GRACE/NLO for LHC

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@Loops & Legs 2008

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Contents

1. Introduction

- LHC Experimental requirement

  New Particle Search/Precision Measurements

LO-QCD Event generator+K-factor

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Obviously not enough!

We need
NLO Event generator!

Difficulties

- Large number of diagrams
- Large number of processes

- Numerical instability due to a collinear singularity
- Double counting between ME and PDF/PS (two categories)

Solutions

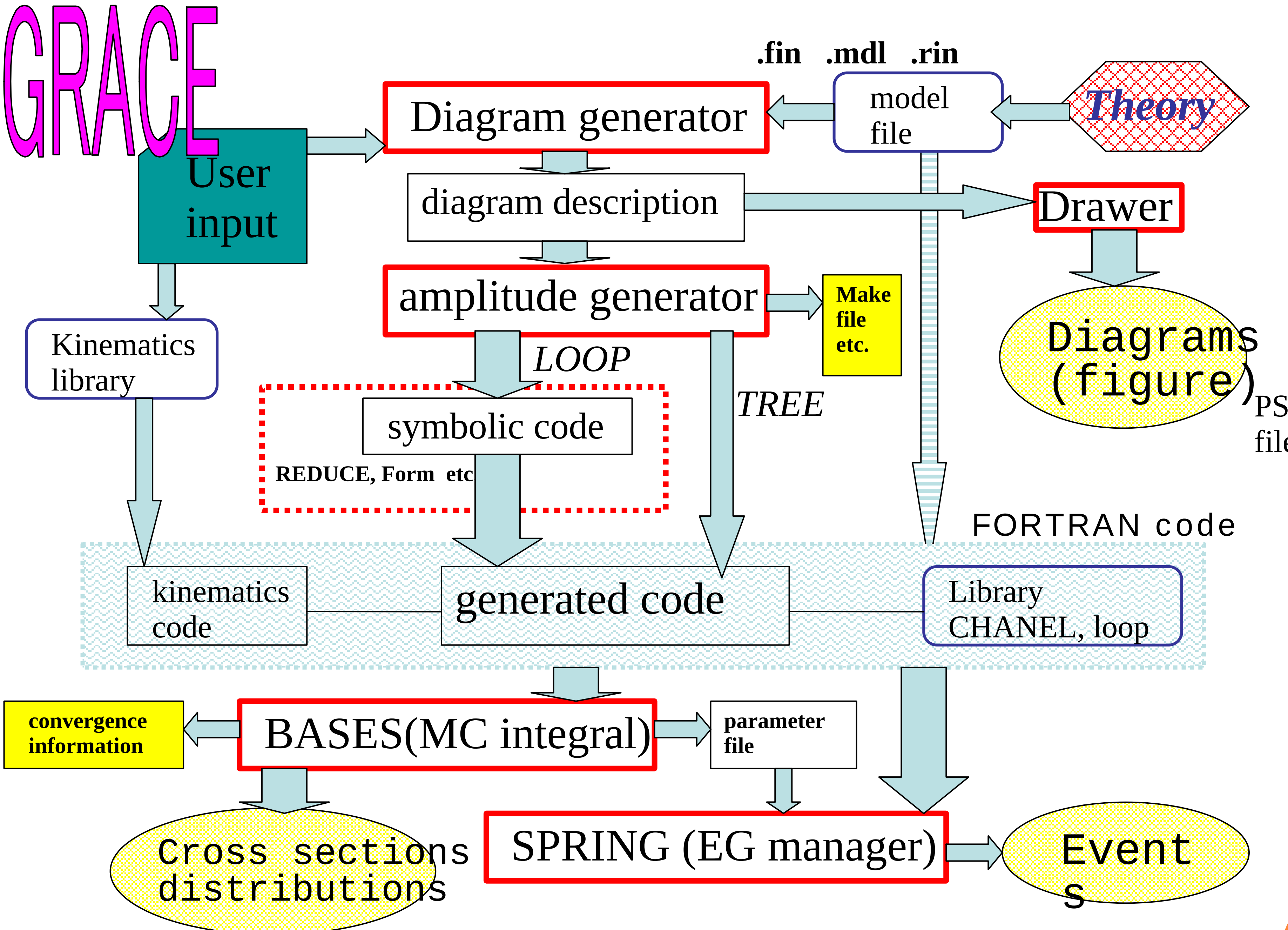
→ GRACE

→ LL-subtraction

2. What is GRACE?

A generator of event generators

- Automatic generation of Feynman diagrams up to 6-loop
- Automatic generation of matrix elements up to 1-loop.
- Kinematics Library
- Loop libraries



GRACE
.fin .mdl .rin
Diagram generator
model file
Theory
User input
diagram description
Drawer
amplitude generator
Make file etc.
Diagrams (figure)
PS file
Kinematics library
LOOP
TREE
symbolic code
REDUCE, Form etc
FORTRAN code
kinematics code
generated code
Library CHANEL, loop
convergence information
BASES(MC integral)
parameter file
Cross sections distributions
SPRING (EG manager)
Events

Matrix Elements: Tree

- Tree diagrams

$M_0(1,2 \rightarrow 1,2 \cdot\cdot\cdot,n)$: Born

$M_R(1,2 \rightarrow 1,2 \cdot\cdot\cdot,n,n+1)$: Real radiation

GRACE: Automatic generation up to n≅6

↑

Diagrams & FORTRAN source code

3. GR@PPA framework

Library of event generators for LHC generated by GRACE

- Treatment of proton/anti-proton
- Treatment of multi-processes
- LHA
- Interface to hadronization program
- Un-weighted events are generated
- At this moment, LO

- pp→many, TEVATRON/LHC, GR@PPA 2.7, S. Tsuno et al., 2006

- W + jets (up to 4 jets) with the subsequent W decay to a fermion pair,
- Z + jets (up to 4 jets) with the subsequent Z decay to a fermion pair,
- QCD multi-jets (up to 4 jets),
- Four bottom quarks via Z and Higgs-boson mediated processes as well as those from pure QCD interactions (same as GR@PPA_4b),
- top-quark pair (+jet)with the subsequent decay to W and b, and the W decay to a fermion pair,
- di-boson(WW, WZ and ZZ)+jets(up to 2jets) with the subsequent W/Z decay to a fermion pair.

N jets	Tevatron Run-II		LHC	
	$W(e\nu_e)$	$Z(e^+e^-)$	$W(e\nu_e)$	$Z(e^+e^-)$
0	$1.576(2) \times 10^3$	$1.598(3) \times 10^2$	$1.116(2) \times 10^4$	$9.57(3) \times 10^2$
1	$1.852(3) \times 10^2$	$1.829(4) \times 10^1$	$2.854(5) \times 10^3$	$2.614(7) \times 10^2$
2	$3.461(7) \times 10^1$	3.485(6)	$1.143(3) \times 10^3$	$1.082(2) \times 10^2$
3	6.29(2)	$6.35(2) \times 10^{-1}$	$4.82(1) \times 10^2$	$4.53(1) \times 10^1$
4	1.201(5)	$1.173(3) \times 10^{-1}$	$2.19(1) \times 10^2$	$2.045(5) \times 10^1$
2 ($\geqslant 1b$)	$3.260(6) \times 10^{-1}$	$9.24(2) \times 10^{-2}$	2.720(7)	8.68(2)
3 ($\geqslant 1b$)	$1.019(2) \times 10^{-1}$	$2.266(5) \times 10^{-2}$	4.305(9)	3.740(7)
4 ($\geqslant 1b$)	$2.947(8) \times 10^{-2}$	$3.817(5) \times 10^{-3}$	3.90(3)	$9.47(2) \times 10^{-1}$
$t\bar{t}+0$	$5.269(9) \times 10^{-4}$	$2.248(3) \times 10^{-4}$	$3.774(7) \times 10^{-2}$	$2.682(6) \times 10^{-2}$
$t\bar{t}+1$	$1.357(2) \times 10^{-4}$	$6.302(9) \times 10^{-5}$	$4.98(2) \times 10^{-2}$	$3.59(1) \times 10^{-2}$
$t\bar{t}+2$	$6.86(1) \times 10^{-5}$	$5.707(5) \times 10^{-7}$	$6.52(2) \times 10^{-2}$	$1.156(2) \times 10^{-2}$

4. Double counting

- Double Counting in tree-level

$$[\sigma_{tree} + \sigma_{vis(cut)}] \otimes \text{PDF/PS}$$

$$= \sigma_{tree} \otimes \text{PDF/PS} + \sigma_{vis(cut)} \otimes \text{PDF/PS}$$

Two sources of Double Counting

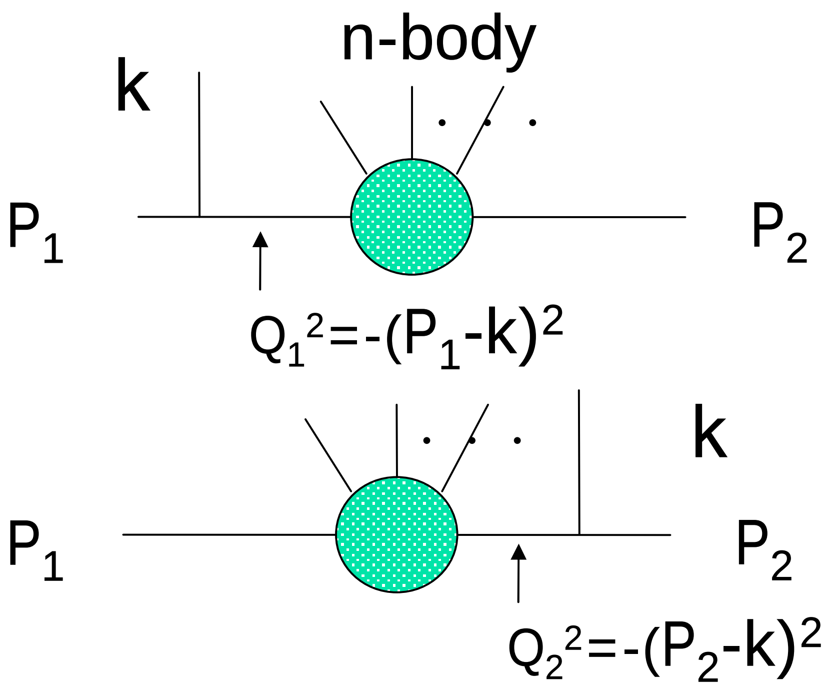
A famous solution is CKKW.

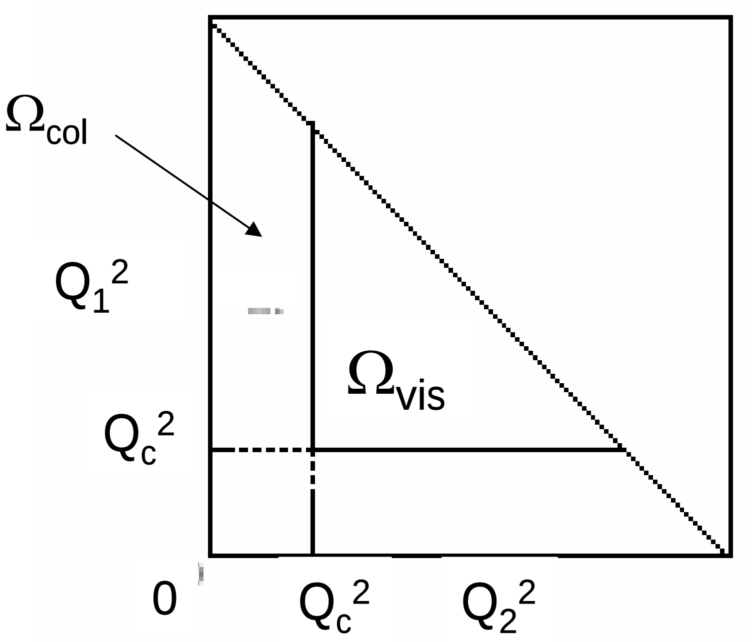
Solutions in GRACE

$$\sigma_{tree} \otimes \text{PDF/PS} + \sigma_{vis} \otimes \text{PDF/PS}$$

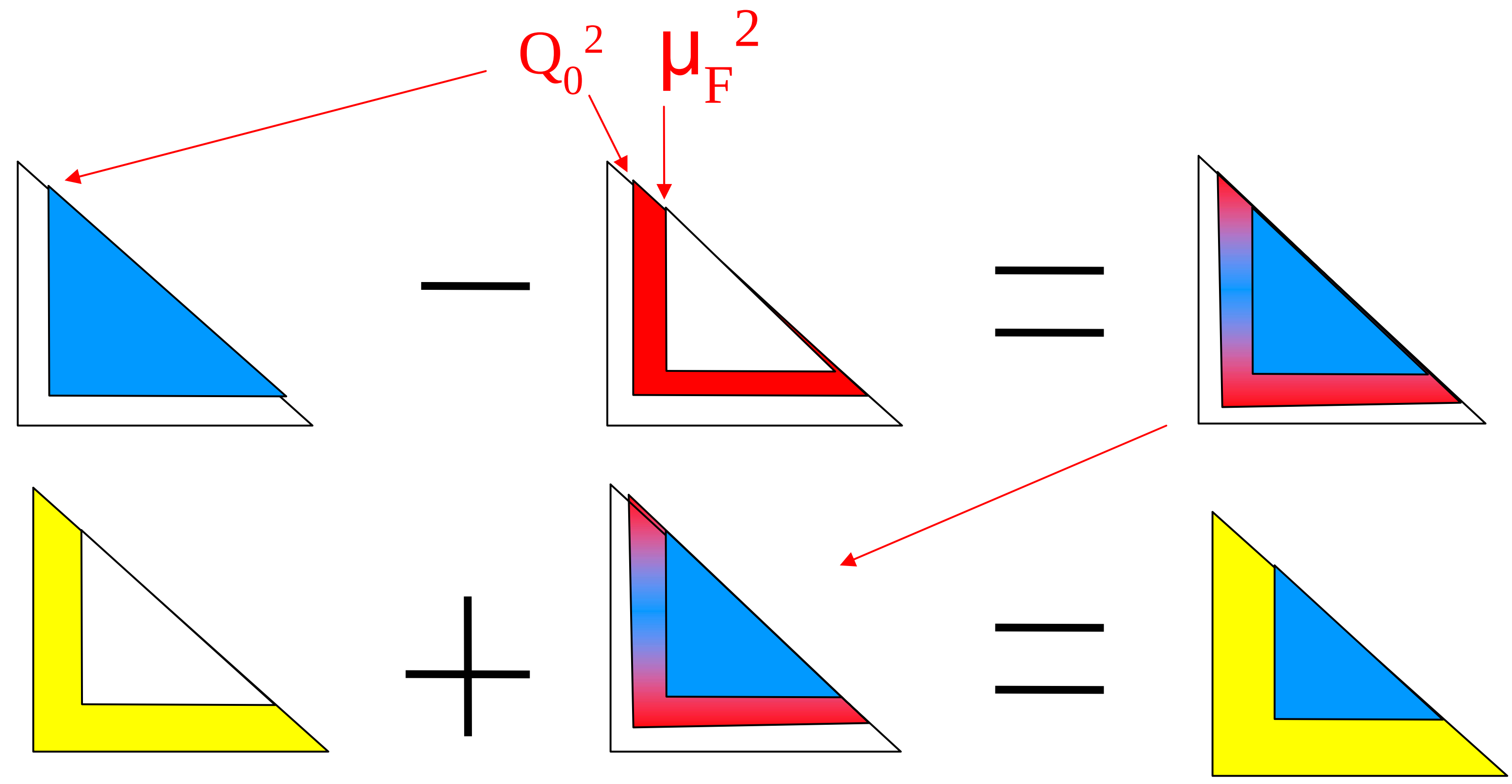
$$\rightarrow \sigma_{tree} \otimes \text{PDF/PS} + (\sigma_{vis} - \sigma_{LL}) \otimes \text{PDF/PS}$$

Leading Log subtraction





Leading Log Subtraction in GRACE



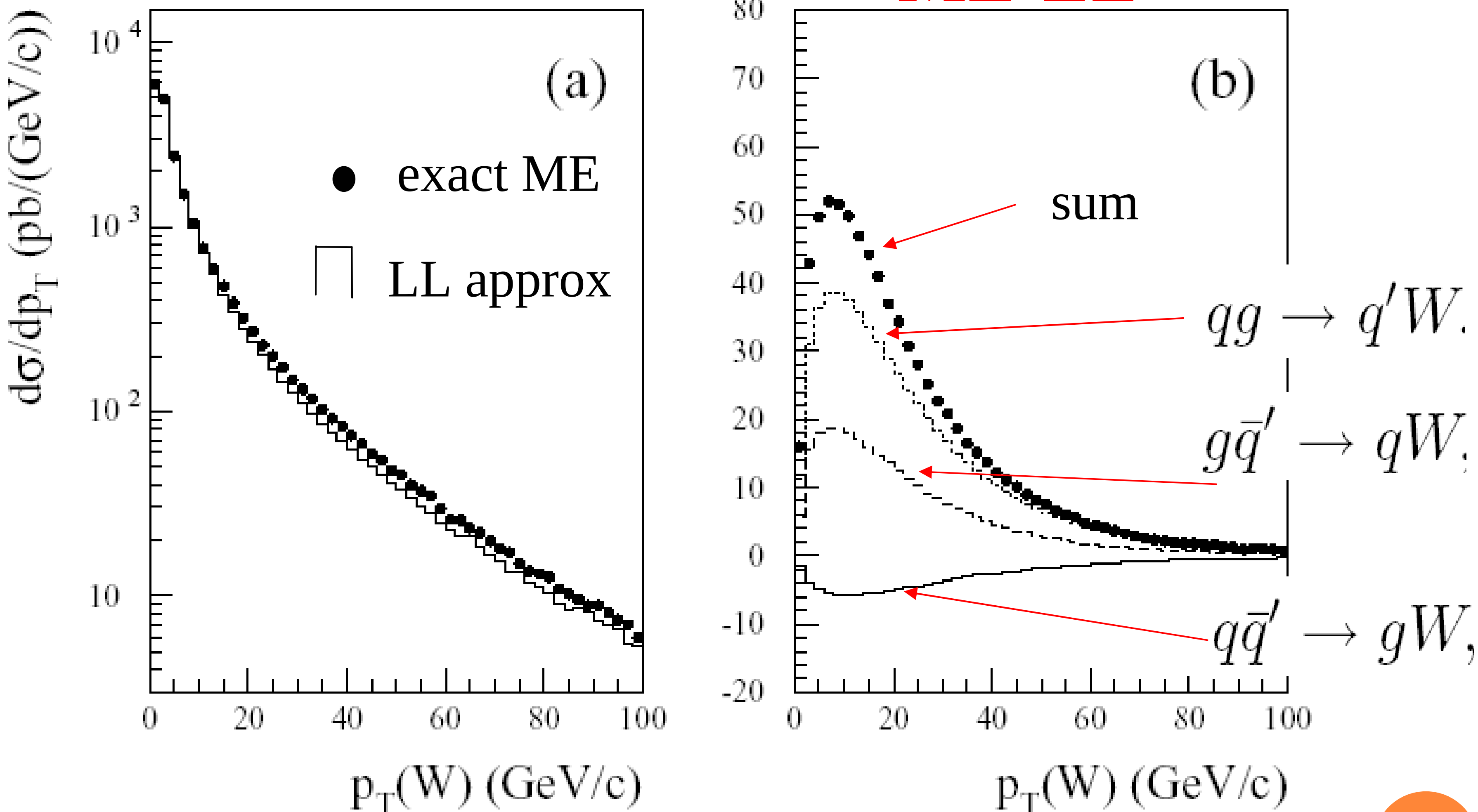
: Collinear (Leading Log) Approx.

: Exact Matrix Elements w/ PS

: Tree ME w/ PS

PP → W+jet

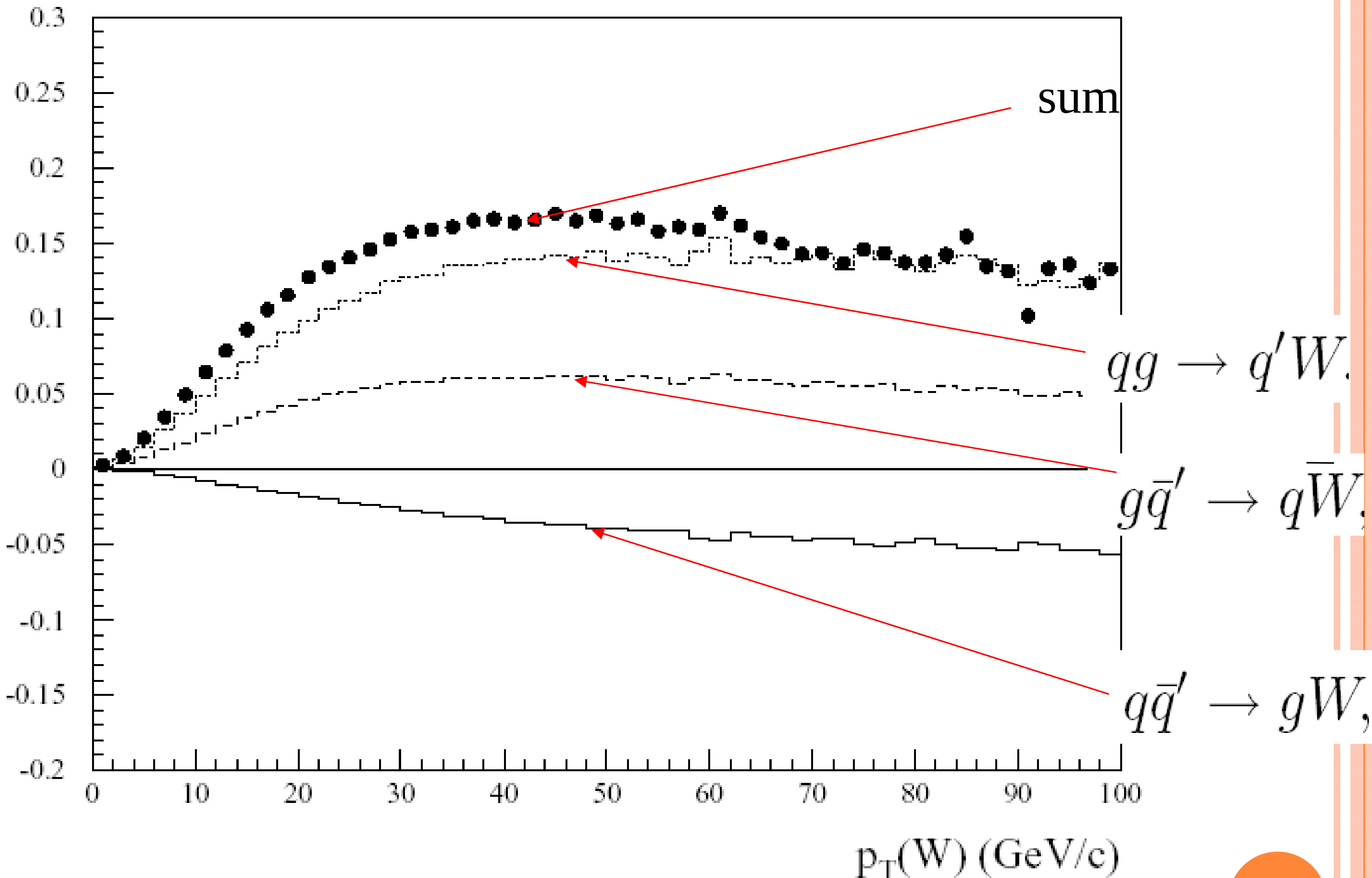
ME-LL

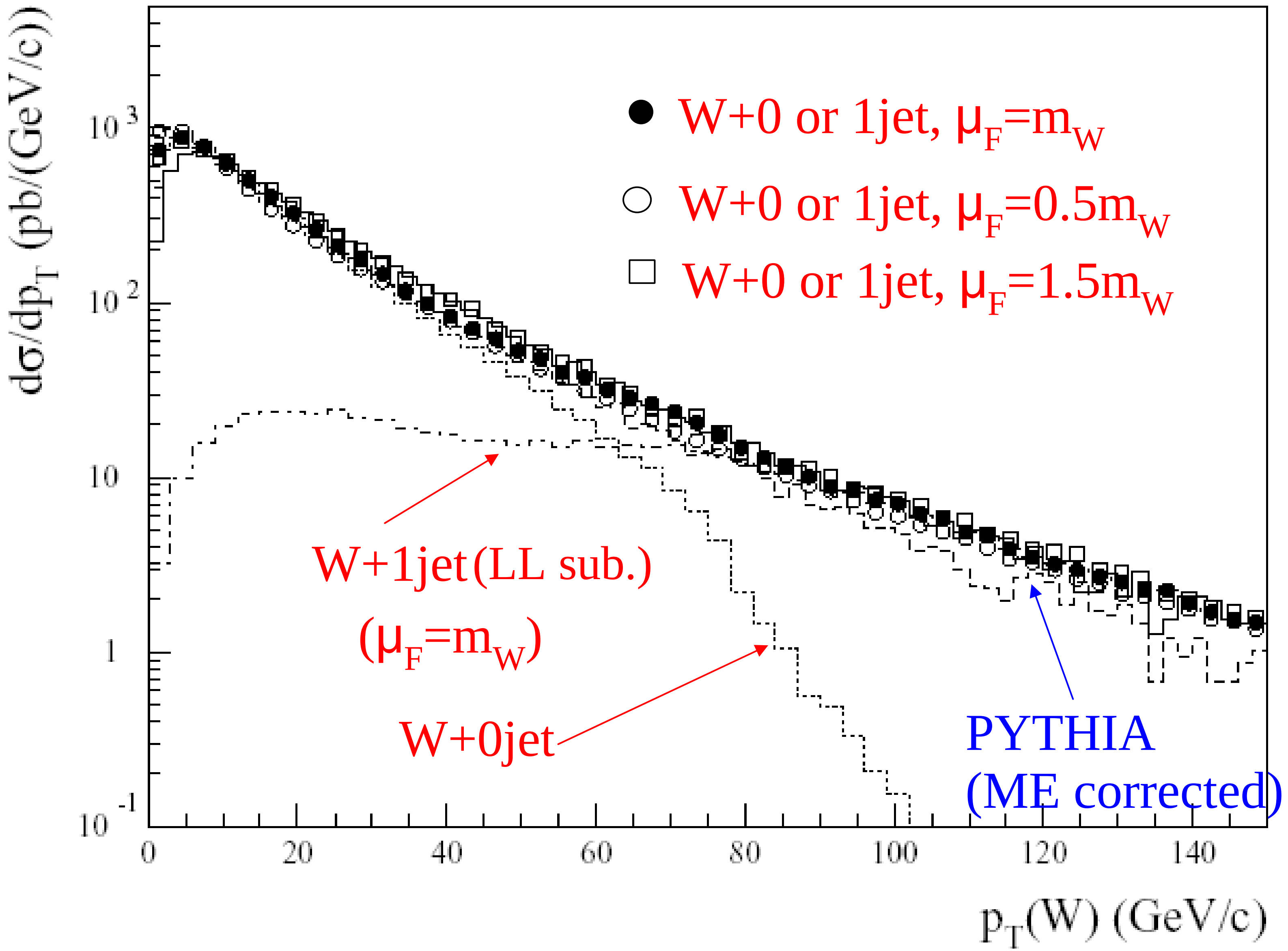


hep-ph/0702138

p_T>1 GeV

$(\sigma_{ME}-\sigma_{LL})/\sigma_{ME}$ S. Odaka





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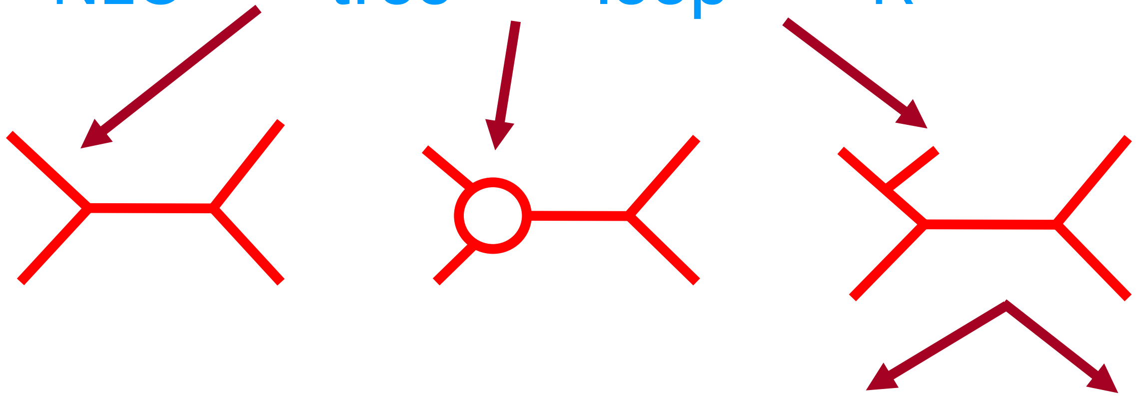
5. GRACE/NLO

- NLO Cross sections

$$\sigma_{NLO} = \sigma_{tree} + \sigma_{loop} + \sigma_R$$



$$= \sigma_{tree}(1 + \delta_V + \delta_{s/c}) + \sigma_{vis}$$

δ_V : Virtual (loop) correction

$\delta_{s/c}$: Soft/Collinear correction

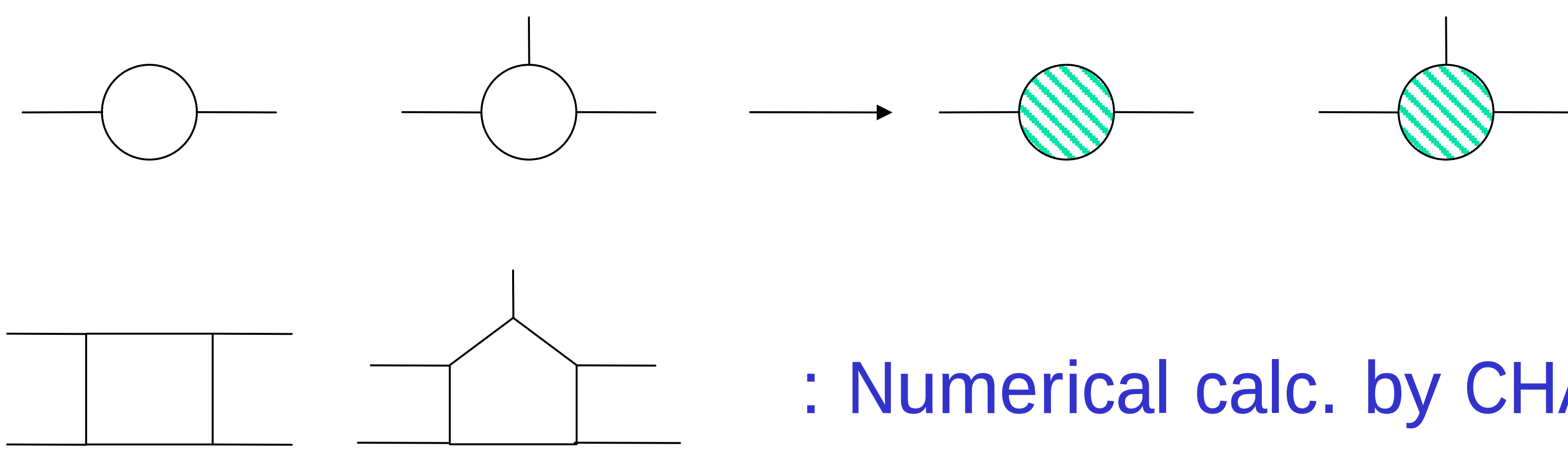
σ_{vis} : Visible jet cross section

Treatment of Loop diagrams

- Loop diagrams

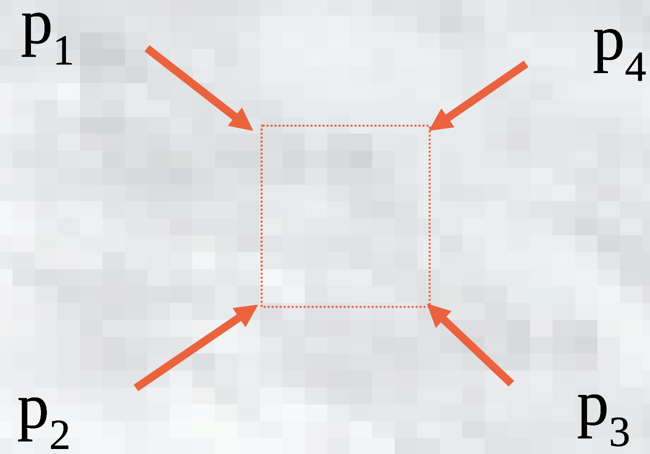
$M_V(1,2\rightarrow1,2\cdot\cdot\cdot,n)$: Effective vertices

(up to three point)



: Numerical calc. by CHANEL

Box Integral



$$J_{(4)}(s,t;p_1^2,p_2^2,p_3^2,p_4^2;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\,\frac{x^{n_x}y^{n_y}z^{n_z}}{D^{2-\varepsilon_{IR}}},$$

$$\begin{aligned}
D &= -s\,xz - t\,yw - p_1^2\,xy - p_2^2\,yz - p_3^2\,zw - p_4^2\,xw - i0,\\
w &= 1-x-y-z,\\
s &= (p_1+p_2)^2,\\
t &= (p_1+p_4)^2.
\end{aligned}$$

Y.Kurihara Eur.Phys.J.C45(2006)427

All on-shell (massless) external legs

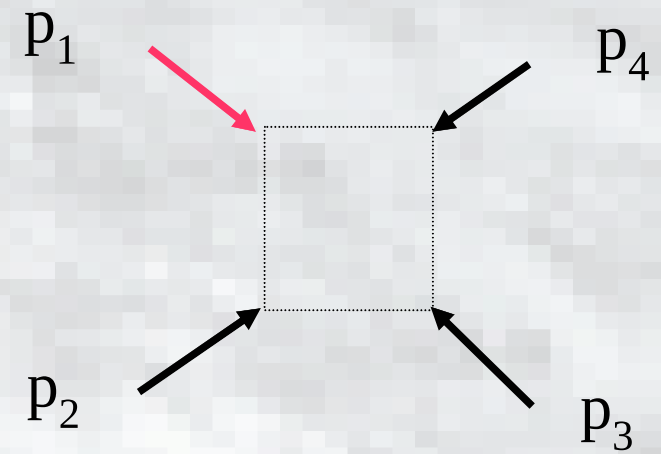
$$J_4(s,t;0,0,0,0;n_x,n_y,n_z) = \frac{1}{(4\pi)^2 s\, t} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$
$$\times \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(\frac{-t}{s}\right)^{n_x} \frac{B(1+n_z, n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})} \right.$$
$$\times\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$
$$+ \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \left(\frac{-s}{t}\right)^{l} \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} B(1+n_y, l+n_z+\varepsilon_{IR})$$
$$\left. \times\ {}_2F_1\left(1+l, l+n_z+\varepsilon_{IR}, 1+l+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right],$$

Scalar Integral

$$J_{(4)}(s,t;0,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t} \frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$
$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right]$$

This result is compared with G. Duplančić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

One off-shell box integral



$$J_4(s,t;p_1^2,0,0,0;n_x,n_y,n_z) =$$

$$\frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\frac{x^{n_x}y^{n_y}z^{n_z}}{\left(-xzs-y(1-x-y-z)t-p_1^2xy-i0\right)^{2-\varepsilon_{IR}}}$$

$$\begin{aligned}
= \quad & \frac{1}{(4\pi)^2 s\ t}B(n_x+\varepsilon_{IR},n_y+n_z+\varepsilon_{IR})n_x!\Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR}) \\
\times \quad & \left[\left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\left(\frac{-t}{s}\right)^{n_x}\frac{B(1+n_z,n_x+n_y+\varepsilon_{IR})}{\Gamma(n_x+\varepsilon_{IR})}\mathcal{I}^{(1)}\right. \\
+ \quad & \left.\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}}\sum_{l=0}^{n_x}\frac{(-1)^l B(1+n_y,l+n_z+\varepsilon_{IR})}{\Gamma(l+\varepsilon_{IR})(n_x-l)!}\mathcal{I}_l^{(2)}\right]
\end{aligned}$$

$$\mathcal{I}^{(1)} = B(1+n_z, n_x+n_y+\varepsilon_{IR})\ {}_2F_1\left(1+n_x, n_x+n_y+\varepsilon_{IR}, 1+n_x+n_y+n_z+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{s}}\right)$$

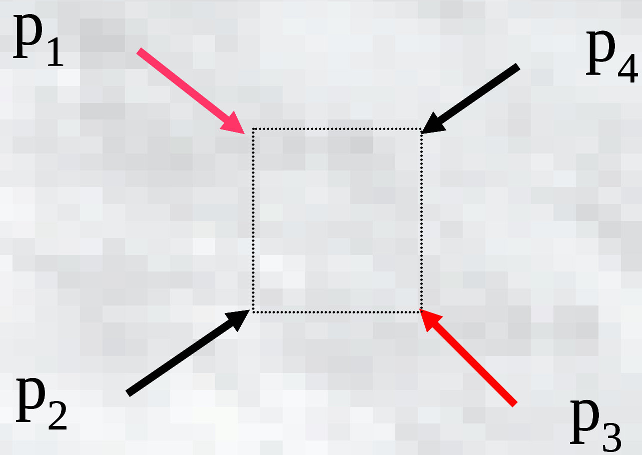
$$\begin{aligned}
\mathcal{I}^{(2)}_l &= \sum_{k_1=0}^{n_z} {}_{n_z}C_{k_1} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \sum_{k_2=0}^{n_y+k_1} {}_{n_y+k_1}C_k\ (-1)^{n_y+k_2} \left(\frac{-t}{s}\right) \\
&\times \int_0^1 dw\ \left(1+\frac{\tilde{u}}{\tilde{s}}w\right)^{-(l+1)} \left(1+\frac{\tilde{t}+\tilde{u}}{\tilde{s}}w\right)^{k_2+l-1+\varepsilon_{IR}} \\
&= \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1}\ {}_{n_y+k_1}C_k\ (-1)^{k_1+k_2} \left(\frac{s}{p_1^2 - s}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(1+\frac{u}{t}\right)^l \\
&\times \left[\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}}{\tilde{t}}\right)\right. \\
&- \left.\left(\frac{\tilde{p_1}^2}{\tilde{s}}\right)^{l+k_2+\varepsilon_{IR}}\ {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, -\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],
\end{aligned}$$

Scalar Integral

$$J_4(s,t;p_1^2,0,0,0;0,0,0) = \frac{1}{(4\pi)^2 s\, t}\frac{B(\varepsilon_{IR},\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})}{\varepsilon_{IR}}$$
$$\times \left[\left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{t}}\right) + \left(\frac{-\tilde{t}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}}{\tilde{s}}\right)\right.$$
$$\left.- \left(\frac{-\tilde{p_1}^2}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} {}_2F_1\left(1,\varepsilon_{IR},1+\varepsilon_{IR},-\frac{\tilde{u}\tilde{p_1}^2}{\tilde{t}\tilde{s}}\right)\right],$$

This result is compared with G. Duplančić, B. Nižić, Eur. Phys. J. C **20**, 357 (2001)

Two off-shell box integral



Easy case

$$J_4(s,t;p_1^2,0,p_3^2,0;n_x,n_y,n_z) = \frac{\Gamma(2-\varepsilon_{IR})}{(4\pi)^2\left(4\pi\mu_R^2\right)^{\varepsilon_{IR}}}$$

$$\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x^{n_x} y^{n_y} z^{n_z}}{\left(-xzs - y(1-x-y-z)t - p_1^2 xy - p_3^2 z(1-x-y-z) - i0\right)^{2-\varepsilon_{IR}}}$$

$$= \frac{1}{(4\pi)^2(s-p_3^2)(t-p_3^2)} B(n_x+\varepsilon_{IR}, n_y+n_z+\varepsilon_{IR}) n_x! \Gamma(\varepsilon_{IR})\Gamma(1-\varepsilon_{IR})$$

$$\times \left[\left(-\frac{\tilde{t}-p_3^2}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \left(-\frac{t-p_3^2}{s-p_3^2}\right)^{n_x} \frac{1}{\Gamma(n_x+\varepsilon_{IR})} \mathcal{I}^{(1)} + \left(\frac{-\tilde{s}}{4\pi\mu_R^2}\right)^{\varepsilon_{IR}} \sum_{l=0}^{n_x} \frac{(-1)^l}{\Gamma(l+\varepsilon_{IR})(n_x-l)!} \mathcal{I}_l^{(2)}\right]$$

$$\mathcal{I}^{(1)} = \frac{1}{n_x+\varepsilon_{IR}} \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1} \; {}_{n_y+k_1}C_{k_2} \, (-1)^{k_1+k_2} \left(\frac{p_3^2-s}{u}\right)^{n_y+k_1} (1-\alpha)^{k_2-n_x-1}$$

$$\times \left[\left(1+\frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\right)^{n_x+\varepsilon_{IR}} {}_2F_1\left(1+n_x-k_2, n_x+\varepsilon_{IR}, 1+n_x+\varepsilon_{IR}, \frac{\bar{u}/(\bar{s}-\bar{p}_3^2)+\alpha}{\alpha-1}\right) \right.$$

$$\left. - \left(\frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\right)^{n_x+\varepsilon_{IR}} {}_2F_1\left(1+n_x-k_2, n_x+\varepsilon_{IR}, 1+n_x+\varepsilon_{IR}, \frac{\alpha}{\alpha-1}\right) \right]$$

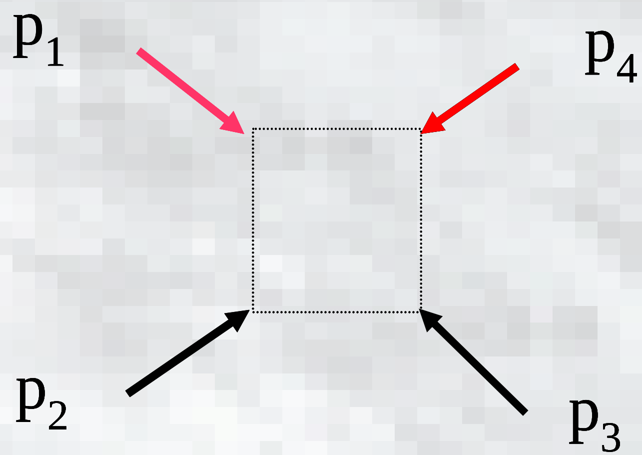
$$\mathcal{I}_l^{(2)} = \sum_{k_1=0}^{n_z} \sum_{k_2=0}^{n_y+k_1} {}_{n_z}C_{k_1} \; {}_{n_y+k_1}C_{k_2} \, (-1)^{k_1+k_2} \left(\frac{s}{s-p_1^2}\right)^{n_y+k_1} \frac{1}{l+k_2+\varepsilon_{IR}} \left(\frac{1}{1-\beta}\right)^{l+1} \left(\frac{t-p_3^2}{t+u-p_3^2}\right) \left(\frac{s}{s-p_3^2}\right)^{l}$$

$$\times \left[{}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, \frac{\beta}{\beta-1}\right) \right.$$

$$\left. - \left(\frac{\bar{p_1}^2}{\bar{s}}\right)^{l+k_2+\varepsilon_{IR}} {}_2F_1\left(1+l, l+k_2+\varepsilon_{IR}, 1+l+k_2+\varepsilon_{IR}, \frac{\beta}{\beta-1}\frac{\bar{p}_1^2}{\bar{s}}\right) \right]$$

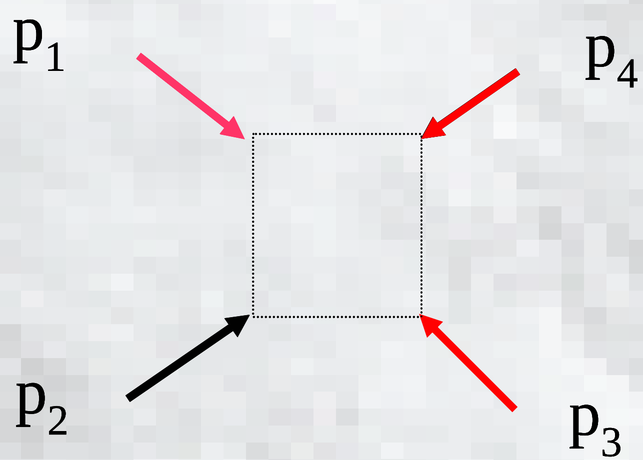
$$\alpha = \frac{\bar{p}_3^2}{\bar{t}-\bar{p}_3^2}\frac{\bar{u}}{\bar{s}-\bar{p}_3^2}, \quad \beta = \frac{\bar{u}}{\bar{s}-\bar{p}_3^2}\frac{\bar{s}}{\bar{t}+\bar{u}-\bar{p}_3^2}$$

Two off-shell box integral



Hard case

Three off-shell box integral



Tensor integral

▼

Analytic expression

by T. Kaneko

Double Counting in NLO

P(x) : Splitting function

Space/time dimension : $d=4+2\varepsilon_{IR}$

Two sources of Double Counting

$$\sigma_{NLO}=\sigma_{tree}(1+\delta_V+\delta_{s/c})\otimes \text{PDF/PS} + \sigma_{vis}\otimes \text{PDF/PS}$$

$1/\varepsilon^2_{IR}, 1/\varepsilon_{IR}$ cancellation

$$\frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x)$$

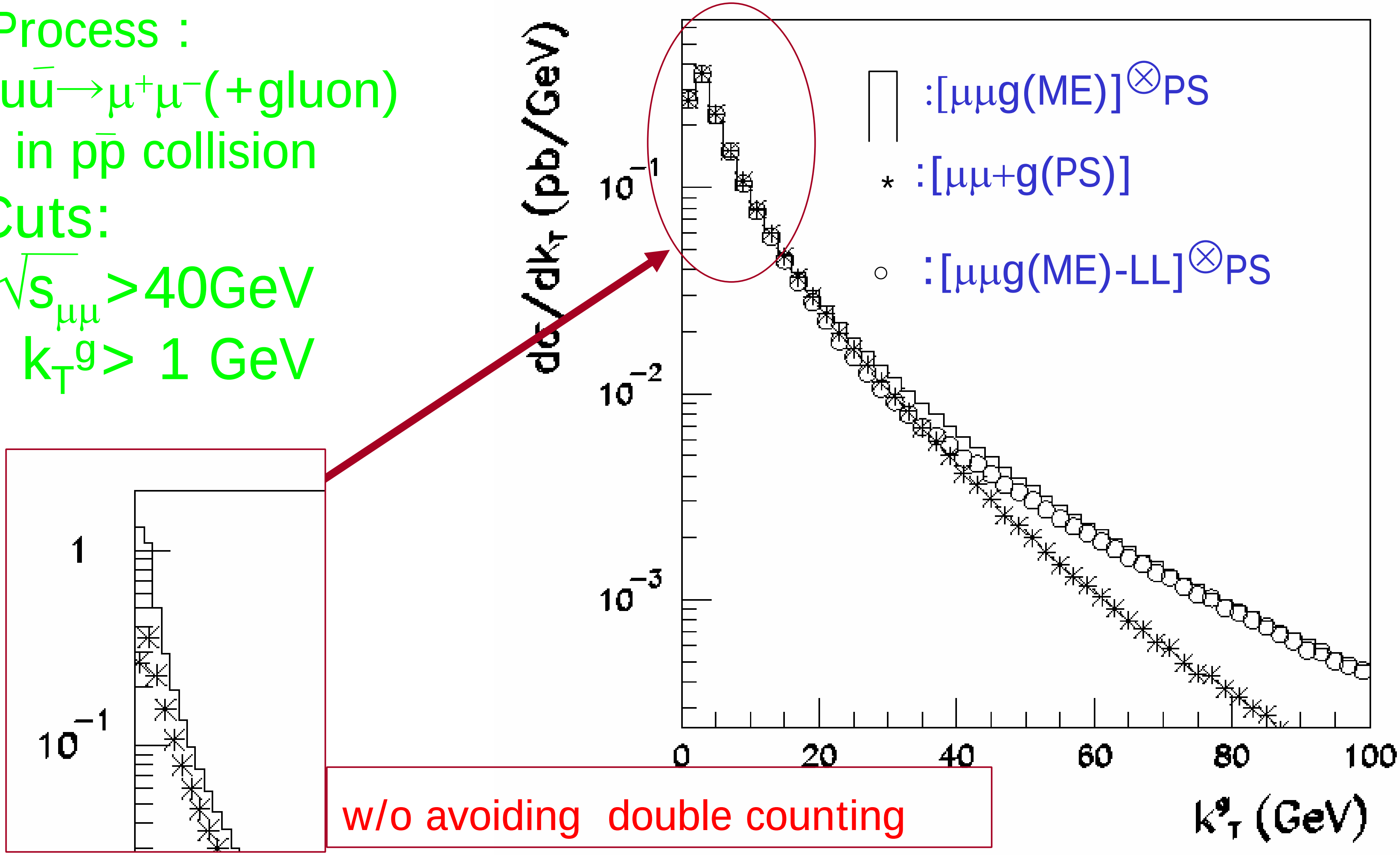
Solutions in GRACE/NLO

$$[\sigma_{tree}(1+\delta_V+\delta_{s/c})\otimes \text{PDF/PS}\ ] + [\sigma_{vis} - \sigma_{LL}\ ]\otimes \text{PDF/PS}$$

Drell-Yan process

$k^g{}_T$ Test

- Process : $u\bar{u} \rightarrow \mu^+\mu^-$(+gluon) in $p\bar{p}$ collision
- Cuts: $\sqrt{s_{\mu\mu}} > 40$GeV, $k_T{}^g > 1$ GeV



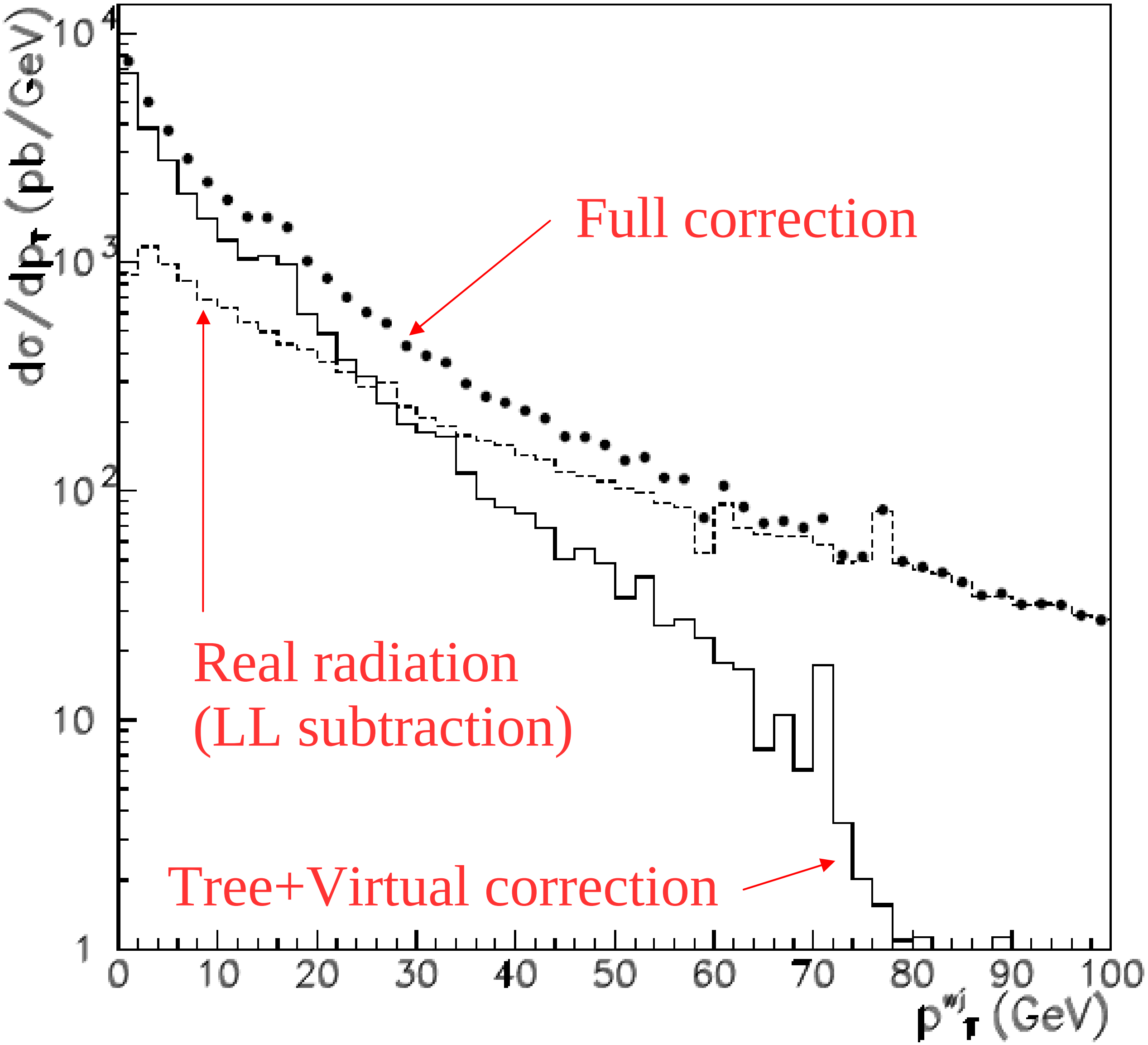
Test process: $PP \to W+jet$

- $\sqrt{s}=14$ TeV
- PDF: CTEQ5L
- Cuts: $p_T^W>20GeV$, $p_T^g>20GeV$
- $\mu_F=\mu_R=m_W=80.2GeV$

IR cancellation

$1/\varepsilon_{IR}^2, 1/\varepsilon_{IR}$ $O(10^{-10})$

Transverse momentum distribution of W-jet



$P^W_T > 20$GeV

• $\sigma_{NLO} = 7.06 \cdot 10^4$ pb

Test process: $PP\rightarrow\gamma\gamma$ under FJPPL collaboration

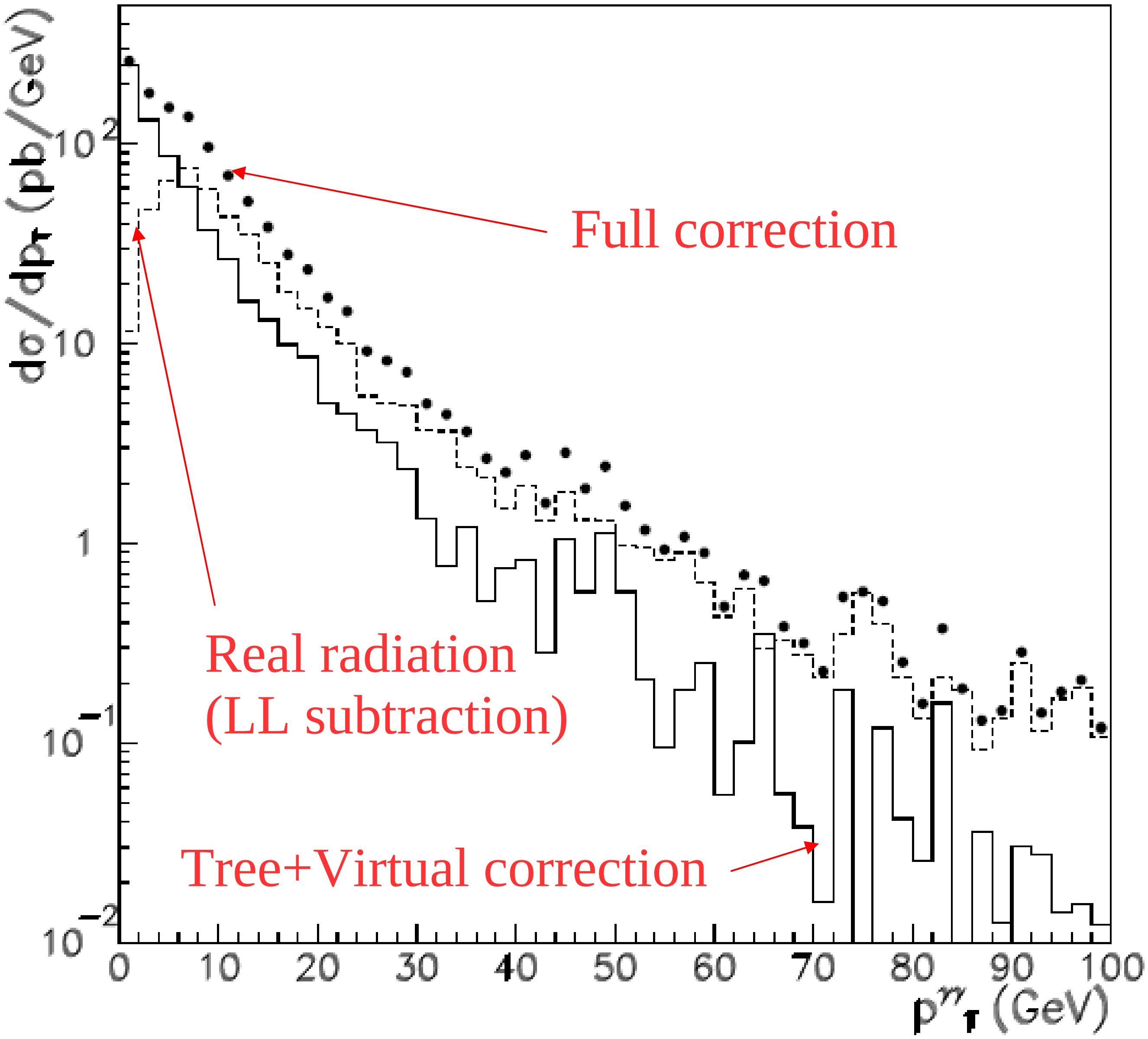
- $\sqrt{s}=14$ TeV
- PDF: CTEQ5L
- Cuts: $E_\gamma>10\text{GeV}, 10^o<\theta_\gamma<170^o, \theta_{\gamma\gamma}>10^o$
- $\mu_F=\mu_R=s_0$

  - σ_{tree} $=7.35\cdot10^2$ pb
  - $\sigma_{tree+virtual}$ $=1.32\cdot10^3$ pb
  - $\sigma_{real\ radiation}$ $=9.32\cdot10^2$ pb

IR cancellation

$1/\varepsilon_{IR}{}^2, 1/\varepsilon_{IR}$ $O(10^{-10})$

Transverse momentum distribution of γγ



- $\sigma_{NLO}=2.11\bullet 10^{3}$ pb

6. Summary

(1) QCD-NLO Matrix Elements
- Automatic generation by GRACE

(2) GR@PPA framework is ready for LHC

(3) Double count treatment
- LL-subtraction method

(4) From GRACE/NLO to GR@PPA/NLO
- Drell-Yan process
- prompt photon
- Di-photon
- V+1jet,2jet, VV+1 jet