Heavy-quark hadro-production at two loops in QCD

Sven-Olaf Moch

Sven-Olaf.Moch@desy.de

DESY, Zeuthen

in collaboration with **P. Uwer** on arXiv:0804.1476v1

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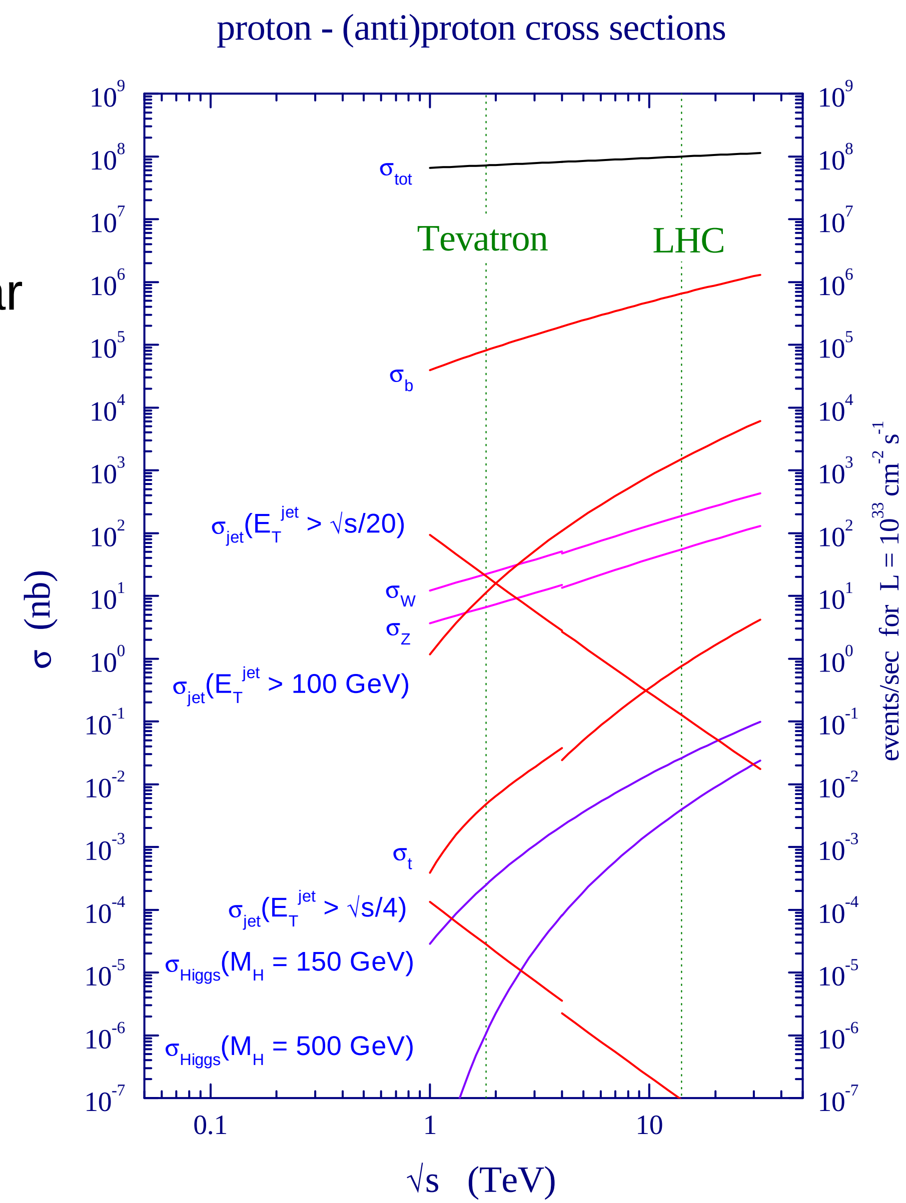
The facts

- LHC will accumulate very high statistics for $t\bar{t}$-pairs
  - low luminosity run: $8 \cdot 10^6$ events/year (high luminosity run: 10 times more)
  - mass measurement $\Delta m_t = \mathcal{O}(1)$GeV (constraints on Standard Model Higgs mass m_h)

Total cross section

- Standard currency for comparisons: $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}(S, m^2)$
- QCD theory improvements: resummation, NNLO, . . .
- Dependence on parton luminosity



Hard scattering at colliders

- QCD theory $\longrightarrow$ **factorization** of cross section
  - separate sensitivity to dynamics form different scales
  - center-of-mass energy S, factorization scale μ

$$\sigma_{\mathrm{pp}}\left(S,m^2\right) = \sum_{\mathrm{ij}} L_{\mathrm{ij}}\left(\mu^2\right) \otimes \hat{\sigma}_{\mathrm{ij}}\left(S,m^2,\mu^2,\alpha_s(\mu)\right)$$

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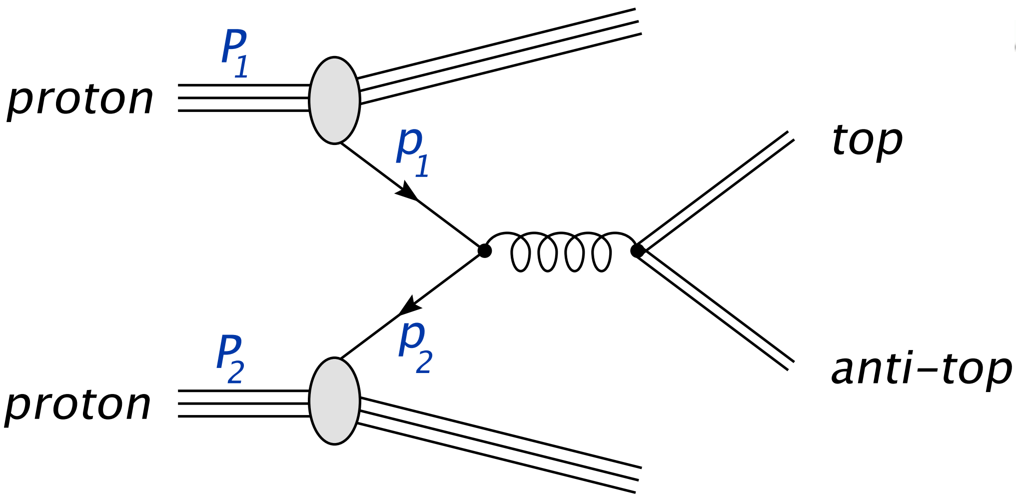
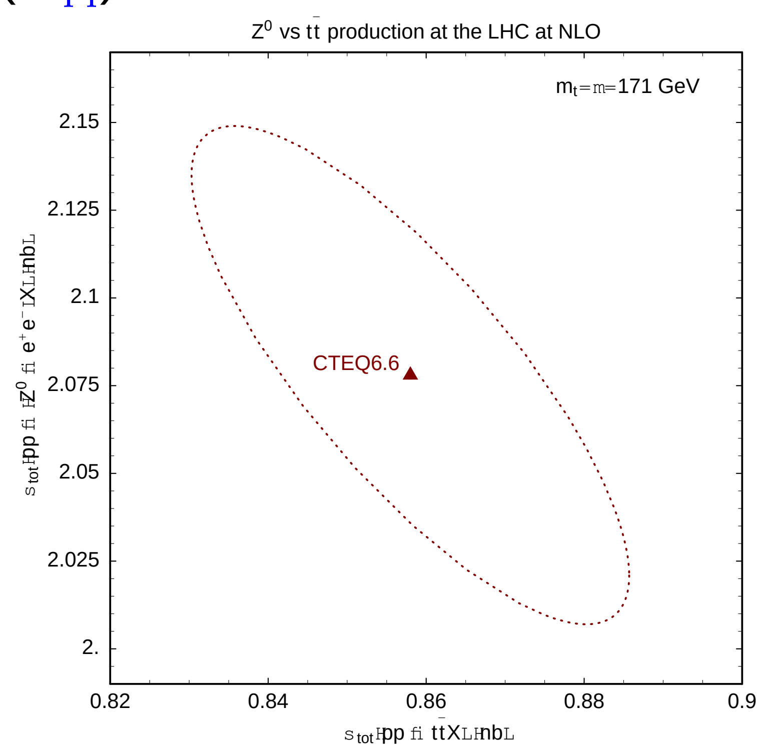


- Parton luminosity
  $L_{\rm ij} = PDF_{\rm i} \otimes PDF_{\rm j}$
- Theory predictions for $\hat{\sigma}_{\rm ij}$ in QCD with partonic channels
  - LO:
    $\rm q + \bar{q} \rightarrow Q + \bar{Q}$
    $\rm g + g \rightarrow Q + \bar{Q}$
  - NLO:
    $\rm q + g \rightarrow Q + \bar{Q} + \mathit{X}(q)$
  - NNLO:
    $\rm q + q \rightarrow Q + \bar{Q} + \mathit{X}(qq)$
    $\rm \bar{q} + \bar{q} \rightarrow Q + \bar{Q} + \mathit{X}(\bar{q}\bar{q})$

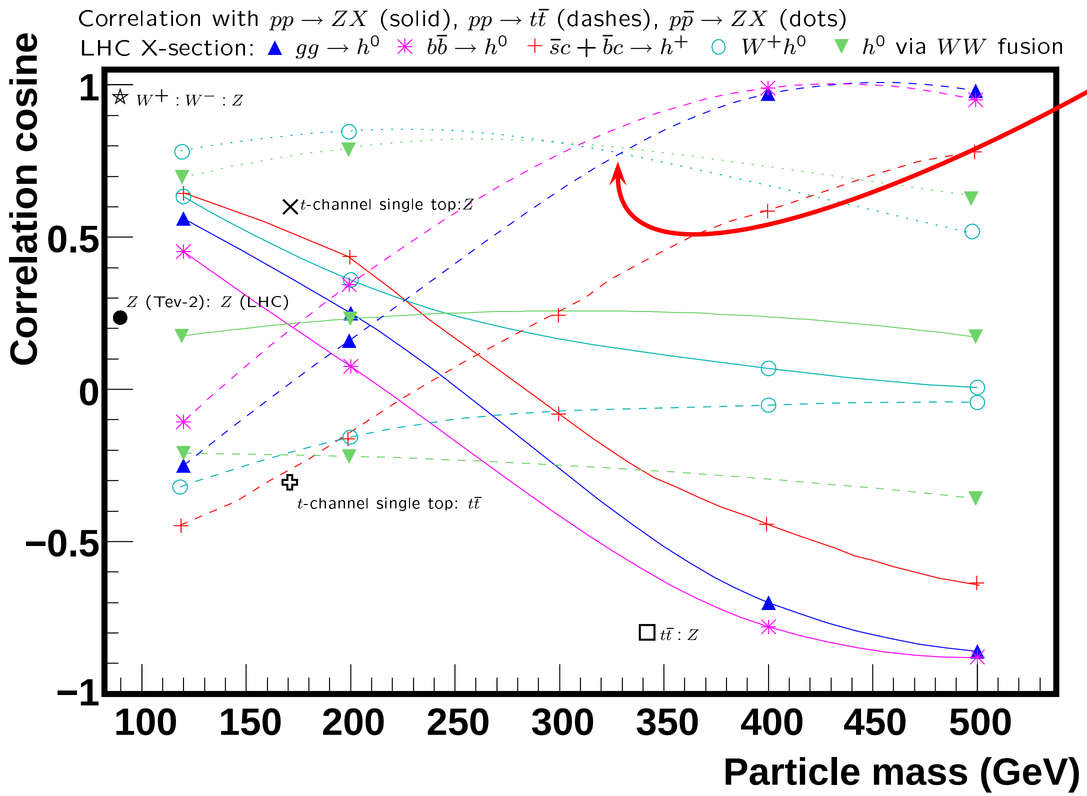
Parton luminosity

- Uncertainties in parton luminosity reduced in ratios of cross sections
  - well-known idea Dittmar, Paus, Zürcher ‘97
  - $W^{\pm}$, Z boson production “standard candle” for $L_{q\bar{q}}$ at LHC
- Drell-Yan process through $q\bar{q}$-annihilation
  - sensitive to quark PDFs at LHC ($L_{q\bar{q}}$)
- Cross section $t\bar{t}$-production at LHC
  - anti-correlated with Z boson production
  - sensitive to gluon PDFs (L_{gg}) CTEQ ‘08



Standard candles

- $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$ at LHC correlated with Higgs boson production (line - - - -) especially for larger Higgs masses



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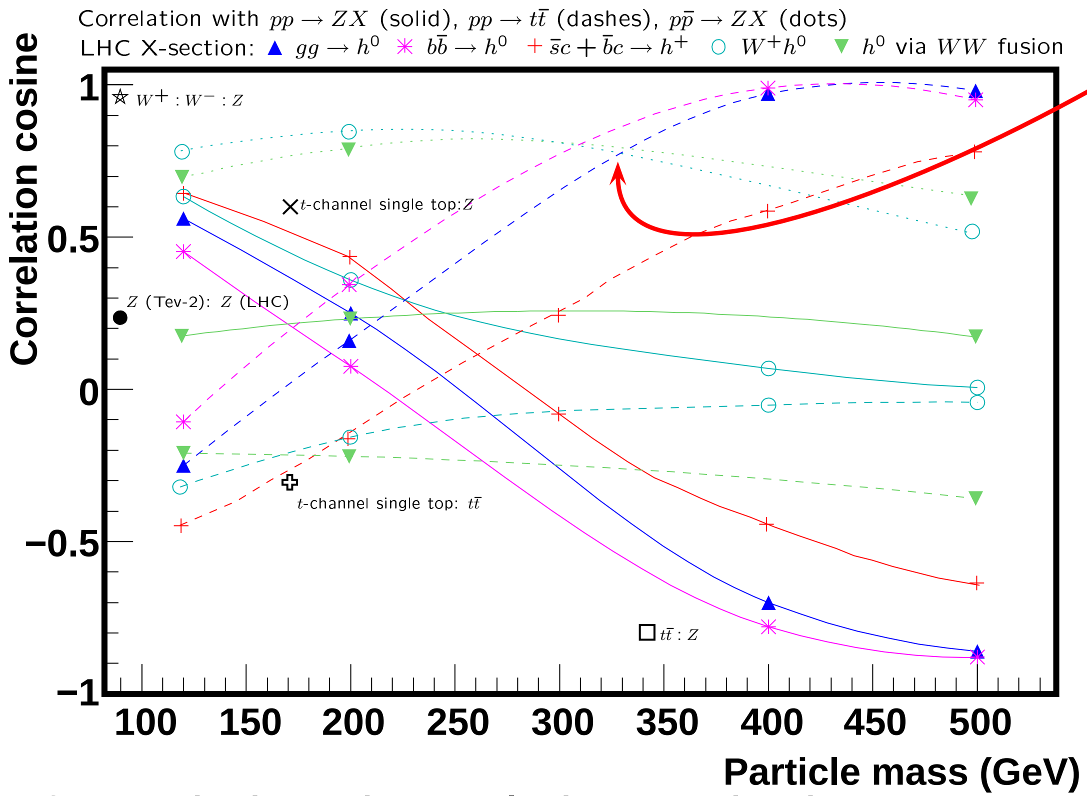

- Proposal of PDF-induced correlation method CTEQ '08
  - $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$ as benchmark for all processes which are anti-correlated with Z boson production
  - however, large theoretical uncertainty for $\sigma_{\mathrm{pp}\to\mathrm{t\bar{t}}}$ at NLO

QCD theory

- Theory predictions for total cross section
  - Plain vanilla NLO QCD
    Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano, Nason, Ridolfi '92; Bernreuther, Brandenburg, Si, Uwer '04; ...

Kinematical limits

- Small-mass limit $m^2 \ll s, t, u$
  - simple multiplicative relation between massive $\mathcal{M}^{(m)}$ and massless $\mathcal{M}^{(m=0)}$ amplitudes to all orders S.M., Mitov '06
  - full result for heavy-quark hadro-production at two loops in QCD in limit $m^2 \ll s, t, u$ S.M., Czakon, Mitov '07
- Threshold at $s \simeq 4m^2$
  - parton cross section exhibit Sudakov-type logarithms $\ln(\beta)$ with velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$ at n$^{\mathrm{th}}$-order

$$\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$$

Threshold resummation

- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$
  - resummation in Mellin space (renormalization group equation)
- Resummed cross section in Mellin space

$$\frac{\hat{\sigma}^N_{ij,\,I}(m^2)}{\hat{\sigma}^{(0),N}_{ij,\,I}(m^2)} = g^0_{ij,\,I}(m^2) \cdot \exp\left(G^{N+1}_{ij,\,I}(m^2)\right) + \mathcal{O}(N^{-1}\ln^n N)$$

  - exponent in singlet-octet color basis decomposition $I = 1, 8$

$$G^N_{q\bar{q}/gg,\,I} \quad = \quad G^N_{\rm DY/Higgs} + \delta_{I,8} G^N_{Q\bar{Q}}$$

- Renormalization group equations for functions $G^N_{\rm DY/Higgs}$ and $G^N_{Q\bar{Q}}$
  - well-known exponentiation from factorization in soft/collinear limit

The radiative factors

- Production of color singlet final state from parton-parton scattering described by $G^N_{\mathrm{DY/Higgs}}$

  $G^N_{\mathrm{DY/Higgs}} =$

  $$\int\limits_0^1 dz\, \frac{z^{N-1} - 1}{1-z} \int_{\mu_f^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2}\, 2\, A_i(\alpha_s(q^2)) + D_i(\alpha_s(4m^2[1-z]^2))$$

  - well known anomalous dimensions A_i (collinear gluon emission) and D_i (process dependent gluon emission at large angles)
    Vogt '00; Catani, Grazzini, de Florian, Nason '03; S.M., Vermaseren, Vogt '05

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- $G^N_{Q\bar{Q}}$ accounts for gluon emission from octet final state
  $$G^N_{Q\bar{Q}} \;=\; \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \, D_{Q\bar{Q}}(\alpha_s([1-z]^2 4m^2))$$
  - anomalous dimensions $D_{Q\bar{Q}}$ (cf. pole of form factor for massive parton)
    $$D^{(1)}_{Q\bar{Q}} = -A^{(1)}_g \,, \qquad\qquad D^{(2)}_{Q\bar{Q}} = -A^{(2)}_g \qquad \longleftarrow \text{new for NNLL}$$
    also consistent with massless case Dixon, Mert Aybat, Sterman '06

Massive form factor

- Renormalization group equation factorizes into functions G and K
  - all Q^2-dependence in finite function G
  - function K dependent on infrared sector (parton mass m)

$$\mu^2 \frac{\partial}{\partial \mu^2} \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right) = \frac{1}{2} K \left(\frac{m^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right) + \frac{1}{2} G \left(\frac{Q^2}{\mu^2}, \alpha_{\mathrm{s}}, \epsilon\right)$$

- Solution for evolution equation S.M., Mitov '06

$$2 \ln \mathcal{F}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s, \epsilon\right) =$$

$$\int\limits_0^{Q^2/\mu^2} \frac{d\xi}{\xi} \left\{ G(\bar{a}(\xi\mu^2, \epsilon)) + K(\bar{a}(\xi\mu^2 m^2/Q^2, \epsilon)) - \int\limits_{\xi m^2/Q^2}^{\xi} \frac{d\lambda}{\lambda} A(\bar{a}(\lambda\mu^2, \epsilon)) \right\}$$

  - Double logarithms $L = \ln(Q^2/m^2)$ from integral over A
  - A and G same functions as in massless calculations
  - K determined matching to fixed order results

Result (massive)

- Massive form factor with logarithms $L = \ln(Q^2/m^2)$
  - expansion in terms of coefficients A, G, K and constant terms C (all finite in m^2 and ϵ)
- Expansion up to two loops Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi '04; S.M., Mitov '06

$$
\begin{aligned}
\mathcal{F}_1 &= \frac{1}{\epsilon}\left\{\frac{1}{2}A_1 L + \frac{1}{2}(G_1 + K_1)\right\} - \frac{1}{4}A_1 L^2 - \frac{1}{2}G_1 L + C_1 \\
\mathcal{F}_2 &= \frac{1}{\epsilon^2}\left\{\frac{1}{8}A_1^2 L^2 + \frac{1}{4}A_1(G_1 + K_1 - \beta_0)L + \frac{1}{8}(G_1 + K_1)(G_1 + K_1 - 2\beta_0)\right\} \\
&\quad + \frac{1}{\epsilon}\left\{-\frac{1}{8}A_1^2 L^3 - \frac{1}{8}A_1(3G_1 + K_1)L^2 + \frac{1}{4}(A_2 - G_1^2 - K_1 G_1 + 2A_1 C_1)L\right. \\
&\quad \left. + \frac{1}{4}(G_2 + K_2) + \frac{1}{2}C_1(G_1 + K_1)\right\} + \frac{7}{96}A_1^2 L^4 \\
&\quad + \frac{1}{24}A_1(7G_1 + K_1 + 2\beta_0)L^3 + \frac{1}{8}G_1(2G_1 + K_1 + 2\beta_0)L^2 \\
&\quad - \frac{1}{4}(A_2 + A_1 C_1)L^2 - \frac{1}{2}(G_2 + G_1 C_1)L + C_2
\end{aligned}
$$

Accuracy under control

- Control over logarithms $\ln(N)$ with $\lambda = \beta_0 \alpha_s \ln(N)$ to N^{k}LL accuracy

$$G^N_{ij\,I} = \ln N \cdot g^1_{ij}(\lambda) + g^2_{ij,\,I}(\lambda) + \alpha_s\, g^3_{ij,\,I}(\lambda) + \dots$$

  - $g^1(\lambda)$: LL

    Laenen, Smith, v.Neerven ‘92; Berger, Contopanagos ‘95; Catani, Mangano, Nason, Trenatedue ‘96
  - $g^2(\lambda)$: NLL

    Bonciani, Catani, Mangano, Nason ‘98; Kidonakis, Laenen, S.M., Vogt ‘01
  - $g^3(\lambda)$: NNLL $\longleftarrow$ new

    S.M., Uwer ‘08
- Resummed G^N predicts fixed orders in perturbation theory
  - generating functional for towers of large logarithms

Results

- NNLO cross section for heavy-quark hadro-production near threshold (all powers of $\ln\beta$ and Coulomb corrections) S.M., Uwer '08
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m$

$$\hat{\sigma}^{(1)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}}\left\{96\ln^2\beta - 9.5165\ln\beta + 35.322 + 5.1698\frac{1}{\beta}\right\}$$

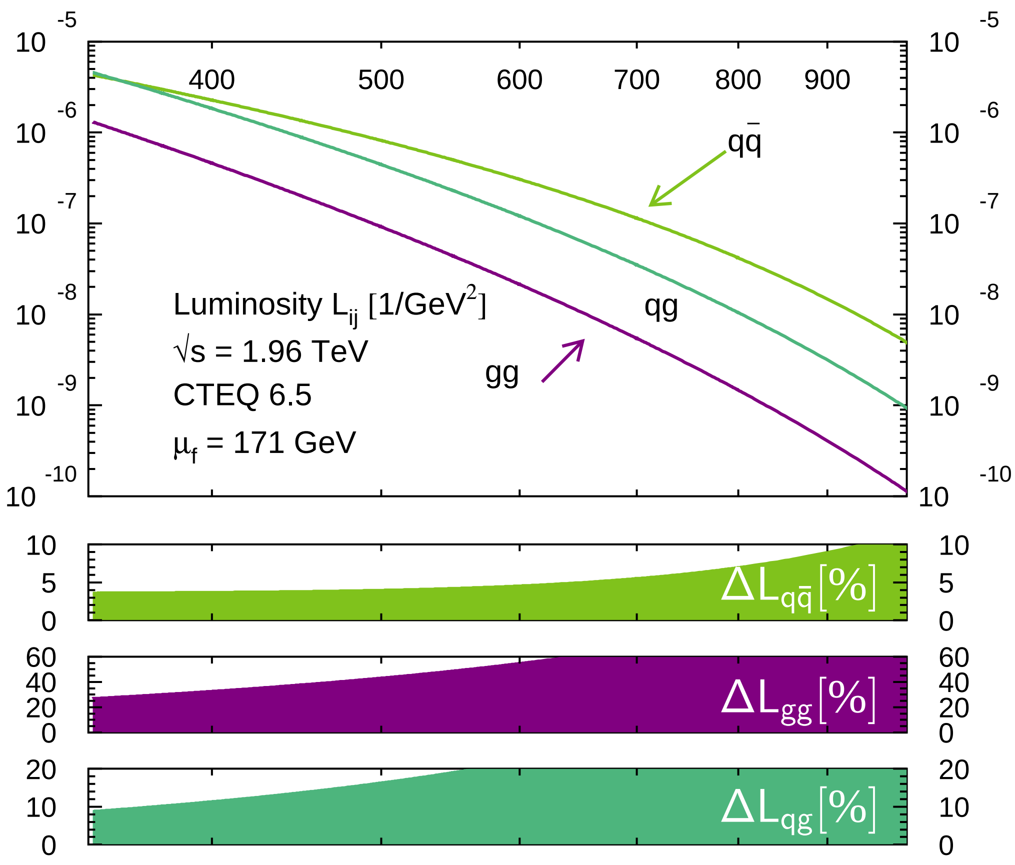
$$\hat{\sigma}^{(2)}_{gg\to t\bar{t}} = \hat{\sigma}^{(0)}_{gg\to t\bar{t}}\left\{4608\ln^4\beta - 1894.9\ln^3\beta + \left(-3.4811 + 496.30\frac{1}{\beta}\right)\ln^2\beta\right.$$
$$\left. + \left(3144.4 + 321.17\frac{1}{\beta}\right)\ln\beta + 45.354\frac{1}{\beta^2} - 140.53\frac{1}{\beta} + C^{(2)}_{gg}\right\}$$

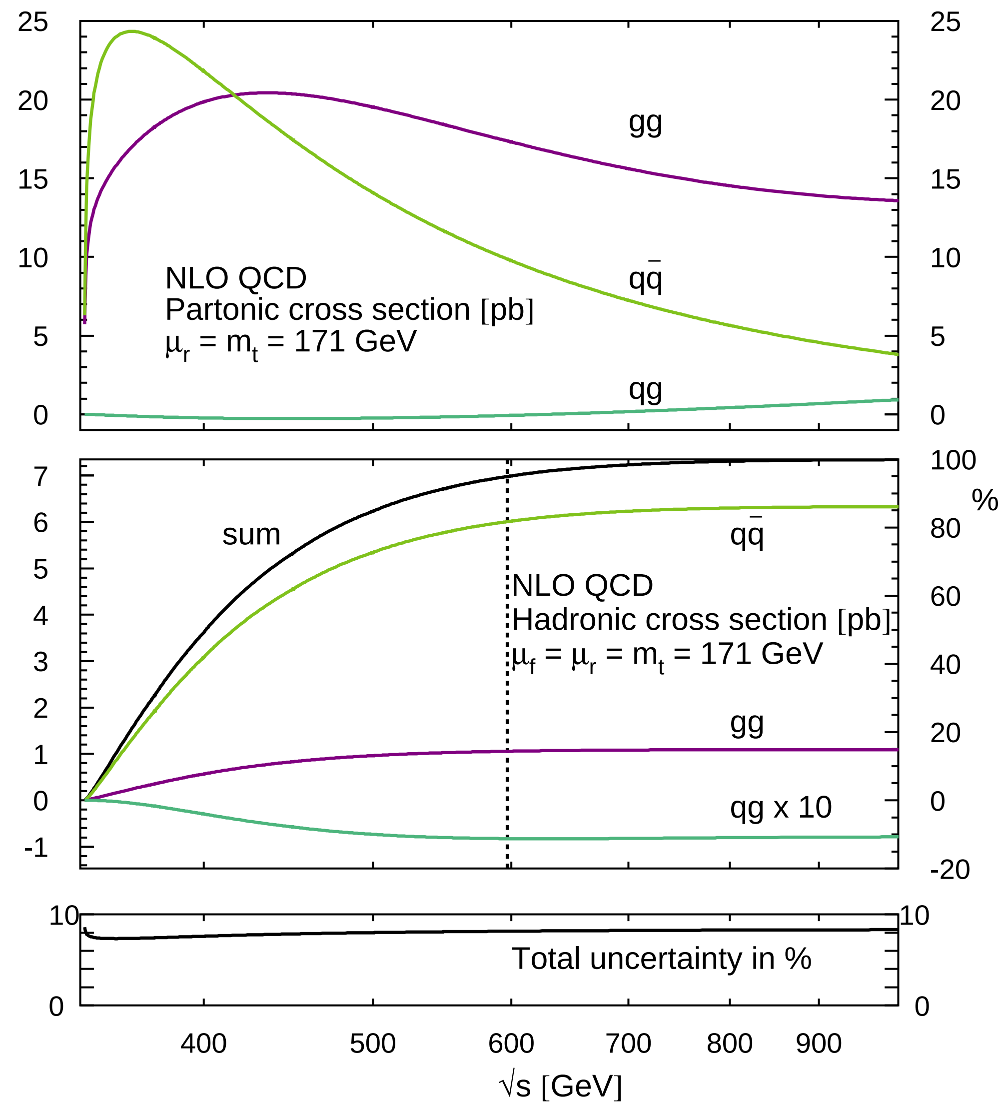
- Add all scale dependent terms
  - $\ln(\mu/m)$-terms exactly known from renormalization group methods

Upshot

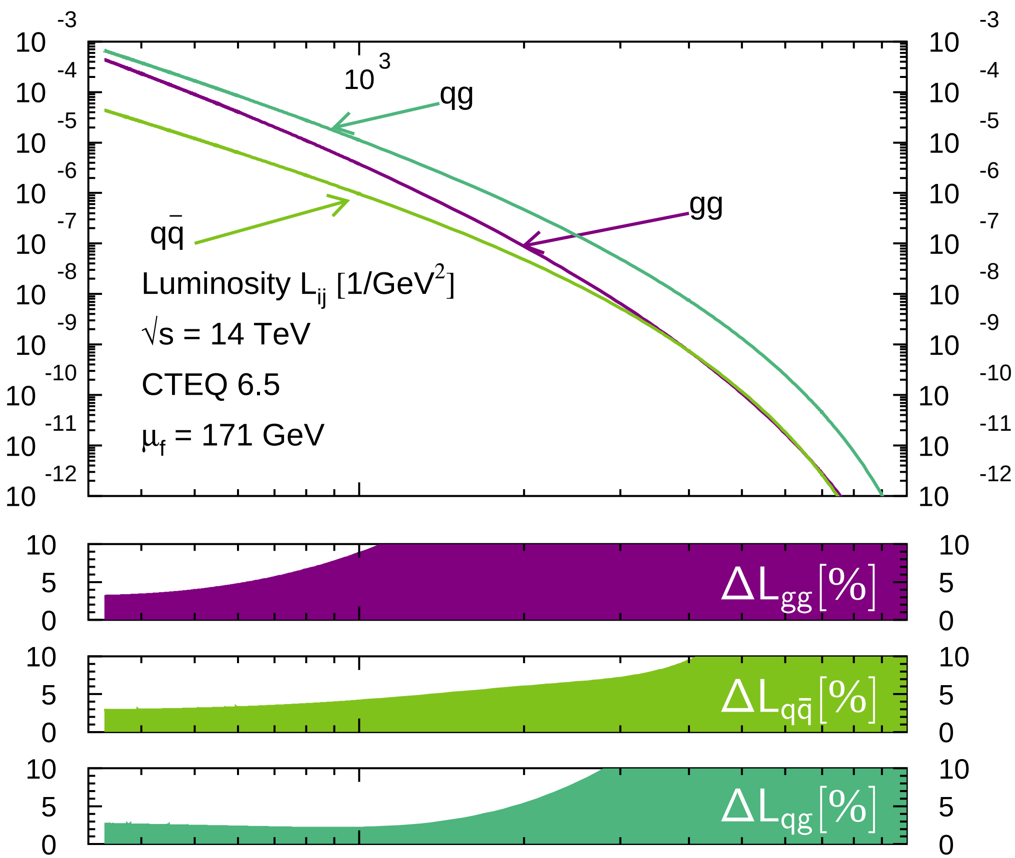
- Best approximation to complete NNLO

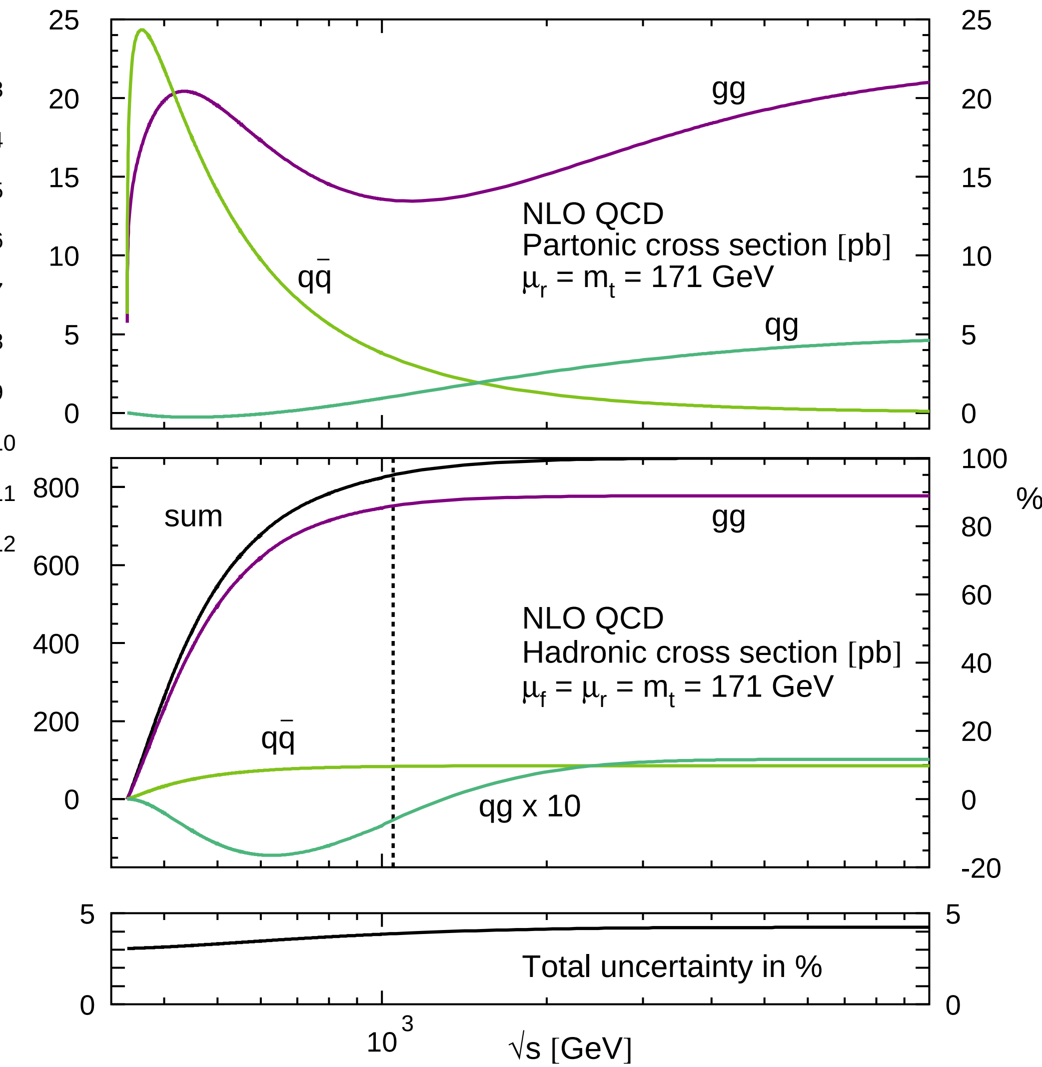
Total cross section at Tevatron





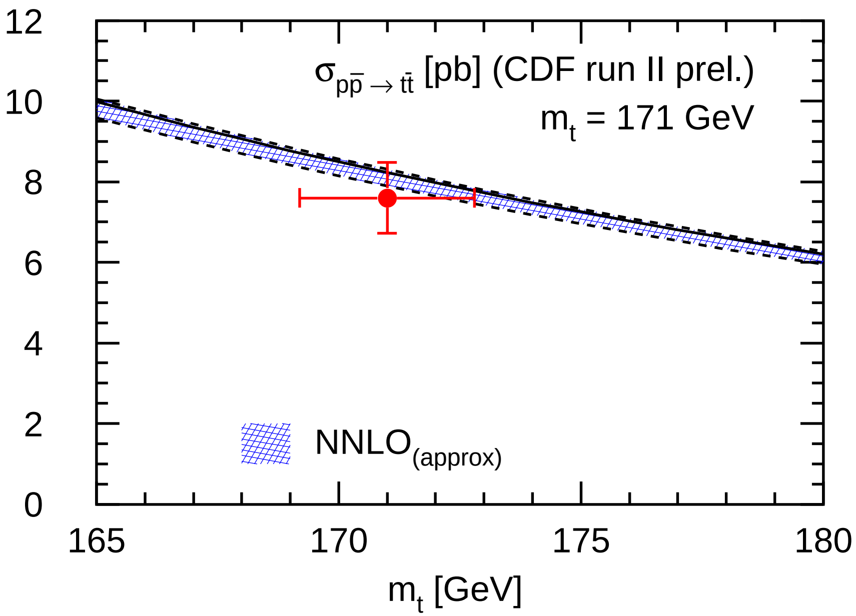
Total cross section at LHC

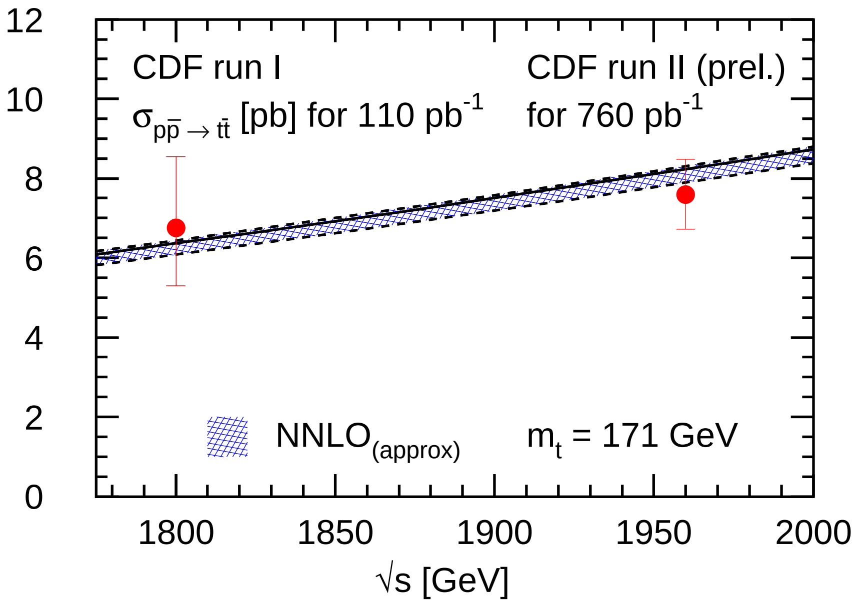




Tevatron results

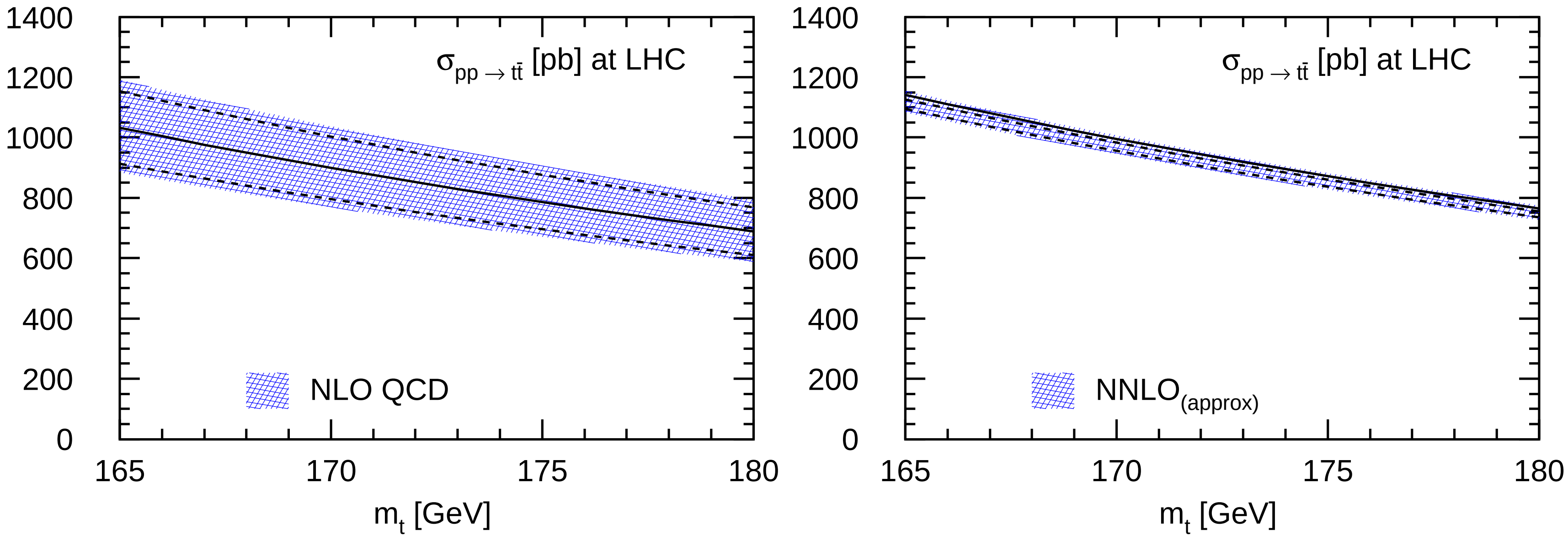
- Total cross section as function of mass m and energy $\sqrt{s}$
- NNLO$_{\rm approx}$ (with MRST2006 PDF set)
  - scale uncertainty $\mathcal{O}(3\%)$ $\oplus$ PDF uncertainty $\mathcal{O}(3\%)$





LHC total cross section

- NLO (with CTEQ6.5 PDF set)
  - scale uncertainty $\mathcal{O}(10\%) \oplus$ PDF uncertainty $\mathcal{O}(5\%)$
- NNLO$_{\mathrm{approx}}$ (with MRST2006 PDF set)
  - scale uncertainty $\mathcal{O}(3\%) \oplus$ PDF uncertainty $\mathcal{O}(2\%)$



- Theory at NNLO matches anticipated experimental precision $\mathcal{O}(10\%)$

Summary

Theory

- Small-mass limit
  - two-loop amplitde in limit $m^2 \ll s, t, u$ from simple relation between massless and massive amplitudes
- Threshold resummation
  - NNLL accuracy for resummed cross section
  - NNLO$_{\rm approx}$ threshold improved and exact scale dependence ($\ln(\mu/m)$-terms)

Phenomenology

- Improved predictions for top-pair production at NNLO in QCD
- Tevatron with MRST2006 set
  - scale uncertainty $\mathcal{O}(3\%)$ $\oplus$ PDF uncertainty $\mathcal{O}(3\%)$
- LHC with MRST2006 set
  - scale uncertainty $\mathcal{O}(3\%)$ $\oplus$ PDF uncertainty $\mathcal{O}(2\%)$