NLO QCD corrections to WW+jet production at hadron colliders

Stefan Kallweit[1]

in collaboration with S. Dittmaier[1] and P. Uwer[2]

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Outline

1. Introduction
2. Calculation of NLO QCD corrections
   - Virtual corrections
   - Real corrections
   - Collinear subtraction counterterm
3. Numerical results
4. Comparison with other groups
5. Conclusions & Outlook

Motivation

Why is $\mathrm{pp/p\bar{p}} \to \mathrm{WW} + \text{jet} + \mathrm{X}$ interesting ?

- Important background process at the LHC (and for Tevatron Higgs searches)
  "Les Houches experimenter's wishlist '05" for important missing NLO predictions:

process	background for
$\mathrm{pp} \to \mathrm{WW} + \mathrm{jet}$	H+jet, $\mathrm{t\bar{t}H}$, new physics (leptons + $\not{E}_{\mathrm{T}}$)
Dittmaier/S.K./Uwer '07, Campbell/Ellis/Zanderighi '07	
Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)	
$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{t\bar{t}} + 2\mathrm{jets}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{VVb\bar{b}}$	VBF $\to \mathrm{H} \to \mathrm{VV}$, $\mathrm{t\bar{t}H}$, new physics
$\mathrm{pp} \to \mathrm{VV} + 2\mathrm{jets}$	VBF $\to \mathrm{H} \to \mathrm{VV}$
VBF: Jäger et al. '06, Bozzi et al. '07	
$\mathrm{pp} \to \mathrm{V} + 3\mathrm{jets}$	$\mathrm{t\bar{t}}$, new physics
$\mathrm{pp} \to \mathrm{VVV}$	SUSY tri-lepton
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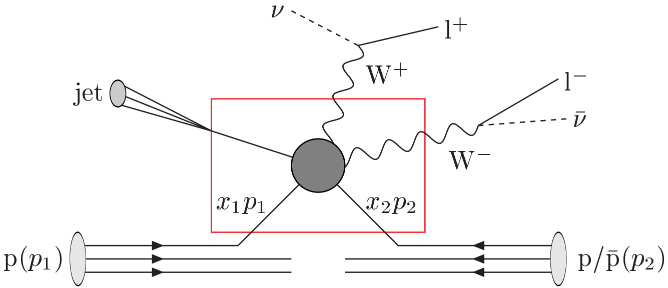
  $\hookrightarrow$ EW gauge-boson coupling analysis
- Process is an important test ground before approaching more complicated many-particle processes at NLO.

Hadronic cross section

Schematic illustration of the hadronic process $\mathrm{pp/p\bar{p}} \to \mathrm{W^+W^-} + \mathrm{jet} + \mathrm{X}$:



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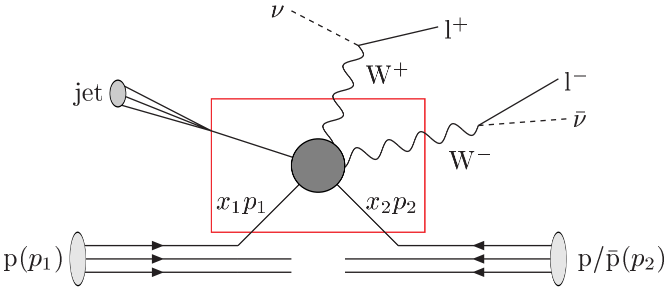

Hadronic cross section:

$$\sigma^{\mathrm{pp}/\mathrm{p\bar{p}}}(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \underbrace{f_{a(\mathrm{p})}(x_1, \mu_\mathrm{F})\, f_{b(\mathrm{p}/\mathrm{\bar{p}})}(x_2, \mu_\mathrm{F})}_{\text{PDF's of parton } a/b \text{ in } \mathrm{p}/\mathrm{\bar{p}}} \underbrace{\hat{\sigma}^{ab}(x_1 p_1, x_2 p_2)}_{\text{partonic cross section}}$$

Subprocesses contributing at leading order

6 partonic channels in LO (12 flavour channels for 2 generations, b quarks negligible):

$$\mathrm{u\bar{u}} \to \mathrm{W^+W^-g}, \quad \mathrm{ug} \to \mathrm{W^+W^-u}, \quad \mathrm{g\bar{u}} \to \mathrm{W^+W^-\bar{u}},$$
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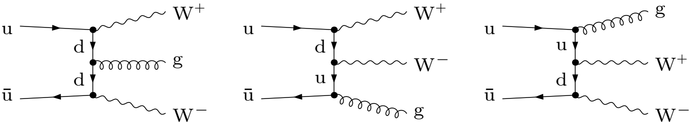
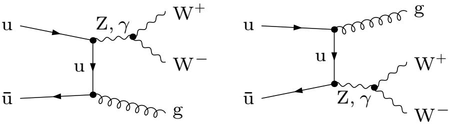
Diagrams for $u\bar{u}$ initial state:





- all light quarks massless
- CKM matrix → Cabibbo matrix

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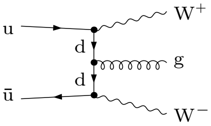

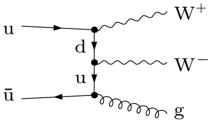

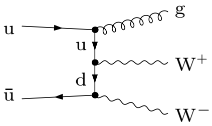

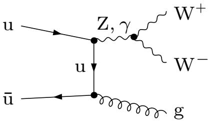

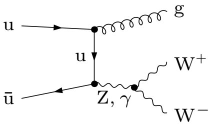

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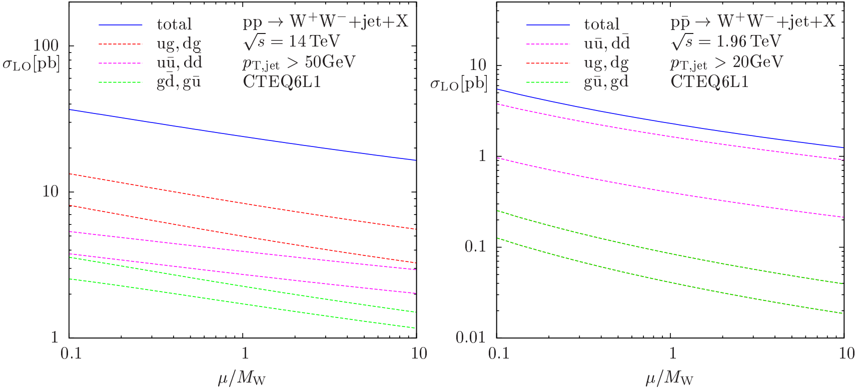
The amplitudes for all other channels are generated by crossing the gluon into the initial state and by SU(2) symmetry ($\mathrm{u} \leftrightarrow \mathrm{d}, \mathrm{W^+} \leftrightarrow \mathrm{W^-}$).

Leading-order prediction

Scale dependence of LO cross section (with $\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):



LHC: Cross section changes by 12% (30%) when scaling μ by a factor of 2 (5).
$\hookrightarrow$ For precise predictions the calculation of NLO QCD corrections is required.

NLO cross section with the dipole subtraction formalism

Schematic formula for the NLO cross section in the situation of two initial-state hadrons (LHC and Tevatron):

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1} d\sigma^{R}}_{\text{real corrections}} + \underbrace{\int_{m} d\sigma^{V}}_{\text{virtual corrections}} + \underbrace{\int_0^1 dx \int_m d\sigma^{C}}_{\text{collinear-subtraction counterterm}}$$

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$$d\sigma^A = \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

$$= \int_{m+1} \left[d\sigma^R - d\sigma^A \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{R} - \mathrm{A}$$

$$+ \int_m \left[d\sigma^V + \sum_{\text{dipoles}} d\sigma^B \otimes V_{\text{dipole}}(1) \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{V} + \mathrm{A}$$

$$+ \int_0^1 dx \int_m \left[d\sigma^C + \sum_{\text{dipoles}} \int_1 d\sigma^B(x) \otimes \left[dV_{\text{dipole}}(x) \right]_+ \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{C} + \mathrm{A}$$

$$dV_{\text{dipole}}(x) = \left[dV_{\text{dipole}}(x) \right]_+ + dV_{\text{dipole}}(1) \delta(1-x)$$

Virtual corrections (V+A term)

For each channel $\mathcal{O}(100)$ 1-loop diagrams contribute, which can be classified as

- "bosonic" corrections (exchange of an additional gluon)
- "fermionic" corrections (closed quark loops)

Renormalization leads to counterterm diagrams contributing on 1-loop level.

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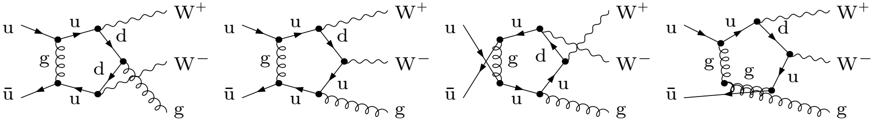

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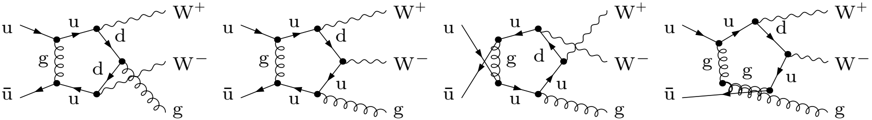
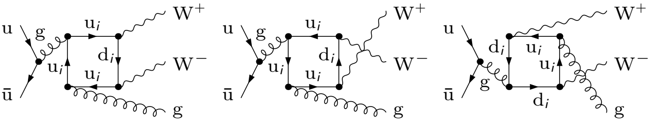


- Fermionic corrections: e. g. box contributions (4-point functions)



+ 3 more

Virtual corrections (V+A term)

Strategy for extracting or translating IR (soft / collinear) singularities

Idea: integrals $I^{(\varepsilon)}$ in $d = 4-2\varepsilon$ dim. $\leftrightarrow$ 4-dim. integrals $I^{(\lambda)}$ with mass regulator λ

Procedure: Consider finite and regularization-scheme-independent difference:

$$\left[I^{(\varepsilon)} - I^{(\varepsilon)}_{\text{sing}} \right]\Big|_{\varepsilon\to 0} = \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}} \right]\Big|_{\lambda\to 0}$$

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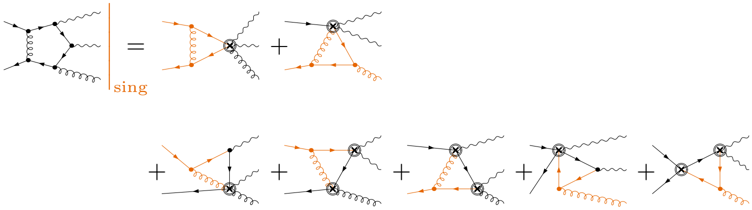
Note: Mass-singular part can be universally constructed from 3-point integrals. [Beenakker et al. '02]

$\hookrightarrow$ general result known explicitly [Dittmaier '03]



$\Big\}$ $\frac{1}{\varepsilon^2}, \frac{1}{\varepsilon}$; $\ln^2\lambda, \ln\lambda$

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Virtual corrections (V+A term)

Two different strategies for evaluation of loop amplitudes
(realized in two independent calculations!)

- Analogous to NLO calculation for $\mathrm{pp} \to \mathrm{t\bar{t}H}$ [Beenakker et al. '02] and $\mathrm{pp} \to \mathrm{t\bar{t}} + \mathrm{jet}$ [Dittmaier, Uwer, Weinzierl '07]
  - diagrams generated with FeynArts 1.0 [Küblbeck, Böhm, Denner '90] and reduced with in-house Mathematica routines → Fortran
  - analytical extraction of IR singularities [Beenakker et al. '02, Dittmaier '03]
  - reduction of 5-point to 4-point integrals according to [Denner, Dittmaier '02]

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- Alternative calculation with available tools
  - diagrams generated with FeynArts 3.2 [Hahn '00]
  - algebraic reduction / numerical evaluation with FormCalc 5.2/LoopTools: [Hahn, Perez-Victoria '98]
    - reduction of 5-point integrals à la [Denner, Dittmaier '02]
    - regular scalar integrals with FF [v.Oldenborgh '91]
  - dimensionally regularized singular integrals implemented into LoopTools:
    - box integrals checked against result of [Bern, Dixon, Kosower '93]

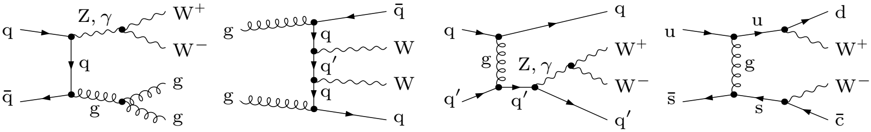
Real corrections (R−A term)

Contributing processes are generated by two types of generic amplitudes

- $0 \to \mathrm{W^+W^-q\bar{q}gg}$
- $0 \to \mathrm{W^+W^-q\bar{q}q'\bar{q}'}$

and crossing any two partons into the initial state.
↪ Large number of contributions (136 flavour channels for 2 generations)!

Sample of real-correction diagrams:



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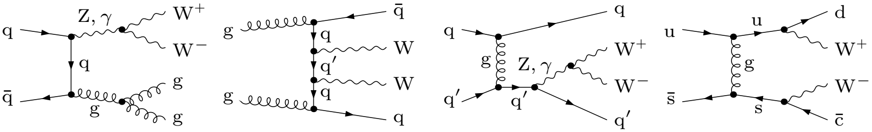

Two independent evaluations of helicity amplitudes:

- application of (4-dimensional) spinor techniques
- alternative evaluation based on MADGRAPH [Stelzer, Long '94]

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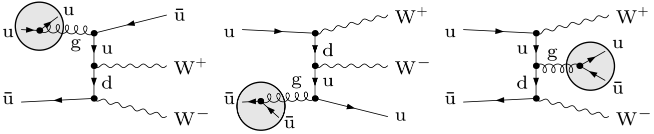
E. g. for $u\bar{u} \to W^+W^- u\bar{u}$ the following subtraction terms contribute:



- process-independent part → dipole terms → IR (soft and collinear) singularities
- process-dependent part → on-shell amplitudes of LO subprocesses

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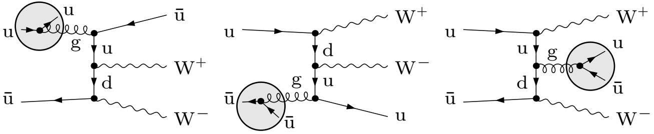

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Two independent versions of Monte Carlo integrators:

- one entirely based on multi-channel MC technique [Berends, Pittau, Kleis '94] [Kleis, Pittau '94]
  - non-singular parts checked against SHERPA 1.0.8 [Gleisberg et al. '03] and WHIZARD 1.50 [Kilian '01] (details on comparison → [diploma thesis of S.K. '06]
  - extra channels included for subtraction terms
- second version based on simple mapping (phase space by sequential splitting)

Real corrections (R−A term)

Subtraction-term phase spaces have the following features:

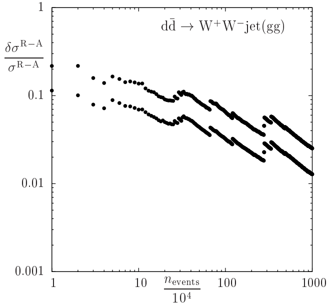
- Momentum conservation is maintained.
- A one-to-one-correspondence between subtraction-term phase space and real-correction phase space exists (degenerate in the relevant limits).

Numerical problem: R − A

R ↙ event passes cuts

A ↘ event is cut away

(or vice versa)



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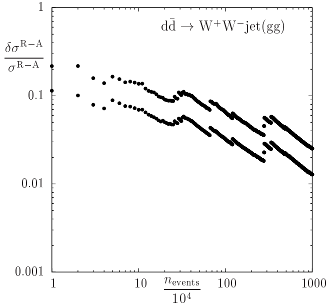
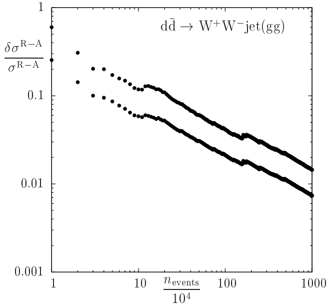


Improved numerical integration by adding of extra channels populating the critical region:



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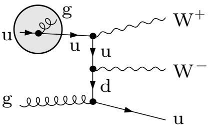
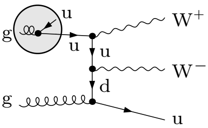
Sample of diagrams showing collinear subtraction counterterms:



$\sim f_{\mathrm{q}}(\frac{x}{z},\mu_{\mathrm{F}})P_{\mathrm{qq}}(z)$



$\sim f_{\mathrm{g}}(\frac{x}{z},\mu_{\mathrm{F}})P_{\mathrm{gq}}(z)$

Corresponding redefinition of PDF's:

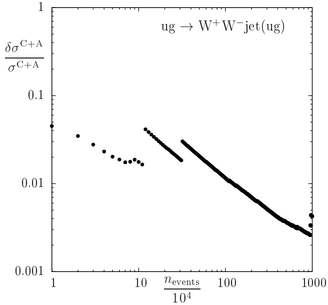
$$
\begin{aligned}
f_{\mathrm{q}}(x,\mu_{\mathrm{F}}) \quad \longrightarrow \quad & f_{\mathrm{q}}(x,\mu_{\mathrm{F}}) \\
& + \frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 \frac{dz}{z} f_{\mathrm{q}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left(\frac{\Gamma(1+\varepsilon)}{\varepsilon}(4\pi)^{\varepsilon} + \ln\frac{\mu^2}{\mu_{\mathrm{F}}^2}\right) C_{\mathrm{F}}\left[P_{\mathrm{qq}}(z)\right]_+ \\
& + \frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 \frac{dz}{z} f_{\mathrm{g}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left(\frac{\Gamma(1+\varepsilon)}{\varepsilon}(4\pi)^{\varepsilon} + \ln\frac{\mu^2}{\mu_{\mathrm{F}}^2}\right) T_{\mathrm{R}} P_{\mathrm{gq}}(z) \\
f_{\mathrm{g}}(x,\mu_{\mathrm{F}}) \quad \longrightarrow \quad & \cdots \text{ analogously}
\end{aligned}
$$

Collinear subtraction counterterm (C+A term)

Meaning of z: Momentum fraction of the radiating parton after splitting

$$\int_0^1 dz\, \hat{\sigma}(z\hat{s})\,[\mathcal{V}(z)]_+ = \int_0^1 dz \left[\hat{\sigma}(z\hat{s}) - \hat{\sigma}(\hat{s})\right] \mathcal{V}(z)$$

- Cancellations between different phase-space points (degenerate only for $z \to 1$)!
- Spoiled cancellations if only one point passes phase-space cuts!



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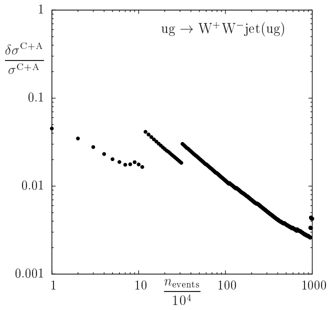
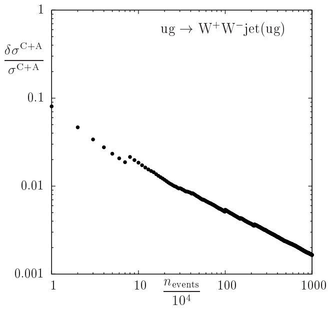


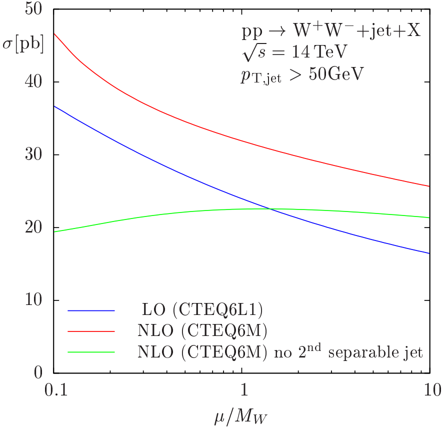
Improved numerical integration by adding of extra channels populating the critical region:

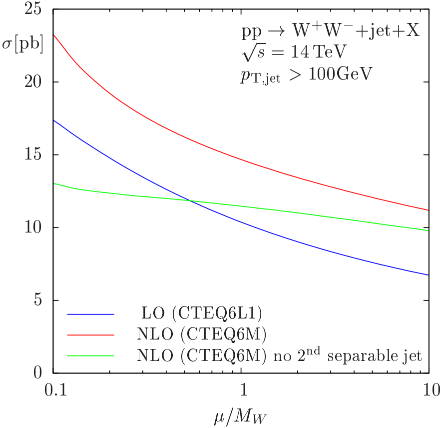


Numerical results

LO versus NLO cross section at the LHC ($\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):

[Dittmaier, S.K., Uwer '07]



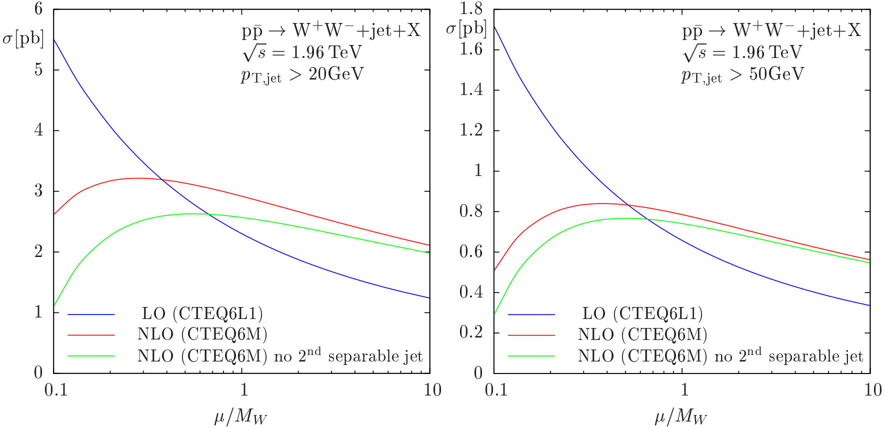


Jet definition: successive combination algorithm (with $R = 1$) [S.D.Ellis, Soper '93]

- Scale dependence stabilizes at NLO for genuine WW+jet production.
- But: Significant scale dependence is introduced by WW+2jets events.

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):

[Dittmaier, S.K., Uwer '07]

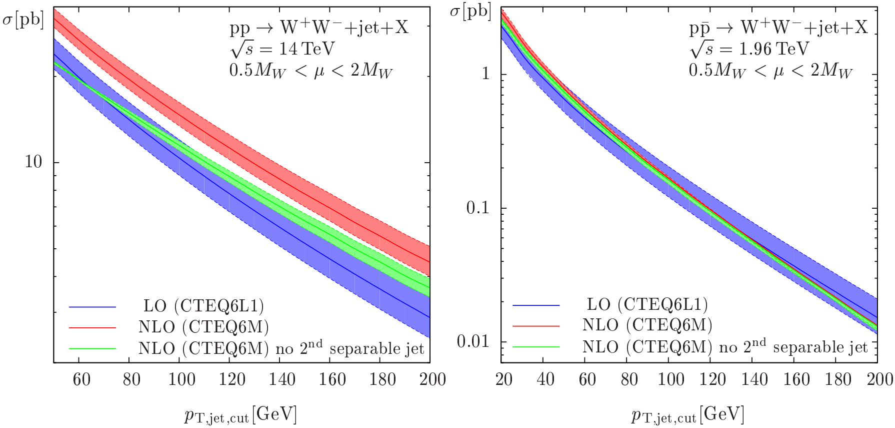


↪ Scale dependence stabilizes at NLO.
(only small influence from WW+2jets production)

Numerical results

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)

[Dittmaier, S.K., Uwer '07]



↪ Significant stabilisation of scale dependence is achieved at NLO (at LHC especially for genuine WW+jet production)

Tuned comparison

Status of tuned comparison between results of [Dittmaier, S.K., Uwer '07], [Campbell, Ellis, Zanderighi '07] and [Binoth, Guillet, Karg, Kauer, Sanguinetti (in progress)]:
↪ [NLM Les Houches report '08]

- Integrated LO results checked:

$\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^- + \mathrm{jet} + \mathrm{X}$	σ_{LO}[fb]
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

- Results for virtual corrections checked at one phase-space point:

$\mathrm{u\bar{u}} \to \mathrm{W}^+\mathrm{W}^-\mathrm{g}$ $\quad |\mathcal{M}_{\mathrm{LO}}|^2/e^4 g_s^2 = 0.9963809154477200 \cdot 10^{-3}$

$$2\mathrm{Re}\{\mathcal{M}_{\mathrm{V}}^* \cdot \mathcal{M}_{\mathrm{LO}}\} = e^4 g_s^2 \Gamma(1+\varepsilon) \left(\frac{4\pi\mu^2}{M_{\mathrm{W}}^2}\right)^{\varepsilon} \left(\frac{1}{\varepsilon^2} c_{-2} + \frac{1}{\varepsilon} c_{-1} + c_0\right)$$

All bosonic contributions:

$\mathrm{u\bar{u}} \to \mathrm{W}^+\mathrm{W}^-\mathrm{g}$	$c_{-2}[\mathrm{GeV}^{-2}]$	$c_{-1}^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$	$c_0^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$
DKU	$-1.080699305508758 \cdot 10^{-4}$	$7.842861905263072 \cdot 10^{-4}$	$-3.382910915425372 \cdot 10^{-3}$
CEZ	$-1.080699305505865 \cdot 10^{-4}$	$7.842861905276719 \cdot 10^{-4}$	$-3.382910915464027 \cdot 10^{-3}$
BGKKS	$-1.080699305508814 \cdot 10^{-4}$	$7.842861905263293 \cdot 10^{-4}$	$-3.382910915616242 \cdot 10^{-3}$

Fermionic contributions for 2 light generations in the loop:

$\mathrm{u\bar{u}} \to \mathrm{W}^+\mathrm{W}^-\mathrm{g}$	$c_{-1}^{\mathrm{ferm1+2}}[\mathrm{GeV}^{-2}]$	$c_0^{\mathrm{ferm1+2}}[\mathrm{GeV}^{-2}]$
DKU	$2.542821895320379 \cdot 10^{-5}$	$4.372323372044527 \cdot 10^{-7}$
CEZ	$2.542821895311753 \cdot 10^{-5}$	$4.372790378087550 \cdot 10^{-7}$
BGKKS	$2.542821895314862 \cdot 10^{-5}$	$4.372324288356448 \cdot 10^{-7}$

Conclusions & Outlook

Conclusions

$\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \text{jet} + X$

- Important background process for Higgs and other searches at Tevatron/LHC
- WW(+jets) production used for EW gauge-boson coupling analysis at the LHC

NEW: $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \text{jet} + X$ at NLO QCD

- Tevatron: NLO correction stabilizes LO cross sections
- LHC: Significant reduction of scale uncertainty for genuine WW+jet production
  But: significant scale dependence via WW+2jets events
- Comparison with [Ellis et al.], [Binoth et al.] in progress

Outlook

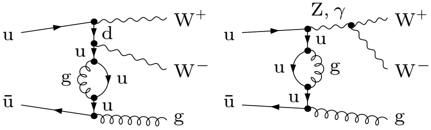
- Implementation of leptonic W decays and distributions
- Calculation of similar processes of the class $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW}/\mathrm{WZ}/\mathrm{ZZ} + \text{jet} + X$
- Methods not yet exhausted $\to$ more complicated applications ($2 \to 4$) feasible!

Backup slides

Virtual corrections (V+A term)

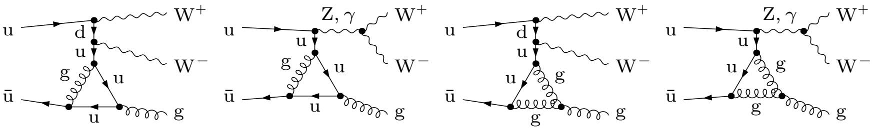
Bosonic corrections can be further subdivided to

- self-energy contributions (2-point functions)



$\mathcal{M}_{\text{self-energy}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu}{(q^2)^2} \sim \int_0^\infty d|q| \rightarrow$ UV-divergent, but IR-finite

- vertex contributions (3-point functions)

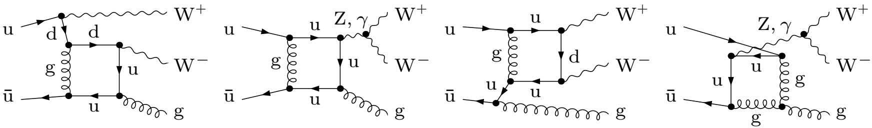


$\mathcal{M}_{\text{vertex}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu}{(q^2)^3} \sim \int_0^\infty d|q|\,|q|^{-1} \rightarrow$ UV-divergent, also IR-divergent

Virtual corrections (V+A term)

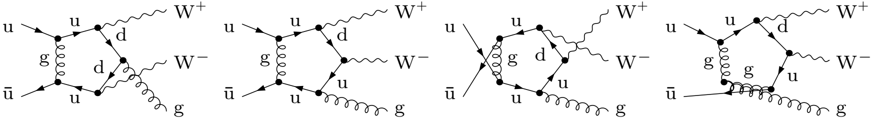
Further bosonic corrections:

- box contributions (4-point functions)



$\mathcal{M}_{\text{box}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu q^\rho}{(q^2)^4} \sim \int_0^\infty d|q|\,|q|^{-2} \rightarrow$ UV-finite, but IR-divergent

- pentagon contributions (5-point functions)

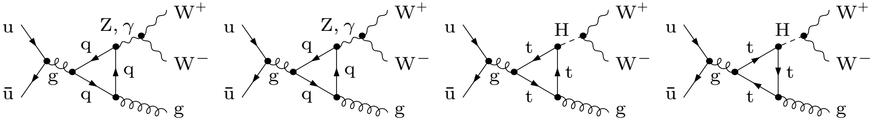


$\mathcal{M}_{\text{pentagon}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu q^\rho q^\sigma}{(q^2)^5} \sim \int_0^\infty d|q|\,|q|^{-3} \rightarrow$ UV-finite, but IR-divergent

Virtual corrections (V+A term)

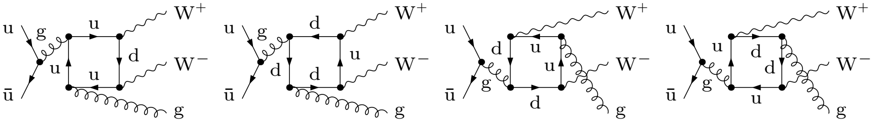
Fermionic corrections can be subdivided to

- vertex contributions (3-point functions)



$\rightarrow$ UV-finite because no $\mathrm{Zgg}, \gamma\mathrm{gg}, \mathrm{Hgg}$ vertex exists in LO, also IR-finite

- box contributions (4-point functions)



$\rightarrow$ UV-finite (no ggWW vertex in LO), also IR-finite

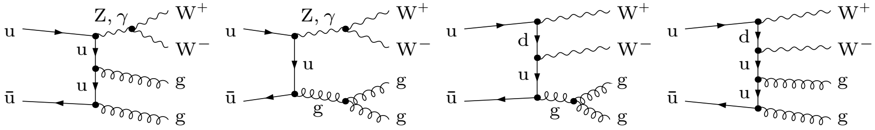
Real corrections (R−A term)

Contributing processes are generated by two types of generic amplitudes,

- $0 \rightarrow \mathrm{W^+W^-q\bar{q}gg}$,
- $0 \rightarrow \mathrm{W^+W^-q\bar{q}q'\bar{q}'}$,

and crossing any two partons into the initial state.
↪ Large number of contributions (136 flavour channels for 2 generations)!

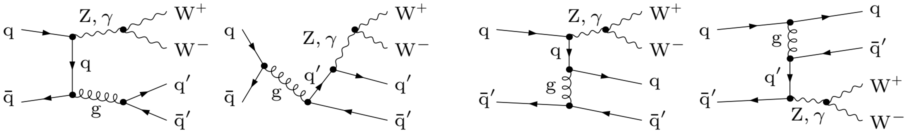
Representative diagrams of the subprocess $\mathrm{u\bar{u} \rightarrow W^+W^-gg}$ (31 diagrams in total):



- Amplitudes for $\mathrm{ug \rightarrow W^+W^-ug}$, $\mathrm{g\bar{u} \rightarrow W^+W^-g\bar{u}}$ and $\mathrm{gg \rightarrow W^+W^-u\bar{u}}$ are achieved from this amplitude by applying crossing symmetry.
- Amplitudes with external d-type quarks are generated via SU(2) symmetry $(\mathrm{u} \leftrightarrow \mathrm{d}, \mathrm{W^+} \leftrightarrow \mathrm{W^-})$.

Real corrections (R−A term)

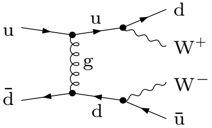
Sample of diagrams of the subprocesses $q\bar{q} \to W^+W^-q'\bar{q}'$ (up to 28 diagrams):



$u\bar{u} \to W^+W^-c\bar{c}$, $u\bar{u} \to W^+W^-d\bar{d}$, ... $u\bar{c} \to W^+W^-u\bar{c}$, $u\bar{d} \to W^+W^-u\bar{d}$, ...

$u\bar{u} \to W^+W^-u\bar{u}$, $d\bar{d} \to W^+W^-d\bar{d}$, ...

Further diagrams contribute in case of both external u-type and d-type quarks:



W bosons couple to different fermion lines.

↪ Subprocesses with up to 4 different quark flavours give non-vanishing contributions (e. g. $u\bar{s} \to W^+W^-d\bar{c}$).

Crossing symmetry leads to amplitudes for the subprocess classes $qq \to W^+W^-qq$ and $\bar{q}\bar{q} \to W^+W^-\bar{q}\bar{q}$.

Treatment of quarks

All light quarks (u, d, s, c, b) are treated as massless particles.
$\hookrightarrow$ IR singularities are treated in dimensional regularization.

For external particles only the two light generations (u, d, s, c) are taken into account.

- initial state: negligible contributions due to small b-quark pdf's
- final state: selection by anti-b-tagging

The CKM matrix reduces to Cabibbo matrix (mixing only between the two light generations). Many subprocesses are not influenced by CKM matrix (unitarity).

The running of α_S is solely generated by light quark and gluon loops.
(The top-quark loop is subtracted at zero momentum.)
$\hookrightarrow$ 5-flavour-running of α_S