Supersymmetry at higher orders

- ***3-loop corrections to the Higgs boson mass***
- α_s ***at the GUT scale to 3-loop accuracy***

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in collaboration with

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3-loop corrections to the Higgs boson mass

Motivation

Experiment LHC: $\delta m_h^{\mathrm{exp}} = 100 - 200\,\mathrm{MeV}$ and ILC: $\delta m_h^{\mathrm{exp}} = 50\,\mathrm{MeV}$

Theory: Higgs sector of the MSSM perturbatively calculable ($m_h \leq 135\,\mathrm{GeV}$)

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- exact 1-loop [Chankowski, Pokorski and Rosiek '92], [Brignole '92], [Dabelstein '94]
- 2-loop $\mathcal{O}(\alpha_t\alpha_s, \alpha_t^2, \alpha_b\alpha_s, \alpha_b\alpha_t)$ in effective potential approximation ($p^2 = 0$)
  [Haber, Hempfling, Hoang '96], [Heinemeyer, Hollik and Weiglein '98], [Degrassi, Slavich, Zwirner '01], [Espinosa and Zang '00], [Brignole, Degrassi, Slavich, Zwirner '02] , [Carena et al '00], [Heinemeyer et al '05] , [S. Martin '03]
- Momentum-dependent corrections ($p^2 = m_h^2$): 2-loop SUSY-QCD [S.Martin '05]
- 3-loop LL and NLL $\mathcal{O}(\alpha_t\alpha_s^2,\, \alpha_t^2\alpha_s,\, \alpha_t^3)$ [S. Martin '07]

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- 3-loop LL and NLL $\mathcal{O}(\alpha_t\alpha_s^2,\, \alpha_t^2\alpha_s,\, \alpha_t^3)$ [S. Martin '07]
- Missing contributions: $\delta m_h^{\rm th} \simeq 3 - 5\,{\rm GeV}$ [G. Degrassi et al '02], [Allanach et al '04]
  - full 2-loop corrections
  - full 3-loop SUSY-QCD corrections

Framework

MSSM:

$$\begin{aligned} V_0 &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ &+ \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \end{aligned}$$

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m_1, m_2, M_{12}: soft SUSY breaking terms

g, g': $SU(2)$ and $U(1)$ gauge couplings

$\epsilon_{12} = -1$

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

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Additional parameters: $\tan\beta = v_2/v_1, \quad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$

$$\mathcal{M}^2_{H,\text{tree}} = \frac{\sin 2\beta}{2} \times \begin{pmatrix} M_Z^2 \cot\beta + M_A^2 \tan\beta & -M_Z^2 - M_A^2 \\ -M_Z^2 - M_A^2 & M_Z^2 \tan\beta + M_A^2 \cot\beta \end{pmatrix}$$

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Higher order corrections

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$\hat{\Sigma}_{\phi_i}$ = renormalized self-energies

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V_{eff}-approximation: $p^2 = 0 \;\Rightarrow\; \hat{\Sigma}_i(0) = \Sigma_i(0) - \delta V_i$

$\Sigma_i(0)$ = bare self-energies

δV_i = Higgs potential counterterms

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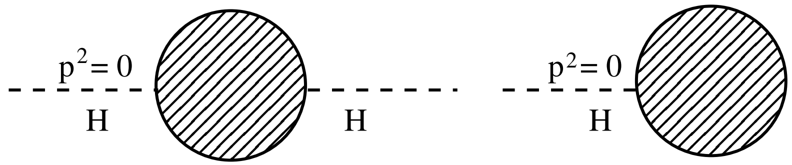

Framework (2)

Approximations:

- $\mathcal{O}(\alpha_t \alpha_s^2)$ corrections $\rightsquigarrow$ $\mathcal{O}(M_{\text{top}}^4)$
- no-mixing in the top-sector $\rightsquigarrow$ $M_{\tilde{t}_1} = M_{\tilde{t}_2}$
- A_t-contributions neglected $\rightsquigarrow$ $A_t = 0$

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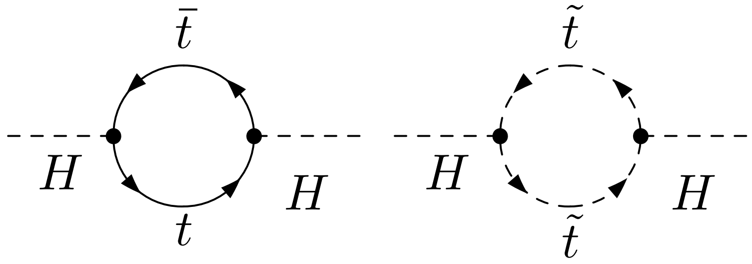

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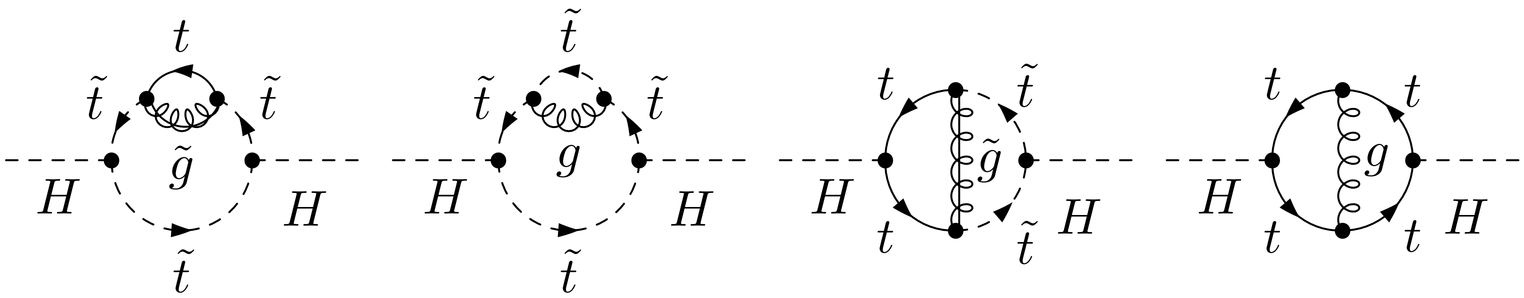

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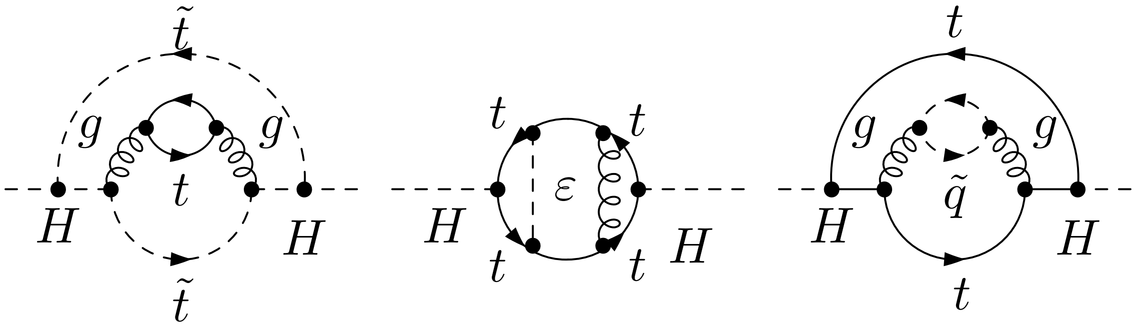

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- $\simeq$ 16.000 diagrams
- Computer programs: QGRAF, PERL, FORM, MINCER, MATAD, EXP, . . .

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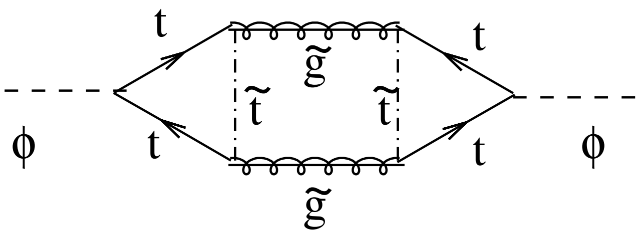

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Aymptotic expansion $\rightsquigarrow$ 3-loop tadpole integrals

Regularization and Renormalization

- Regularization: Dimensional Reduction $\rightsquigarrow$ ε-scalars
  - anti-commuting γ_5
- Renormalization
  - α_s in $\overline{\text{DR}}$ - scheme to 1-loop
  - $M_t,\ M_{\tilde{t}_1},\ M_{\tilde{t}_2}$ in OS - scheme to 2-loops
    [Bednyakov, Onishchenko, Velizhanin and Veretin '02], [S. Martin '03,'05]
    re-computed for specific mass hierarchies
  - $M_{\tilde{g}}$ in OS - scheme to 1-loop [Pierce, Bagger, Matchev and Zahng '96]
  - M_ε in OS - scheme to 1-loop [Bednyakov, Onishchenko, Velizhanin and Veretin '02]
    $M_\varepsilon = 0 \Leftrightarrow \overline{\text{DR}}'$
  - $\theta_{\tilde{t}}$ in OS - scheme to 1-loop [Pierce, Bagger, Matchev and Zahng '96]
  - $A_t \rightsquigarrow 2M_tA_t = (M^2_{\tilde{t}_1} - M^2_{\tilde{t}_2})\sin 2\theta_t + 2M_t\mu_{\text{SUSY}}\cot\beta$

Results

- Cross Checks
  - exact 2-loop results: agreement with [FeynHiggs], [Degrassi, Slavich, Zwirner '01]
  - 2- and 3-loop results: gauge independent
  - SUSY-limit ($M_t = M_{\tilde{t}},\; M_{\tilde{g}} = M_q = M_{\tilde{q}} = 0)$: $\delta M_h^{(3)} = 0$

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\begin{aligned}
\hat{\Sigma}_{\phi_2} \quad = \quad & \frac{3 G_F M_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \Bigg\{ L_{tS} + \frac{\alpha_s}{\pi}\left[-4L_{tS} + 2L_{tS}^2\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[\frac{671}{324} + \frac{1}{27}\pi^2 + \frac{1}{9}\zeta_3 \\
& + \left(-\frac{1591}{108} - 3L_{\mu t} + \frac{1}{3}\pi^2 - \frac{4}{9}\pi^2 \ln 2 + \frac{55}{18} L_{t\tilde{q}} + \frac{5}{6} L_{t\tilde{q}}^2 \right) L_{tS} \\
& + \left(\frac{13}{18} + \frac{3}{2} L_{\mu t} - \frac{5}{3} L_{t\tilde{q}} \right) L_{tS}^2 + \frac{53}{18} L_{tS}^3 \\
& + \left(\frac{475}{108} - \frac{5}{9}\pi^2 \right) L_{t\tilde{q}} - \frac{25}{36} L_{t\tilde{q}}^2 - \frac{5}{18} L_{t\tilde{q}}^3 + \mathcal{O}\left(\frac{M_{\mathrm{SUSY}}^2}{M_{\tilde{q}}^2} \right) \Bigg] \Bigg\}
\end{aligned}
$$

$$
L_{\mu t} = \ln(\mu^2/M_t^2), \quad L_{tS} = \ln(M_t^2/M_{\mathrm{SUSY}}^2), \quad L_{t\tilde{q}} = \ln(M_t^2/M_{\tilde{q}}^2)
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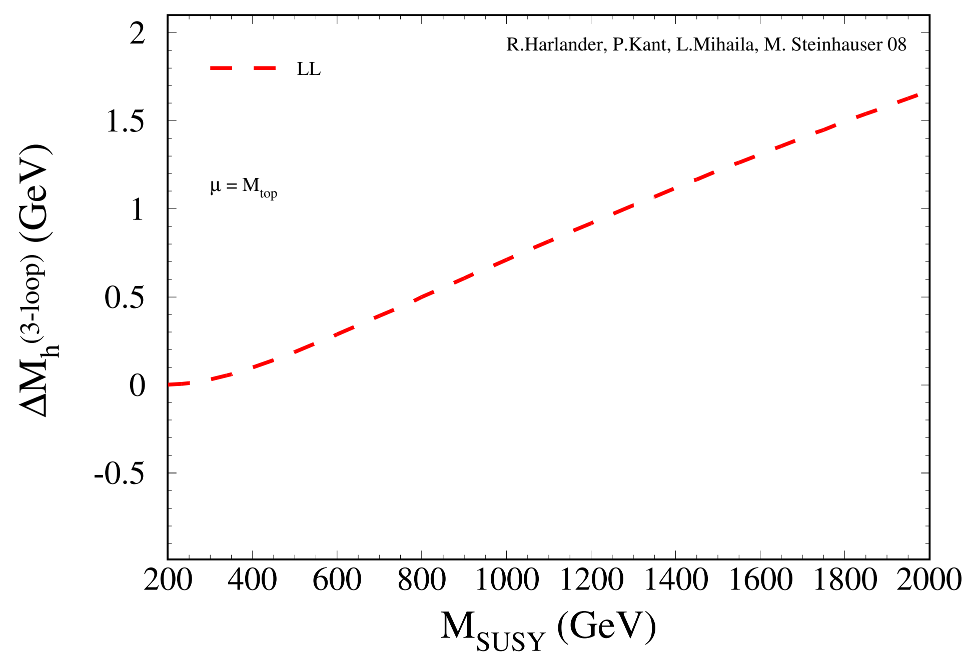

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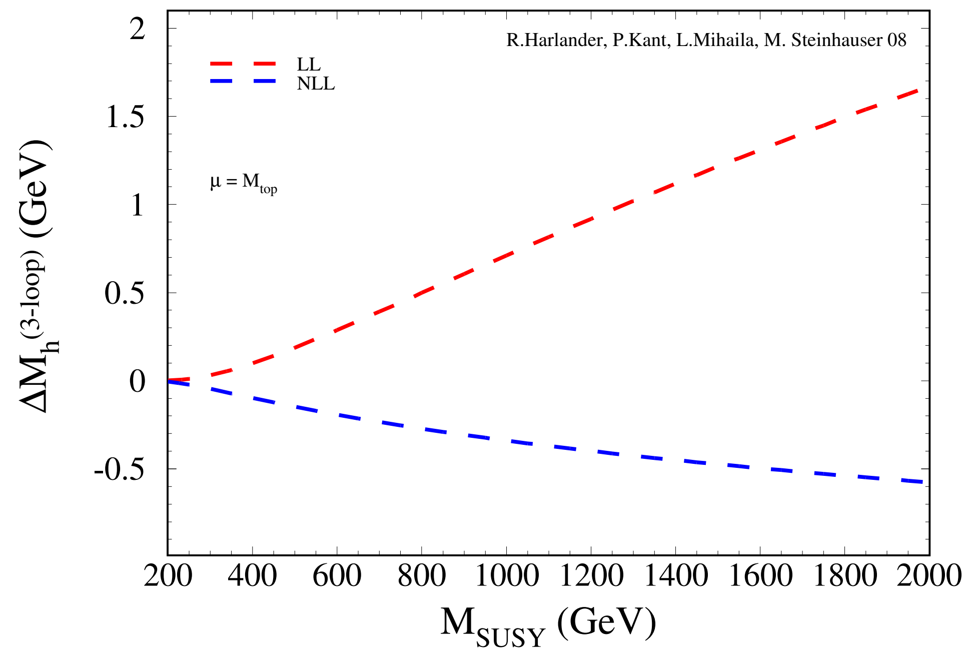

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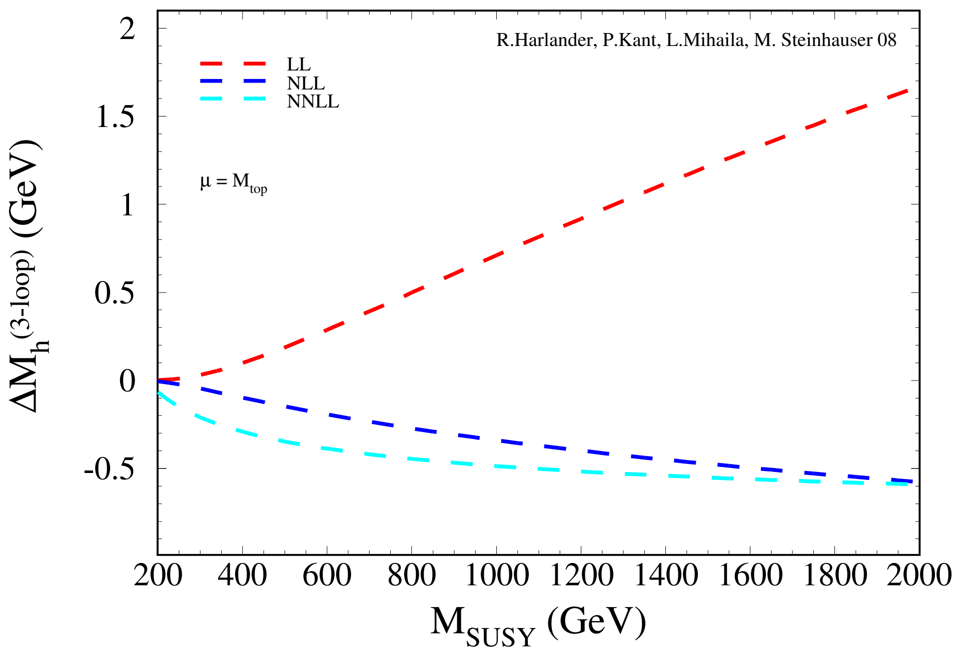

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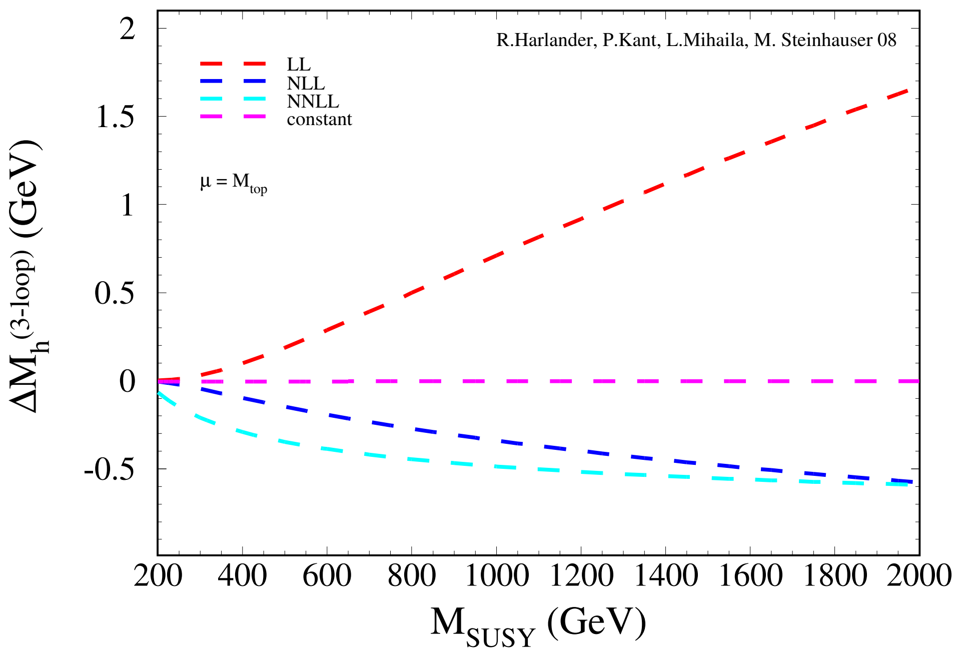

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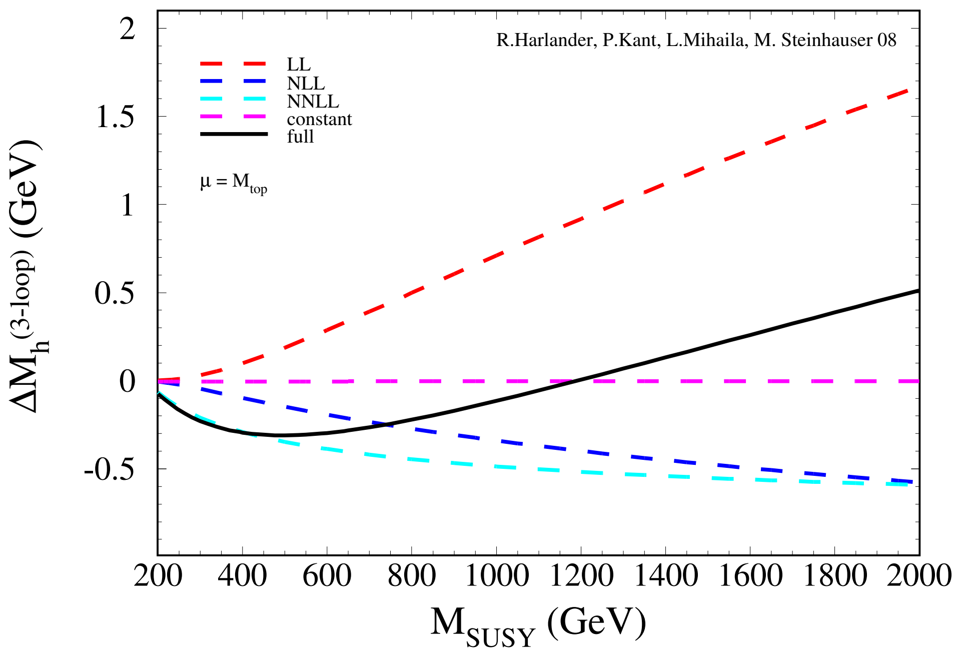

Numerical Results

Input parameters:

$\mu = M_t = 170.9$ GeV	$G_F = 1.16637 \times 10^{-5}$ GeV^{-2}	
$M_Z = 91.1876$ GeV	$\alpha_s^5(M_Z) = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$	
$M_A = 1$ TeV	$\tan\beta = 40$	$M_{\tilde{q}} = 2$ TeV

$$\Delta M_h^{(n)} \equiv M_h^{(n-\mathrm{loop})} - M_h^{\mathrm{tree}}$$

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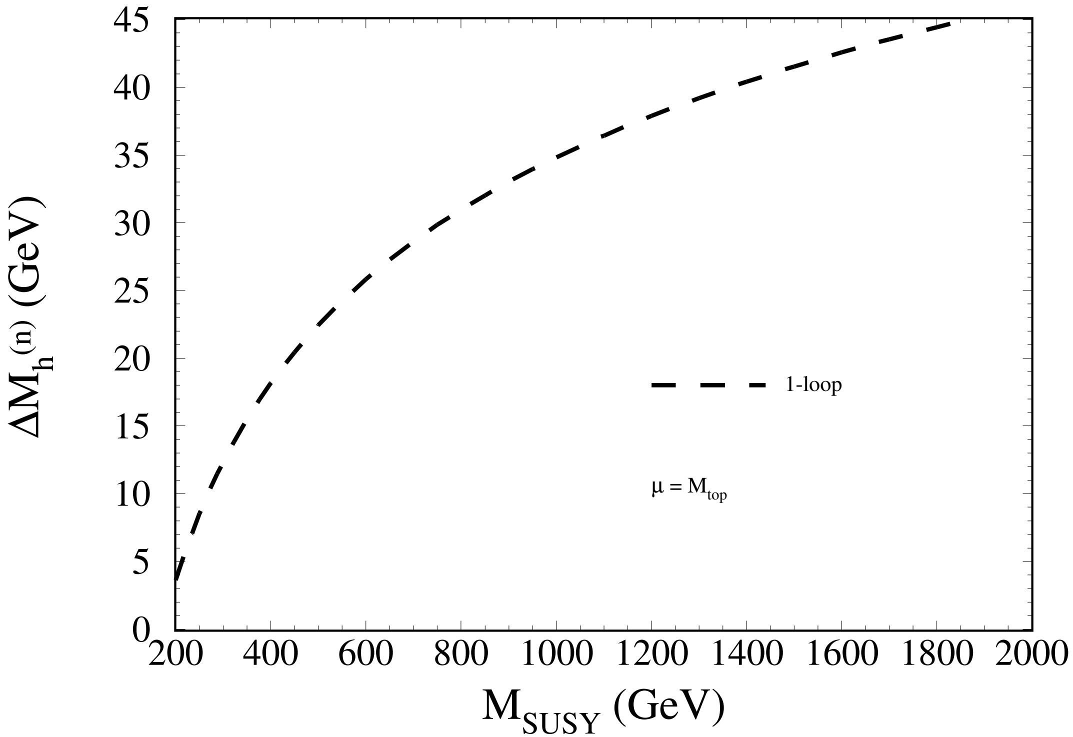

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Input parameters: $\mu \ = M_t = 170.9$ GeV $\quad G_F \ = 1.16637 \times 10^{-5}$ GeV^{-2}

$M_Z = 91.1876$ GeV $\quad \alpha_s^5(M_Z) \ = 0.1189 \Rightarrow \alpha_s(M_t) = 0.0926$

$M_A = 1$ TeV $\quad \tan\beta \ = 40 \quad M_{\tilde{q}} \ = 2$ TeV

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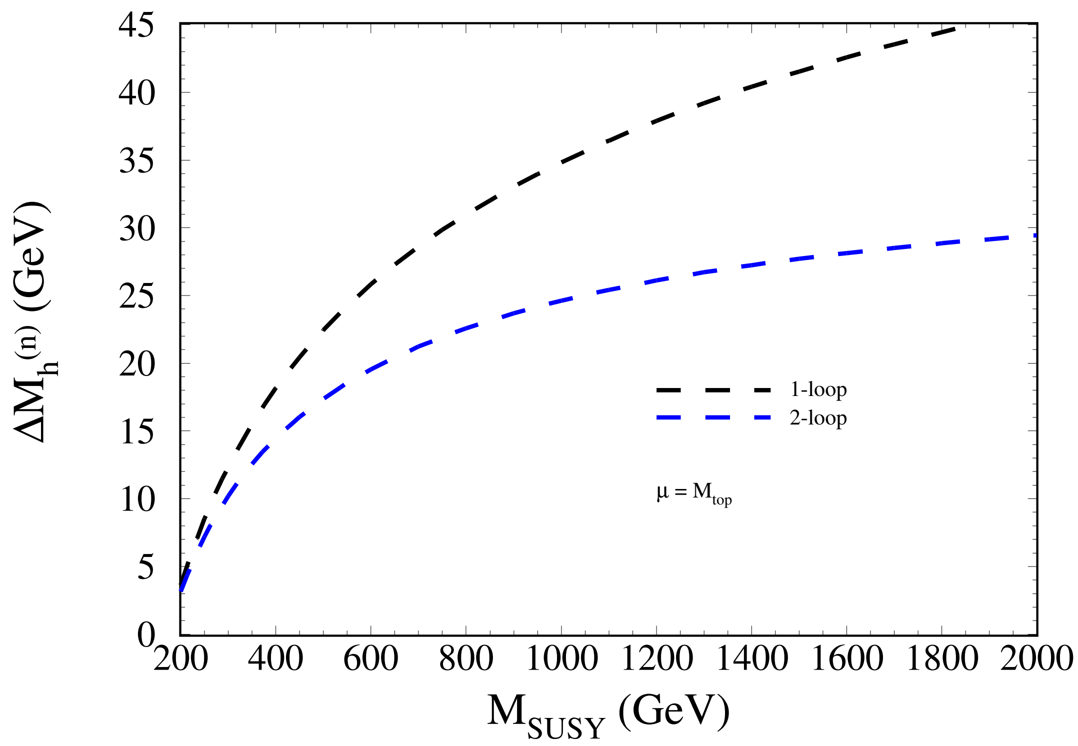

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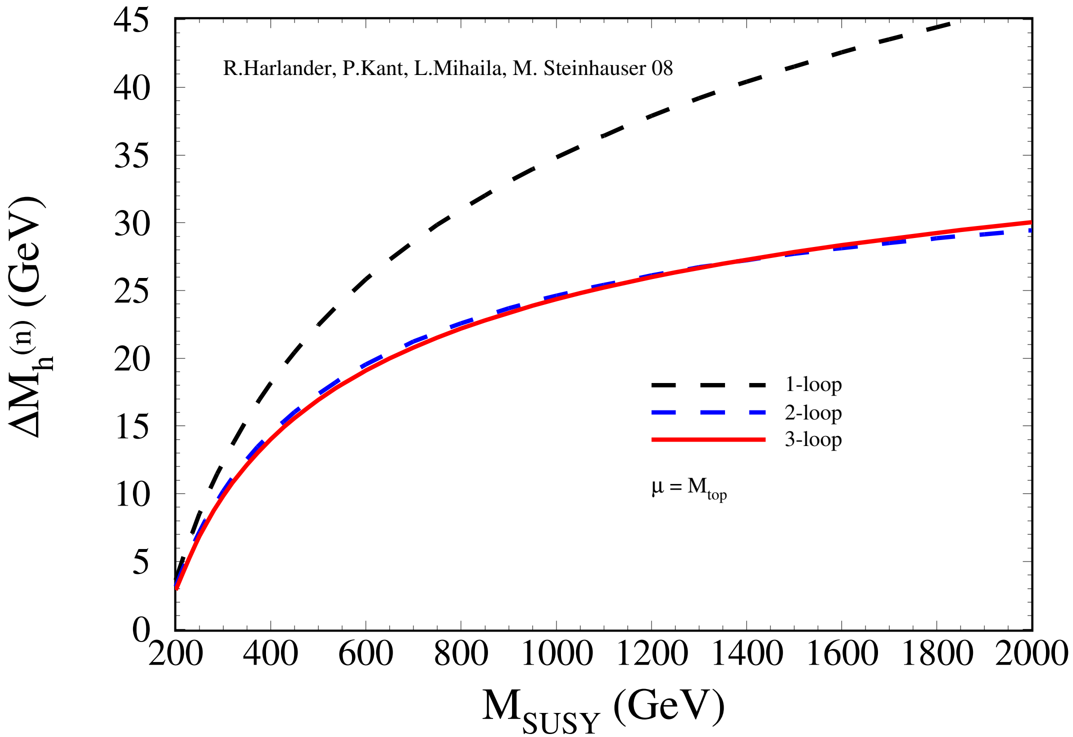

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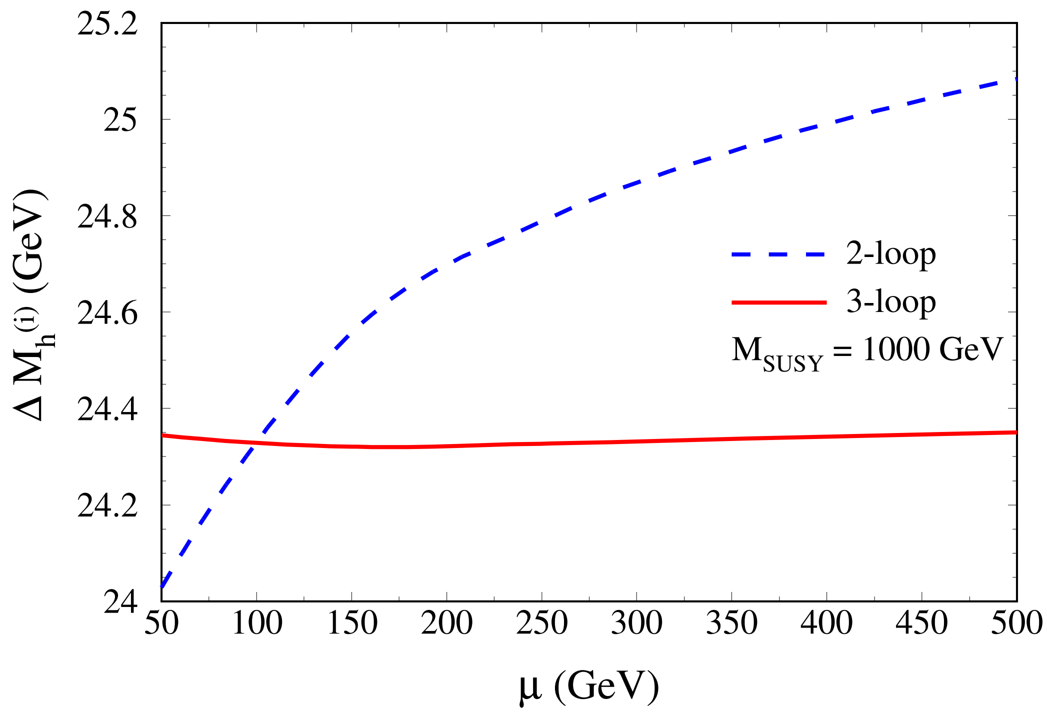
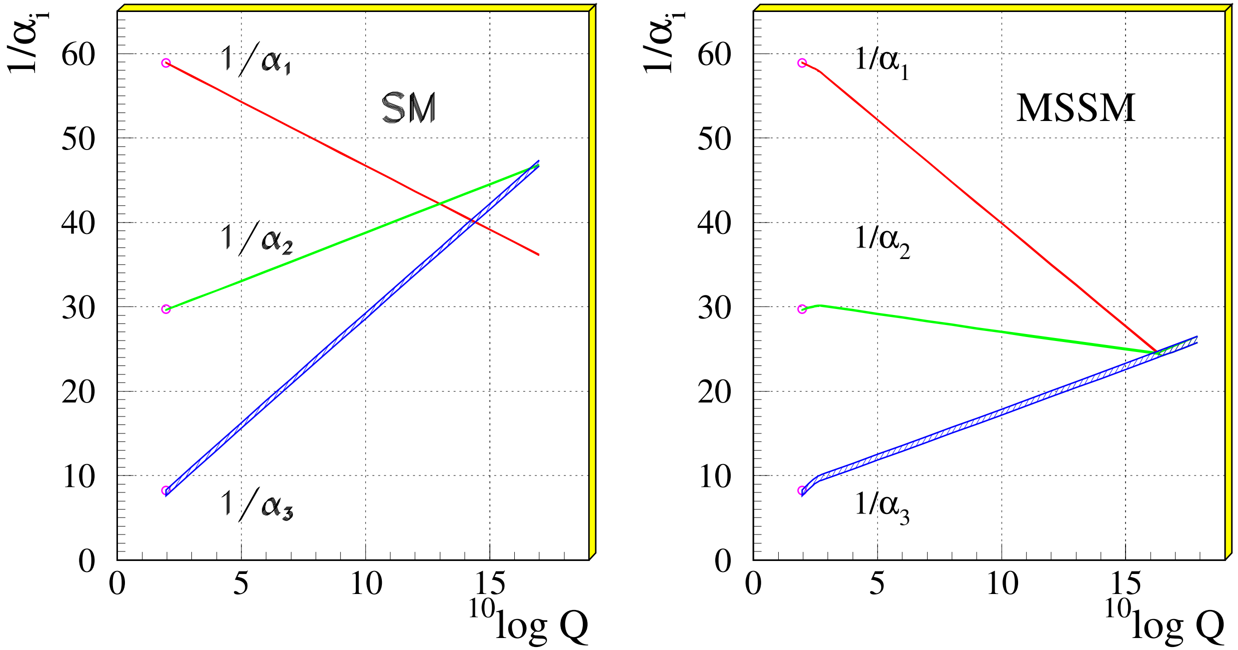
Renormalization scale dependence



α_s at the GUT scale to 3-loop accuracy

MSSM and LEP data

Unification of the Coupling Constants
in the SM and the minimal MSSM



[Amaldi, Furstenau, de Boer]

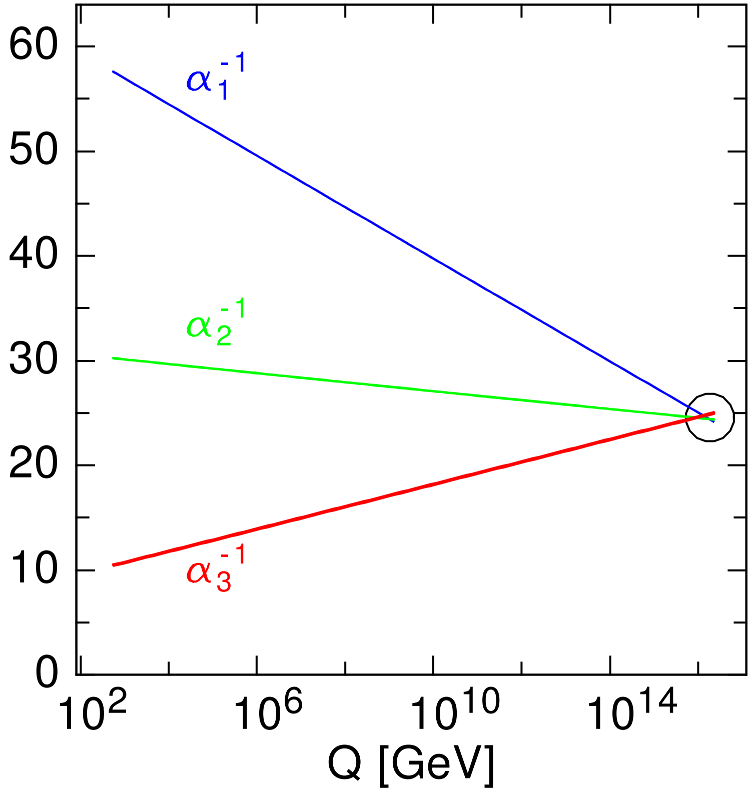
[Langacker, Luo]

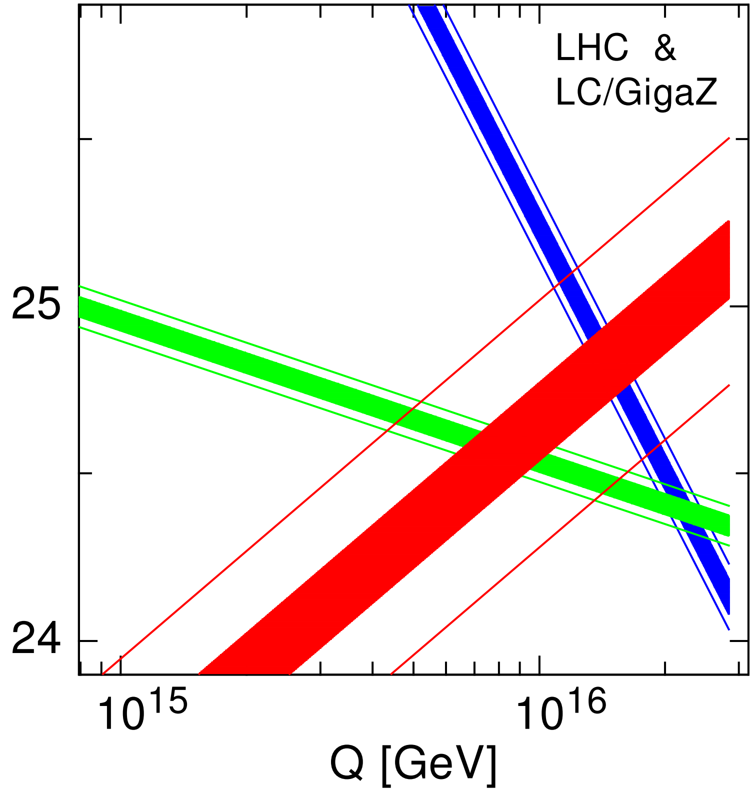
[Ellis, Kelley, Nanopoulos]

- Gauge Coupling unification within SM excluded by about $12\,\sigma$.
- Gauge coupling Unification within SUSY GUTs works extremely well: it fits within 3σ the present low energy data.

High precision data

[Blair, Porod, Zerwas '04]

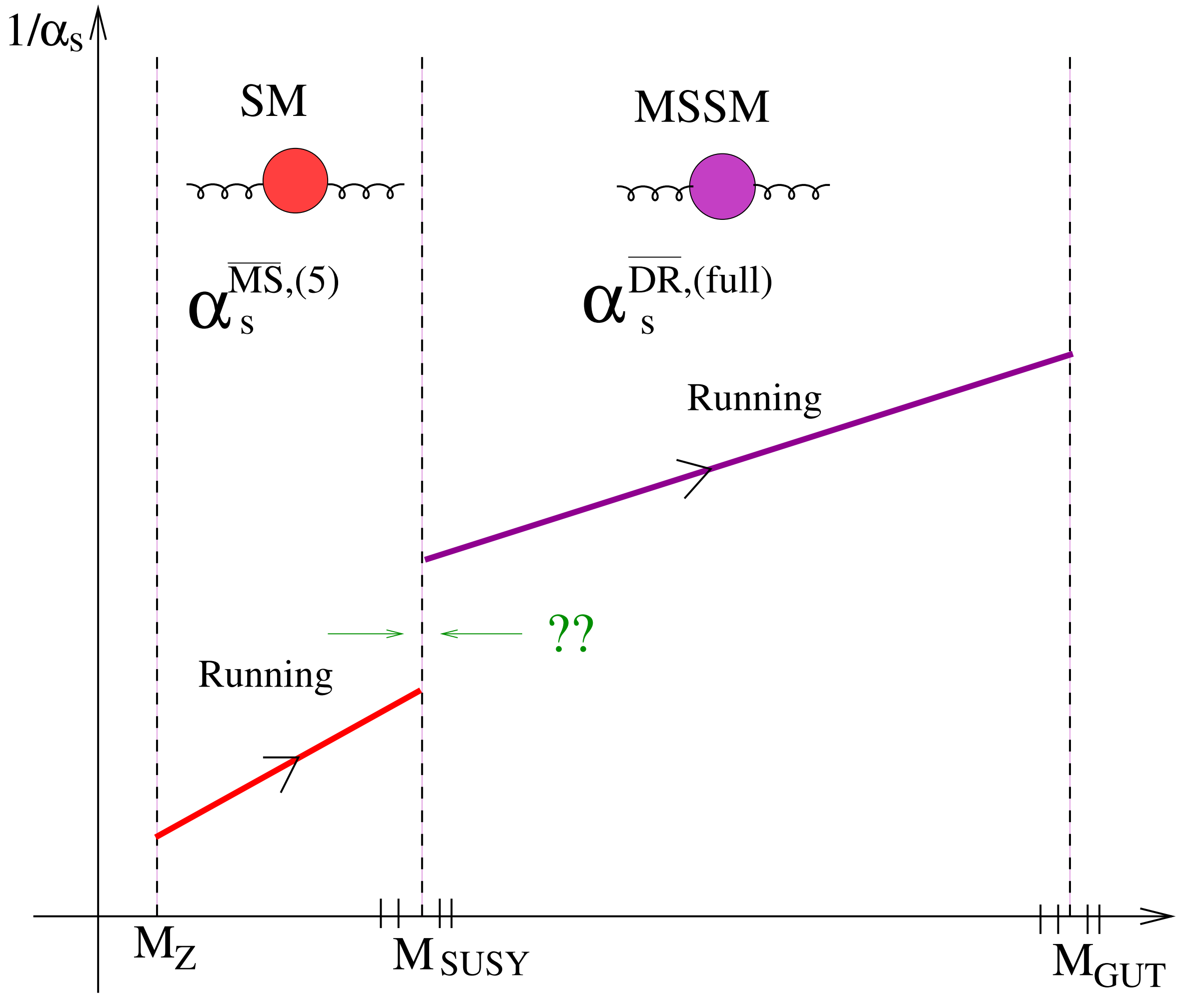




- Computation: common SUSY mass scale $\simeq$1 TeV
  2-loop Renormalization Group Running 1-loop threshold corrections at the weak scale (M_Z)
- Our aim: improve theoretical accuracy on $\alpha_s(M_{\rm GUT})$ calculated from $\alpha_s(M_Z)$

Evolution of the strong coupling

- Input parameter: $\alpha_s^{\overline{\mathrm{MS}},(5)}(\mathrm{M_Z})$ $\Rightarrow$ Output parameter: $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mathrm{M_{GUT}})$



Running of α_s

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$$

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  [v. Ritbergen, Vermaseren, Larin '97], [Czakon '05]

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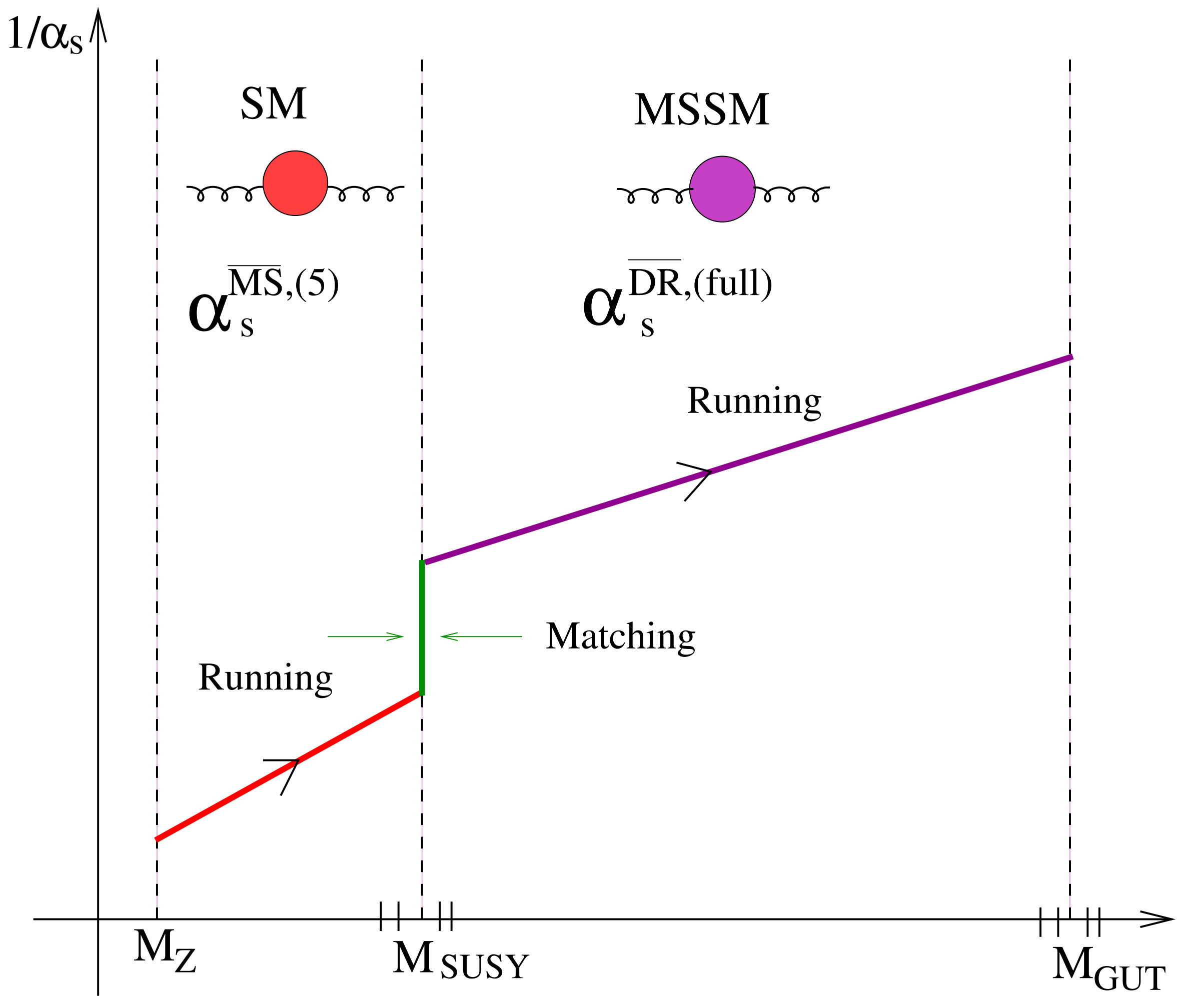
- 3-loop β_s in the MSSM
  - $\simeq$ 10.000 diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, ...

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

Matching

- Effective Field Theory:



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$$\mathcal{L}_{\rm MSSM}(\alpha_s^{(\rm full)}, \ldots) \quad \rightarrow \quad \mathcal{L}(\alpha_s^{(5)}, \ldots) \quad \text{at energy } \mu$$

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$$\alpha_s^{(5)} = \zeta_s \, \alpha_s^{(\rm full)}$$

$$\vdots$$

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- Relate Green functions in the full and effective theory for $p^2 = 0$
  - 1-loop ζ_s in MSSM [Pierce et al '95]
  - 2-loop ζ_s in MSSM [R. Harlander, L. M., M. Steinhauser '05], [A. Bauer, L. M., J. Salomon in progress]

Matching

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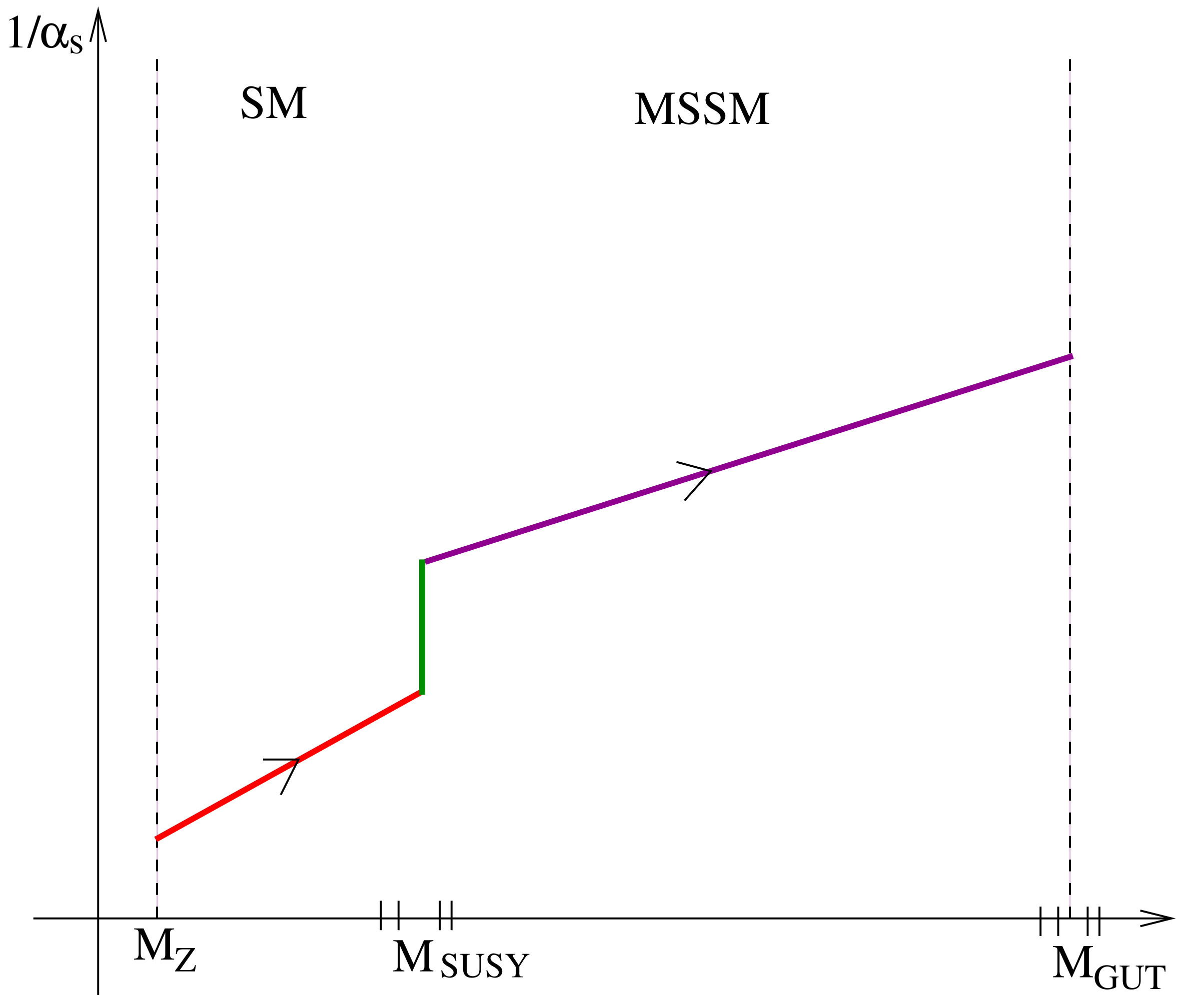

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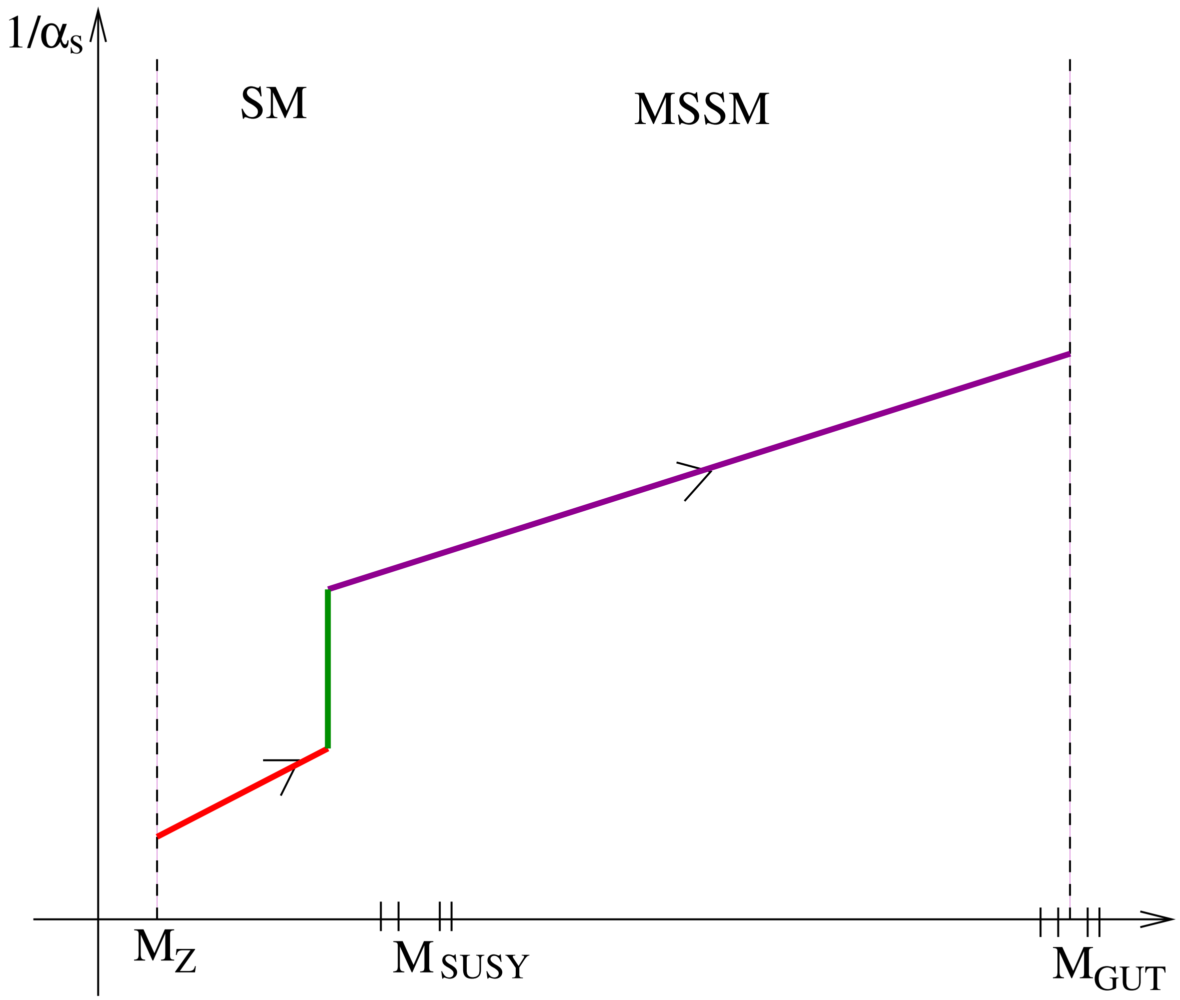

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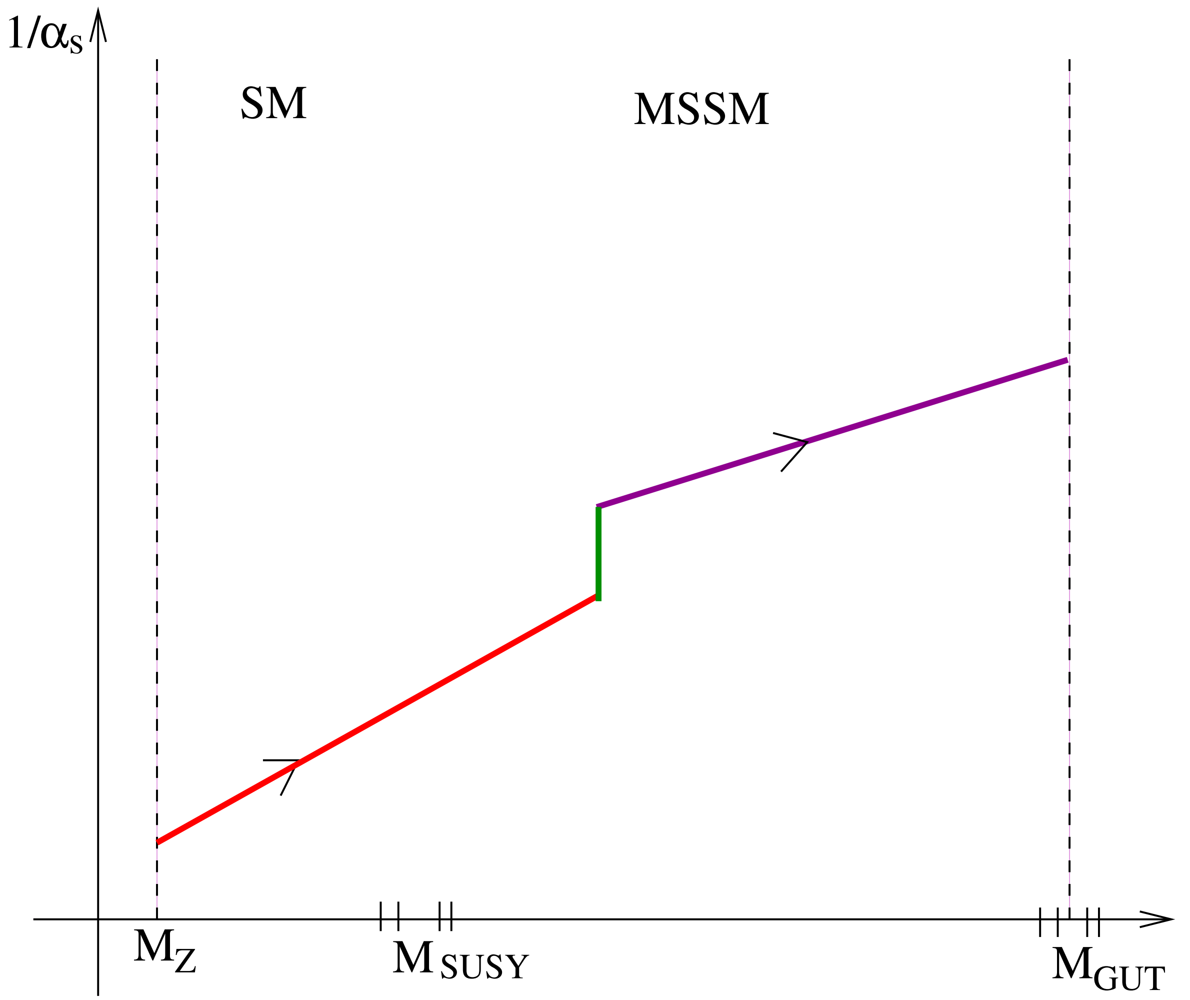

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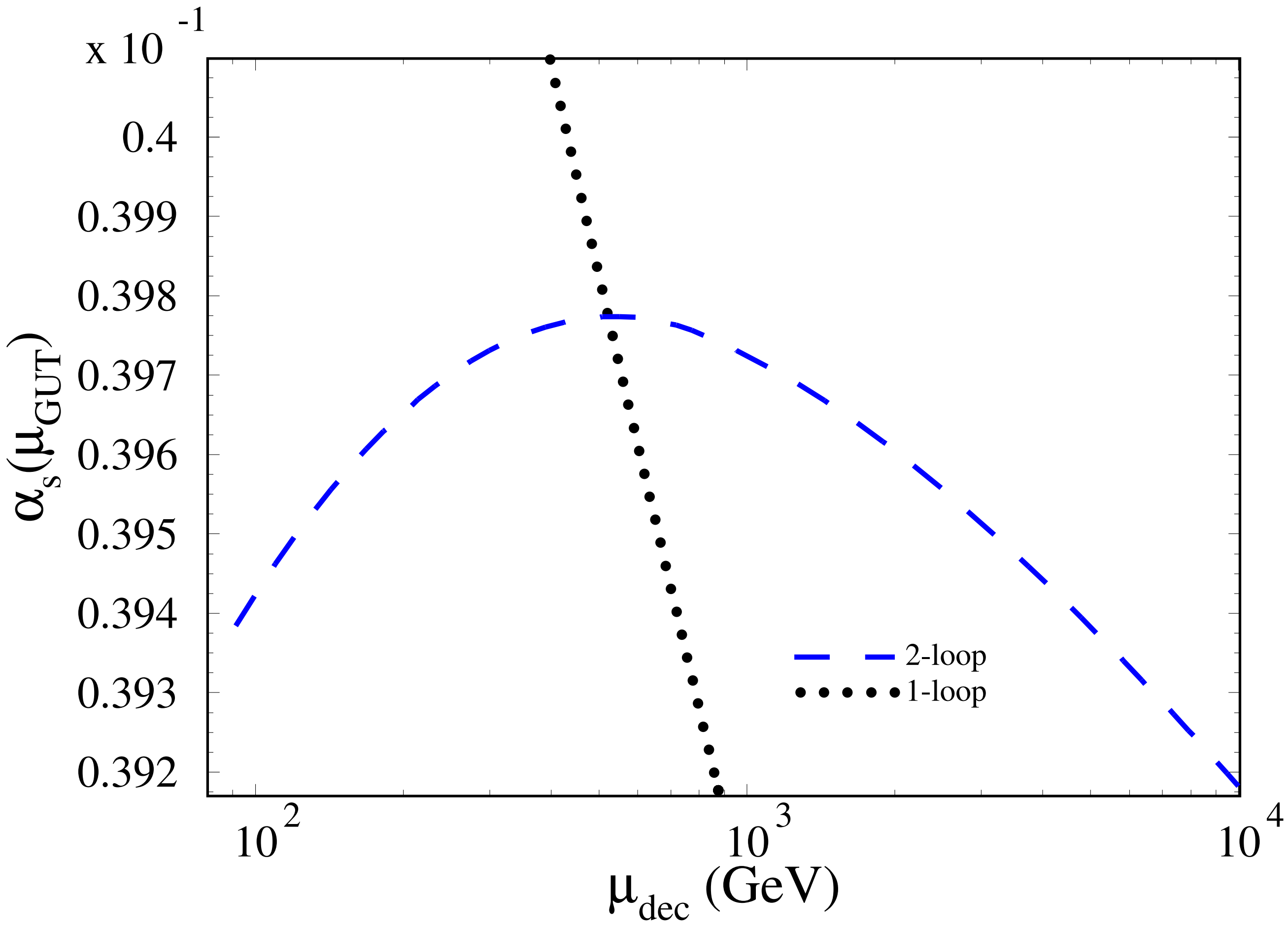

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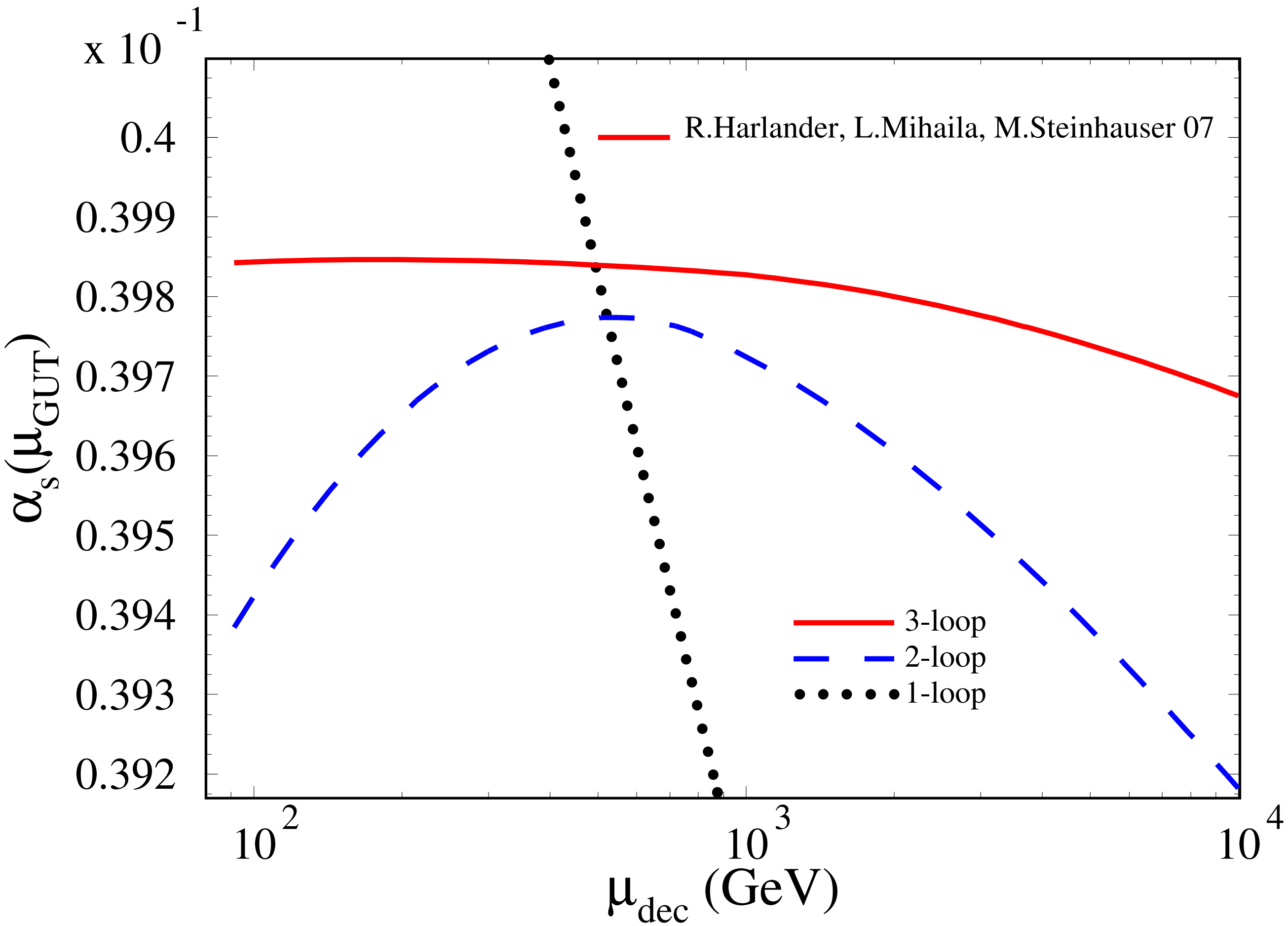

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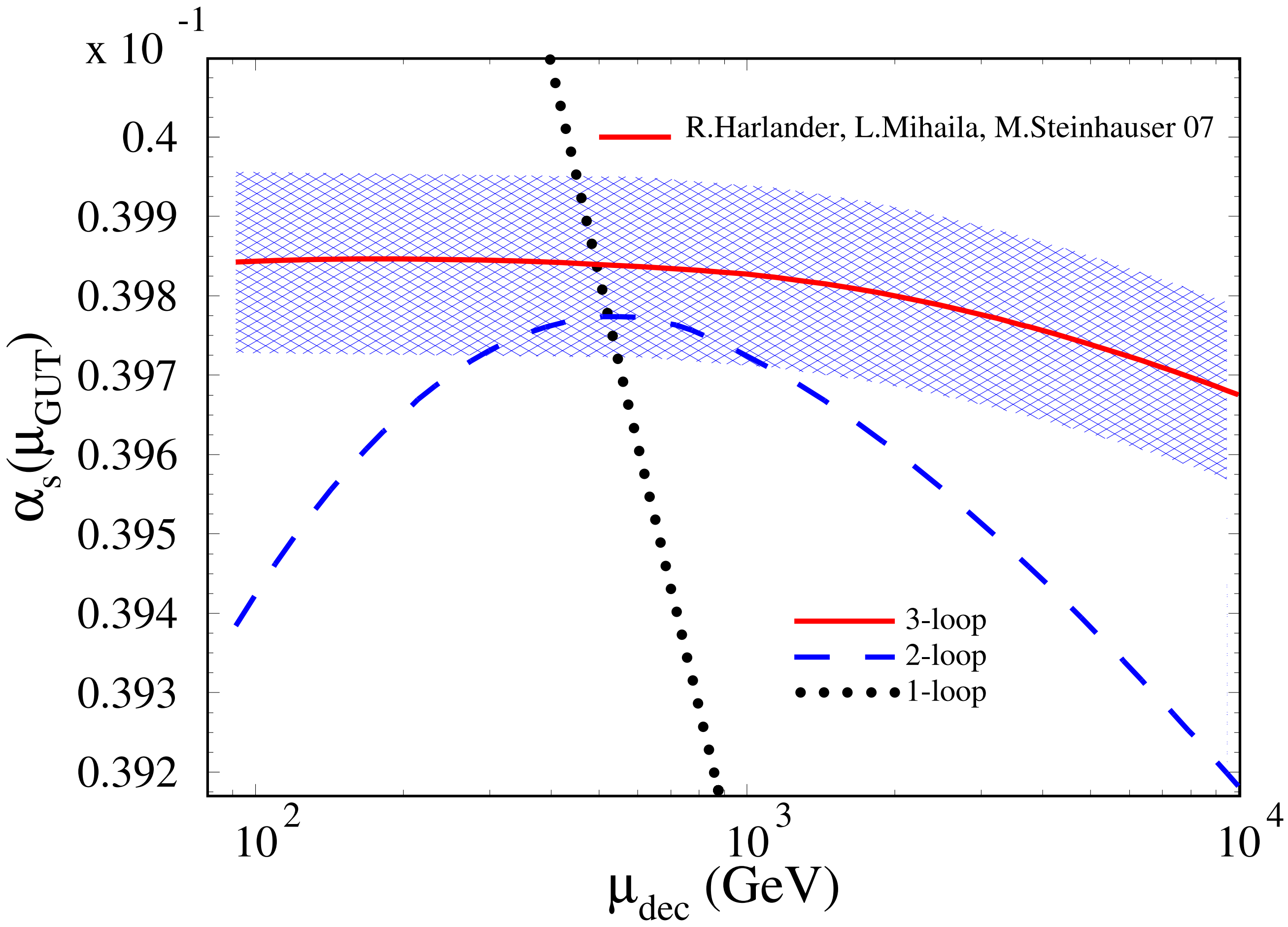
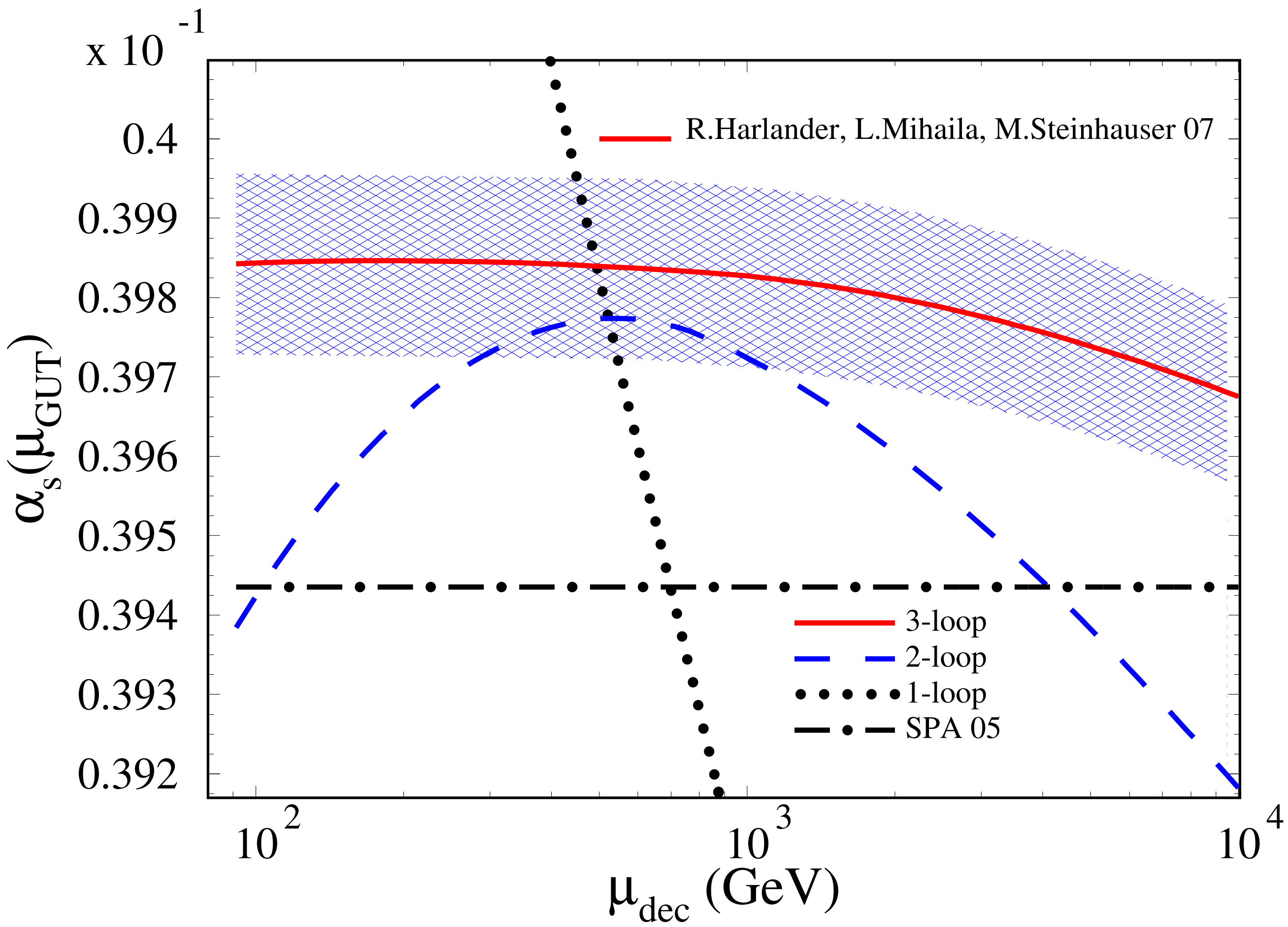


$\alpha_s(M_{\mathrm{GUT}})$

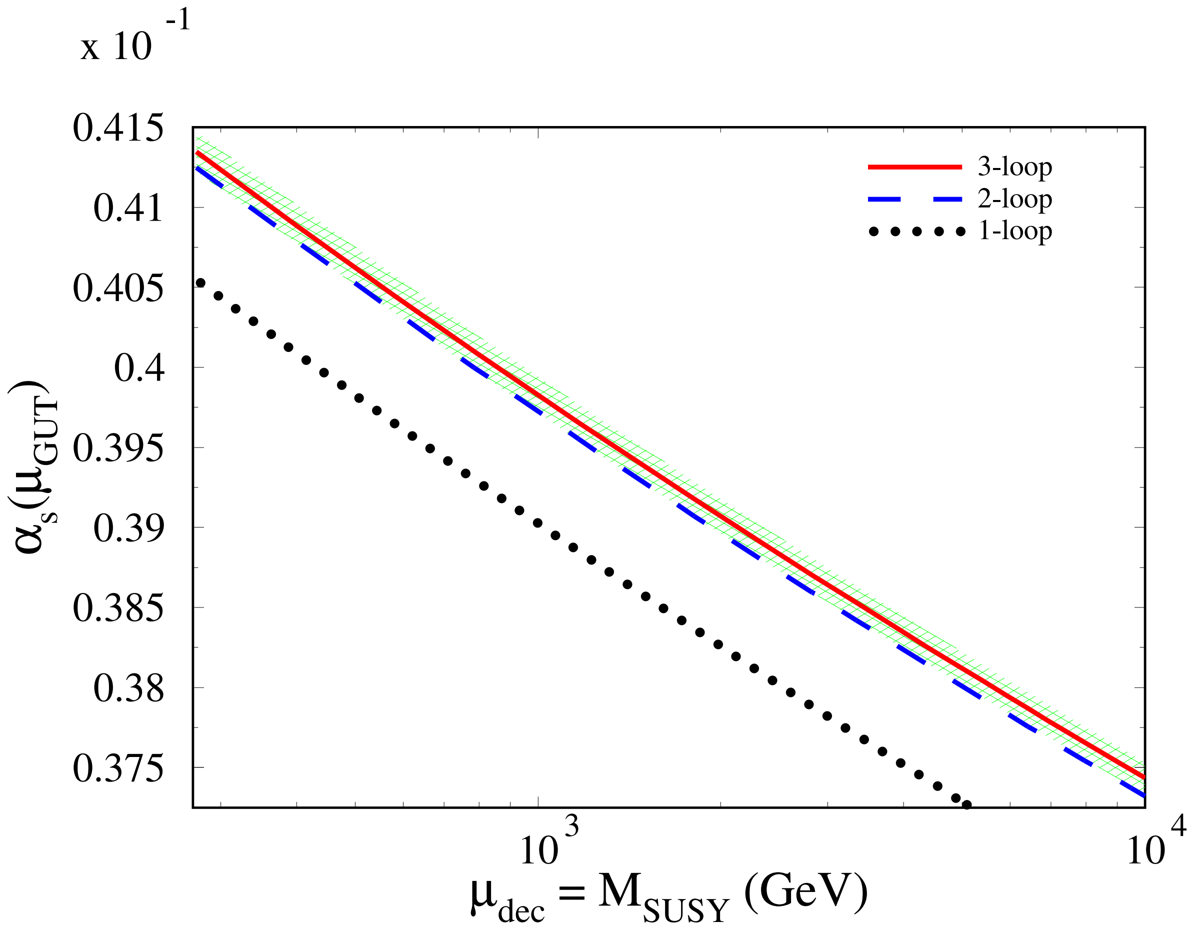
- Comparison with the Leading-Log Approximation SPA-Convention'05



SPA: state of the art at < 2007

$M_{\rm SUSY}$

- Sensitivity of $\alpha_s(M_{\rm GUT})$ to SUSY-mass scale:



Conclusions

- m_h to 3-loop accuracy
  - 3-loop effects larger than experimental accuracy expected at LHC & ILC
  - 3-loop corrections stabilize the perturbative series

- $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ to 3-loop accuracy
  - 3-loop effects comparable with experimental accuracy on α_s
  - $\alpha_{\rm s}(M_{\rm GUT})$ very sensitive to SUSY-mass scale