Matched Showers with Vincia

W.G., Kosower, Skands: hep-ph/0707.3652

Frederix, W.G., Kosower, Skands: hep-ph/0803.0494

- The Vincia Parton Shower Monte Carlo
- From Matrix Elements to Sudakov
- From Sudakov to Shower
- From Showers to Matrix Elements: Matching
- Vincia Status and Plans

Loops & Legs 2008
Sondershausen
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The Vincia Shower Monte Carlo

- Matching is the driving principle behind VINCIA:
  - A shower MC which can be added to a fixed order LO, NLO, NNLO calculation.
  - A shower MC which can easily “absorb” LO/NLO/NNLO matrix elements to further improve its predictive power of the fully exclusive final state

- Vincia fully embedded in PYTHIA 8

Sjöstrand, Mrenna, Skands, hep-ph/0710.3820.

  - It replaces the parton shower part and matrix element generators inside PYTHIA.
  - It uses the PYTHA non-perturbative framework
  - It uses the PYTHIA interface (if you have PYTHIA up and running, VINCIA can be plugged in and can be used instantly).

From Matrix Elements to Sudakov

- Calculating an observable in fixed order:

“Experimental” distribution of observable O in production of X:

Fixed Order (all orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

k : legs $\quad$ ℓ : loops $\quad$ {p} : momenta

- For a NLO parton level generator a subtraction formalism is used to cancel the soft/collinear divergences between virtual and real radiation

Subtracted NLO

$$\left.\frac{d\sigma}{dO}\right|_{\mathrm{NLO}} = \int d\Phi_X \left(\boxed{\left|M_X^{(0)}\right|^2 + 2\mathrm{Re}\left[M_X^{(0)} M_X^{(1)*}\right]} + \boxed{\left(\int \frac{d\Phi_{X+1}}{d\Phi_X} A\right) \times \left|\hat{M}_X^{(0)}\right|^2} \right) \delta(O - O(\{p\}_X))$$
$$+ \int d\Phi_{X+1} \left(\boxed{\left|M_{X+1}^{(0)}\right|^2} - \boxed{A \left|M_X^{(0)}\right|^2} \right) \delta(O - O(\{p\}_{X+1})) + \cdots$$

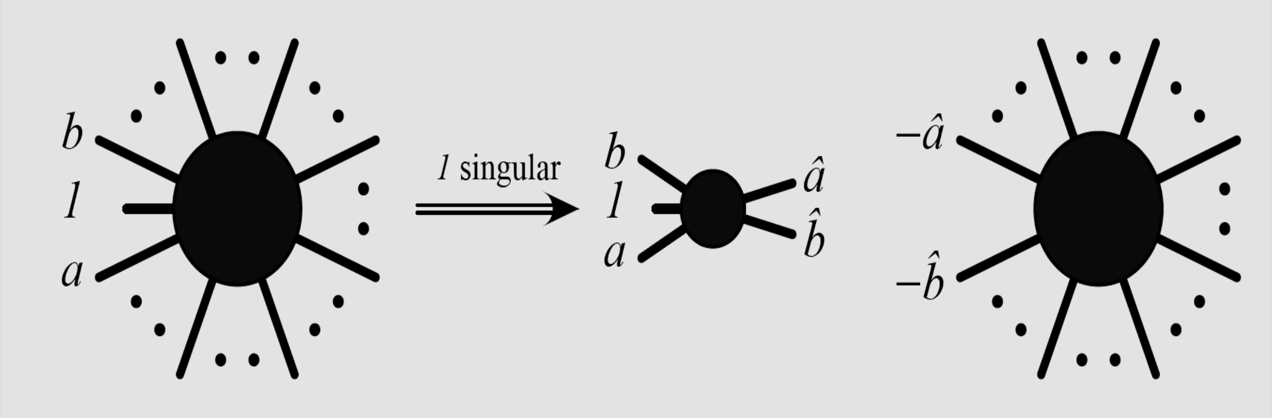
From Matrix Elements to Sudakov

- The subtraction function contains the correct soft/collinear singular structure

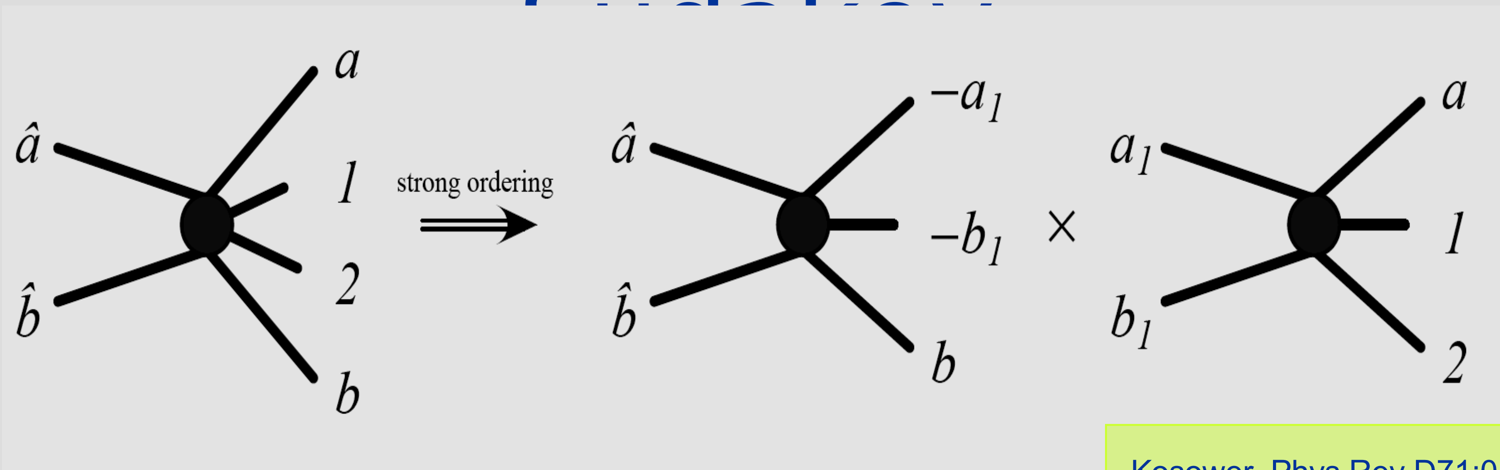
$$\boxed{\left|M_{X+1}^{(0)}\right|^2 \xrightarrow{\text{unresolved}} A \times \left|M_X^{(0)}\right|^2}$$

- Color ordered amplitude: dipole/antenna factorization

Kosower PRD57(1998)5410; Campbell,Cullen,Glover EPJC9(1999)245.
(see also Gustafson, PLB175(1986)453; Lönnblad (ARIADNE), CPC71(1992)15.
Azimov, Dokshitzer, Khoze, Troyan, PLB165B(1985)147.)



From Matrix Elements to Sudakov



Kosower, Phys.Rev.D71:045016,2005.

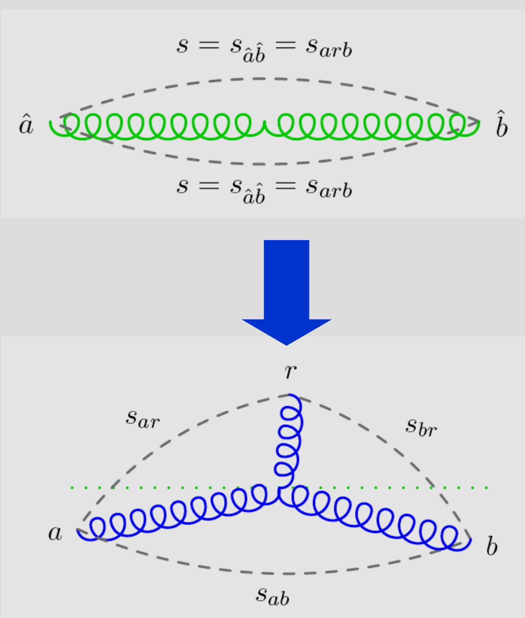
- The antenna functions have the expected strong ordering behavior which captures the leading logarithms
- As a result we can resum the antenna functions into a Sudakov factor which will form the basis for the shower
- It will also intertwine the NLO parton generator with the shower.

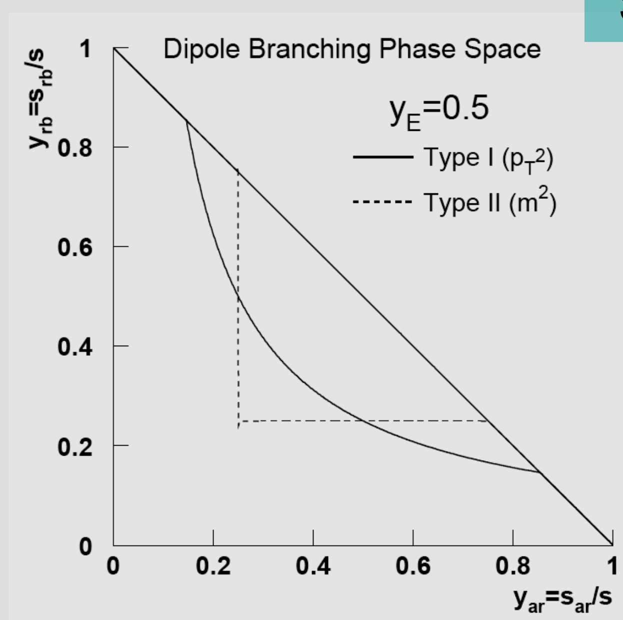
From Matrix Elements to Sudakov

- The Sudakov function reflect the integrated probability the system does not change state between time t_1 and t_2 ($t=1/Q_{resolution}$)

$$\Delta(t_{\rm n}, t_{\rm end}; \{p\}_n) = \exp\left(- \int_{t_{\rm n}}^{t_{\rm end}} {\rm d}t_{n+1} \sum_{i\in\{n\to n+1\}} \int \frac{{\rm d}\Phi_{n+1}^{[i]}}{{\rm d}\Phi_n} \delta(t_{n+1} - t^{[i]}(\{p\}_{n+1})) A_i(\{p\}_n \to \{p\}_{n+1}) \right)$$

Summed over all dipoles





From Sudakov to Shower

- The Sudakov is the first step towards a shower

Fixed Order $$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left|\sum_{\ell=0} M_{X+k}^{(\ell)}\right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Matched Shower $$\left.\frac{d\sigma}{dO}\right|_{\mathrm{matched}} = \sum_{k=0}^{K} \int d\Phi_{X+k} \left|\sum_{l=0}^{L} w_{X+k}^{(l)}\right| \times S(\{p\}_{X+k}, O)$$

- The unitary shower function, or evolution operator *S*, "evolves" the phase space point $X \to X+1 \to \ldots \to X+N$ as a function of resolution
- The matching condition defines the parton generators *W* (expanding the shower function in branchings and α_s ***must*** agree with the fixed order expansion)

From Sudakov to Shower

Pure Shower (all orders) $$\left.\frac{d\sigma_X}{d\mathcal{O}}\right|_{\text{PS}} = \int d\Phi_X \, w_X \, S(\{p\}_X, \mathcal{O})$$

- Using the Sudakov $\Delta(t_1,t_2)$ we can define the evolution operator through a Markov Chain.
- Subsequently we can implement the Markov Chain in a numerical algorithm.

$$S(\{p\}_X, \mathcal{O}) = \underbrace{\delta\left(\mathcal{O} - \mathcal{O}(\{p\}_X)\right) \Delta(t_X, t_{\text{had}})}_{X+0 \text{ exclusive above } 1/t_{\text{had}}}$$

← "X + nothing"

"X+something"

$$+ \underbrace{\int_{t_X}^{t_{\text{had}}} dt_{X+1} \sum_i \int \frac{d\Phi_{X+1}^{[i]}}{d\Phi_X} \delta(t_{X+1} - t^{[i]}(\{p\}_{X+1})) \Delta(t_X, t_{X+1}) A_i(\ldots) S(\{p\}_{X+1}, \mathcal{O})}_{X+1 \text{ inclusive above } 1/t_{\text{had}}}$$

From Sudakov to Shower

- The final answer depends on
  - The choice of evolution variable
  - The dipole/antenna functions (finite/sub-leading terms not fixed)
  - The phase space map ($d\Phi_{n+1}/d\Phi_n$)
  - The renormalization scheme (argument of α_s)
  - The infrared cutoff contour (Hadronization cutoff)
- Step 1: Quantify these uncertainty
  - Vary these within reasonable limits
- Step 2, Systematically reduce uncertainties

  Understand the importance of each and how it is canceled by
  - Matching to fixed order matrix elements, at LO, NLO, NNLO, …
  - Sub-leading logarithms, sub-leading color, etc.

From Sudakov to Shower

- Starting point: “GGG” antenna functions, e.g., gg→ggg:

Gehrmann-De Ridder, Gehrmann, Glover, JHEP 09 (2005) 056

$$f_3^0(p_a,p_r,p_b) = \frac{1}{s_{[i]}}\left[(1-y_{ar}-y_{rb})\left(\underbrace{\frac{2}{y_{ar}y_{rb}}}_{\text{“soft”}} + \underbrace{\frac{y_{ar}}{y_{rb}} + \frac{y_{rb}}{y_{ar}}}_{\text{“collinear”}}\right) + \frac{8}{3}\right]$$

$$y_{ar} = s_{ar} / s_i$$

s_i = invariant mass of i’th dipole-antenna

- Generalize to arbitrary double Laurent series:

Frederix, WG, Kosower, Skands : Les Houches NLM, arxiv:0803.0494

$$A(s_{ar},s_{rb}) = \frac{4\pi\alpha_s(\mu_R)N_C}{s_{arb}} \sum_{\alpha,\beta\geq -1} C_{\alpha\beta}\frac{s_{ar}^{\alpha}s_{rb}^{\beta}}{s_{arb}^{\alpha+\beta}}$$

Singular parts fixed, finite terms arbitrary

- Can make shower systematically “softer” or “harder”

  We will see later how this variation is explicitly canceled by matching

  - quantification of uncertainty
  - quantification of improvement by matching

From Sudakov to Shower

$$A_{ggg} \sim \frac{P_{gg}(z)}{y};\; z = y_{ar} + y_{rb};\; y^{-1} = y_{ar}^{-1} + y_{rb}^{-1}$$

$$A(s_{ar}, s_{rb}) = \frac{4\pi\alpha_s(\mu_R) N_C}{s_{arb}} \sum_{\alpha,\beta \geq -1} C_{\alpha\beta} \frac{s_{ar}^{\alpha} s_{rb}^{\beta}}{s_{arb}^{\alpha+\beta}}$$

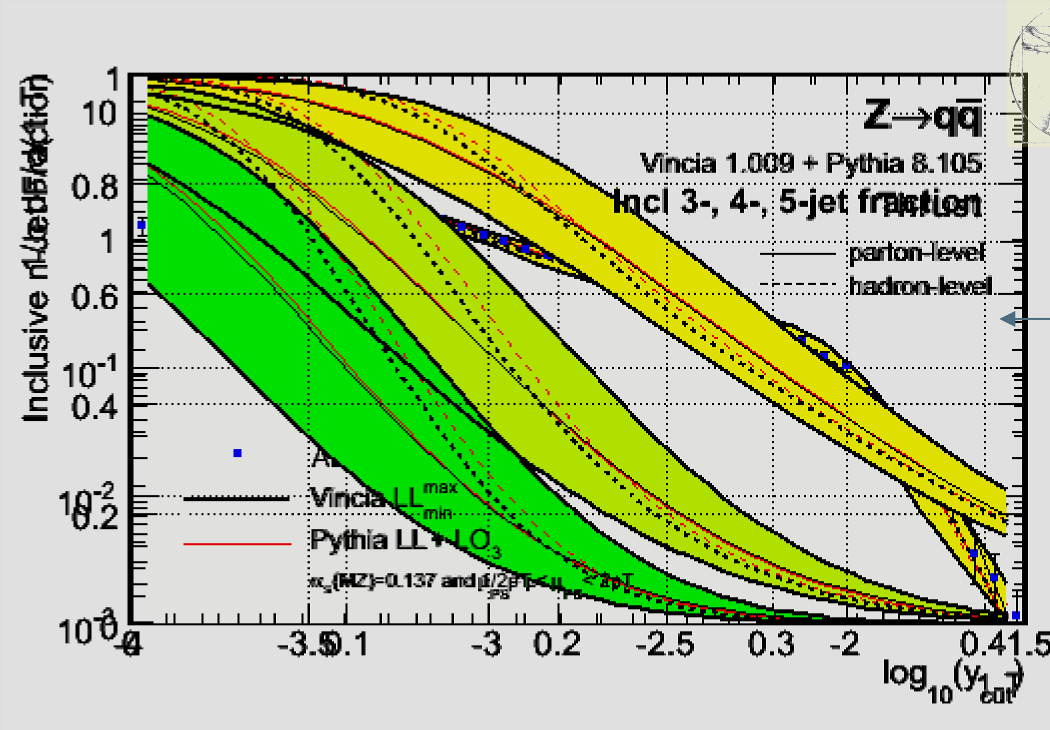
Frederix, Giele, Kosower, PS : Les Houches 'NLM', arxiv:0803.0494

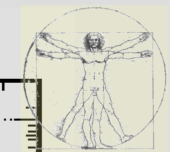
	C_{-1-1}	C_{-10}	C_{0-1}	C_{-11}	C_{1-1}	C_{-12}	C_{2-1}	C_{00}	C_{10}	C_{01}
GGG										
$q\bar{q} \to qg\bar{q}$	2	-2	-2	1	1	0	0	0	0	0
$qg \to qgg$	2	-2	-2	1	1	0	-1	$\frac{5}{2}$	-1	$\frac{3}{2}$
$gg \to ggg$	2	-2	-2	1	1	-1	-1	$\frac{8}{3}$	-1	-1
$gg \to q\bar{q}'q'$	0	0	$\frac{1}{2}$	0	-1	0	1	$-\frac{1}{2}$	1	0
$gg \to g\bar{q}q$	0	0	$\frac{1}{2}$	0	-1	0	1	-1	1	$\frac{1}{2}$
ARIADNE										
$q\bar{q} \to qg\bar{q}$	2	-2	-2	1	1	0	0	0	0	0
$qg \to qgg$	2	-2	-3	1	3	0	-1	0	0	0
$gg \to ggg$	2	-3	-3	3	3	-1	-1	0	0	0
$gg \to q\bar{q}'q'$	0	0	$\frac{1}{2}$	0	-1	0	1	-1	1	$\frac{1}{2}$
$gg \to g\bar{q}q$	0	0	$\frac{1}{2}$	0	-1	0	1	-1	1	$\frac{1}{2}$
ARIADNE2 (reparametrization of ARIADNE functions à la GGG, for comparison)										
$q\bar{q} \to qg\bar{q}$	2	-2	-2	1	1	0	0	0	0	0
$qg \to qgg$	2	-2	-2	1	1	0	-1	-1	0	0
$gg \to ggg$	2	-2	-2	1	1	-1	-1	$-\frac{4}{3}$	-1	-1

Table 1: Laurent coefficients for massless LL QCD antennae ($\hat{a}\hat{b} \to arb$). The coefficients with at least one negative index are universal (apart from a reparametrization ambiguity for gluons). For "GGG" (the defaults in VINCIA), the finite terms

From Sudakov to Shower

- The unknown finite terms are a major source of uncertainty
  - DGLAP has some, GGG have others, ARIADNE has yet others, etc…
  - They are arbitrary (and in general process-dependent → don't tune!)





Varying finite terms only

with

$\alpha_s(M_Z) = 0.137$,

$\mu_{PS} = p_T$,

$p_{Thad} = 0.5$ GeV

(huge variation with μ_{PS} from <u>pure</u> LL point of view, but NLL tells you using p_T at LL → (N)LL.)

From Showers to Matrix Elements

WG, Kosower, Skands : hep-ph/0707.3652

- Subtracted matching is based on expanding out the Sudakov functions in the Markov chain (an expansion in virtuality)

$$S(\{p\}_X,\mathcal{O}) = \underbrace{\delta\left(\mathcal{O}-\mathcal{O}(\{p\}_X)\right)\Delta(t_X,t_{\mathrm{had}})}_{X+0\text{ exclusive above }1/t_{\mathrm{had}}}$$

$$\Delta(t)\to 1-\frac{\alpha_S N_C}{2\pi}\int_t \frac{d\Phi_3}{d\Phi_2}A(\{p\}_2\to\{p\}_3)+O(\alpha_S^2)$$

$$+\underbrace{\int_{t_X}^{t_{\mathrm{had}}}\mathrm{d}t_{X+1}\sum_i\int\frac{\mathrm{d}\Phi_{X+1}^{[i]}}{\mathrm{d}\Phi_X}\delta(t_{X+1}-t^{[i]}(\{p\}_{X+1}))\Delta(t_X,t_{X+1})A_i(\ldots)S(\{p\}_{X+1},\mathcal{O})}_{X+1\text{ inclusive above }1/t_{\mathrm{had}}}$$

- The parton generators in this matched shower (including simultaneous tree- and 1-loop matching for any number of legs) are determined by equating the expanded shower with the fixed order (known) matrix elements:

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{MS}} = \sum_{k=0}^{n}\int\mathrm{d}\Phi_{X+k}\left(w_{X+k}^{(R)}+w_{X+k}^{(V)}\right)S(\{p\}_{X+k},\mathcal{O})$$

$$\equiv\sum_{k=0}^{n}\int d\Phi_{X+k}\left|M_{X+k}^{(0)}+M_{X+k}^{(1)}\right|^2\delta\left(O-O(\{p\}_{X+k})\right)+\text{higher order}+\text{higher branchings}$$

From Showers to Matrix Elements

- The result of matching to a NLO calculation is that the parton level generators of the shower are the antenna/dipole subtracted matrix elements:

$$w_X^{(V)} = 2\,\mathrm{Re}[M_X^{(0)} M_X^{*(1)}] + |M_X^{(0)}|^2 \sum_i \int_{\mathrm{all}\,t} \frac{\mathrm{d}\Phi_{X+1}^{[i]}}{\mathrm{d}\Phi_X} A_i(\ldots) + \mathcal{O}(t_X/t_{\mathrm{had}})$$

$$w_{X+1}^{(R)} = |M_{X+1}^{(0)}|^2 - \sum_{i \in X \to X+1} A_i(\ldots) |M_{X+0}^{(0)}(\{\hat{p}_i\}_{X+0})|^2$$

$$A(s_{ar}, s_{rb}) = \frac{4\pi\alpha_s(\mu_R) N_C}{s_{arb}} \sum_{\alpha,\beta \geq -1} C_{\alpha\beta} \frac{s_{ar}^{\alpha} s_{rb}^{\beta}}{s_{arb}^{\alpha+\beta}}$$

- Given a NLO calculation using a subtraction scheme one can easily add the shower with the identical subtraction function in the Sudakov (i.e. matched).

$$\left.\frac{d\sigma}{dO}\right|_{\text{NLO Matched}} = \int d\Phi_X (w_X^{(R)} + w_X^{(V)}) \times S(\{p\}_X, O) + \int d\Phi_{X+1} w_{x+1}^{(R)} \times S(\{p\}_{X+1}, O)$$

From Showers to Matrix Elements

- E.g. I want to make predictions for a 4 jet observable at LEP using the one-loop Z→ 4 parton and tree level Z→ 5 parton matrix elements.
- Fixed order parton level generator:

$$\left.\frac{d\sigma}{dO}\right|_{\mathrm{NLO}} = \int d\Phi_4 \left(\left|M_4^{(0)}\right|^2 + 2\mathrm{Re}\left[M_4^{(0)} M_4^{(1)*}\right] + \left(\int \frac{d\Phi_5}{d\Phi_4} A\right) \times \left|\hat{M}_4^{(0)}\right|^2 \right) \delta\left(O - O\left(\{p\}_4\right)\right)$$
$$+ \int d\Phi_5 \left(\left|M_5^{(0)}\right|^2 - A\left|M_4^{(0)}\right|^2 \right) \delta\left(O - O\left(\{p\}_5\right)\right)$$

- I can now add the matched parton shower to get the showered and hadronized prediction of my observable:

$$\left.\frac{d\sigma}{dO}\right|_{\mathrm{NLO\ Matched}} = \int d\Phi_4 \left(w_4^{(R)} + w_4^{(V)} \right) \times S\left(\{p\}_4, O\right) + \int d\Phi_5 w_5^{(R)} \times S\left(\{p\}_5, O\right)$$

Or:

- Calculate the NLO parton generator in a subtraction scheme
- Replace the delta functions with the shower operator
- Equate the antenna function used in the subtraction with the one used in Sudakov of the VINCA shower MC

From Showers to Matrix Elements

- This is how fixed order calculators would think of using a shower MC.
- However, the shower MC community is much more ambitious:
  - They view the shower as a fully exclusive simulation of real events (i.e. it should predict *any* observable).
  - Any known fixed order tree-level and one-loop amplitude should just be inserted into the shower MC to further enhance the predictive power.
  - An implementation through a subtraction scheme becomes difficult

$$\boxed{\left.\frac{d\sigma}{dO}\right|_{\text{LO Matched}} = \sum_{k=2}^{K} \int d\Phi_{X+k}\, w_{x+k}^{(R)} \times S\left(\left\{p\right\}_{X+k}, O\right) \qquad w_{X+k}^{(R)} = \left|M_{X+k}^{(0)}\right|^2 - \sum_i A_i(\cdots)\left|M_{X+k+1}^{(0)}\right|^2}$$

  - We now suddenly have un-subtracted sub-leading logarithms all over. An additional regulator on these logarithms has to be introduced…
  - Can we do better?

From Showers to Matrix Elements

- In general one wants a shower MC where we can insert any set of tree-level, one-loop,… calculated matrix elements
  - Subtraction method problematic. One needs to “regulate” color suppressed logs, non-leading logs,…
- Example: match the shower to Z → 2,3,4 partons matrix elements at LO
  - Some other method desirable → re-weighed matching
  - Veto branchings with probability

Sjöstrand, Be

Norrbin, S

$$w_{\text{accept}} = \frac{|M_n|^2}{|M_{\text{shower}}|^2} = \frac{|M_n|^2}{A\times|M_{\text{n-1}}|^2} \quad \left(\text{with } A\times|M_{\text{n}}\right.$$

  - This modifies the antenna function such that

$$w_n = |M_n|^2 - \hat{A}\times|M_{n-1}|^2 = 0 \;\; \text{for} \;\; n = 3,4,\ldots$$

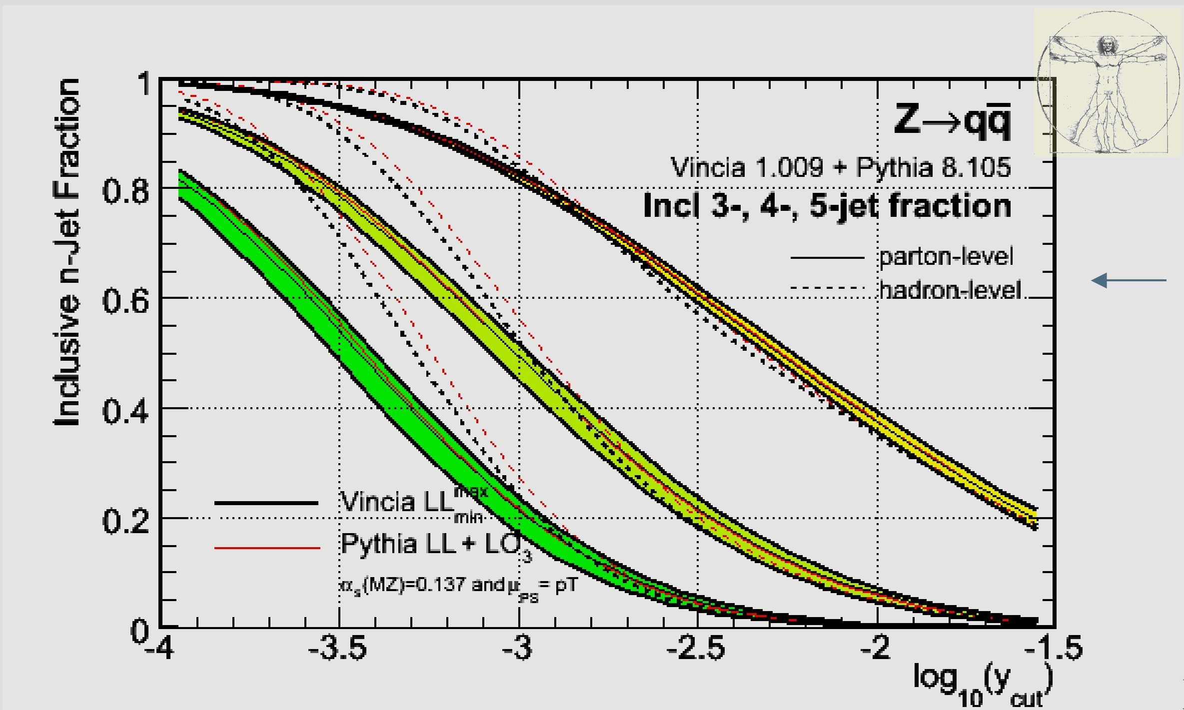
  - The re-weighting matched shower is now simply

$$\frac{d\sigma}{dO} = \int d\,\Phi_2 w_2(Z \to q\bar{q})\times S(\{p_1, p_2\}, O)$$

- Exponentiates the whole matrix element
- This regulated the non-leading and color suppressed logs by exponentiation
- Can easily be extended for one-, two-,… loop matching
- Unitary shower, so we maintain LO normalization

From Showers to Matrix Elements

- The unknown finite terms are a major source of uncertainty
  - DGLAP has some, GGG have others, ARIADNE has yet others, etc…
  - They are arbitrary (and in general process-dependent)



Varying finite terms only

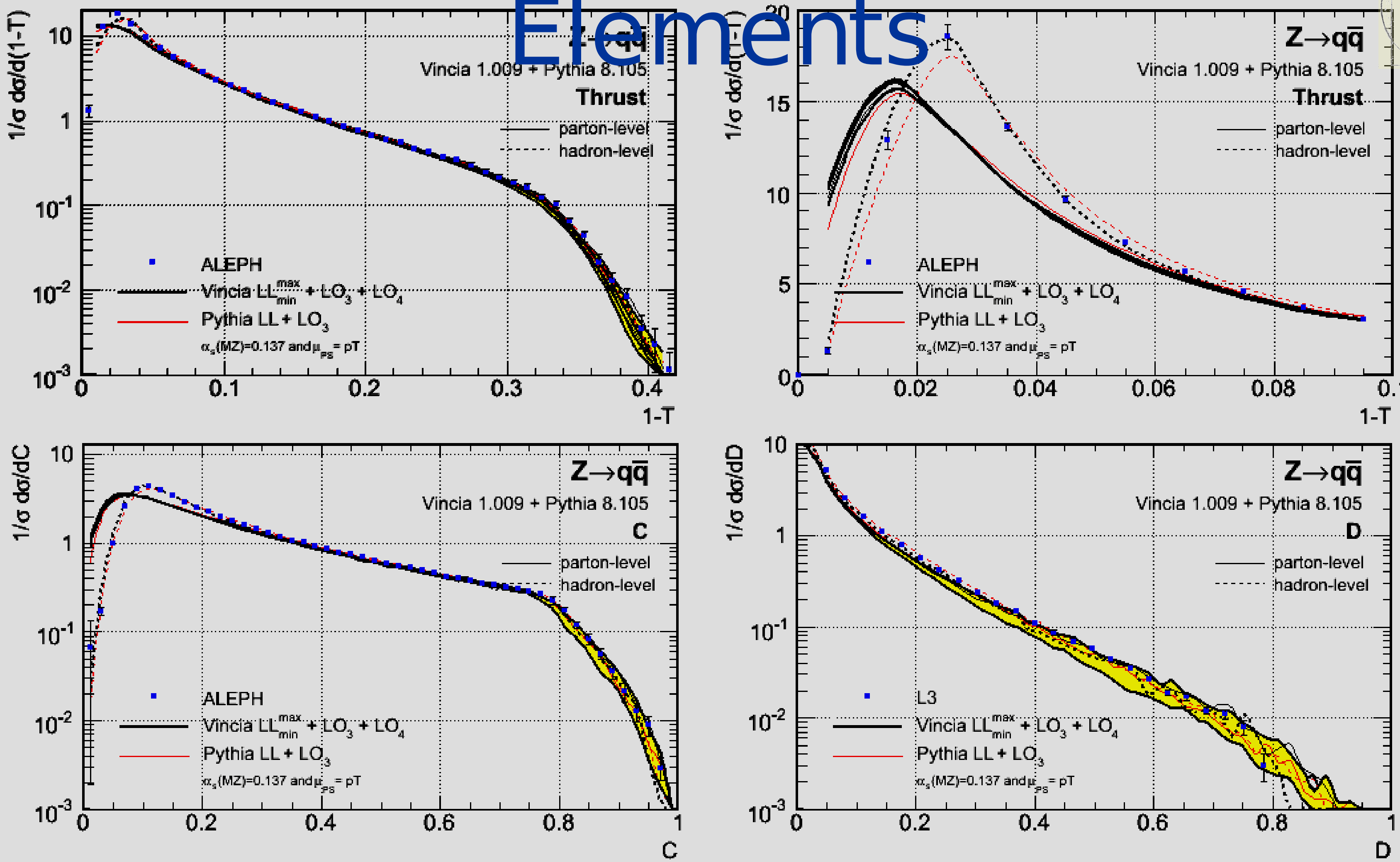
with $\alpha_s(M_Z)$=0.137,

$\mu_R = p_T$,

p_{Thad} = 0.5 GeV

Proper matching removes the ambiguity associated with the freedom of choosing the finite part of the antenna functions

From Showers to Matrix Elements



Still with $\alpha_s(M_Z)=0.137$…

From Showers to Matrix Elements

- This matching automatically exponentiates (and thus regulates) any sub-leading logs
- It is easily extendible to one-loop matrix elements
  - I.e. we can incorporate in the shower any mix of tree-level and one-loop matrix elements
- The cross section/total width is guaranteed to be normalized to the inclusive LO/NLO/… Z→parton cross section/width
- Because the antenna functions are known at one-loop we can start constructing a NLL shower with NNLO matched matrix elements.

Vincia status and plans

- Up to now we build a time-like parton shower which
  - Is highly customizable by design
  - Keeps all uncertaities/choices explicit and changeable
  - Can be used as an “add-on” to an existing NLO parton level MC
  - Can be used as a “true” event generator and integrate tree-level and one-loop amplitudes to improve the predictions
  - Is fully integrated within PYTHIA8
- Comparisons to LEP data are very promising
- We are working on
  - Massive particles in the shower
  - Extension to space-like showers (and TEVATRON/LHC phenomenology)
  - Started building in the NLO antenna functions as a first step towards a next-to-leading log shower (attacks scale choices)