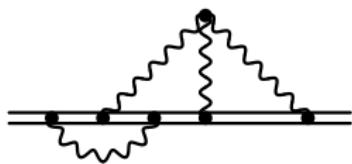
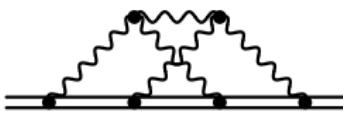
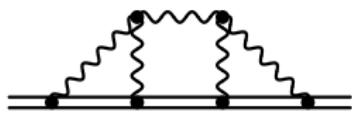
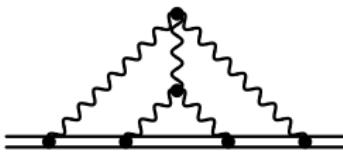
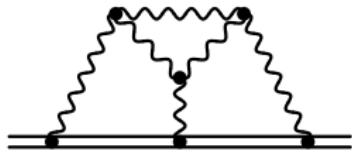
Three-loop results in HQET

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Institut für Theoretische Teilchenphysik
Universität Karlsruhe

Propagator diagrams



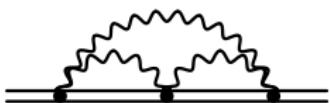
Grinder (REDUCE), AG (2000)

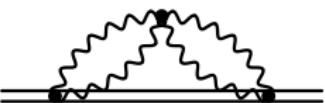
Master integrals

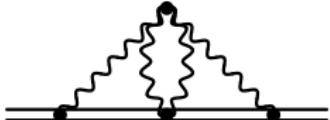

$$= I_1^3$$


$$= I_1 I_2$$

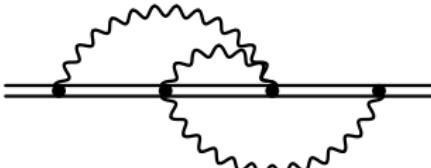

$$= I_3$$

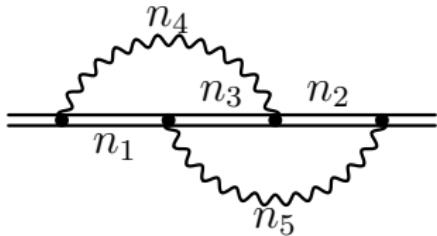

$$\sim I_3 \frac{I_2}{I_1^2}$$


$$\sim I_3 \frac{G_1^2}{G_2}$$


$$= G_1 I(1, 1, 1, 1, \varepsilon)$$

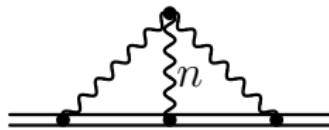

$$= G_1 I(1, 1, 1, 1, \varepsilon)$$


$$= I_1 J(1, 1, -1 + 2\varepsilon, 1, 1)$$



$$\begin{aligned}
 J(n_1, n_2, n_3, n_4, n_5) &= \Gamma(n_1 + n_2 + n_3 + 2(n_4 + n_5 - d)) \\
 &\times \frac{\Gamma(n_1 + n_3 + 2n_4 - d)\Gamma(d/2 - n_4)\Gamma(d/2 - n_5)}{\Gamma(n_1 + n_2 + n_3 + 2n_4 - d)\Gamma(n_4)\Gamma(n_5)\Gamma(n_1 + n_3)} \\
 &\times {}_3F_2 \left(\begin{array}{c} n_1, d - 2n_5, n_1 + n_3 + 2n_4 - d \\ n_1 + n_3, n_1 + n_2 + n_3 + 2n_4 - d \end{array} \middle| 1 \right)
 \end{aligned}$$

AG (2000)



$$\begin{aligned} I(1, 1, 1, 1, n) &= \frac{\Gamma\left(\frac{d}{2} - 1\right) \Gamma\left(\frac{d}{2} - n - 1\right)}{\Gamma(d - 2)} \\ &\times \left[2 \frac{\Gamma(2n - d + 3) \Gamma(2n - 2d + 6)}{(n - d + 3) \Gamma(3n - 2d + 6)} \right. \\ &\quad \times {}_3F_2 \left(\begin{array}{c} n - d + 3, n - d + 3, 2n - 2d + 6 \\ n - d + 4, 3n - 2d + 6 \end{array} \middle| 1 \right) \\ &\quad \left. - \Gamma(d - n - 2) \Gamma^2(n - d + 3) \right] \end{aligned}$$

M. Beneke, V. Braun (1994)

Inversion

$$\begin{array}{c} \text{Diagram:} \\ \text{A horizontal line with vertices } n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8. \text{ A wavy line connects } n_3 \text{ to } n_7, n_7 \text{ to } n_8, \text{ and } n_8 \text{ to } n_4. \\ = \\ \text{Diagram:} \\ \text{The same horizontal line with vertices } n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8. \text{ A wavy line connects } d - n_1 - n_3 - n_5 - n_7 \text{ to } n_7, n_7 \text{ to } n_8, \text{ and } n_8 \text{ to } d - n_2 - n_4. \end{array}$$

$$\begin{array}{c} \text{Diagram:} \\ \text{A horizontal line with vertices } n_1, n_3, n_4, n_5, n_6, n_7, n_8. \text{ A wavy line connects } n_4 \text{ to } n_6, n_6 \text{ to } n_7, \text{ and } n_7 \text{ to } n_5. \\ = \\ \text{Diagram:} \\ \text{The same horizontal line with vertices } n_1, n_3, n_4, n_5, n_6, n_7, n_8. \text{ A wavy line connects } d - n_1 - n_4 - n_6 \text{ to } n_6, n_6 \text{ to } n_7, \text{ and } n_7 \text{ to } d - n_2 - n_5. \end{array}$$

Inversion

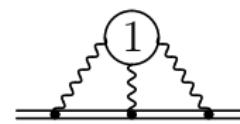
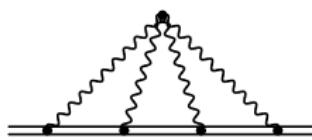
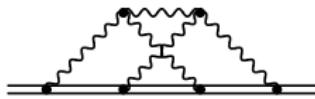
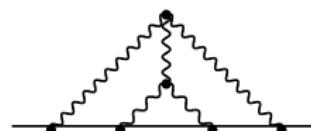
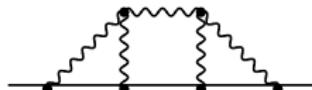
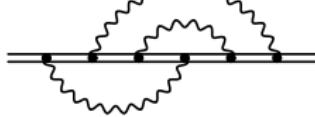
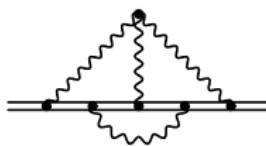
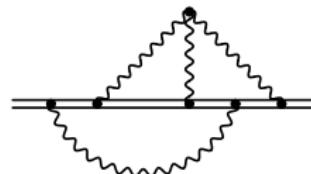
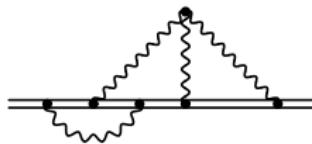
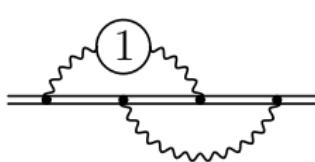
$$\begin{array}{c} \text{Diagram:} \\ \text{A wavy line with vertices } n_3, n_7, n_8, n_6, n_5, n_4 \text{ attached to a horizontal line with vertices } n_1, n_2. \\ \text{Diagram:} \\ \text{Two wavy lines with vertices } d-n_1-n_3-n_5-n_7, n_7, n_8, d-n_2-n_4-n_5-n_8 \text{ attached to a horizontal line with vertices } n_1, n_2. \end{array} =$$

$$\begin{array}{c} \text{Diagram:} \\ \text{A wavy line with vertices } n_4, n_6, n_7, n_8 \text{ attached to a horizontal line with vertices } n_1, n_3, n_2. \\ \text{Diagram:} \\ \text{Two wavy lines with vertices } d-n_1-n_3-n_6-n_7-n_8, d-n_2-n_5-n_6-n_7 \text{ attached to a horizontal line with vertices } n_1, n_3, n_2. \end{array} =$$

$$\begin{array}{c} \text{Diagram:} \\ \text{A wavy line with vertices } n_1, n_2, n_3, n_4 \text{ attached to a horizontal line.} \\ \text{Diagram:} \\ \text{A wavy line with vertices } n_1, n_2, n_3, n_4, n_5 \text{ attached to a horizontal line.} \end{array} = -5\zeta_5 + 12\zeta_2\zeta_3$$

A. Czarnecki, K. Melnikov (2002)

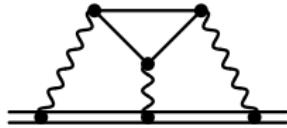
Heavy-quark propagator



Heavy-light current (K. Chetyrkin, AG 2003)

$$\begin{aligned}\tilde{\gamma}_j = & -3C_F \frac{\alpha_s}{4\pi} \\ & + C_F \left[C_F \left(-16\zeta_2 + \frac{5}{2} \right) + C_A \left(4\zeta_2 - \frac{49}{6} \right) + \frac{10}{3} T_F n_l \right] \left(\frac{\alpha_s}{4\pi} \right)^2 \\ & + C_F \left[C_F^2 \left(-80\zeta_4 - 36\zeta_3 + 64\zeta_2 - \frac{37}{2} \right) \right. \\ & \quad \left. + C_F C_A \left(-16\zeta_4 + \frac{142}{3}\zeta_3 - \frac{1184}{9}\zeta_2 - \frac{655}{36} \right) \right. \\ & \quad \left. + C_A^2 \left(-24\zeta_4 - \frac{22}{3}\zeta_3 + \frac{260}{9}\zeta_2 + \frac{1451}{108} \right) \right. \\ & \quad \left. + C_F T_F n_l \left(-\frac{176}{3}\zeta_3 + \frac{448}{9}\zeta_2 + \frac{470}{9} \right) \right. \\ & \quad \left. + C_A T_F n_l \left(\frac{152}{3}\zeta_3 - \frac{112}{9}\zeta_2 - \frac{512}{27} \right) + \frac{140}{27} (T_F n_l)^2 \right] \left(\frac{\alpha_s}{4\pi} \right)^3 + \dots\end{aligned}$$

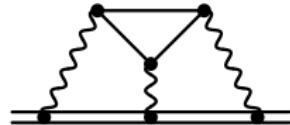
On-shell diagrams with mass 1



Gröbner bases

AG, A. Smirnov, V. Smirnov (2006)

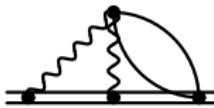
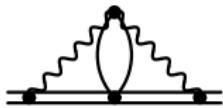
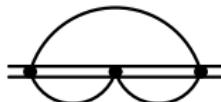
On-shell diagrams with mass 1



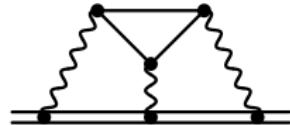
Gröbner bases

AG, A. Smirnov, V. Smirnov (2006)

Apparently even



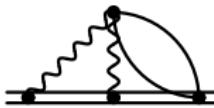
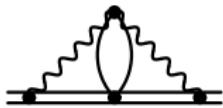
On-shell diagrams with mass 1



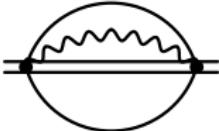
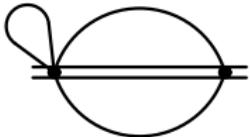
Gröbner bases

AG, A. Smirnov, V. Smirnov (2006)

Apparently even



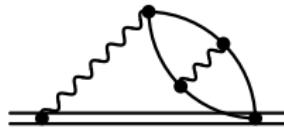
Apparently odd



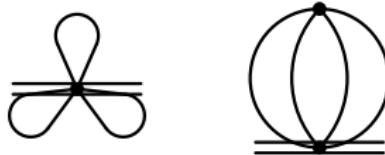
On-shell diagrams with mass 2



On-shell diagrams with mass 2



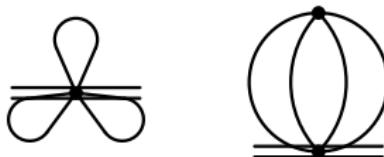
Apparently even



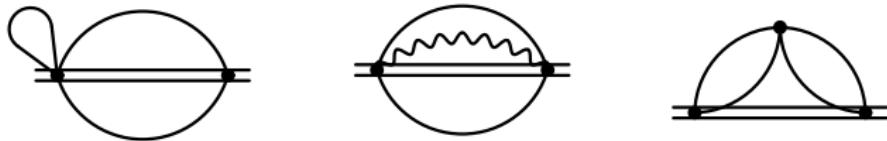
On-shell diagrams with mass 2

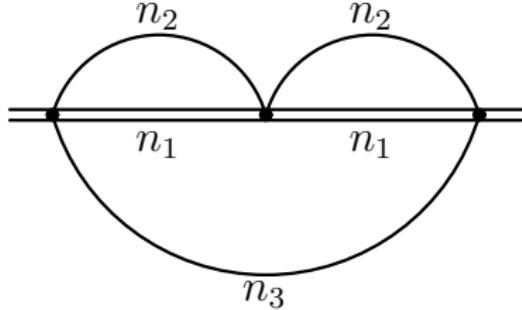


Apparently even



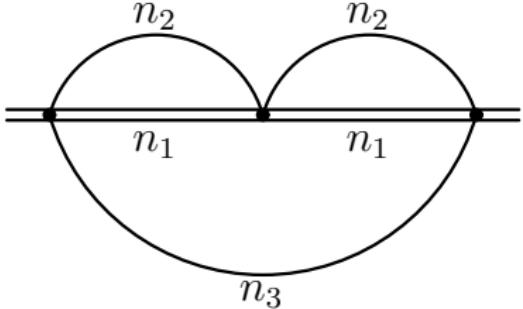
Apparently odd





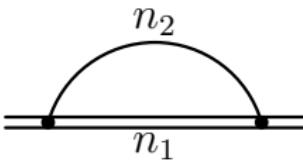
AG, T. Huber, D. Maître (2007)

$$I_{n_1 n_2 n_3} = \frac{\Gamma(n_3 - 3/2 + \varepsilon)}{\pi^{1/2} \Gamma(n_3)} \int_{-\infty}^{+\infty} I_{n_1 n_2}^2(ip_{E0}) (1 + p_{E0}^2)^{3/2 - n_3 - \varepsilon} dp_{E0}$$

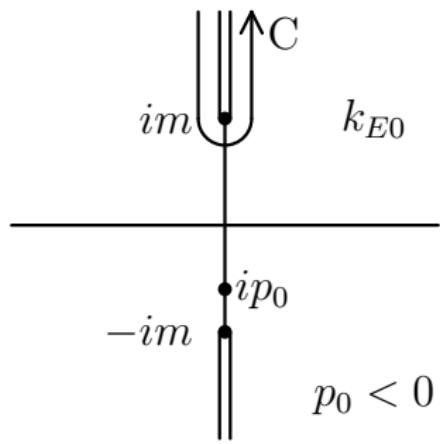


AG, T. Huber, D. Maître (2007)

$$I_{n_1 n_2 n_3} = \frac{\Gamma(n_3 - 3/2 + \varepsilon)}{\pi^{1/2} \Gamma(n_3)} \int_{-\infty}^{+\infty} I_{n_1 n_2}^2(ip_{E0}) (1 + p_{E0}^2)^{3/2 - n_3 - \varepsilon} dp_{E0}$$



$$\begin{aligned} I_{n_1 n_2}(p_0) &= \frac{1}{i\pi^{d/2}} \int \frac{dk_0 d^{d-1}\vec{k}}{[-2(k_0 + p_0) - i0]^{n_1} [m^2 - k^2 - i0]^{n_2}} \\ &= \frac{\Gamma(n_2 - (d-1)/2)}{\pi^{1/2} \Gamma(n_2)} \int_{-\infty}^{+\infty} dk_{E0} \frac{(k_{E0}^2 + m^2)^{(d-1)/2 - n_2}}{(-2p_0 - 2ik_{E0})^{n_1}} \end{aligned}$$



$$I_{n_1 n_2}(p_0) = 2 \frac{\Gamma(n_2 - (d-1)/2)}{\pi^{1/2} \Gamma(n_2)} \cos \left[\pi \left(\frac{d}{2} - n_2 \right) \right] \\ \times \int\limits_m^{\infty} dk \frac{(k^2 - m^2)^{(d-1)/2 - n_2}}{(2k - 2p_0)^{n_1}}$$

$$\begin{aligned}
I_{n_1 n_2}(p_0) &= \frac{\Gamma(n_1 + n_2 - 2 + \varepsilon)\Gamma(n_1 + 2n_2 - 4 + 2\varepsilon)}{\Gamma(n_2)\Gamma(2(n_1 + n_2 - 2 + \varepsilon))} \\
&\quad \times {}_2F_1 \left(\begin{array}{c} n_1, n_1 + 2n_2 - 4 + 2\varepsilon \\ n_1 + n_2 - \frac{3}{2} + \varepsilon \end{array} \middle| \frac{1}{2} \left(1 + \frac{p_0}{m} \right) \right) m^{d-n_1-2n_2} \\
&= \frac{\Gamma(n_1 + n_2 - 2 + \varepsilon)\Gamma(n_1 + 2n_2 - 4 + 2\varepsilon)}{\Gamma(n_2)\Gamma(2(n_1 + n_2 - 2 + \varepsilon))} \\
&\quad \times {}_2F_1 \left(\begin{array}{c} \frac{1}{2}n_1, \frac{1}{2}n_1 + n_2 - 2 + \varepsilon \\ n_1 + n_2 - \frac{3}{2} + \varepsilon \end{array} \middle| 1 - \frac{p_0^2}{m^2} \right) m^{d-n_1-2n_2}
\end{aligned}$$

$$\begin{aligned}
I_{n_1 n_2}(p_0) &= \frac{\Gamma(n_1 + n_2 - 2 + \varepsilon)\Gamma(n_1 + 2n_2 - 4 + 2\varepsilon)}{\Gamma(n_2)\Gamma(2(n_1 + n_2 - 2 + \varepsilon))} \\
&\quad \times {}_2F_1 \left(\begin{array}{c} n_1, n_1 + 2n_2 - 4 + 2\varepsilon \\ n_1 + n_2 - \frac{3}{2} + \varepsilon \end{array} \middle| \frac{1}{2} \left(1 + \frac{p_0}{m} \right) \right) m^{d-n_1-2n_2} \\
&= \frac{\Gamma(n_1 + n_2 - 2 + \varepsilon)\Gamma(n_1 + 2n_2 - 4 + 2\varepsilon)}{\Gamma(n_2)\Gamma(2(n_1 + n_2 - 2 + \varepsilon))} \\
&\quad \times {}_2F_1 \left(\begin{array}{c} \frac{1}{2}n_1, \frac{1}{2}n_1 + n_2 - 2 + \varepsilon \\ n_1 + n_2 - \frac{3}{2} + \varepsilon \end{array} \middle| 1 - \frac{p_0^2}{m^2} \right) m^{d-n_1-2n_2}
\end{aligned}$$

$m = 0$:

$$I_{n_1 n_2}(p_0) = \frac{\Gamma(n_1 + 2n_2 - d)\Gamma(d/2 - n_2)}{\Gamma(n_1)\Gamma(n_2)} (-2p_0)^{d-n_1-2n_2}$$

$$\begin{aligned}
& \frac{I_{122}}{\Gamma^3(1+\varepsilon)} = -\frac{1}{2\varepsilon^2} \left[\frac{1}{1+2\varepsilon} {}_4F_3 \left(\begin{array}{c} 1, \frac{1}{2}-\varepsilon, 1+\varepsilon, -2\varepsilon \\ \frac{3}{2}+\varepsilon, 1-\varepsilon, 1-2\varepsilon \end{array} \middle| 1 \right) \right. \\
& - \frac{2}{1+4\varepsilon} \frac{\Gamma^2(1-\varepsilon)\Gamma^3(1+2\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)\Gamma(1+4\varepsilon)} {}_3F_2 \left(\begin{array}{c} \frac{1}{2}, 1+2\varepsilon, -\varepsilon \\ \frac{3}{2}+2\varepsilon, 1-\varepsilon \end{array} \middle| 1 \right) \\
& \left. + \frac{1}{1+6\varepsilon} \frac{\Gamma^2(1-\varepsilon)\Gamma^4(1+2\varepsilon)\Gamma(1-2\varepsilon)\Gamma^2(1+3\varepsilon)}{\Gamma^4(1+\varepsilon)\Gamma(1+4\varepsilon)\Gamma(1-4\varepsilon)\Gamma(1+6\varepsilon)} \right]
\end{aligned}$$

$$\begin{aligned}
\frac{I_{122}}{\Gamma^3(1+\varepsilon)} &= -\frac{1}{2\varepsilon^2} \left[\frac{1}{1+2\varepsilon} {}_4F_3 \left(\begin{array}{c} 1, \frac{1}{2}-\varepsilon, 1+\varepsilon, -2\varepsilon \\ \frac{3}{2}+\varepsilon, 1-\varepsilon, 1-2\varepsilon \end{array} \middle| 1 \right) \right. \\
&- \frac{2}{1+4\varepsilon} \frac{\Gamma^2(1-\varepsilon)\Gamma^3(1+2\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)\Gamma(1+4\varepsilon)} {}_3F_2 \left(\begin{array}{c} \frac{1}{2}, 1+2\varepsilon, -\varepsilon \\ \frac{3}{2}+2\varepsilon, 1-\varepsilon \end{array} \middle| 1 \right) \\
&\left. + \frac{1}{1+6\varepsilon} \frac{\Gamma^2(1-\varepsilon)\Gamma^4(1+2\varepsilon)\Gamma(1-2\varepsilon)\Gamma^2(1+3\varepsilon)}{\Gamma^4(1+\varepsilon)\Gamma(1+4\varepsilon)\Gamma(1-4\varepsilon)\Gamma(1+6\varepsilon)} \right]
\end{aligned}$$

Expansion up to ε^7 agrees with

$$\frac{I_{122}}{\Gamma^3(1+\varepsilon)} = \frac{\pi^2}{3} \frac{\Gamma^3(1+2\varepsilon)\Gamma^2(1+3\varepsilon)}{\Gamma^6(1+\varepsilon)\Gamma(2+6\varepsilon)}$$

$$b(\varepsilon) = \frac{\Gamma(1-\varepsilon)\Gamma(1+2\varepsilon)}{\Gamma(1+\varepsilon)} \quad g_n(\varepsilon) = \frac{b^n(\varepsilon)}{b(n\varepsilon)(1+2n\varepsilon)}$$

$$g_1(\varepsilon) {}_4F_3 \left(\begin{array}{c} 1, \frac{1}{2}-\varepsilon, 1+\varepsilon, -2\varepsilon \\ \frac{3}{2}+\varepsilon, 1-\varepsilon, 1-2\varepsilon \end{array} \middle| 1 \right)$$

$$- 2g_2(\varepsilon) {}_3F_2 \left(\begin{array}{c} \frac{1}{2}, 1+2\varepsilon, -\varepsilon \\ \frac{3}{2}+2\varepsilon, 1-\varepsilon \end{array} \middle| 1 \right) + g_3(\varepsilon) = 0$$

Mellin–Barnes

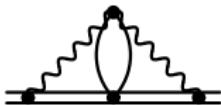
$$\begin{aligned} & \text{Diagram: A loop with two external lines. The top line is labeled } n_1 \text{ and the bottom line is labeled } n_2. \\ & = \frac{1}{i\pi^{d/2}} \int \frac{d^d k}{[m^2 - k^2 - i0]^{n_1} [m^2 - (k+p)^2 - i0]^{n_2}} \\ & = \frac{m^{d-2(n_1+n_2)}}{\Gamma(n_1)\Gamma(n_2)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(-z) \\ & \times \frac{\Gamma(n_1+z)\Gamma(n_2+z)\Gamma(n_1+n_2-d/2+z)}{\Gamma(n_1+n_2+2z)} m^{-2z} \text{Diagram: Two wavy lines meeting at a point, with a minus sign between them.} \end{aligned}$$



$$\begin{aligned} &= \frac{\Gamma^2(2\varepsilon)\Gamma(3\varepsilon - 1)}{4\Gamma(4\varepsilon)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \\ &\frac{\Gamma(1+z)\Gamma(1/2 + \varepsilon + z)\Gamma(1 + \varepsilon + z)\Gamma(-2\varepsilon - z)\Gamma(-\varepsilon - z)\Gamma(-z)}{\Gamma(3/2 + \varepsilon + z)\Gamma(1 - 2\varepsilon - z)} \\ &= -\Gamma^3(1 + \varepsilon) \left[\frac{\pi^2}{9\varepsilon^2} - \frac{6\zeta_3 - 5\pi^2}{9\varepsilon} + \frac{11}{270}\pi^4 - \frac{10}{3}\zeta_3 + \frac{19}{9}\pi^2 \right. \\ &\quad \left. + \left(-\frac{8}{3}\zeta_5 + \frac{8}{9}\pi^2\zeta_3 + \frac{11}{54}\pi^4 - \frac{38}{3}\zeta_3 + \frac{65}{9}\pi^2 \right) \varepsilon + \dots \right] \end{aligned}$$



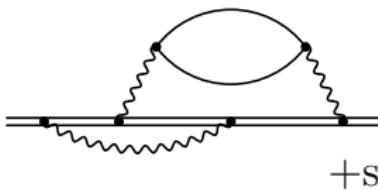
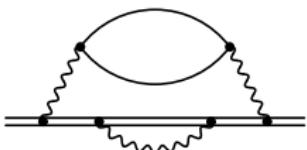
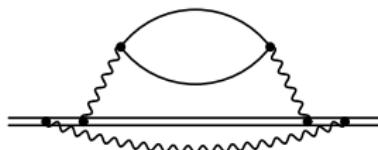
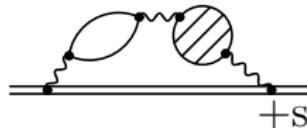
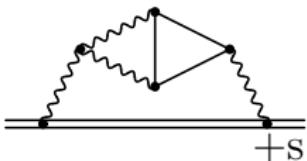
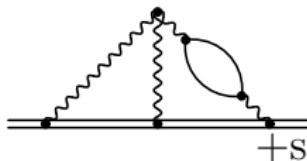
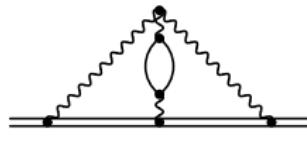
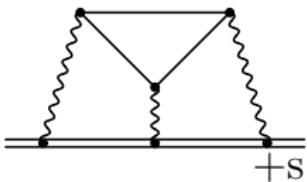
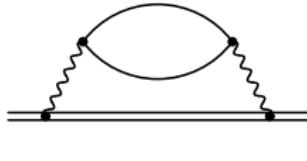
$$\begin{aligned}
 &= \frac{\Gamma^2(2\varepsilon)\Gamma(3\varepsilon - 1)}{4\Gamma(4\varepsilon)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \\
 &\frac{\Gamma(1+z)\Gamma(1/2 + \varepsilon + z)\Gamma(1 + \varepsilon + z)\Gamma(-2\varepsilon - z)\Gamma(-\varepsilon - z)\Gamma(-z)}{\Gamma(3/2 + \varepsilon + z)\Gamma(1 - 2\varepsilon - z)} \\
 &= -\Gamma^3(1 + \varepsilon) \left[\frac{\pi^2}{9\varepsilon^2} - \frac{6\zeta_3 - 5\pi^2}{9\varepsilon} + \frac{11}{270}\pi^4 - \frac{10}{3}\zeta_3 + \frac{19}{9}\pi^2 \right. \\
 &\quad \left. + \left(-\frac{8}{3}\zeta_5 + \frac{8}{9}\pi^2\zeta_3 + \frac{11}{54}\pi^4 - \frac{38}{3}\zeta_3 + \frac{65}{9}\pi^2 \right) \varepsilon + \dots \right]
 \end{aligned}$$



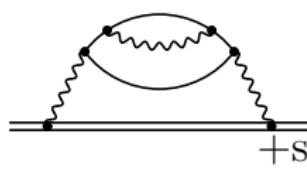
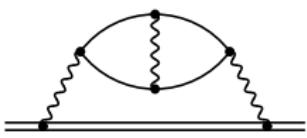
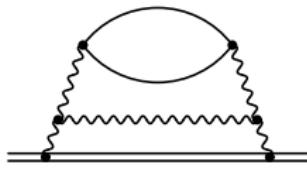
$$\begin{aligned}
 &= \frac{\Gamma(1/2 - \varepsilon)\Gamma(-\varepsilon)\Gamma^2(2\varepsilon)\Gamma(1 + \varepsilon)\Gamma(3\varepsilon - 1)}{4\Gamma(3/2 - \varepsilon)\Gamma(4\varepsilon)} \\
 &\times [\psi(1/2 - \varepsilon) + \psi(1 - \varepsilon) - 2\ln 2 + 2\gamma_E]
 \end{aligned}$$



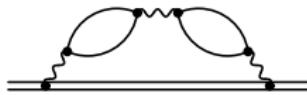
$$\begin{aligned}
 &= \frac{\pi^{3/2}}{4^\varepsilon \Gamma(3/2 - \varepsilon)} \\
 &\times \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \frac{\Gamma(1+z)\Gamma(3/2-\varepsilon+z)\Gamma(\varepsilon+z)}{\Gamma(3/2+z)\Gamma(\varepsilon-z)} \\
 &\quad \times \Gamma(-1/2+\varepsilon-z)\Gamma(-3/2+2\varepsilon-z)\Gamma(-z) \\
 &= -\frac{32}{3}\pi^2 + \dots
 \end{aligned}$$



+s



+s



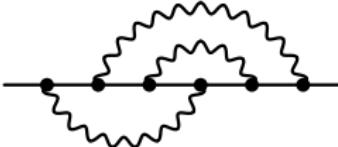
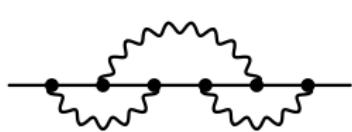
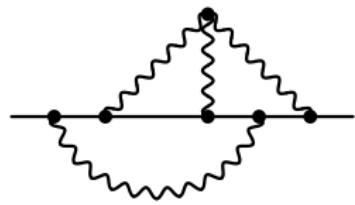
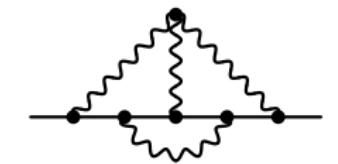
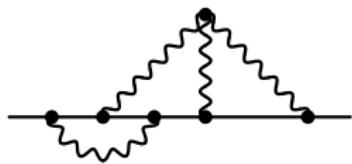
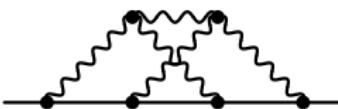
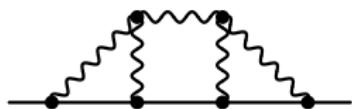
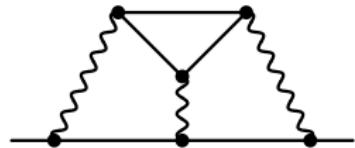
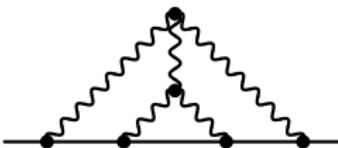
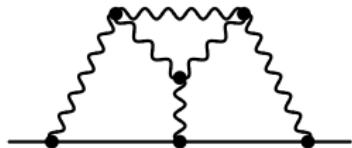
Decoupling for the heavy-light current

$$\begin{aligned}\tilde{j}(\mu_c) = & \tilde{j}'(\mu_c) \left\{ 1 + \frac{89}{36} C_F T_F \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \right. \\ & + C_F T_F \left[\left(8B_4 - \frac{86}{405}\pi^4 + \frac{1427}{27}\zeta_3 + \frac{1600}{243}\pi^2 - \frac{7219}{162} \right) C_F \right. \\ & - \left(4B_4 - \frac{43}{81}\pi^4 + \frac{1471}{54}\zeta_3 + \frac{400}{243}\pi^2 + \frac{3845}{486} \right) C_A \\ & \left. \left. - \frac{2}{9} \left(32\zeta_3 - \frac{1327}{27} \right) T_F n_l + \frac{1}{9} \left(112\zeta_3 - \frac{1685}{27} \right) T_F \right] \left(\frac{\alpha_s(\mu)}{4\pi} \right)^3 \right\}\end{aligned}$$

$$m_c(\mu_c) = \mu_c$$

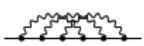
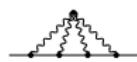
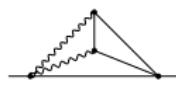
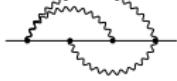
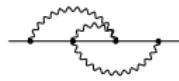
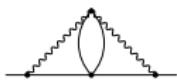
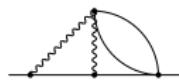
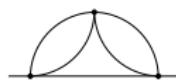
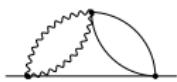
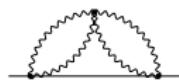
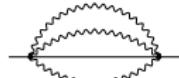
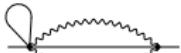
$$B_4 = 16 \operatorname{Li}_4 \left(\frac{1}{2} \right) + \frac{2}{3} \ln^2 2 \left(\ln^2 2 - \pi^2 \right) - \frac{13}{180} \pi^4$$

Massive on-shell diagrams



K. Melnikov, T. van Ritbergen (2000)

Master integrals

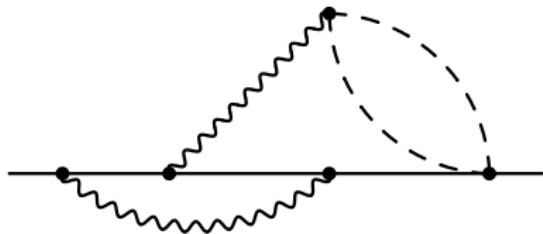
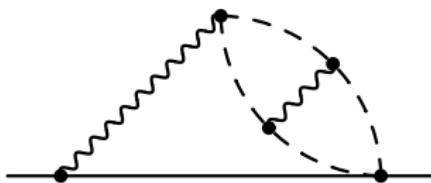
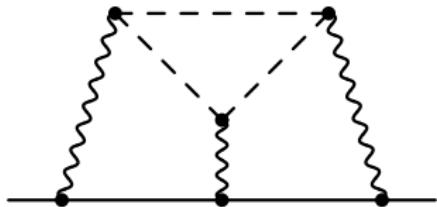


Chromomagnetic interaction

$$\begin{aligned}\gamma = & \frac{\alpha_s}{\pi} \frac{1}{2} C_A + \left(\frac{\alpha_s}{\pi} \right)^2 C_A \left(\frac{17}{36} C_A - \frac{13}{36} T_F n_l \right) \\ & + \left(\frac{\alpha_s}{\pi} \right)^3 \left\{ \left(\frac{1}{8} \zeta_3 + \frac{899}{1728} \right) C_A^3 + \frac{1}{2} \pi^2 \frac{d_F^{abcd} d_A^{abcd}}{C_F N_F} \right. \\ & - \left[\left(\frac{1}{2} \zeta_3 + \frac{65}{216} \right) C_A^2 - \left(\frac{1}{2} \zeta_3 - \frac{49}{96} \right) C_A C_F + \frac{1}{36} C_A T_F n_l \right] T_F n_l \\ & \left. - \frac{2}{3} \pi^2 \frac{d_F^{abcd} d_F^{abcd}}{C_F N_F} n_l \right\}\end{aligned}$$

AG, P. Marquard, J. Piclum, M. Steinhauser (2008)

Massive on-shell diagrams with 2 masses



Master integrals

