Towards the next-to-next-to-leading order evolution of polarised parton distributions

Andreas Vogt (University of Liverpool)

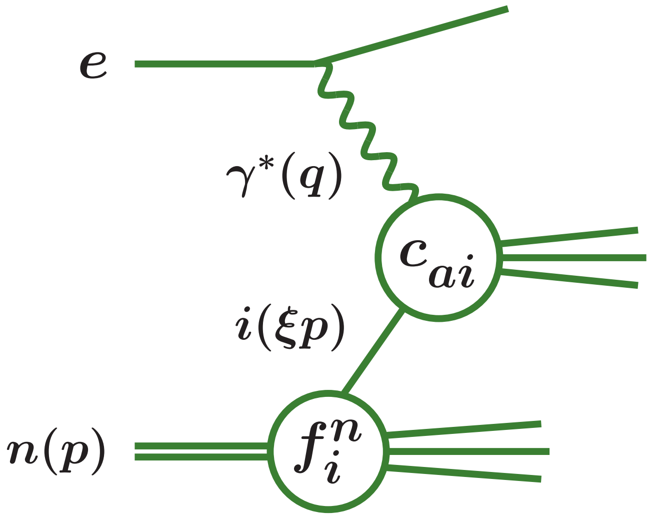
Collaboration with Sven Moch, Mikhail Rogal and Jos Vermaseren

- Introduction and previous second-order calculations
- A three-loop computation via forward Compton amplitudes
- Present results: two-loop checks, $\Delta P_{\rm qq}$ and $\Delta P_{\rm qg}$ to NNLO

Loops and Legs 2008, Sondershausen, 23-04-08

Hard processes in perturbative QCD (I)

Example: inclusive deep-inelastic electron-nucleon scattering (DIS)



Kinematic variables

$$Q^2 = -q^2$$

$$x = Q^2/(2P\cdot q)$$

Lowest order : $x = \xi$

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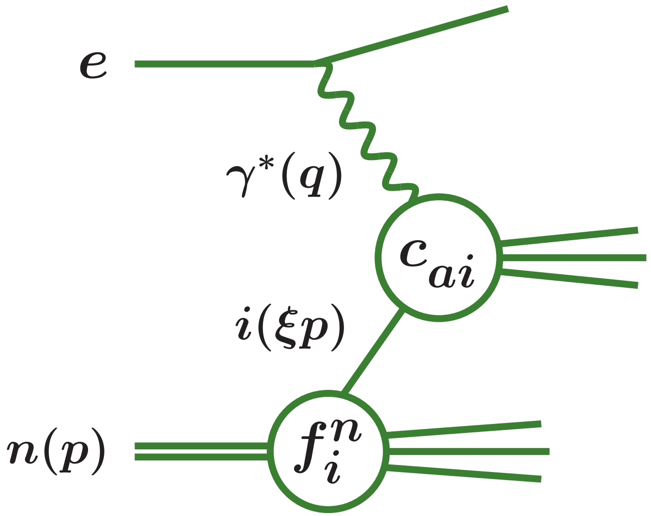

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Polarisation difference $\sigma_{\vec{e}\,\vec{n}} - \sigma_{\vec{e}\,\overleftarrow{n}}$ **: structure functions** $g_1^n, \ldots$

$$g_1^n(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{g_1,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) \Delta f_i^n(\xi, \mu^2)$$

Coefficient functions: renormalization/factorization scale $\mu = \mathcal{O}(Q)$

Hard processes in perturbative QCD (II)

Polarised parton densities $\Delta f_i = f_{i\rightarrow} - f_{i\leftarrow}$: evolution equations

$$\frac{d}{d\ln\mu^2}\Delta f_i(\xi,\mu^2) \;=\; \sum_k \left[\Delta P_{ik}(\alpha_s(\mu^2)) \otimes \Delta f_k(\mu^2)\right](\xi)$$

Initial conditions incalculable in pert. QCD. Lattice: low moments

$\Rightarrow$ **predictions: fit-analyses of reference processes, universality**

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General observable a: splitting functions P, coefficient functions c_a

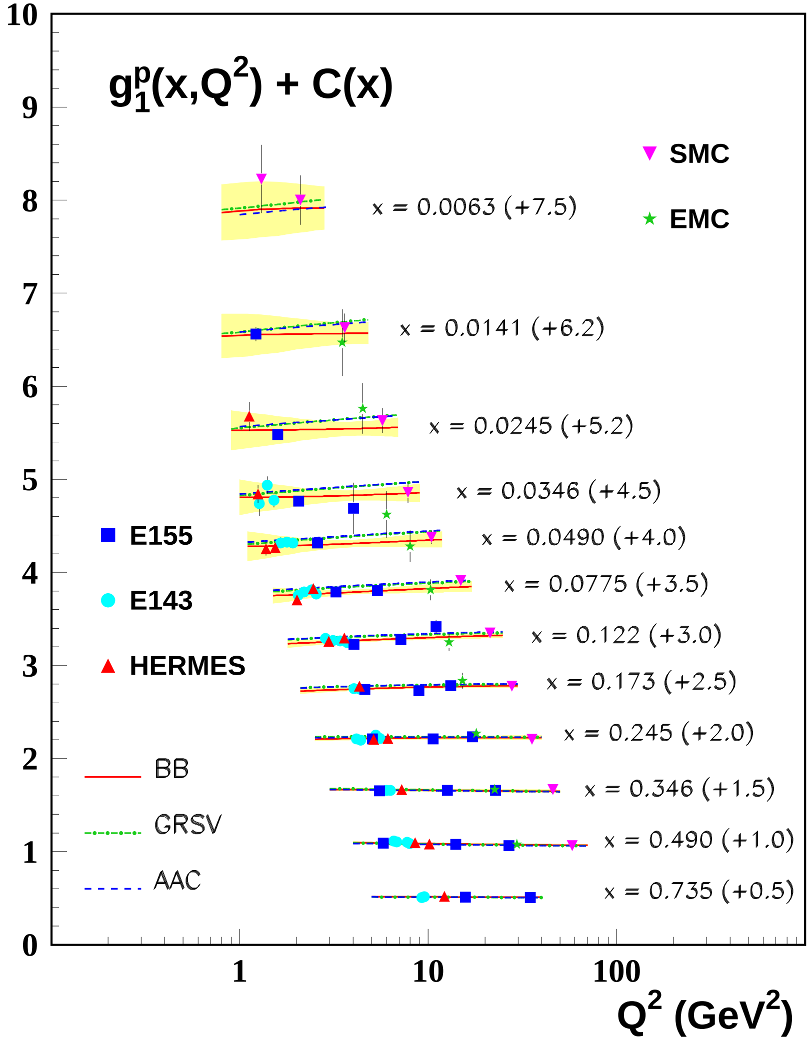
$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \underbrace{\alpha_s^{n_a}\left[c_a^{(0)} + \alpha_s c_a^{(1)}\right.} + \alpha_s^2 c_a^{(2)} + \ldots \left.\right] \end{aligned}$$

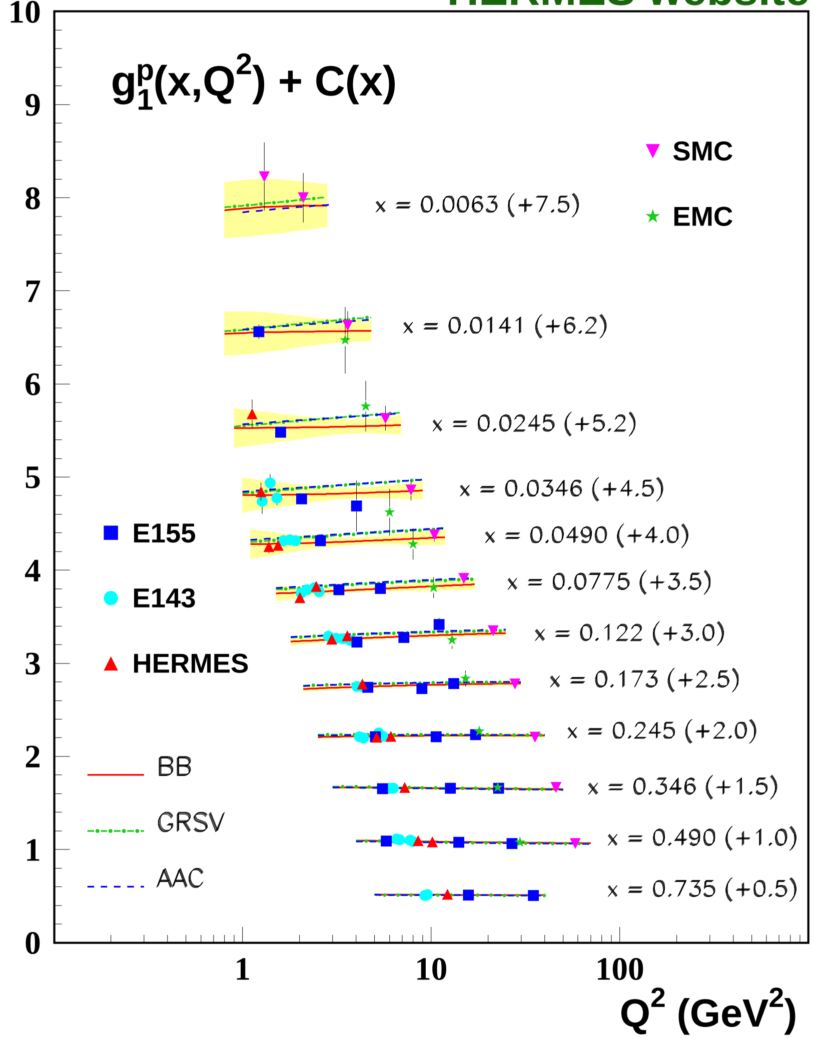
NLO: standard, but no serious error estimate, ...

Next-to-next-to-leading order (NNLO): $P^{(2)}$, $c_a^{(2)}$

2006 world data on proton and deuteron g_1

HERMES website





Rather low Q^2 $\Rightarrow$ rather large pQCD corrections $\Rightarrow$ precision needs NNLO

Two-loop calculations of polarised DIS

Helicity-dependent splitting functions ΔP and coefficient functions for g_1

- Structure function g_1 analogous to $F_{2,3,L}$: $\Delta P^{(1)}_{\mathrm{qq}}$, $\Delta P^{(1)}_{\mathrm{qg}}$, $c^{(2)}_{g_1,\,\mathrm{q/g}}$

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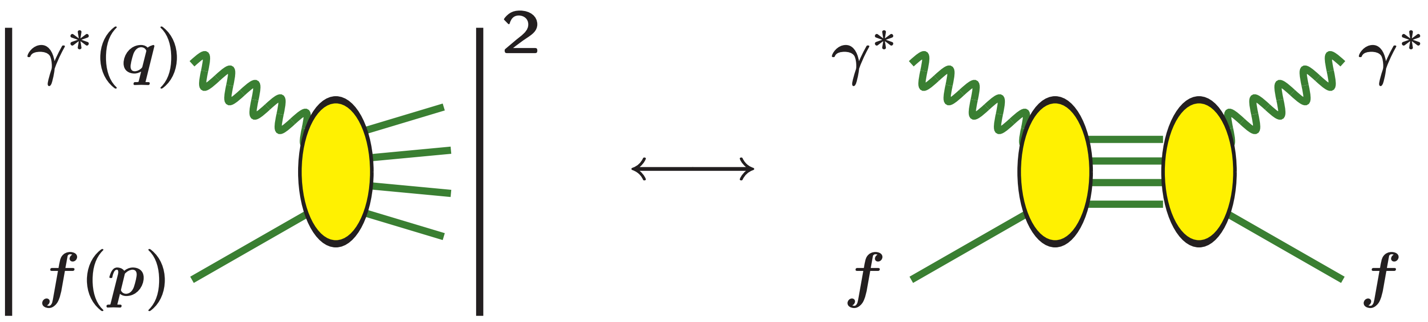
- All NLO splitting functions $\Delta P^{(1)}_{\rm ff'}$ using axial gauge   Vogelsang (95/6)

  γ_5: direct HVBM scheme, checks with Larin and reading point

Usually add. renormalization/factorization required, cf. Matiounine et al. (98)

Third order via forward Compton amplitudes

Optical theorem: $\gamma^* f$ **total cross sections** $\leftrightarrow$ **forward amplitudes**



Dispersion relation in x **: coefficient of** $(2p\cdot q)^N$ $\leftrightarrow$ N**-th moment**

$$A^N \;=\; \int_0^1 dx\, x^{N-1} A(x)$$

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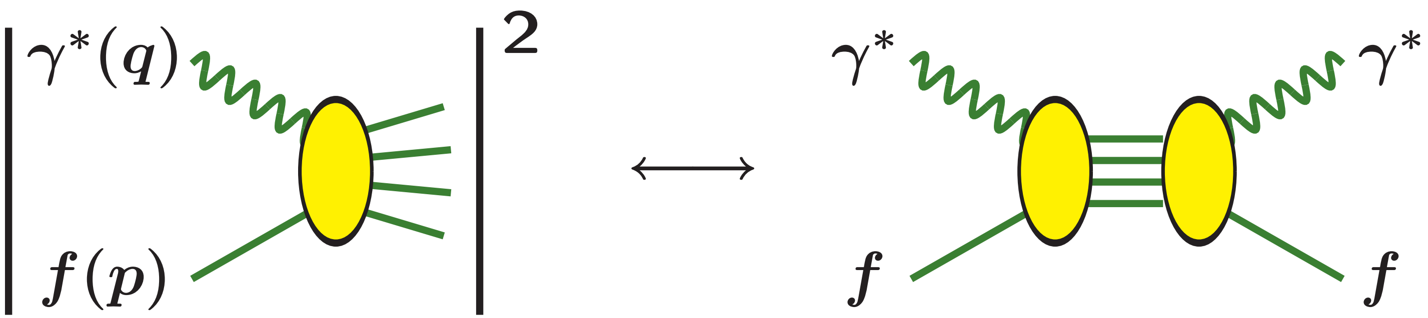

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Projection of partonic tensor on $\hat{g}_1$ in D dimensions ($D = 4 - 2\varepsilon$)

$$\hat{g}_1 \;=\; 2\,[(D-2)(D-3)(p{\cdot}q)]^{-1}\,\varepsilon_{\mu\nu pq}\,\widehat{W}_A^{\mu\nu}$$

$1/\varepsilon$ poles: spin splitting functions, ϵ^0 parts: g_1 coefficient functions

Treatment of the forward-Compton integrals

Combine identities: integration by parts, scaling, Passarino-Veltman type

$\Rightarrow$ Difference equations for $I(N)$ [recall: coefficient of $(2p\cdot q)^N$]

$$a_0(N)I(N) - \ldots - a_n(N)I(N-n) = I_0(N)$$

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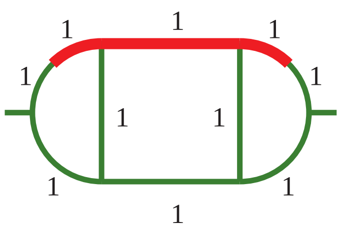
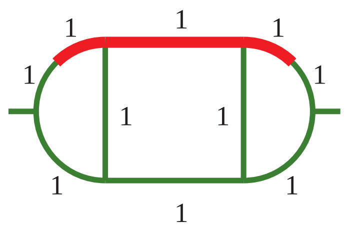
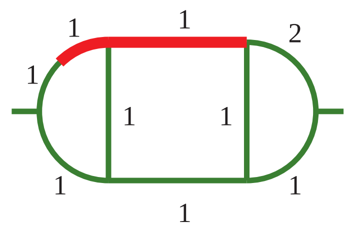
Simple scalar example [red line: flow of massless parton momentum p]



$$+ \quad \frac{N+3+3\epsilon}{N+2}\,\frac{2p\cdot q}{q^2}$$



$$= \quad \frac{2}{N+2}$$



Successive reduction to simpler (lower topologies or ‘less red’) integrals

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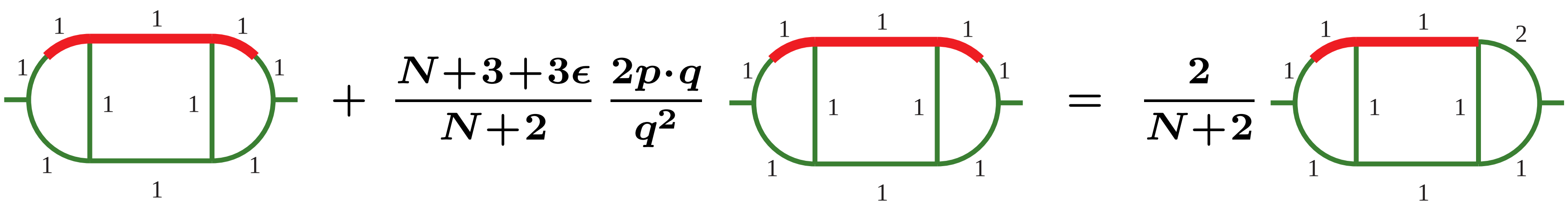


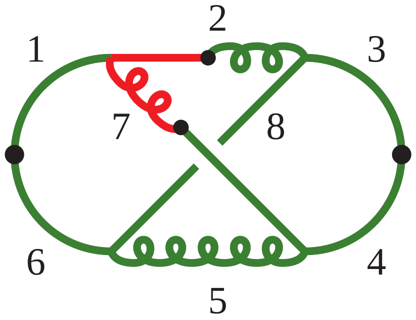
Successive reduction to simpler (lower topologies or ‘less red’) integrals

Essential: non-symbolic case for low N can be done via Mincer

Gorishny, Larin, Tkachov (84, 89); Larin, Tkachov, Vermaseren (91)

Check of new code and results at all stages: $I(N=2,3,4,\ldots) = ?$

Numerators for a non-planar NO_{27} diagram



$$\nu \quad \mu \sim \frac{\gamma(i_1, p_7, i_2, p_4, \mu, p_3, i_3, p_8, i_2, p_6, \nu, p_1, i_1, p+p_2, i_3)}{p_1^2 \ldots p_8^2 \, (p+p_2)^2 (p+p_7)^2}$$

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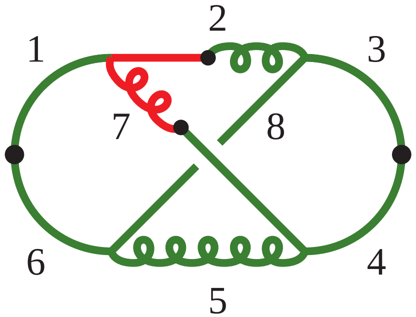

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Unpolarised splitting functions: $\cdot\, \gamma(p)\, \delta_{\mu\nu}$ **sufficient,**

no $(p_2^2)^{-1}$**, num.** $(p_2 \cdot p)^{k_2} (p_3 \cdot p)^{k_3} (p_2 \cdot q)^{k_9}$ **with** $k_2 + k_3 + k_9 \leq 3$

Coefficient functions for F_2/F_L**: need also** $\cdot\, \gamma(p)\, p_\mu p_\nu / (p \cdot q)^2$,

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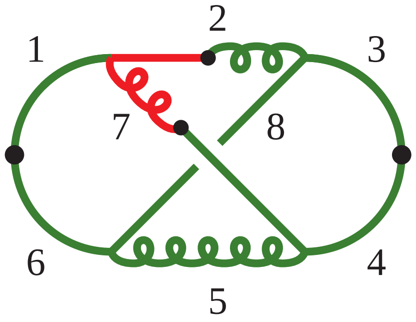

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Spin splitting functions, structure function g_1: $\cdot\, \gamma(p, 5)\, \varepsilon_{\mu\nu pq} / (p \cdot q)$,

also $(p_2^2)^{-1}$, numerators $(p_3 \cdot p)^{k_3} (p_2 \cdot q)^{k_9}$ with $k_3 + k_9 \leq 5$

Similar cases: NO_{12}, BE_{68}, LA_{17}, ... $\Rightarrow$ many new tensor integrals needed

Treatment of γ_5 and two-loop checks

Quark helicity-difference projector: $\not{p}\,\gamma_{5,L} = -\frac{1}{6}\,\varepsilon_{p\mu\nu\rho}\,\gamma_\mu\gamma_\nu\gamma_\rho$

Larin, Vermaseren (91), Larin (93)

$\Rightarrow$ perform scheme tranformation after factorization, as in Vogelsang (95/6)

$$g_1 = c_{g_1,L}\,\Delta f_L = c_{g_1,L}\,Z^{-1} Z\,\Delta f_L = c_{g_1,\overline{\mathrm{MS}}}\,\Delta f_{\overline{\mathrm{MS}}}$$

with $Z_{ij} = 1 + a_{\mathrm{s}}\,z^{(1)}_{qq}\,\delta_{qq} + a_{\mathrm{s}}^2\big(z^{(2)}_{qq,\mathrm{ns}} + z^{(2)}_{qq,\mathrm{ps}}\big)\delta_{qq}$ to NNLO

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Transformation of the parton $\rightarrow$ quark splitting functions

$$\delta[\Delta P^{(1)}_{qq}] \;=\; -\beta_0 z^{(1)}_{qq} \quad , \quad \delta[\Delta P^{(2)}_{qq}] \;=\; \beta_0[(z^{(1)}_{qq})^{\otimes 2} - 2z^{(2)}_{qq}] - \beta_1 z^{(1)}_{qq}$$

$$\delta[\Delta P^{(1)}_{qg}] \;=\; z^{(1)}_{qq} \otimes \Delta P^{(0)}_{qg}\,, \quad \delta[\Delta P^{(2)}_{qg}] \;=\; z^{(2)}_{qq} \otimes \Delta P^{(0)}_{qg} + z^{(1)}_{qq} \otimes \Delta P^{(1)}_{qg,L}$$

Non-singlet: $c_{g_1} \leftrightarrow c_{F_3}$ in νn DIS. Pure singlet $z^{(2)}_{qq,\rm ps}$: Matiounine et al. (98)

After transformation: agree with previous second-order results listed above

Checks and features of the three-loop results

- ε^{-3}, ε^{-2} terms in mass factorization formulae: check of also $\Delta P_{\mathrm{gf}}^{(1)}(x)$

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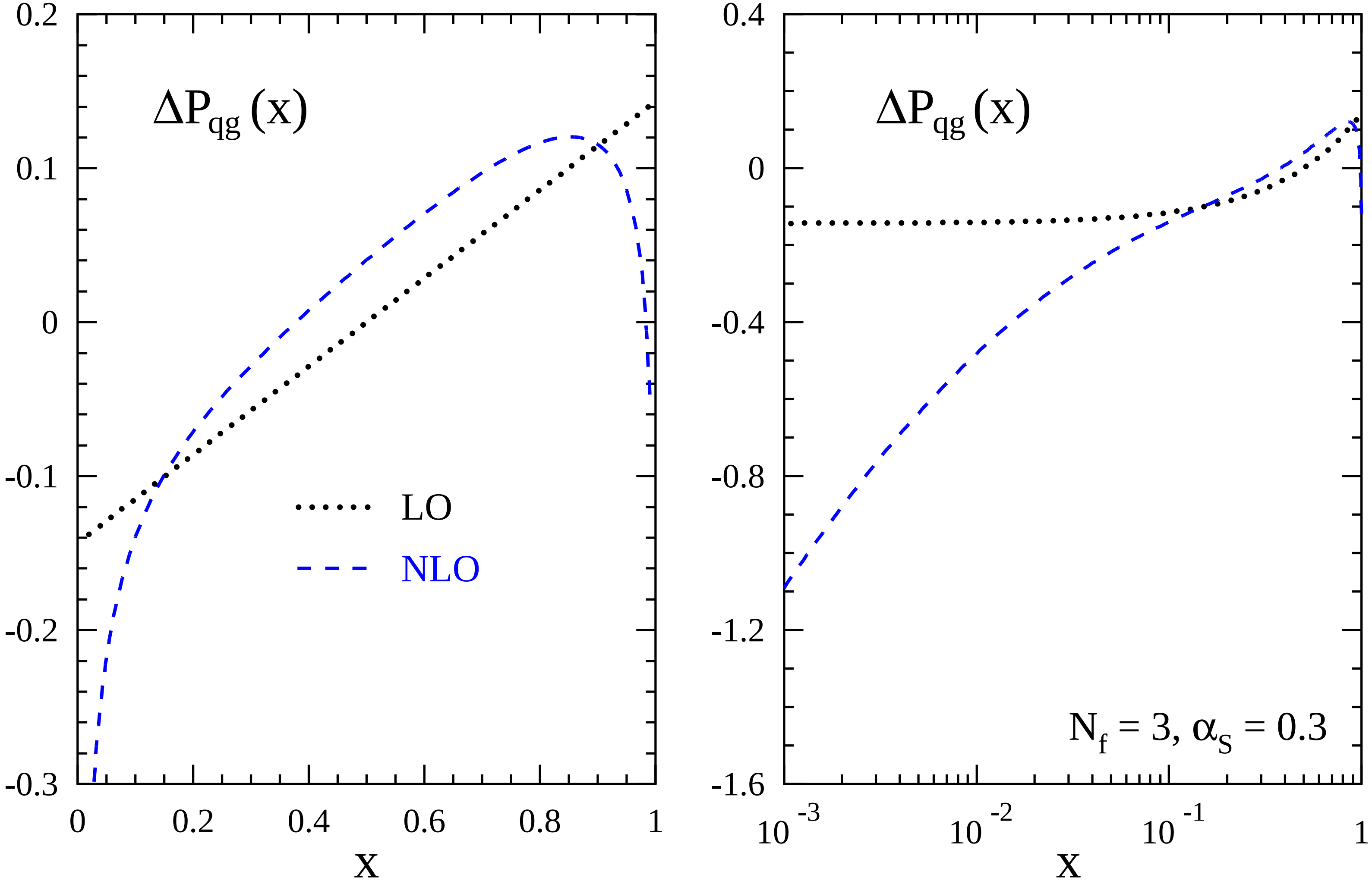
- Small x : $\Delta P^{(2)}_{\rm ps} = -C_F n_f \,(2\, C_A + 8/3\, C_F) \ln^4 x + \ldots,$ as in
  Blümlein, A.V. (96) based on Bartels, Ermolaev, Ryskin (96)
  No direct agreement for $C_F C_A n_f, C_F^2 n_f$ of $\Delta P^{(2)}_{\rm qg}$ – scheme again?

The spin splitting function ΔP_{qg} to NNLO



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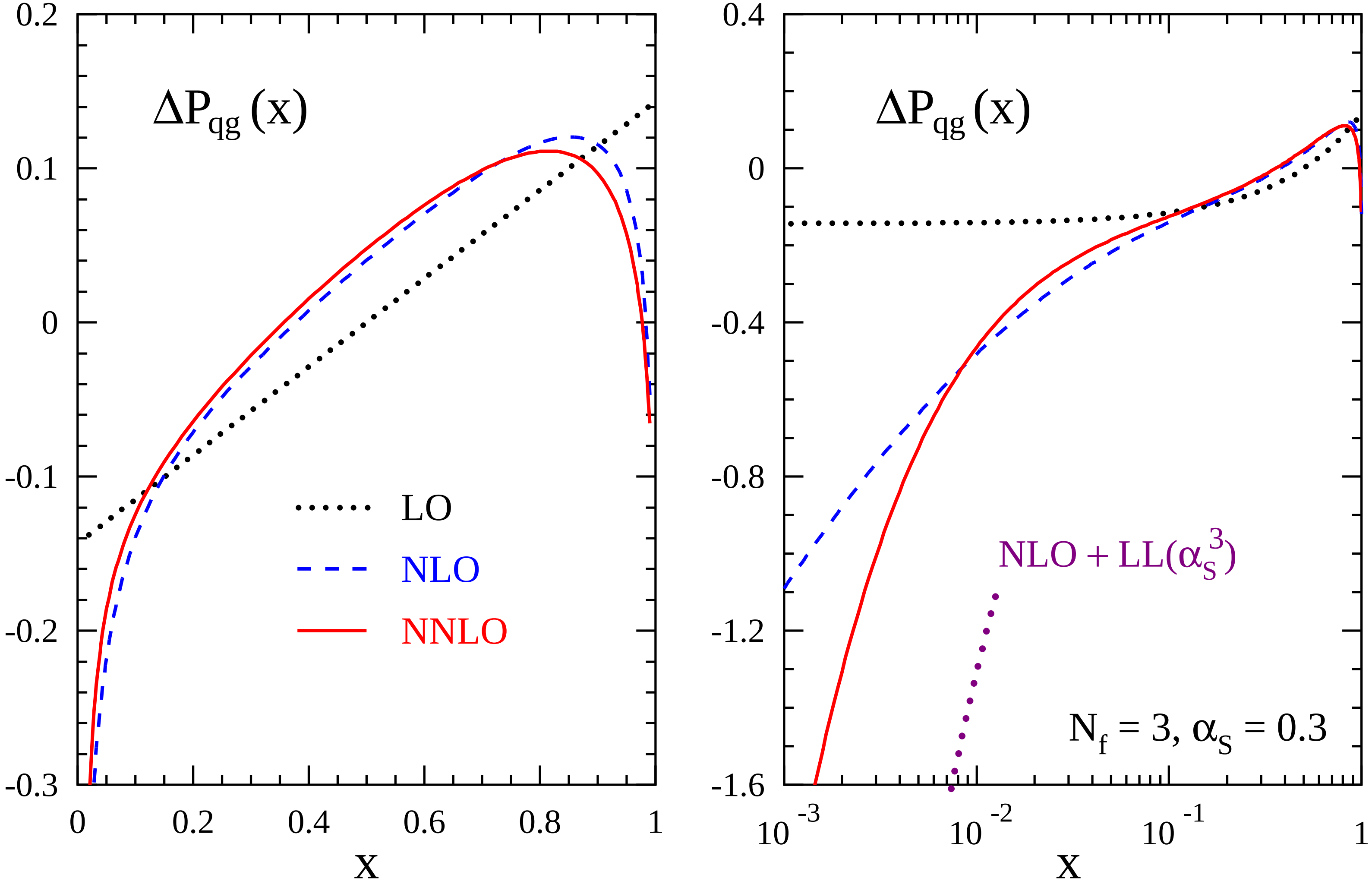
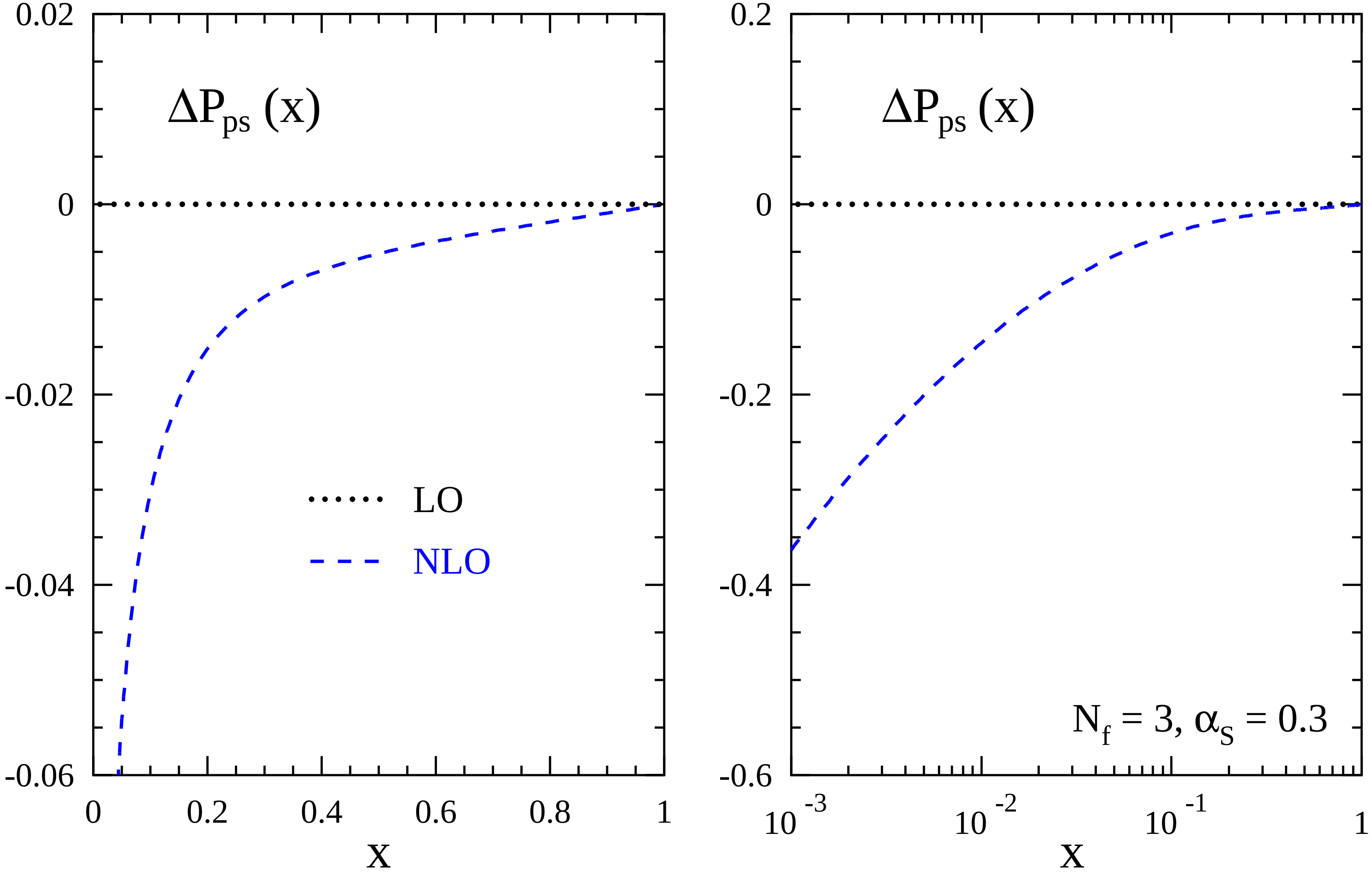


NNLO corr. $\leq$ 15% for 0.005 $\leq x <$ 0.9 . Leading log $\alpha_s^3 \ln^4 x$ insufficient

The spin splitting function ΔP_{ps} to NNLO



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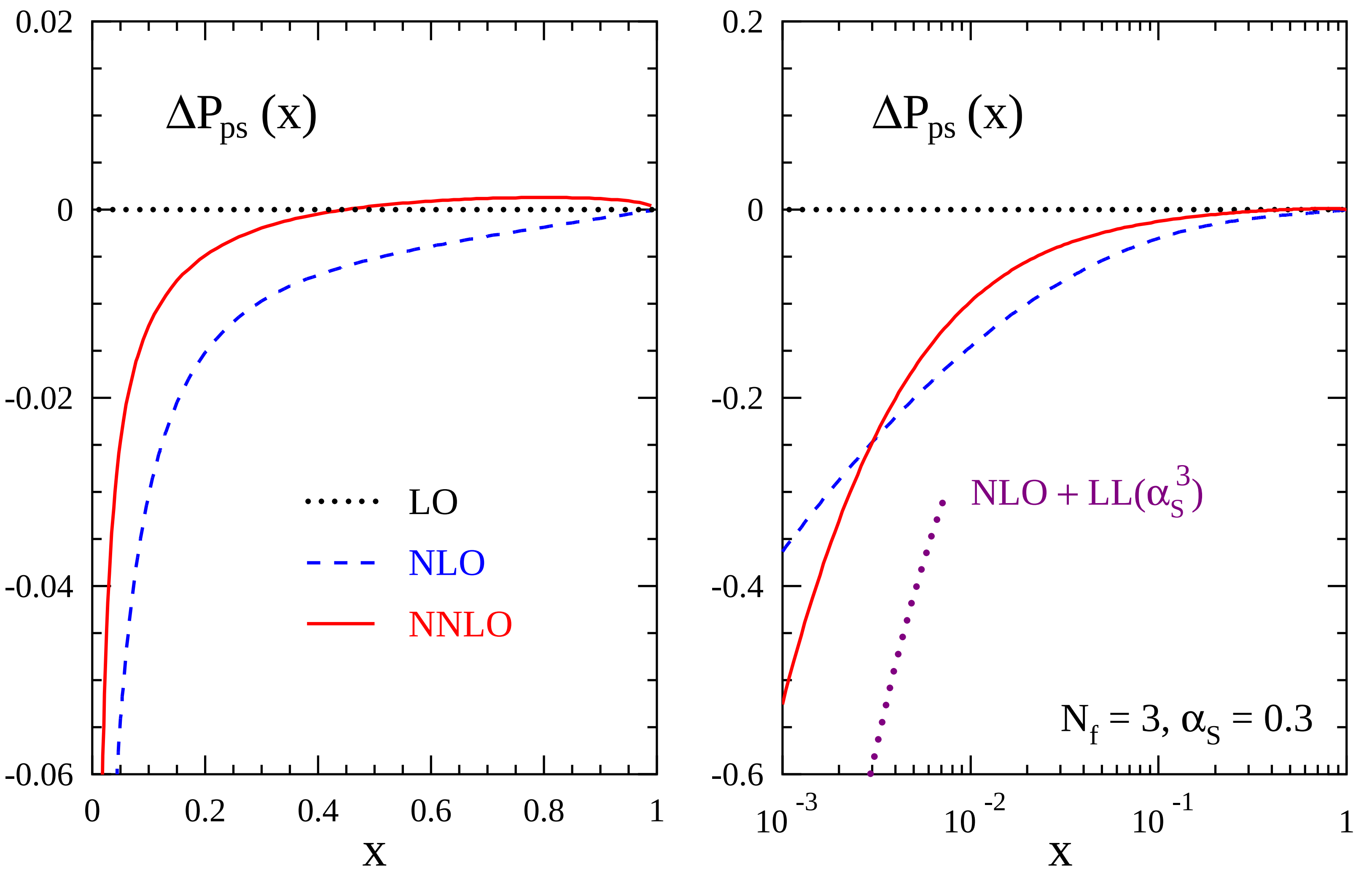

For ΔP_{qq} add non-singlet part = P_{ns}^{-} : pure singlet negligible at large x

Summary and outlook

Upper row calculated of matrix of third-order spin splitting functions

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- **γ_5: Larin prescription + $z_{\rm qq}$ scheme transformation. Two-loop $c^{(2)}_{g_1,\rm f}$ $\surd$**
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Work on lower row started: diagram files, gauge checks (Mincer), . . .

- DIS with exchange of a pseudoscalar (Higgs) χ coupling to $\widetilde{G}^{a}_{\mu\nu} G^{\mu\nu}_{a}$
- Many more diagrams: $\approx$ 30000 in prelim. $g\phi g\chi$ database . . . TFORM
- Final results hopefully NOT at LL 2010 . . . for being 'old news' by then