Mini–review on the status of Monte Carlo programs for Bhabha scattering

Guido Montagna

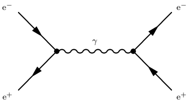
Dipartimento di Fisica Nucleare e Teorica, Università di Pavia
Istituto Nazionale Fisica Nucleare, Sezione di Pavia

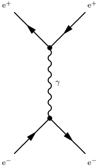
with G. Balossini, C.M. Carloni Calame, O. Nicrosini, F. Piccinini

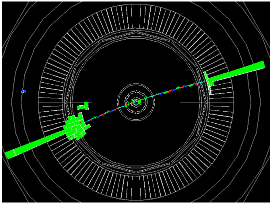
and with many thanks to C. Bignamini, R. Bonciani, H. Czyż, A. Denig, S. Eidelman, A. Ferroglia, F. Jegerlehner, A. Hafner, S. Muller, F. Nguyen, A. Penin, G. Venanzoni...

Bhabha scattering and luminosity

Homi J. Bhabha (1909 – 1966)
Proc. Roy. Soc. **A154** (1936) 195







Bhabha tracks at the B-factory PEP-II

- Precision measurements at e^+e^- colliders require a precise knowledge of the accelerator luminosity

$$\int \mathcal{L}\, dt \;=\; N_{\rm obs}/\sigma_{\rm th}$$

- Precise knowledge of the luminosity needs a reference process with clean topology, high statistics and calculable with high theoretical accuracy $\rightarrow$ Bhabha!

  A. Denig and F. Nguyen, KLOE Note n. 202 (2006)

- High theoretical accuracy and comparison with data require precision Monte Carlo tools, including radiative corrections at the per mille level

Leading Logs & QED Parton Shower

C.M. Carloni Calame *et al.*, Nucl. Phys. **584** (2000) 459 & Nucl. Phys. Proc. Suppl. **131** (2004) 48
as in `BabaYaga v3.5`

- Various approaches, such as analytical Structure Functions (SF) and YFS exponentiation, can be used to account for leading log corrections
- The QED Parton Shower (PS) is a Monte Carlo (MC) solution of the QED DGLAP equation for the electron structure function $D(x, Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t}, Q^2)$$

- The solution can be cast in the form

$$D(x, Q^2) \;=\; \Pi(Q^2) \sum_{n=0}^{\infty} \int \frac{\delta(x - x_1 \cdots x_n)}{n!} \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i) \; L \; dx_i \right]$$

  ★ $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ is the Sudakov form factor $I_+ \equiv \int_0^{1-\epsilon} P(x) dx$
  $L \equiv \log \frac{Q^2}{m^2}$ with ϵ soft/hard separator and Q^2 virtuality scale

- The PS MC algorithm exactly reproduces this solution

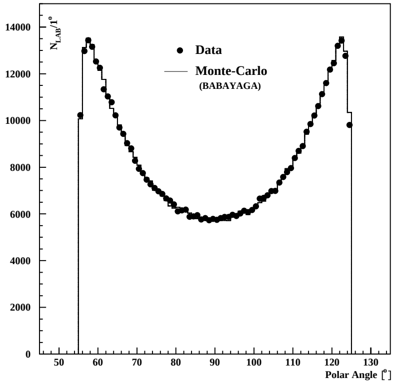
  ★ momenta of the final–state particles can be exclusively generated through Altarelli–Parisi kinematics or its improvements

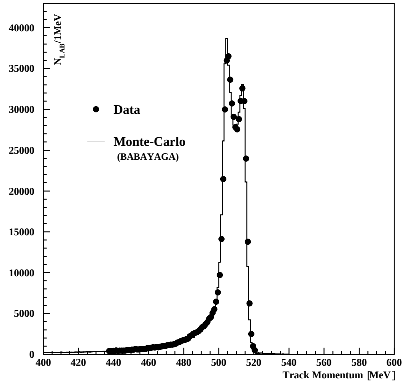
  C.M. Carloni Calame, Phys. Lett. **520** (2001) 16

  ★ the resulting cross section has a leading log accuracy $\longrightarrow$ th. uncertainty mainly due to missing $\mathcal{O}(\alpha)$ non–log contributions

KLOE Bhabha data vs BabaYaga v3.5

F. Aloisio *et al.*, [KLOE Coll.], Phys. Lett. **B606** (2005) 12





Good agreement with data...but till $\sim$ one year ago @KLOE

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.5\%\,({\rm th}) = 0.6\%$$

F. Ambrosino *et al.*, [KLOE Coll.], Eur. Phys. J. **C47** (2006) 589

How to improve the accuracy of BabaYaga?

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227
as in `BabaYaga@NLO`

Exact $\mathcal{O}(\alpha)$ soft+virtual (SV) corrections and hard bremsstrahlung (H) matrix elements can be combined with QED PS through a matching procedure

- $d\sigma^{\infty}_{LL} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$
- $d\sigma^{\alpha}_{LL} = [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma^{SV}_{LL}(\varepsilon) + d\sigma^{H}_{LL}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm exact} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 \equiv d\sigma^{SV}_{\rm exact}(\varepsilon) + d\sigma^{H}_{\rm exact}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV} \, \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$$

in such a way that

- ⋆ $[\sigma^{\infty}_{\rm matched}]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm exact}$, avoiding double counting and preserving resummation of leading logs
- ⋆ theoretical error shifted to $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections

Status of Bhabha generators

⋆ For large–angle Bhabha ⋆

Generator	Processes	Theory	Accuracy	Ref.
Bagenf	e^+e^-	$\mathcal{O}(\alpha)$ Berends&Kleiss	0.5%	INFN-AE-97-48
BabaYaga v3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$0.5 \div 1\%$	hep-ph/0003268
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + PS	$\sim 0.1\%$	hep-ph/0607181
MCGPJ	$e^+e^-, \mu^+\mu^- \dots$	$\mathcal{O}(\alpha)$ + SF	$< 0.2\%$	hep-ph/0504233
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP)	hep-ph/9608412

- Estimate of the technical precision → tuned cross–checks between independent codes
- Estimate of the theoretical uncertainty → comparisons with available $\mathcal{O}(\alpha^2)$ calculations
- At $\mathcal{O}(\alpha^2)$, infrared–enhanced photonic $\mathcal{O}(\alpha^2 L)$, $L = \ln Q^2/m^2$, corrections through factorization $\mathcal{O}(\alpha L) \times \mathcal{O}(\alpha)_{\rm non-log}$

G. M., O. Nicrosini and F. Piccinini, Phys. Lett. **B385** (1996) 348

Large–angle Bhabha: size of radiative corrections

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

Selection criteria – ϕ and B factories

- (a) $\sqrt{s} = 1.02\,\text{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\text{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- (b) $\sqrt{s} = 1.02\,\text{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\text{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$
- (c) $\sqrt{s} = 10\,\text{GeV},\ E_{\text{min}}^{\pm} = 4\,\text{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- (d) $\sqrt{s} = 10\,\text{GeV},\ E_{\text{min}}^{\pm} = 4\,\text{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$

Relative corrections (in %)

set up	a.	b.	c.	d.
δ_α	-13.06	-17.16	-19.10	-24.35
$\delta_\alpha^{\text{non-log}}$	-0.39	-0.66	-0.41	-0.70
δ_{HO}	0.43	0.93	0.87	1.76
$\delta_{\alpha^2 L}$	0.04	0.09	0.06	0.11
δ_{VP}	1.73	2.43	4.59	6.03

- Vacuum polarization included in lowest–order and one–loop diagrams with $\Delta\alpha_{\text{had}}^{(5)}$ contribution through `HADR5N` routine, returning a data–driven error estimate

F. Jegerlehner, Nucl. Phys. Proc. Suppl. **131** (2004) 213

⋆ Both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarization) necessary for 0.1% theoretical precision ⋆

Large–angle Bhabha: tuned comparisons at flavour factories

Without vacuum polarization, to compare consistently

At the Φ–factories (cross sections in nb)

set up	BabaYaga@NLO	BHWIDE	LABSPV	$\delta_{BBH}(\%)$	$\delta_{BL}(\%)$
a.	$6086.6(1)$	$6086.3(2)$	$6088.5(3)$	0.005	0.030
b.	$455.85(1)$	$455.73(1)$	$456.19(1)$	0.030	0.080

★ Agreement within 0.1%! ★

- Now at KLOE: $\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.1\%\,({\rm th}) = 0.3\%$

F. Ambrosino *et al.*, [KLOE Coll.], `arXiv:0707.4078 [hep-ex]`

At BABAR (cross sections in nb)

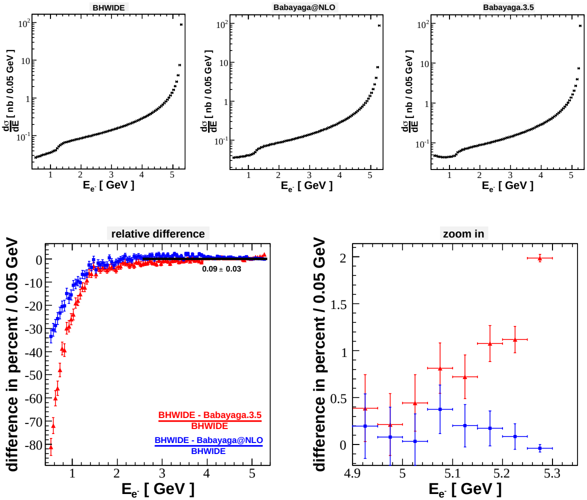
From talks by A. Denig and A. Hafner @LNF

angular range (c.m.s.)	BabaYaga@NLO	BHWIDE	$\delta_{BBH}(\%)$
$15^\circ \div 165^\circ$	$119.5(1)$	$119.53(8)$	0.025
$40^\circ \div 140^\circ$	$11.67(3)$	$11.660(8)$	0.086
$50^\circ \div 130^\circ$	$6.31(3)$	$6.289(4)$	0.332
$60^\circ \div 120^\circ$	$3.554(6)$	$3.549(3)$	0.141

★ Agreement at $\sim$ 0.1% level! ★

BabaYaga@NLO vs BHWIDE at BABAR

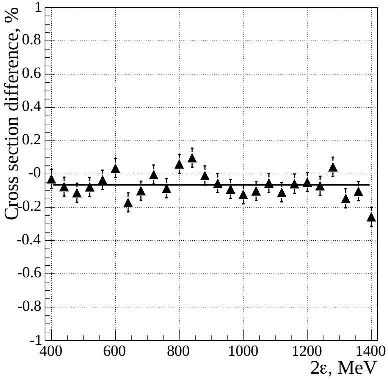
From talks by A. Denig and A. Hafner @LNF

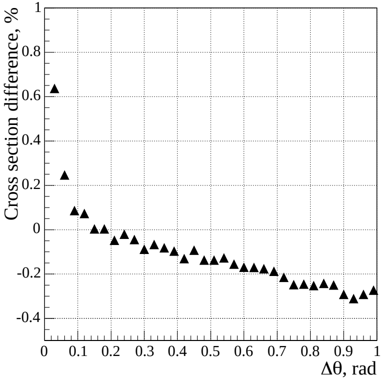


- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for differential distributions

MCGPJ vs BHWIDE at VEPP–2M

A.B. Arbuzov *et al.*, Eur. Phys. J. **C46** (2006) 689
See also A.B. Arbuzov *et al.*, JHEP **9710** (1997) 001
with typical selection cuts for luminosity at VEPP–2M





- Agreement at $0.1 \div 0.2\%$ level for integrated cross section. Agreement well within 1% for distributions
- Comparison between `BabaYaga@NLO` and `MCGPJ` planned for the near future

How to establish the theoretical accuracy?

1. By comparing with two–loop calculations available in the literature
2. By estimating the size of unaccounted higher–order corrections (*e.g.* light pair corrections)

By expanding the matched PS formula up to $\mathcal{O}(\alpha^2)$, the (approximate) `BabaYaga@NLO` second–order cross section can be cast in the form

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

where

- $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual corrections up to $\mathcal{O}(\alpha^2)$ $\longrightarrow$ compared with (a subset of) available NNLO QED calculations
- $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ estimated relying on existing (partial) results
- $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section obtained through the `ALPHA` algorithm, to register really negligible differences (at 1×10^{-5} level)

F. Caravaglios and M. Moretti, Phys. Lett. **B358** (1995) 332

NNLO QED calculations $\longrightarrow$ [See talks by R. Bonciani & J. Gluza]

- Massless two–loop virtual corrections

  Z. Bern, L. Dixon and A. Ghinculov, Phys. Rev. **D63** (2001) 053007

- Exact coefficient of next–to–leading second order $\mathcal{O}(\alpha^2 L)$ corrections, w/o and with two–loop box contributions, plus soft bremsstrahlung

  A.B. Arbuzov, E.A. Kuraev and B.G. Shaikhatdenov, Mod. Phys. Lett. **A13** (1998) 2305
  E.W. Glover, J.B. Tausk and J.J. van der Bij, Phys. Lett. **B516** (2001) 33

- Complete two–loop virtual photonic corrections (in the limit $Q^2 \gg m_e^2$) plus real soft–photon radiation, up to non–logarithmic accuracy

  A.A. Penin, Phys. Rev. Lett. **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185

- Two–loop $N_F = 1$ [only electron loops] fermionic corrections, with finite mass terms, plus soft bremsstrahlung and real pair corrections

  R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280
  R. Bonciani and A. Ferroglia, Phys. Rev. **D72** (2005) 056004

- Two–loop leptonic corrections, with finite mass terms (for $Q^2 \gg m_l^2$)

  S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26
  T. Becher and K. Melnikov, JHEP **0706** (2007) 084

- Two–loop hadronic and heavy flavour corrections (eventually) combined with available non–fermionic contributions

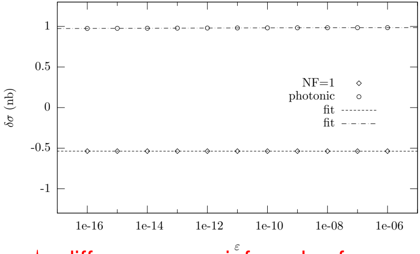
  S. Actis, M. Czakon, J. Gluza and T. Riemann, Phys. Rev. Lett. **100** (2008) 131602
  R. Bonciani, A. Ferroglia and A.A. Penin, JHEP **0802** (2008) 080
  & Phys. Rev. Lett. **100** (2008) 131601

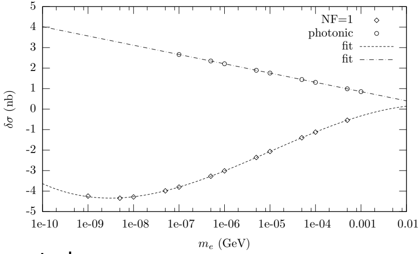
Comparison with NNLO calculations: Bonciani *et al.* & Penin

Thanks to R. Bonciani, A. Ferroglia and A. A. Penin!
Plots for set up a. at the Φ–factories

Comparison of $\sigma_{SV}^{\alpha^2}$ expansion of `BabaYaga@NLO` with

- Penin (photonic): switching off vacuum polarization contributions in `BabaYaga@NLO`
- Bonciani *et al.* ($N_F = 1$): switching off real soft + virtual pair corrections in their formulae [estimated independently]





★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$ $\qquad$ $\delta\sigma(N_F = 1)/\sigma_0 \propto \alpha^2 L^2$

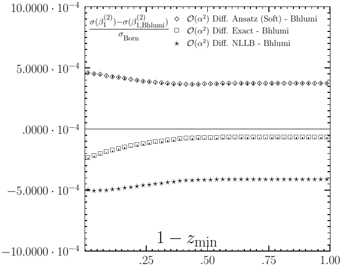
★ Numerically, for all the selection criteria at Φ and B factories

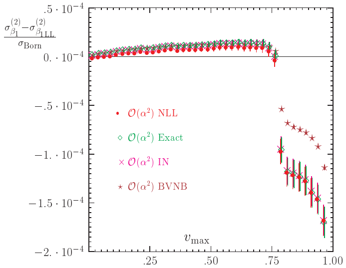
$$\delta\sigma(\text{photonic}) + \delta\sigma(N_F = 1) < 0.02\% \times \sigma_0$$

One–loop corrections to single hard bremsstrahlung

The exact perturbative expression of $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ for full $s+t$ Bhabha scattering is *not* known in the literature. It has been calculated and evaluated for

1. small–angle Bhabha scattering [NNLBHA, BHLUMI]
   A.B. Arbuzov *et al.*, Nucl. Phys. **B485** (1997) 457
   S. Jadach, M. Melles, B.F.L. Ward and S. Yost, Phys. Lett. **B377** (1996) 168 & Phys. Lett. **B450** (1999) 262
2. large–angle s–channel processes [KKMC, PHOKHARA]
   M. Igarashi and N. Nakazawa, Nucl. Phys. **B288** (1987) 301
   F.A. Berends, W.L. van Neerven and G.J.H. Burgers, Nucl. Phys. **B297** (1988) 429
   S. Jadach, M. Melles, B.F.L. Ward and S. Yost, Phys. Rev. **D65** (2002) 073030
   G. Rodrigo, H. Czyż, J.H. Kühn and M. Szopa, Eur. Phys. J. **C24** (2002) 71





★ Relying on these results, the uncertainty of BabaYaga@NLO for $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ is safely estimated 0.05% ★

Summary of theoretical accuracy

...Putting all together, for large–angle Bhabha at meson factories

$\vert\delta^{\rm err}\vert$ (%)	a.	b.	c.	d.
$\vert\delta^{\rm err}_{\rm HH}\vert$	0.00	0.00	0.00	0.00
$\vert\delta^{\rm err}_{{\rm phot}+N_F=1}\vert$	0.01	0.01	0.00	0.01
$\vert\delta^{\rm err}_{\rm SV,H}\vert$	0.05	0.05	0.05	0.05
$\vert\delta^{\rm err}_{\rm pairs}\vert$	0.02	0.03	0.03	0.04
$\vert\delta^{\rm err}_{\rm VP}\vert$	0.01	0.00	0.02	0.04
$\vert\delta^{\rm err}_{\rm tot}\vert$	0.09	0.09	0.10	0.14

Table: Summary of the sources of theoretical uncertainty in `BabaYaga@NLO`

- The accuracy achieved by precision large–angle Bhabha tools for luminosity measurement at meson factories is now *comparable* with that reached about ten years ago for luminosity monitoring through small–angle Bhabha scattering at LEP

A.B. Arbuzov *et al.*, Phys. Lett. **B383** (1996) 238
S. Jadach, `hep-ph/0306083`

Conclusions

- Recent progress in reducing the theoretical precision of Bhabha generators for presently running e^+e^- colliders down to $\sim$ 0.1%

★ Exact $\mathcal{O}(\alpha)$ and multiple photon corrections are necessary ingredients for 0.1% theoretical accuracy

- At least three generators for large–angle Bhabha scattering (`BabaYaga@NLO`, `BHWIDE`, `MCGPJ`) agree within ~ 0.1% for integrated cross sections and $\sim 1\%$ (or better) for distributions
- Precision generators also available for $\gamma\gamma$ production (`BabaYaga@NLO`) and $\mu^+\mu^-$ final states (`BabaYaga@NLO`, `KKMC`, `MCGPJ`)

★ NNLO QED calculations are important to establish the theoretical accuracy and, if necessary, to improve it below the per mille level

- One–loop corrections to single hard bremsstrahlung should be calculated for full Bhabha scattering, to get a better control of the theoretical precision
- For ILC/GigaZ (assuming 10^{-4} accuracy) MC Bhabha programs need to be improved by the inclusion of weak & two–loop QED corrections, beamstrahlung and new $\Delta\alpha^{(5)}_{\rm had}$ parameterizations.

Backup Slides

Higher orders missing in BabaYaga@NLO: light pairs

- They contribute at $\mathcal{O}(\alpha^2)$, where virtual pair corrections (VPC) and real pair corrections (RPC) largely cancel [with exact cancellation of leading $\alpha^2 L^3$ terms]
- Their effect has been evaluated by considering the dominant t–channel contribution and using results for
  - ⋆ real electron pair radiation in soft approximation

    A.B. Arbuzov, E. A. Kuraev, N.P. Merenkov and L. Trentadue, Phys. Atom. Nucl. **60** (1997) 591
  - ⋆ virtual electron pair corrections as in

    R. Barbieri, J.A. Mignaco and E. Remiddi, Nuovo Cimento **A11** (1972) 824

    G.J.H. Burgers, Phys. Lett. **B164** (1985) 167 & B.A. Kniehl, Phys. Lett. **B237** (1990) 127

set up	$\sigma^{\alpha^2}_{\rm VPC}$ (nb)	$\sigma^{\alpha^2}_{\rm RPC}$ (nb)	$\delta^{\rm err}_{\rm pairs}$ (%)
a.	-4.605(3)	3.305(3)	-0.019
b.	-0.5698(1)	0.4375(1)	-0.025
c.	-0.1385(1)	0.1154(1)	-0.032
d.	-0.01542(1)	0.01320(1)	-0.040

⋆ They contribute at the level of a few 0.01% ⋆

- The effect of heavier pairs was studied for small–angle Bhabha at LEP and found to be $\sim 30\%$ of the electron pairs contribution

  G. Montagna *et al.*, Nucl. Phys. **B547** (1999) 39

My personal update of LEP1 theoretical error, Febr. 2003 (red/magenta)

Type of correction/error	Ref.[1]	Ref. [2]	Ref. [3]	My update
Technical precision	—	(0.030%)	(0.030%)	0.030%
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.040%	0.025%
Light pairs	0.03%	0.03%	0.010%	0.010%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.11%	0.061% (0.068)	0.054% (0.061)	0.53%

[1] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

[2] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

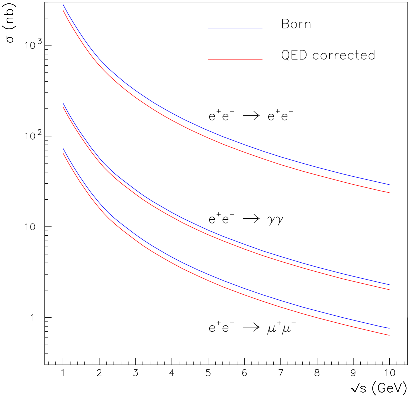
[3] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649

★ the impressive th accuracy has been achieved through a hard work of different groups and comparing independent calculations/codes [BHLUMI, NNLBHA, SABSPV...]

QED corrections to luminosity processes

by BabaYaga v3.5

$E_{\rm min}^{1,2} = 0.8\,E_{\rm beam}, \ \vartheta_{1,2} = 20^\circ \div 160^\circ, \ \xi_{\rm max} = 5^\circ$

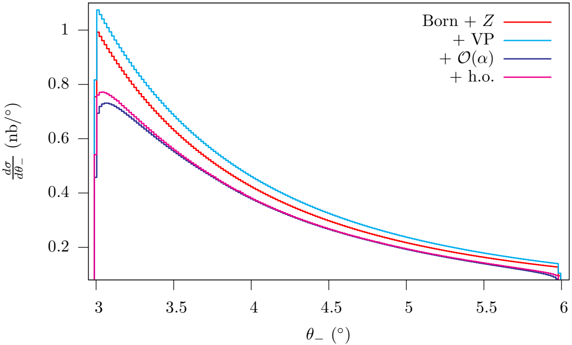


- For typical event selection criteria, the correction due to (leading logarithmic) QED radiation is $\sim -10\%$ at the Φ–factories and $\sim -20\%$ at the B–factories
- In the energy range of meson factories, pure weak corrections are irrelevant (well below 0.1%)

BabaYaga@NLO for Bhabha at ILC

- BabaYaga@NLO can run also at ILC, in the small angle regime (QED)
- *e.g.*, $E_{\rm cm} = 500$ GeV, $3^\circ < \theta_- < 6^\circ$, $174^\circ < \theta_+ < 177^\circ$, $E_\pm > 200$ GeV

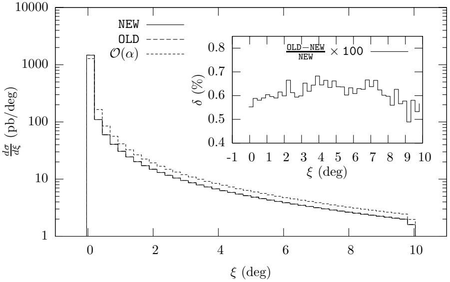
	Born	$+Z$	+VP	$+\mathcal{O}(\alpha)$	+ h.o.
σ (nb)	1.13762	1.13757	1.23816	0.97689	0.99550
%	-	-0.004	+8.84	-12.98	+1.91

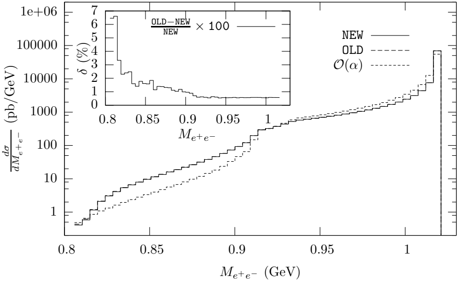


BabaYaga@NLO vs BabaYaga v3.5 at DAΦNE

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

$\sqrt{s} = 1.02\,\mathrm{GeV}, \; E^{\pm}_{\text{min}} = 0.408\,\mathrm{GeV}, \; \vartheta_{\mp} = 55^\circ \div 125^\circ, \; \xi_{\text{max}} = 10^\circ$

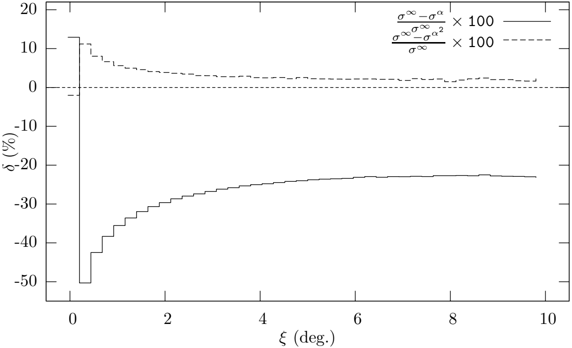




- `BabaYaga@NLO` differs from `BabaYaga v3.5` at ~ 0.5 % level in the statistically dominant regions for luminosity monitoring at the Φ–factories
- Higher–order – beyond $\mathcal{O}(\alpha)$ – leading log corrections amount to several per cent on distributions and are essential for precision luminosity studies

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

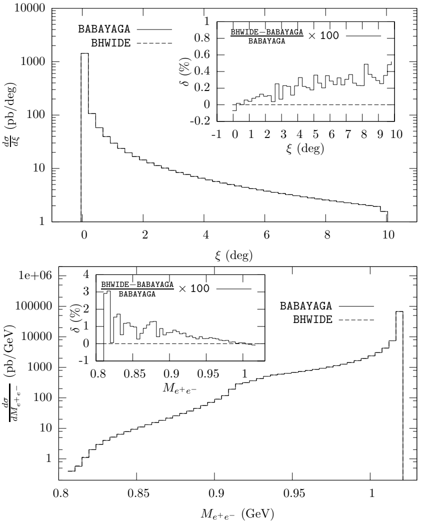
With a complete two–loop generator at hand, leading–log resummation beyond α^2 could be neglected?



- Resummation beyond α^2 still important for Bhabha!

BabaYaga@NLO vs BHWIDE at DAΦNE

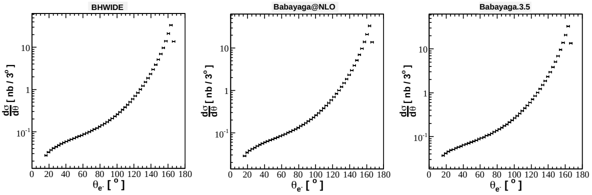
G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

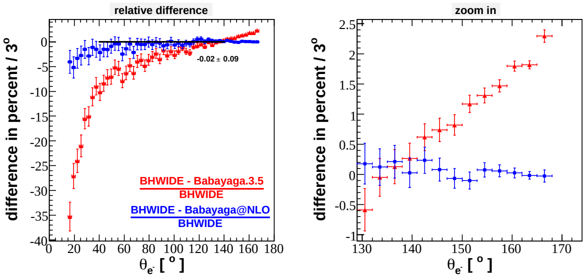


- Agreement within a few 0.1%, a few % only in the hard tails

BabaYaga@NLO vs BHWIDE at BABAR

From talks by A. Denig and A. Hafner @LNF
with realistic selection cuts for luminosity at BABAR

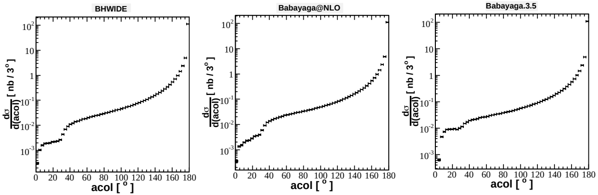


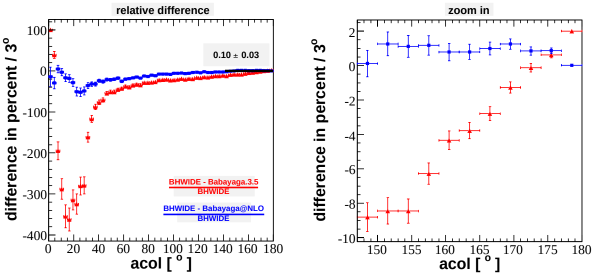


- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for differential distributions

BabaYaga@NLO vs BHWIDE at BABAR

From talks by A. Denig and A. Hafner @LNF
with realistic selection cuts for luminosity at BABAR

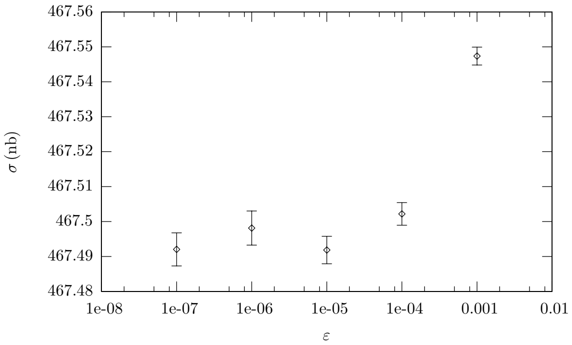




- BabaYaga@NLO and BHWIDE well agree (at a few per mille level) also for differential distributions

Technical test of BabaYaga@NLO: ϵ independence

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227



- Independence of the matched PS cross section from variations of the soft–hard separator ϵ successfully checked!

$\gamma\gamma$ production in BabaYaga@NLO

- Matching also applied to $\gamma\gamma$ and muon pair production
- Tuned comparison with BKQED for the relative $\mathcal{O}(\alpha)$ corrections to the inclusive $e^+e^- \to \gamma\gamma$ cross section

F.A. Berends and R. Kleiss, Nucl. Phys. **B186** (1981) 22

$\sqrt{s}(\mathrm{GeV})$	6	10	20
$\delta_{\mathrm{T}}^{\mathrm{BKQED}}(\%)$	13.8	15.3	17.4
$\delta_{\mathrm{T}}^{\mathrm{BabaYaga@NLO}}(\%)$	13.81(1)	15.30(1)	17.51(10)

- Relative corrections to $e^+e^- \to \gamma\gamma$ cross section with realistic cuts $(E_{\mathrm{min}}^{\gamma_{m.e.},\gamma_{n.m.e}} = 0.3\sqrt{s},\ \vartheta_{\gamma_{m.e.},\gamma_{n.m.e}} = 45^\circ \div 135^\circ,\ \xi_{\mathrm{max}} = 10^\circ)$

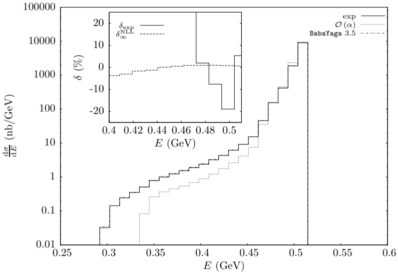
G. Balossini *et al.*, `arXiv:0801.3360 [hep-ph]`

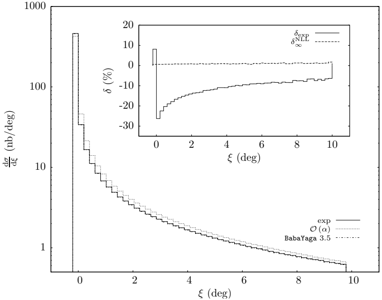
$\sqrt{s}(\mathrm{GeV})$	1	3	10
$\delta_\alpha(\%)$	-5.87	-7.00	-8.24
$\delta_\alpha^{\mathrm{non-log}}(\%)$	0.70	0.71	0.73
$\delta_{\mathrm{HO}}(\%)$	0.24	0.37	0.51

★ Like for Bhabha, both non–log $\mathcal{O}(\alpha)$ and higher–order corrections necessary for 0.1% theoretical precision to $\gamma\gamma$ production ★

$\gamma\gamma$ production in BabaYaga@NLO: distributions

G. Balossini *et al.*, `arXiv:0801.3360 [hep-ph]`

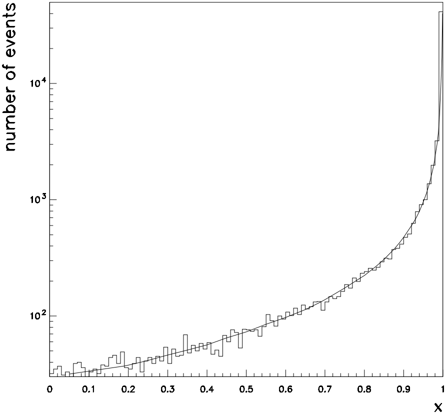




- Exponentiation & NLO important for $\gamma\gamma$ differential distributions

Technical test of BabaYaga: $D(x, Q^2)$

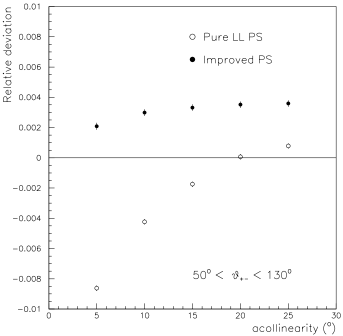
C.M. Carloni Calame *et al.*, Nucl. Phys. **B584** (2000) 459



- Parton Shower reconstruction (histogram) of the x distribution of the electron Structure Function $D(x, Q^2)$ (solid line)

Theoretical accuracy of BabaYaga v3.5

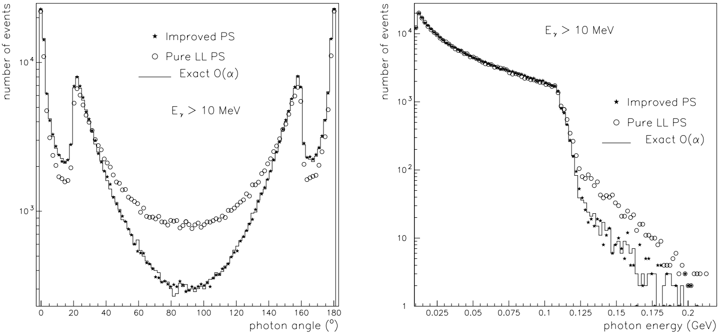
C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16



- Relative difference between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ Bhabha cross section, as a function of the acollinearity cut, for two angular acceptances at $\sqrt{s} = 1$ GeV

Improved PS algorithm in BabaYaga v3.5

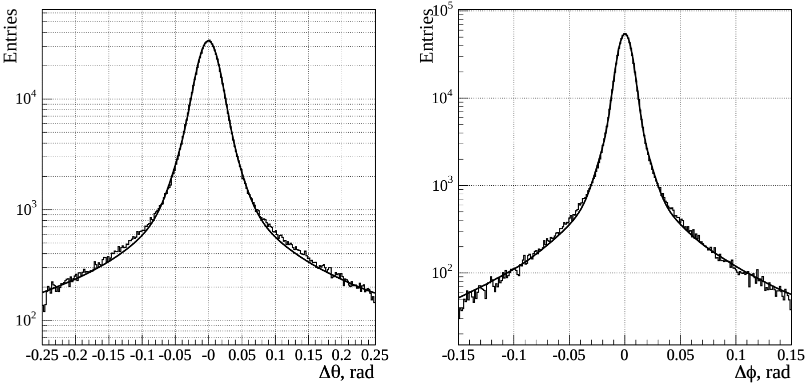
C.M. Carloni Calame, Phys. Lett. **B520** (2001) 16



- Comparison between the $\mathcal{O}(\alpha)$ BabaYaga predictions (original LL version and improved 3.5 version) and the exact $\mathcal{O}(\alpha)$ matrix element for the angular and energy photon distributions

MCGPJ vs CMD–2 data @ VEPP–2M

A.B. Arbuzov *et al.*, Eur. Phys. J. **C46** (2006) 689



- Good agreement with data!

Why precision luminosity at meson factories...?

DAΦNE, VEPP–2M, BEPC, CESR, KEK–B and PEP–II

...Because important parameters for precision tests of the Standard Model, *i.e.*

$$a_\mu \equiv (g-2)_\mu/2 = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$$

$$\alpha(q^2) = \alpha/(1 - \Delta\alpha_{\rm lep}(q^2) - \Delta\alpha_{\rm top}(q^2) - \Delta\alpha^{(5)}_{\rm had}(q^2))$$

are affected by uncertainties totally dominated by hadronic contributions, which

- are not calculable with perturbative QCD at low virtualities
- rely on dispersion relations containing experimental data of $e^+e^- \to {\rm hadrons}$ – $\sigma_{\rm had}(s)$ – at low energies as input

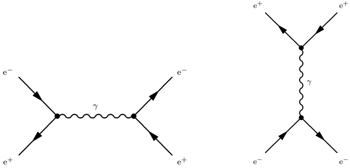
$$a_\mu^{\rm had} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, \sigma^0_{\rm had}(s) K(s)$$

$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{q^2}{4\pi^2\alpha} P \int_{m_\pi^2}^{\infty} ds \frac{\sigma^0_{\rm had}(s)}{s - q^2}$$

★ More and more precise measurements of the hadronic cross section in e^+e^- annihilation at meson factories continuously demanded! ★

The luminosity monitoring processes

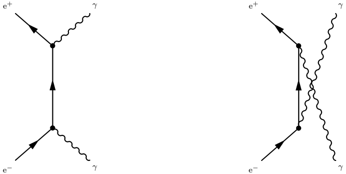
- $e^+e^- \to e^+e^-$ (Bhabha scattering)



Homi J. Bhabha (1909-1966)

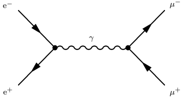
$$|M|^2 \propto \alpha^2 \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} + \frac{2u^2}{ts} \right)$$

- $e^+e^- \to \gamma\gamma$



$$|M|^2 \propto \alpha^2 \left(\frac{u}{t} + \frac{t}{u} \right)$$

- $e^+e^- \to \mu^+\mu^-$



$$|M|^2 \propto \alpha^2 \frac{t^2+u^2}{s^2}$$