Massive Particle Production to NNLO in QCD

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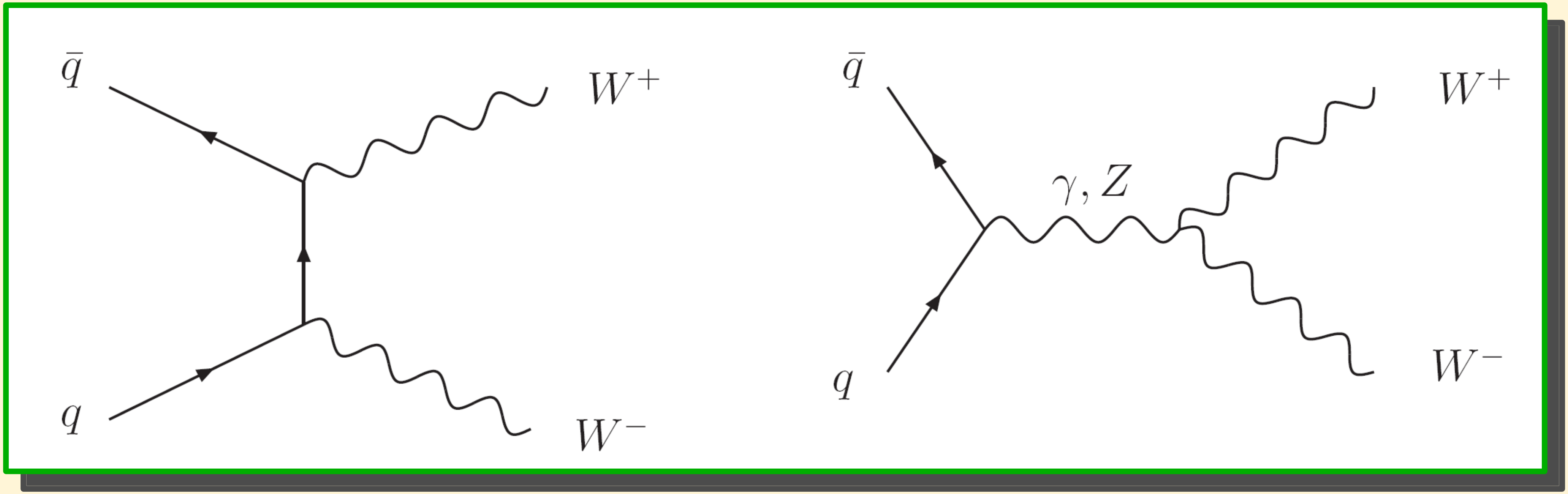
Outline

- Focus on hadronic W pair production:
  Virtual two-loop and one-loop squared amplitudes
- Motivation for studying $\mathrm{q\ q \rightarrow W\ W}$ accurately
- Results: NNLO Virtual Corrections
- Outlook $\rightarrow$ Transition to Top Pair Production

- Power corrections
- Full mass dependence
- Outlook (... Time-reversed)

The process



Motivation I

W pair production important as a **signal** in searches for **New Physics.** Testing ground for non-abelian structure of SM, triple gauge couplings, γWW, ZWW

$$\sigma(p\bar{p} \rightarrow W^+W^-) =$$

$$14.6^{+5.8}_{-5.1}(\mathrm{stat})\ {}^{+1.8}_{-3.0}(\mathrm{syst}) \pm 0.9(\mathrm{lum})\ \mathrm{pb}$$ CDF

$$13.8^{+4.3}_{-3.8}(\mathrm{stat})\ {}^{+1.2}_{-0.9}(\mathrm{syst}) \pm 0.9(\mathrm{lum})\ \mathrm{pb}$$ DØ

$\sqrt{s} = 2$ TeV ($p\bar{p}$)	W^+W^-	
	MRS98	CTEQ5
Born [pb]	10.0	10.3
Full [pb]	13.0	13.5

$\sqrt{s} = 14$ TeV (pp)	W^+W^-	
	MRS98	CTEQ5
Born [pb]	81.8	86.7
Full [pb]	120.6	127.8

?

Tevatron Campbell, Ellis ('99) LHC

Motivation II

The 'elusive' Higgs boson

Higgs:

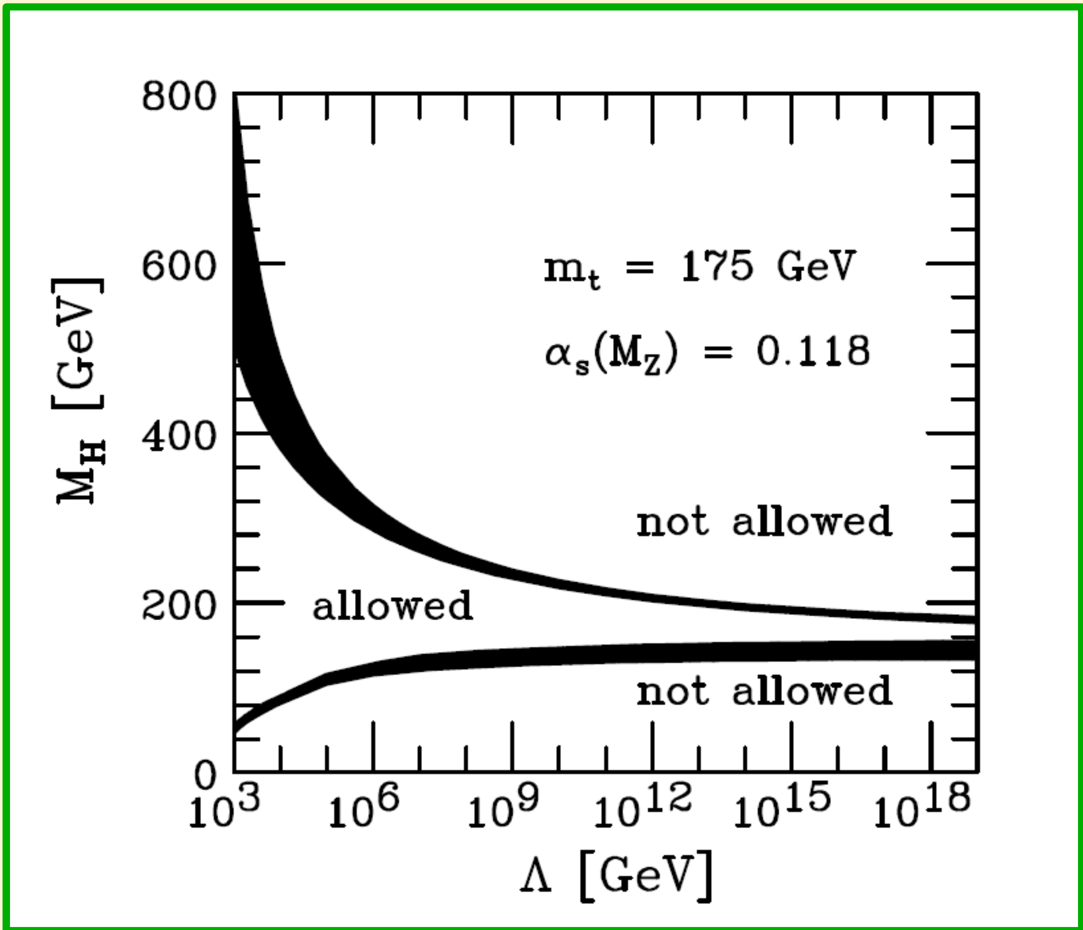
- Only constituent of the SM not experimentally observed yet.
- Electroweak symmetry breaking
- Description of particle masses

Discovery by itself is not enough!
Properties of the Higgs needed
to exclude or verify
alternative models

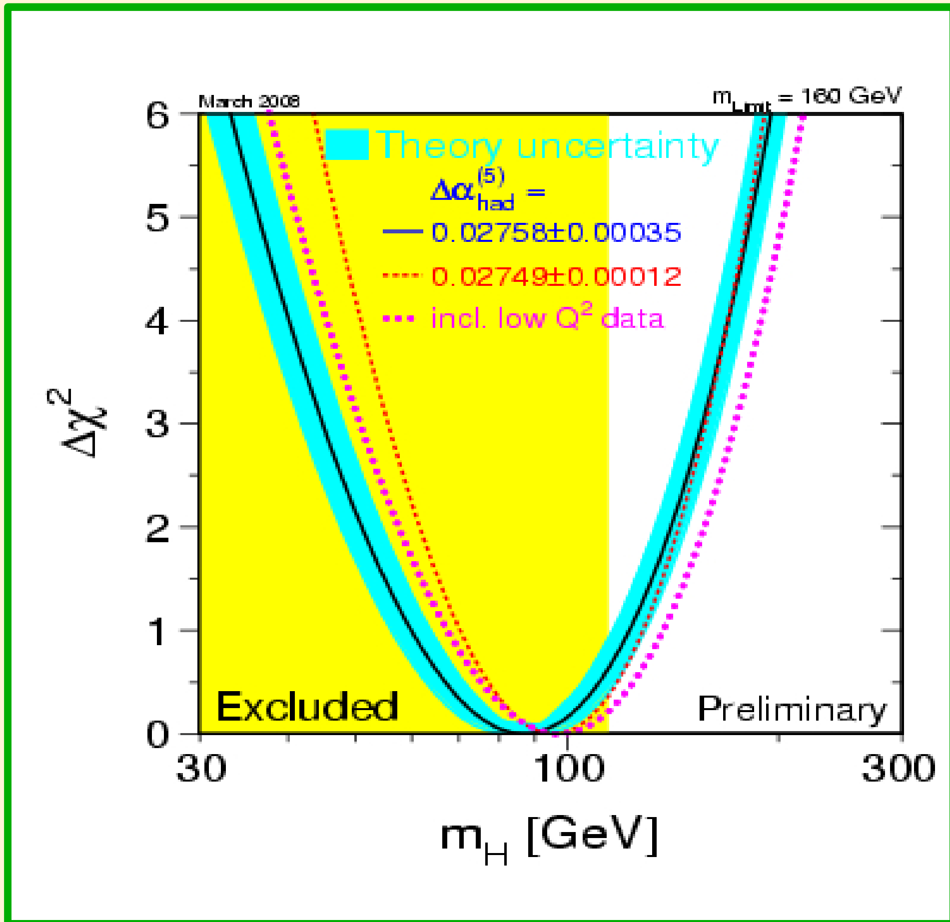
Motivation II

Soon in the LHC era

VIP: Higgs! LHC has the energy and luminosity required to discover the Higgs in all the allowed range, $114 < M_H < 600$ GeV

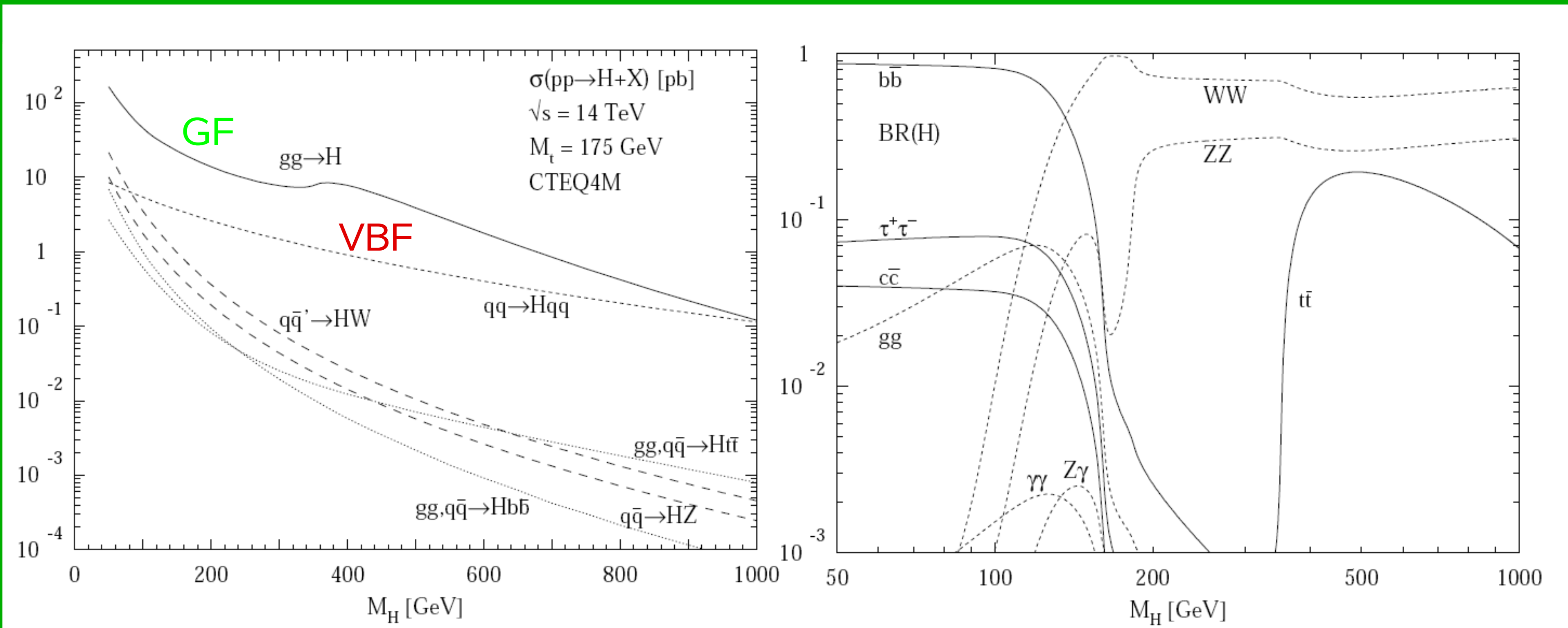


Hambye, Riesselmann ('97)



LEP EW working Group

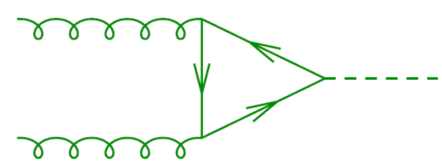
Motivation II



Spira '97

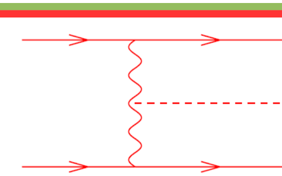
LHC Higgs production ...

Gluon Fusion channel is the dominant production mechanism up to $M_H \sim 1$ TeV : $g\, g \rightarrow H$



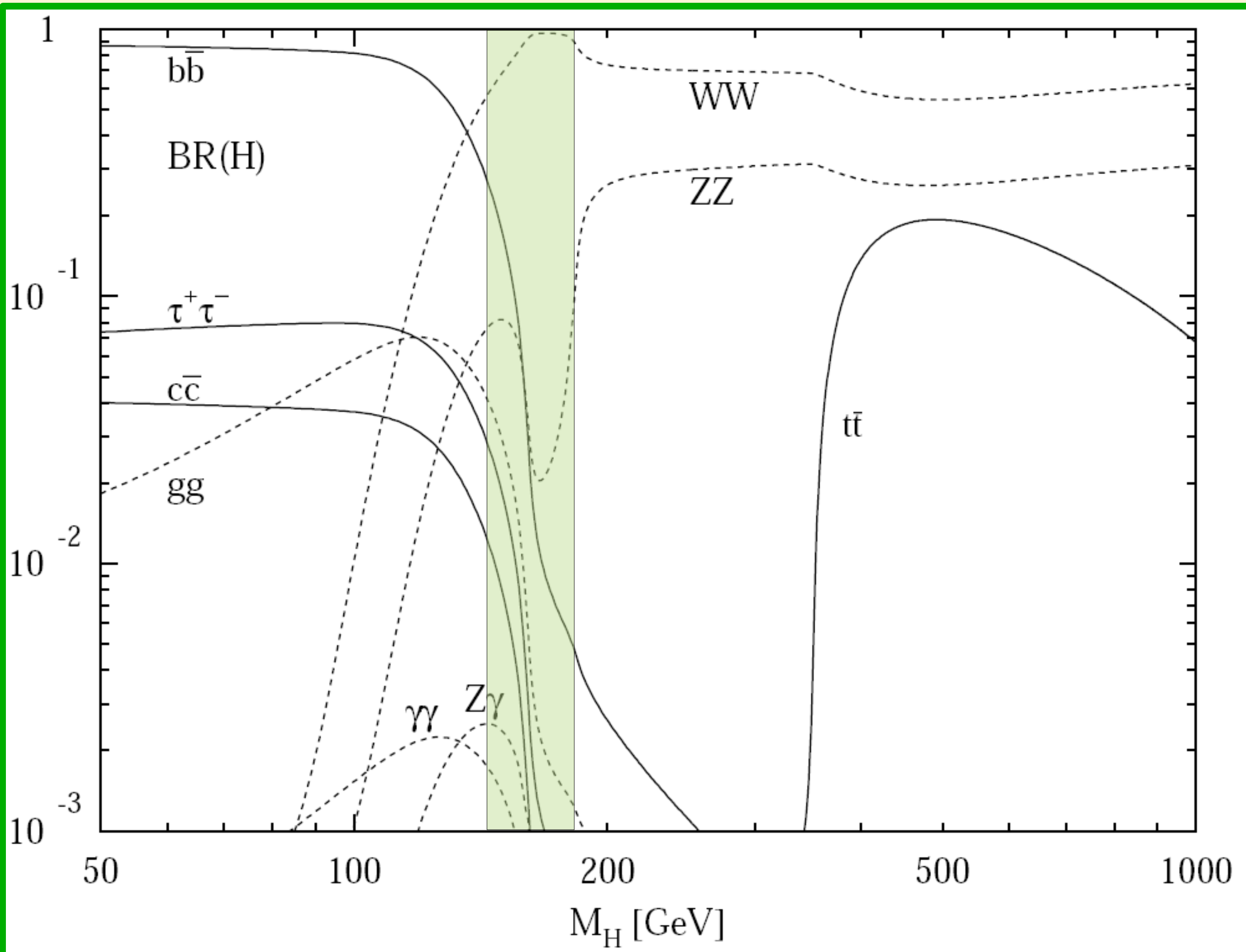
gluon fusion (GF)

Sub-dominant production process is
Vector Boson Fusion: $q\, q \rightarrow V\, V \rightarrow q\, q\, H$



vector boson fusion (VBF)

Motivation II



… and decay

Once the Higgs is produced it will eventually decay into different particles depending on its mass. In the Higgs mass range $140 - 180\ \mathrm{GeV}$ the main decay mode is into W pairs

Motivation II

Going after the Higgs: Main discovery Channels

M_H : 114 – 140 GeV
$H \to \gamma\,\gamma$

M_H : 180 – 600 GeV
$H \to Z\,Z \to 4\,l$

M_H: 140 – 180 GeV
$H \to W\,W \to 2\,l$ + missing Energy E_T

- Pick up the signal process
- Avoid or suppress the usually large **background**
- Accurate theoretical predictions for both signal and background

Main background (irreducible): W pair production

All these are nice but ...

Do we really need to go up to NNLO?

State of the art:

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

Glover

All these are nice but ...

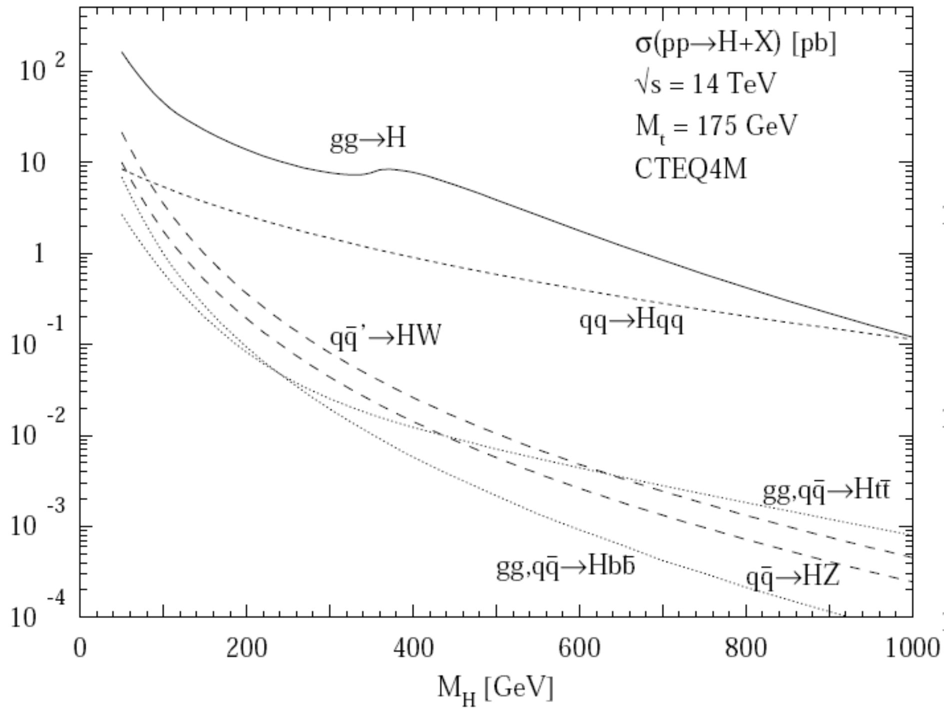
So ???

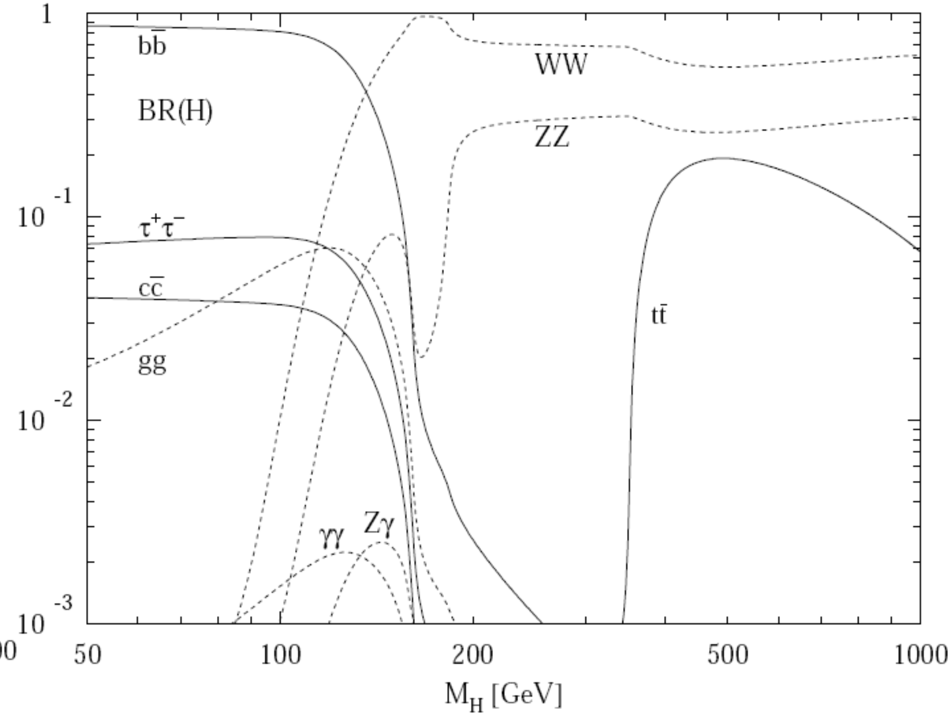
Do we really need to go up to NNLO?

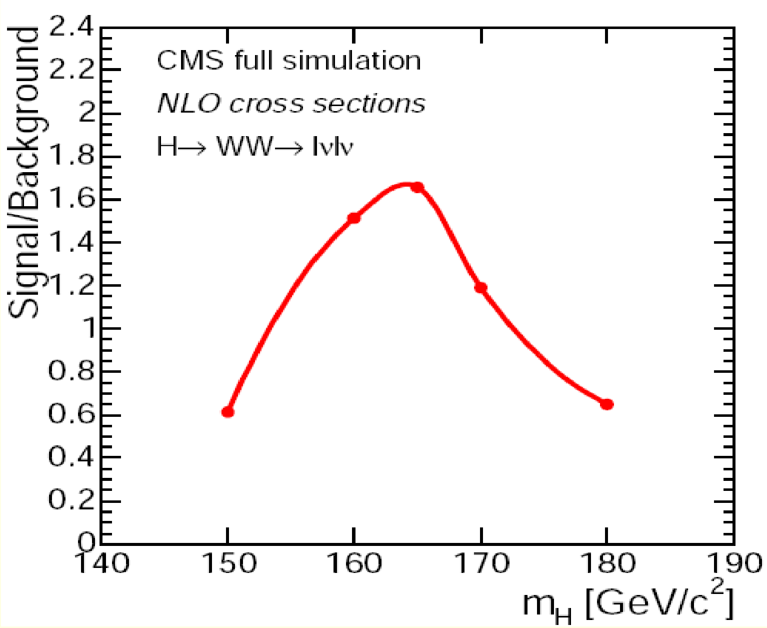
The answer is

YES!

Signal/background







Signal – background ratio: of order unity in the mass range 150 – 180 GeV

Signal known to NNLO

QCD corrections to :

$\underline{g\, g \rightarrow\ H}$

NLO: Contribute ~ 70%

Dawson ('91); Djouadi, Graudenz, Spira, Zerwas ('95)

NNLO: Contribute an additional 20% for LHC

Harlander, Kilgore ('02); Anastasiou, Melnikov ('02)

Ravindran, Smith, van Neerven ('03)

With a Jet veto at NNLO: corrections ~ 85%

Catani, de Florian, Grazzini ('02)

Davatz, Dissertori, Dittmar, Grazzini, Pauss ('04)

Anastasiou, Melnikov, Petrielo ('04)

NNLO $\underline{H \rightarrow W\, W \rightarrow l\, \nu\, l\, \nu}$

Anastasiou, Dissertori, Stöckli, Webber ('08)

Grazinni ('08)

Background

- $qq \rightarrow WW$

Background

- $q\bar{q} \rightarrow WW$

  70% enhancement at NLO. With a jet veto the enhancements fall to 20-30%

  Dixon, Kunszt, Signer ('98, '99)

Background

- qq→WW

  70% enhancement at NLO. With a jet veto the enhancements fall to 20-30%

  Dixon, Kunszt, Signer ('98, '99)

- loop induced gg→WW

Background

- qq→WW

  70% enhancement at NLO. With a jet veto the enhancements fall to 20-30%

  Dixon, Kunszt, Signer ('98, '99)

- loop induced gg→WW

  Contributes to the quark annihilation channel at $\mathcal{O}(\alpha_s^2)$.
  Enhanced by the **large gluon flux.** After Higgs search cuts it increases the background by 30%, with no cuts by 5%

  Glover, van der Bij ('89); Kao, Dicus ('91)
  Binoth, Ciccolini, Kauer, Krämer ('05)
  Duhrssen, Jackobs, v. d. Bij, Marquard ('05)

- EW corrections

  Accomando, Denner, Kaiser('05)

Necessity of NNLO calculation for a few % level accuracy

W Pair Production

1980 1990 2000 Present

LO — Brown, Mikaelian ('79)

NLO — Ohnemus ('91); Frixione ('93);
Ohnemus('94);
Dixon, Kunszt, Signer ('98,'99);
Campbell, K. Ellis ('99)

NNLO massless — Anastasiou, Glover, Tajeda-Yeomans ('02)

resummation — Grazzini ('06)

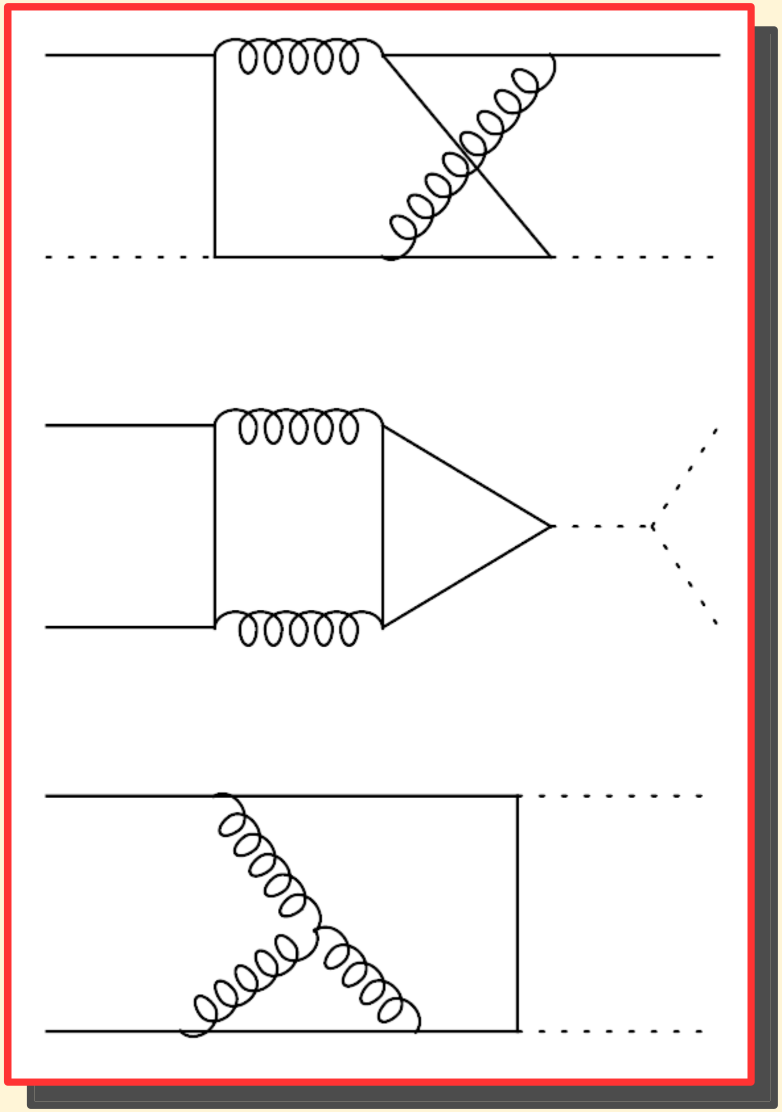
NNLO

The p p $\rightarrow$ W W story till recently...

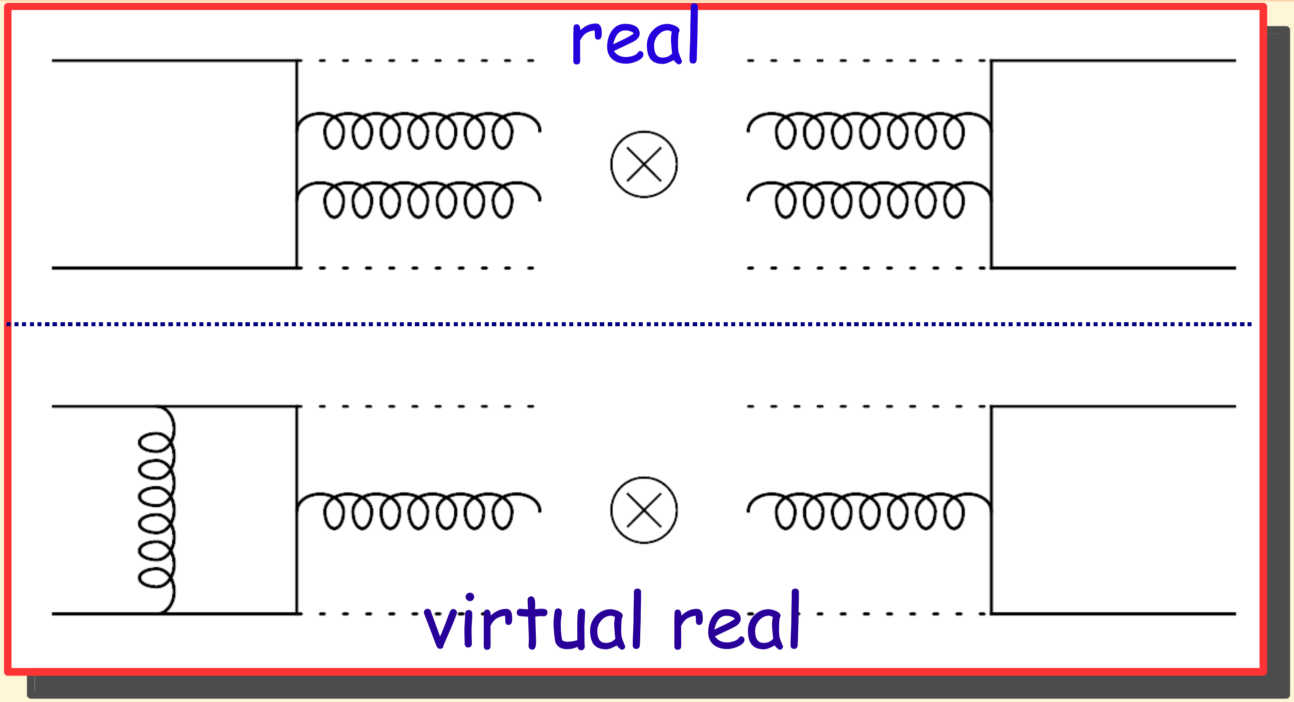
We would now like...

- ... Cross sections for W Pair production at NNLO with full mass dependence
- ... And of course we have to start with the amplitudes
- ... And the difficult part on the amplitude level is the virtual corrections, in particular the two-loop diagrams contracted with the Born

At present (... many diagrams)



2 loop virtual



(one loop) ⊗ (one loop)*

So what is it now at stake?

- A NNLO (4 legs, 2 loops) calculation of a process with massive particles (similar features to the recent "heavy quark production") Czakon, Mitov, Moch ('07)
Czakon ('08)
- Color and spin averaged amplitudes
- Kinematical region: all kinematical invariants large compared to the mass of W:

$$M_W^{\;2} \ll s, t, u$$

- We expand with respect to $m_s = M_W^{\;2}/s$
- Exact analytic result (up to terms suppressed by powers of m_s)

Mellin-Barnes

Starting from the Feynman parameters representation of a diagram one goes along the following

Steps:

- produce representations (**MBrepresentations.m**)
- analytically continue in ε to the vicinity of 0 (**MB.m**)
- expand in mass (**MBasymptotics.m, ** Czakon)
- perform as many as possible integrations using Barnes lemmas (**BarnesRoutines.m**, Kosower)
- resum the remaining integrals by transforming into harmonic series (**Xsummer**)
- resum remaining constants by high-precision numerical evaluation (**quadprec.m**) and fit them to a transcendental basis (**PSLQ**)

Software

MBrepresentations.m (GC, Czakon)

Produces representations for any **multi**-loop, **planar** or **non-planar**, **scalar** or **tensor** integral of any **rank**!

MB.m (Czakon)

Determination of contours, analytic continuation, expansion in a chosen parameter, numerical integration

XSummer (Moch, Uwer)

Evaluation of harmonic sums

PSLQ (Bailey)

Fitting to a transcendental basis

quadprec.m (Czakon)

High precision numerical evaluation with up to 64 digits

Catani's recipe: An important test

One loop: The IR pole structure of the renormalized amplitude can be known by only knowing the tree level amplitude:

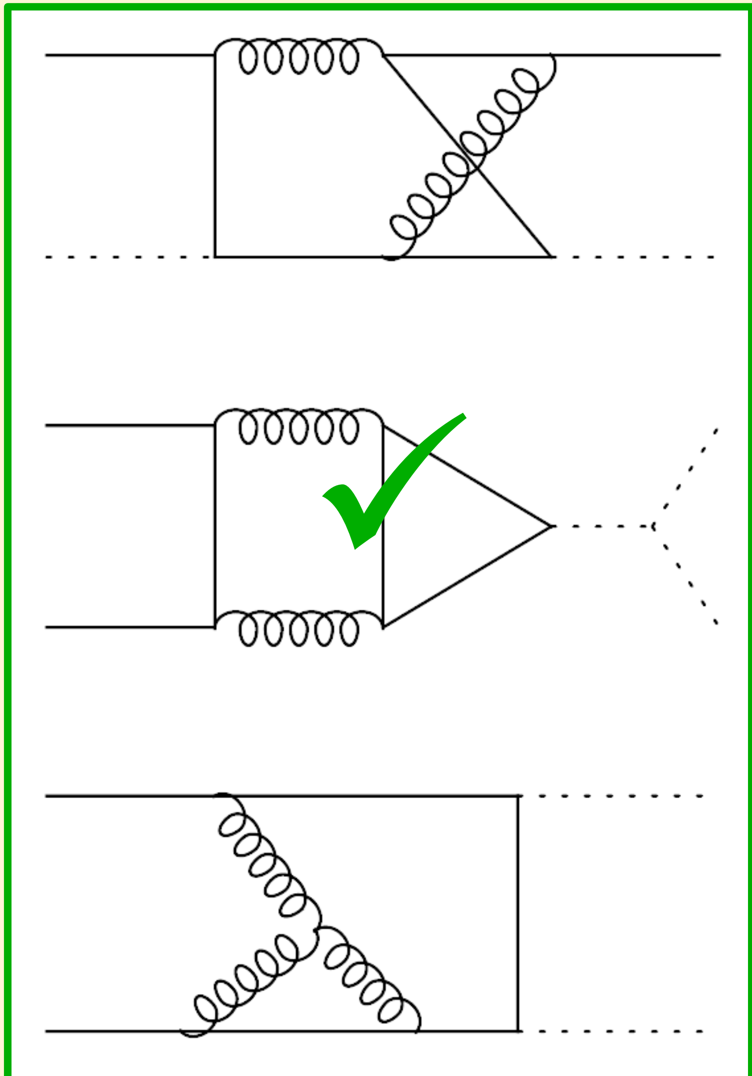
$$|\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm R.S.} = \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\;|\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm R.S.} + |\mathcal{M}_m^{(1)\,{\rm fin}}(\mu^2;\{p\})\rangle_{\rm R.S.}$$

Two loop: Now you need tree and one loop level amplitude:

$$\begin{aligned} |\mathcal{M}_m^{(2)}(\mu^2;\{p\})\rangle_{\rm R.S.} &= \boldsymbol{I}^{(1)}(\epsilon,\mu^2;\{p\})\;|\mathcal{M}_m^{(1)}(\mu^2;\{p\})\rangle_{\rm R.S.} \\ &+ \boldsymbol{I}^{(2)}_{\rm R.S.}(\epsilon,\mu^2;\{p\})\;|\mathcal{M}_m^{(0)}(\mu^2;\{p\})\rangle_{\rm R.S.} + |\mathcal{M}_m^{(2){\rm fin}}(\mu^2;\{p\})\rangle_{\rm R.S.} \end{aligned}$$

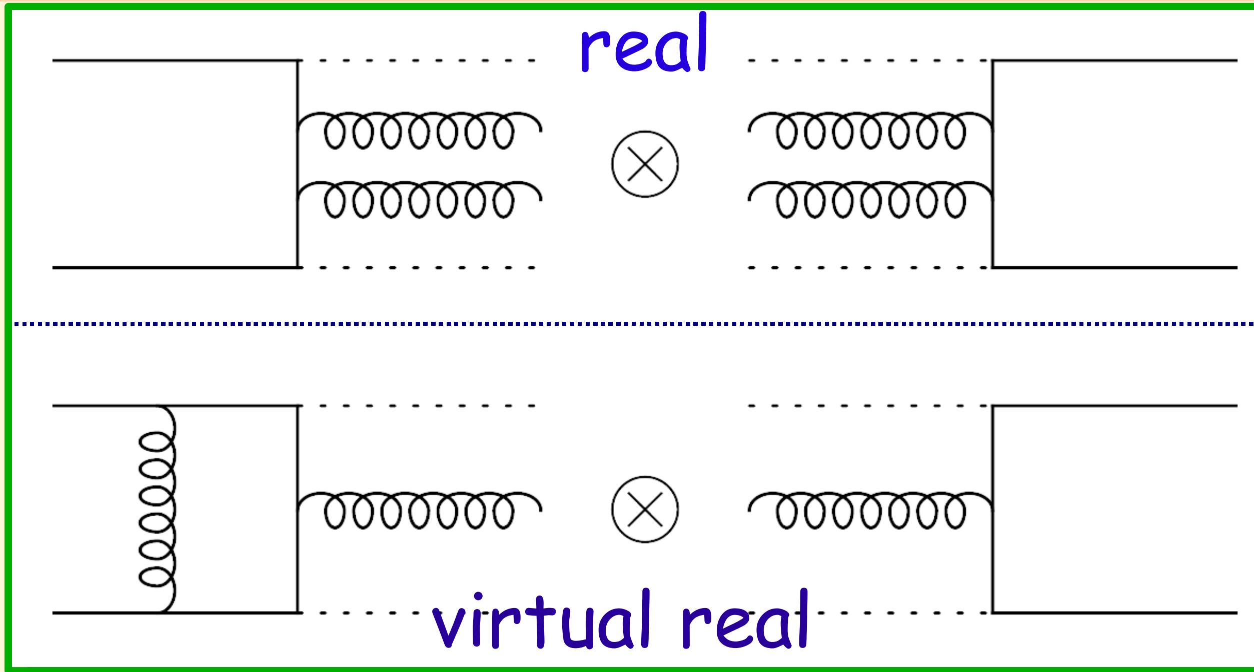
Singular dependence embodied in the operators $I^{(1)}$ and $I^{(2)}$

Check list:



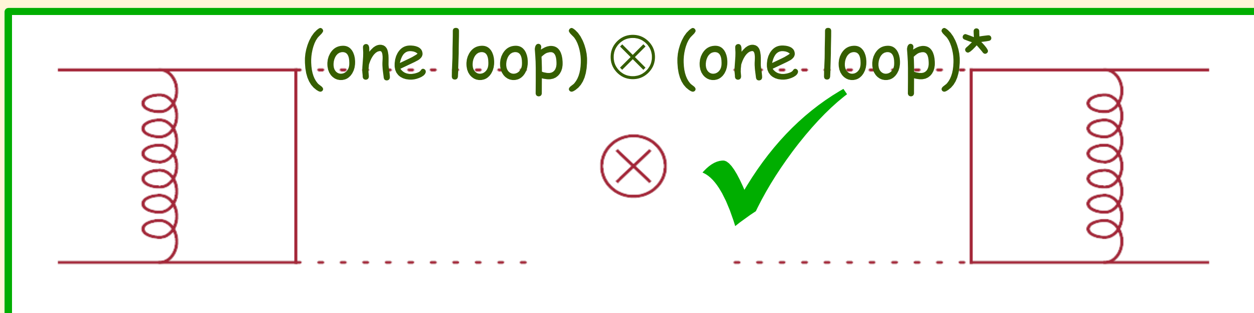
2 loop virtual

G.C., Czakon, Eiras ('08)



Campbell, Ellis, Zanderighi ('07)

Dittmaier, Kallweit, Uwer ('08)



G.C., Czakon, Eiras (to be published)

Conclusions (so far)

- We have finally the full (virtual) result up to $O(m_s^{0})$
- Mellin Barnes representations approach is a powerful technique
- Not an easy one though (especially for the non-planar graphs)
- Very soon to come are higher power corrections (they are actually being produced at the moment)

Next step: Full mass dependence

Similar to M. Czakon [arXiv:0803.1400]

- Deep expansion in mass around the high energy limit.
- Numerical Differential Equation method (more in the talk of T. Schutzmeier)

Caffo, Czyz, Laporta, Remiddi ('98)

Top Pair Production

Czakon, arXiv:0803.1400

Baernreuther, Czakon (in preparation)

And again few words on motivation

- Top mass measurement
- Production cross section to better than 10%
- Decay mechanisms
- Searches for resonances
- Background to new physics searches

The story so far ...

- NLO corrections

  Nason, Dawson, Ellis ('88)
- Implemented in MCFM

  Campbell, Ellis
- LL resummation

  Laenen, Smith, van Neerven ('92)
- NLL resummation

  Bonciani, Catani, Mangano, Nason ('98)
- NNL resummation + ...

  Kidonakis, Vogt ('03)
- NNNL resummation

  Moch, Uwer ('08)
- NNLO PDF evolution

  Moch, Vermaseren, Vogt ('04)
- 1-loop squared

  Koerner, Merebashvili, Rogal ('06)
- NLO tT + jet

  Dittmaier, Uwer, Weinzierl ('07)

High energy asymptotics of two-loop amplitudes

quark annihilation Czakon, Mitov, Moch ('07)
gluon fusion Czakon, Mitov, Moch ('07)

Full mass dependence of two-loop amplitudes

quark annihilation Czakon ('08)
gluon fusion Baernreuter, Czakon (in preparation)

Structure of the result

- color and spin averaged amplitudes (can be changed)
- color decomposition for the annihilation channel

$$2(N^2-1)\left(N^2 A + B + \frac{1}{N^2} C + N n_l D_l + N n_h D_h + \frac{n_l}{N} E_l + \frac{n_h}{N} E_h + (n_l + n_h)^2 F\right)$$

direct computation

common
(factorization and
direct computation)

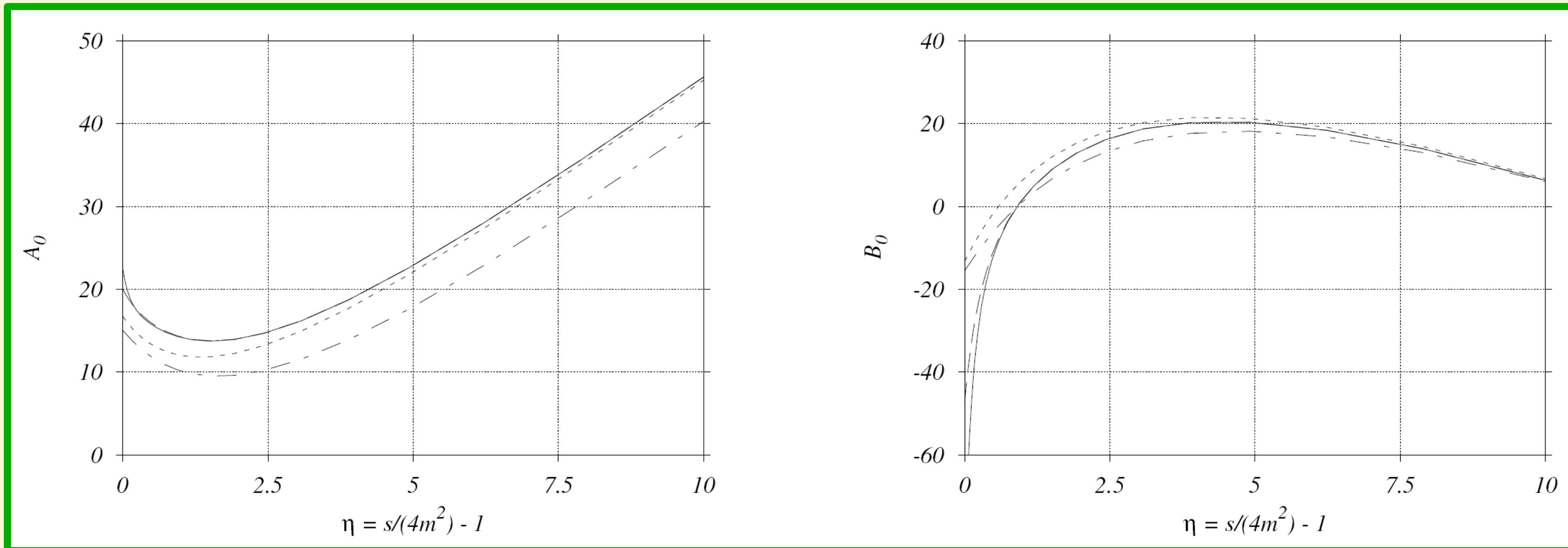
- and the fusion channel

$$2(N^2-1)\left(N^3 A + N B + \frac{1}{N} C + \frac{1}{N^3} D + N^2 n_l E_l + N^2 n_h E_h + n_l F_l + n_h F_h\right.$$
$$\left. + \frac{n_l}{N^2} G_l + \frac{n_h}{N^2} G_h + N n_l^2 H_l + N n_h n_l H_{lh} + N n_h^2 H_h + \frac{n_l^2}{N} I_l + n_l \frac{n_h}{N} I_{lh} + \frac{n_h^2}{N} I_h\right)$$

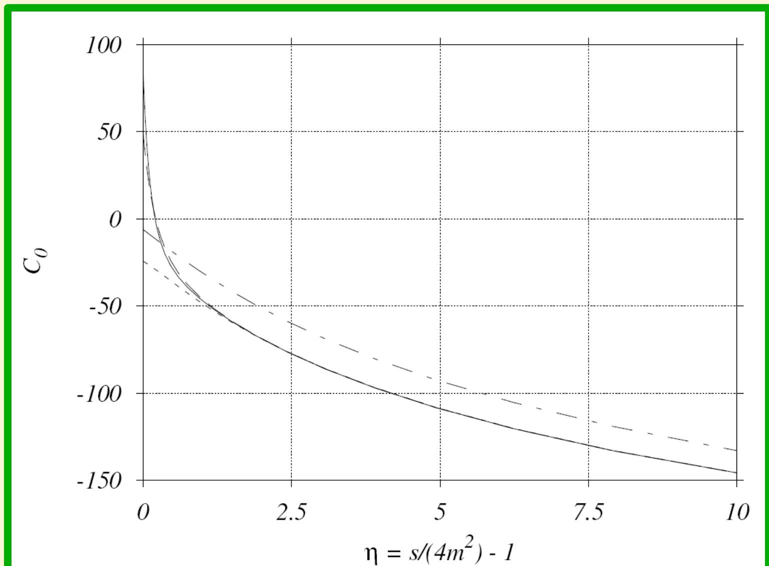
Power corrections

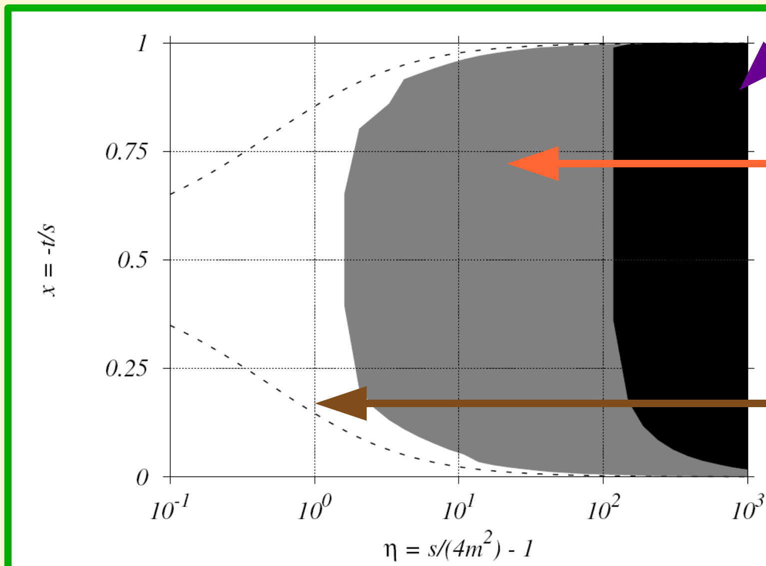
- expansion in m_s (11 terms)

Czakon ('08)



1 % accuracy region of the leading asymptotics





1 ‰ accuracy region of the expansion

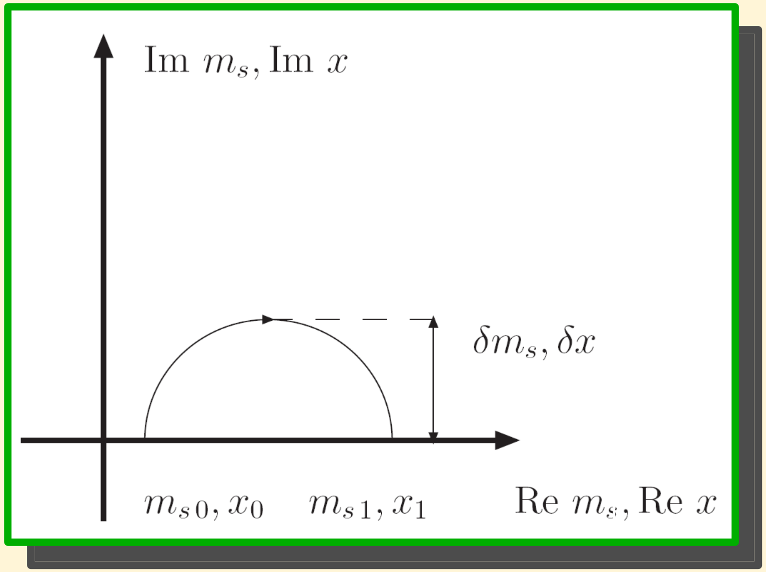
Kinematic boundaries

Towards a numerical solution

- Compute the high energy asymptotics of the master integrals obtaining the leading behaviour of the amplitude
- Determine the coefficients of the mass expansions using differential equations in m_s obtaining the power corrections

$$m_s \frac{d}{d m_s} M_i(m_s, x, \epsilon) = \sum_j C_{ij}(m_s, x, \epsilon) M_j(m_s, x, \epsilon)$$

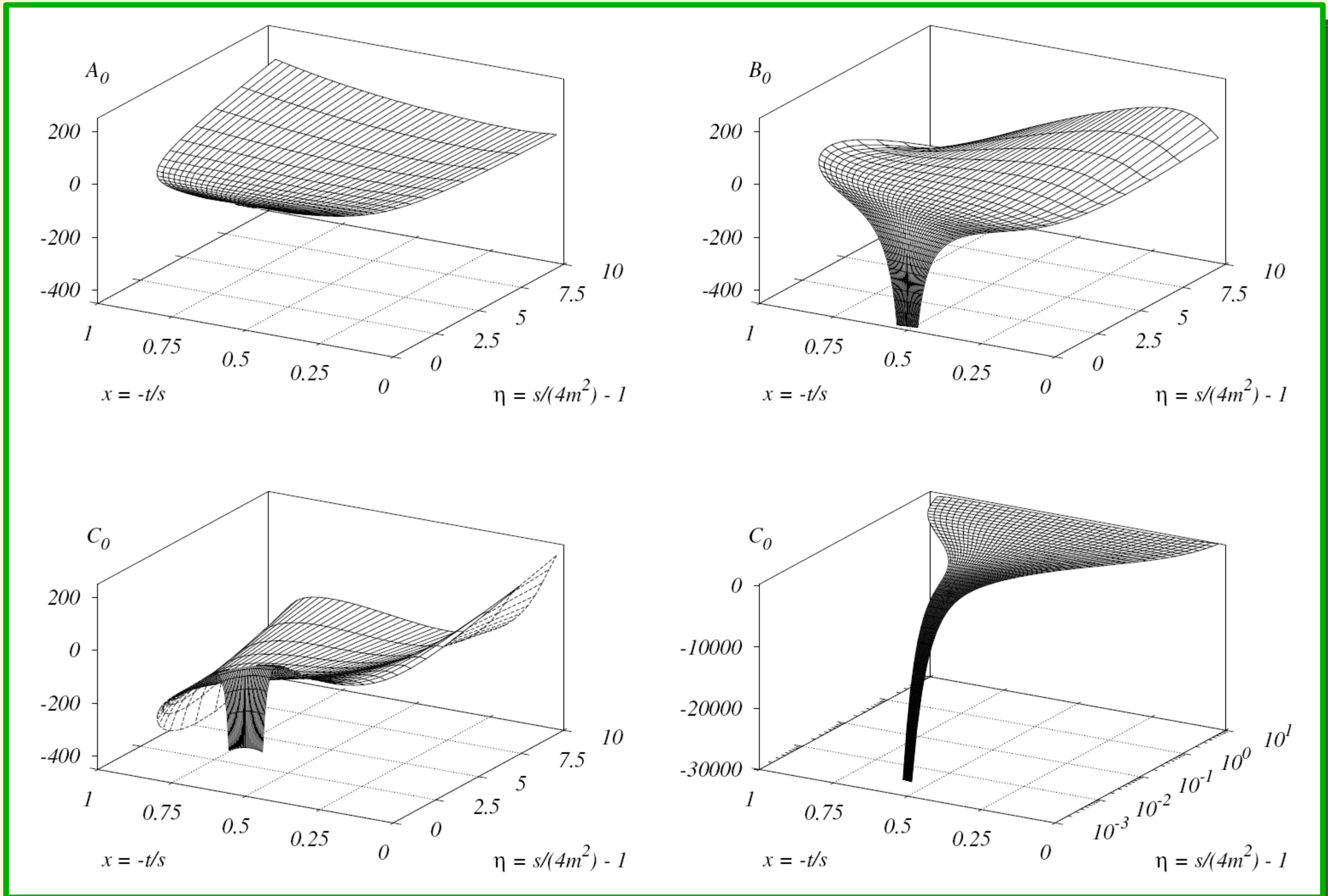
- Evaluate the expansions for $m_s \ll 1$ to obtain the desired numerical precision of the boundaries
- Evolve the functions from the boundary point with differential equations first in m_s and then in x (**ZVODE**)



Full mass dependence

- Numerical solution of differential equations

Czakon ('08)



Example point

$m^2 = .2\ s,\ x = 0.45$

	leading color			full color		
number of masters	36			145		
number of functions	155			595		
precision	quadruple		double	quadruple		double
evolution in m^2/s						
requested local error	10^{-20}	10^{-12}	10^{-12}	10^{-20}	10^{-12}	10^{-12}
contour deformation	0.1	0.1	0.1	0.1	0.1	0.1
number of steps taken	2319	618	534	2932	777	1302
jacobian evaluation time [ms]	3.4	3.4	0.2	37	37	4.9
evolution in x						
requested local error	10^{-18}	10^{-10}	10^{-10}	10^{-18}	10^{-10}	10^{-10}
contour deformation	0.1	0.1	0.1	0.1	0.1	0.1
number of steps taken	545	139	139	739	174	432
jacobian evaluation time [ms]	8.3	8.3	0.4	150	150	17
total evaluation time [s]	49	13	< 1	957	243	26

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
A	0.22625	1.391733154	-2.298174307	-4.145752449	17.37136599
B	-0.4525	-1.323646320	8.507455541	6.035611156	-35.12861106
C	0.22625	-0.06808683395	-18.00716652	6.302454931	3.524044913
D_l		-0.22625	0.2605057339	-0.7250180282	-1.935417247
D_h			0.5623350684	0.1045606449	-1.704747998
E_l		0.22625	-0.3323207300	7.904121951	2.848697837
E_h			-0.5623350684	4.528240788	12.73232424
F_l					-1.984228442
F_{lh}					-2.442562819
F_h					-0.07924540546

Outlook

- The ultimate goal is to have, for both the Top Pair and the Gauge Boson Pair Production, a

NNLO Monte Carlo generator

- The first steps in this direction (and probably not the easier ones) have been done and many more to follow