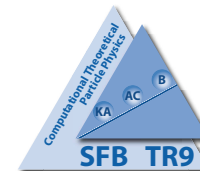


Quark Masses and the Strong Coupling Constant

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I.

Quark Masses

based on Low Moments

WHY precise masses?

B-decays:

$$\Gamma(B \rightarrow X_u l \bar{\nu}) \sim G_F^2 m_b^5 |V_{ub}|^2$$

$$\Gamma(B \rightarrow X_c l \bar{\nu}) \sim G_F^2 m_b^5 f(m_c^2/m_b^2) |V_{cb}|^2$$

moments of $\frac{dN}{dE_l}$, $\frac{dN}{dm(l\bar{\nu})}$,

moments of m_{had}^2 ,

$$B \rightarrow X_s \gamma$$

Υ -spectroscopy:

$$m(\Upsilon(1s)) = 2M_b - \left(\frac{4}{3}\alpha_s\right)^2 \frac{M_b}{4} + \dots$$

sum rules:

$$\int \frac{ds}{s^{n+1}} R_Q(s) \sim \frac{1}{m_Q^{2n}}$$

Higgs decay (ILC)

$$\Gamma(H \rightarrow b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(M_H) \tilde{R}$$

$$\tilde{R} = 1 + 5.6667 a_S + 29.147 a_S^2 + 41.758 a_S^3 - 825.7 a_S^4$$

rapidly increasing coefficients! $\left(a_S \equiv \frac{\alpha_S}{\pi}\right)$

a_S^4 -term = 5-loop calculation (Baikov,...)

Yukawa Unification

$$\lambda_\tau = \lambda_b \quad \text{or} \quad \lambda_\tau = \lambda_b = \lambda_t$$

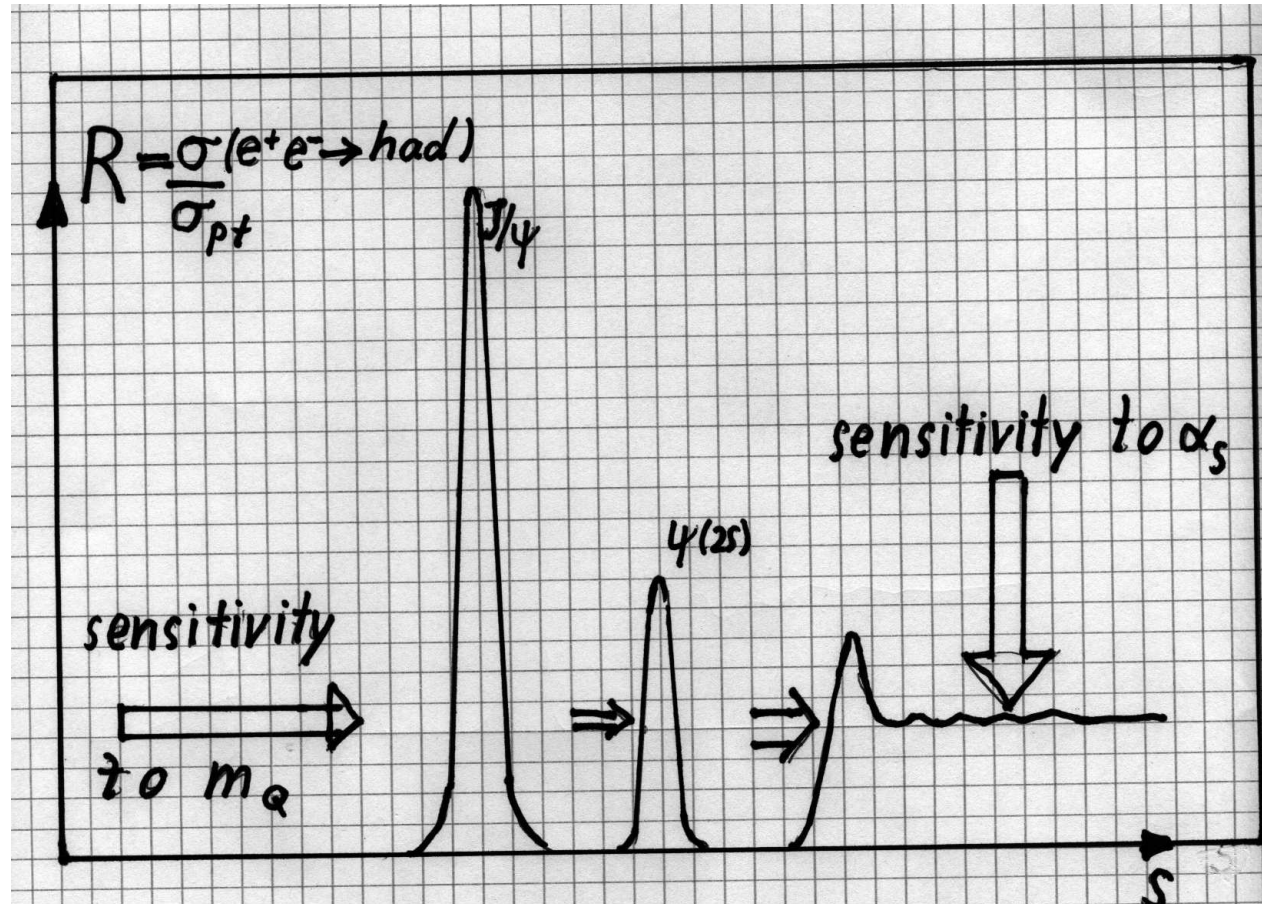
identical coupling to Higgs boson(s) at GUT scale

top-bottom $\rightarrow m_t/m_b \sim$ ratio of vacuum expectation values

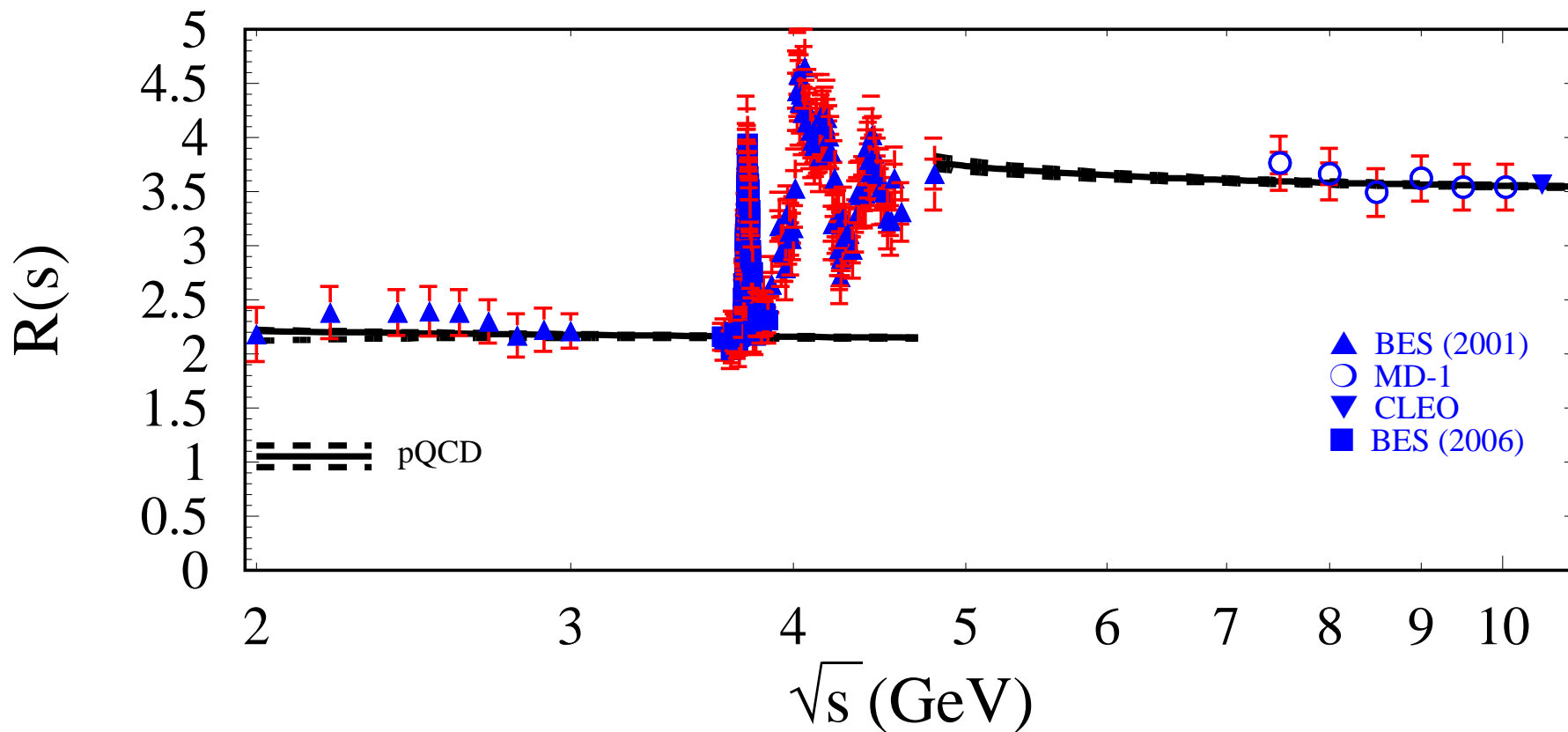
$$\text{request } \frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t}$$

$$\delta m_t \approx 1 \text{ GeV} \Rightarrow \delta m_b \approx 25 \text{ MeV}$$

Main Idea (SVZ)



Data



pQCD and data agree well in the regions
2 – 3.73 GeV and 5 – 10.52 GeV

m_Q from SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) = 12\pi \operatorname{Im} [\Pi(q^2 = s + i\epsilon)]$$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu) \Pi(q^2) \equiv i \int dx e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ

Taylor expansion:
$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\text{MS}}$ mass.

Coefficients \bar{C}_n in order α_S^2 up to $n = 8$

[Chetyrkin, JK, Steinhauser, 1996]

up to high n (~ 30); VV, AA, PP, SS correlators

[Czakon et al., 2006], [Maierhöfer, Maier, Marquard, 2007]

Coefficients \bar{C}_n in order α_S^3 up to $n = 1$

⇒ reduction to master integrals through Laporta algorithm

[Chetyrkin, JK, Sturm], [Boughezal, Czakon, Schutzmeier]

evaluation of master integrals numerically through difference equations
(30 digits) or Padé method or analytically in terms of transcendentals

[Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,
Laporta, Broadhurst, Kniehl et al.]

\bar{C}_2 would be desirable!

Define the moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_c(q^2) \Big|_{q^2=0} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2} \right)^n \bar{C}_n$$

$$\mathcal{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

$$\Leftrightarrow m_c$$

Ingredients

experiment:

- $\alpha_s = 0.1187 \pm 0.0020$
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ from BES

theory:

- N³LO for n=1
- N³LO - estimate for n = 2,3,4
- include condensates

$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s \bar{b}_n}{\pi} \right)$$

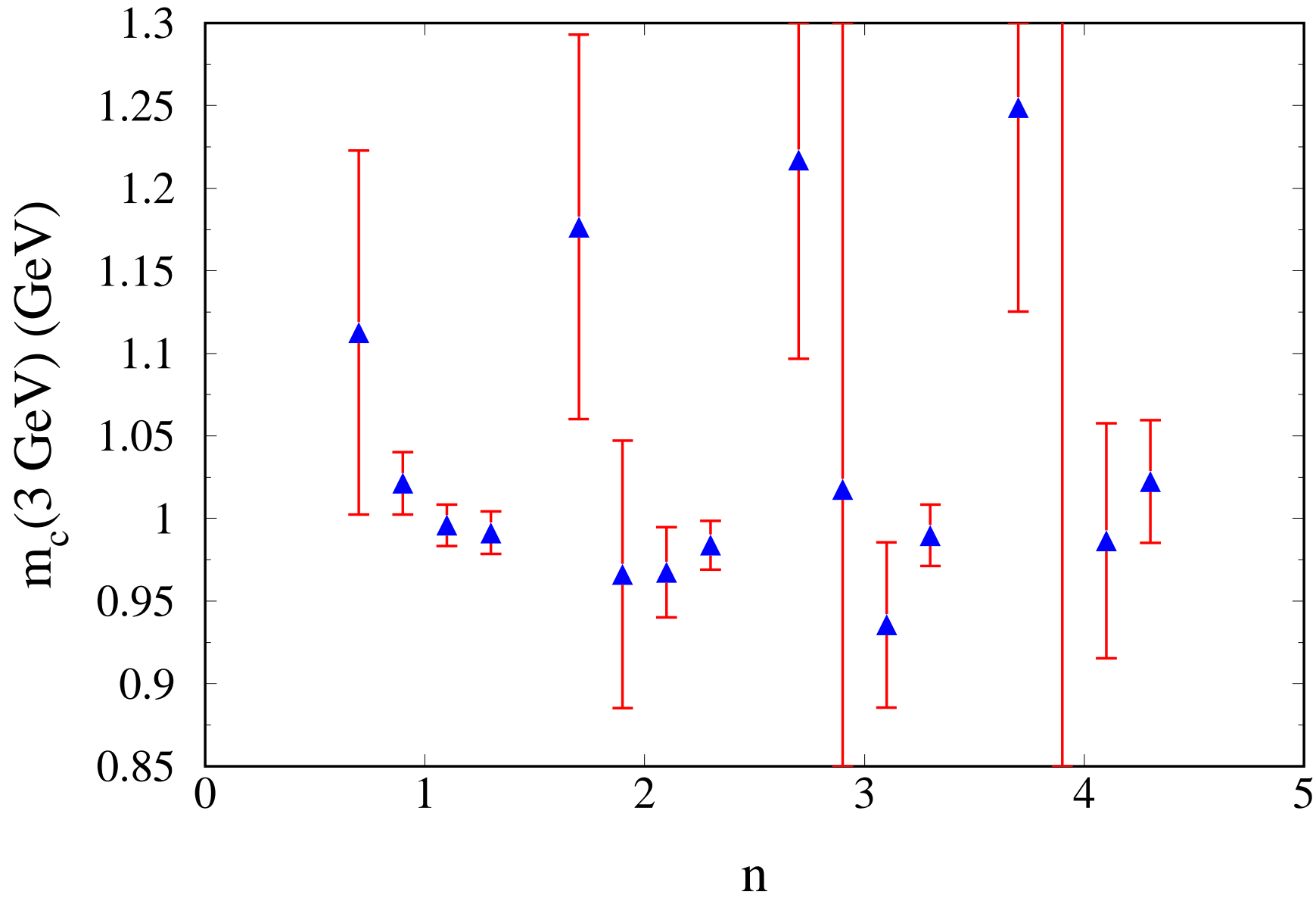
- estimate of non-perturbative terms
(oscillations, based on [Shifman](#))
- careful extrapolation of R_{uds}
- careful definition of R_c

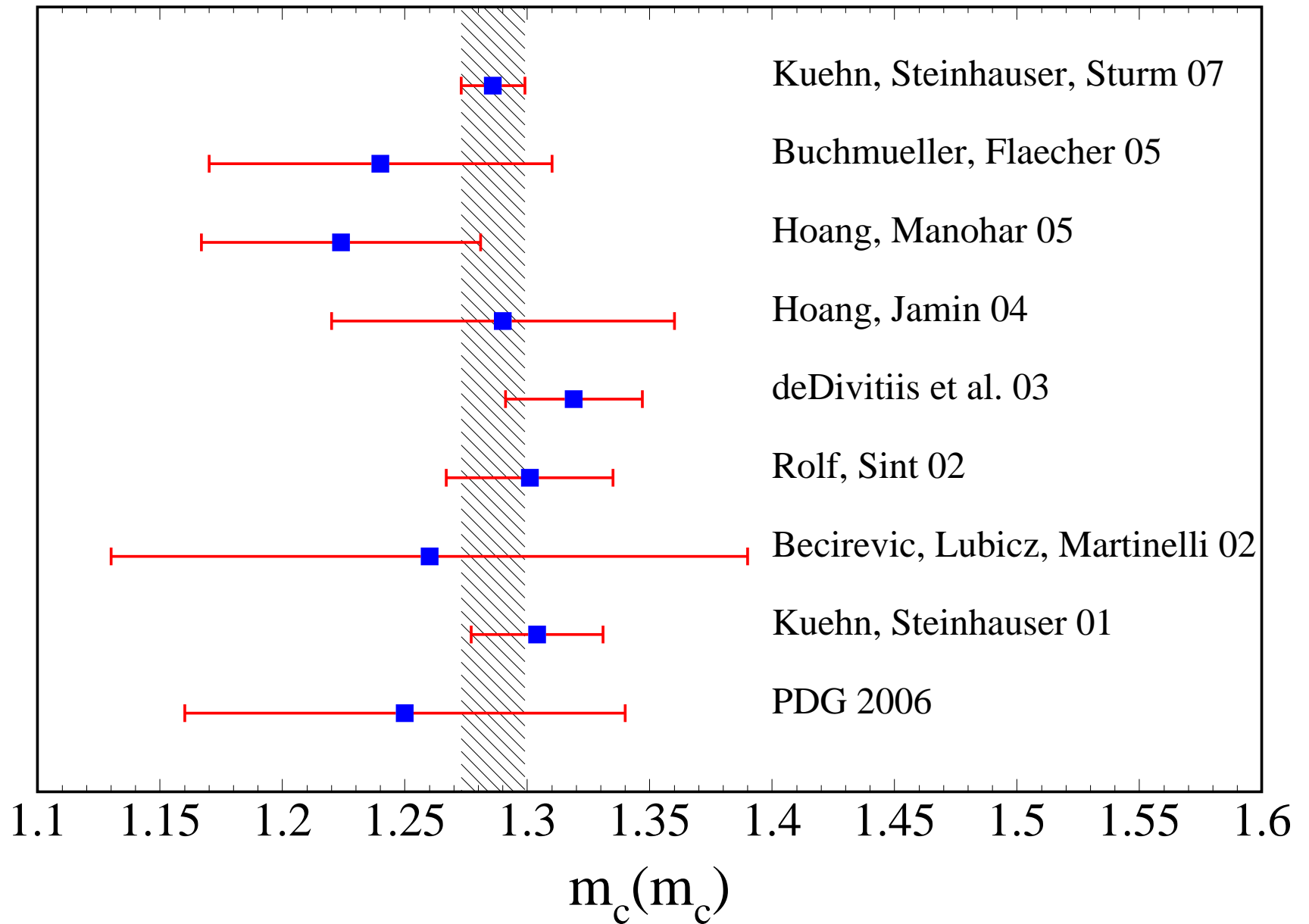
Results (m_c)

n	$m_c(3 \text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

- $m_c(3 \text{ GeV}) = 986 \pm 13 \text{ MeV}$
- $m_c(m_c) = 1286 \pm 13 \text{ MeV}$





Results (m_b)

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_b(m_b) = 4164 \pm 25 \text{ MeV}$
- $m_b(10\text{GeV}) = 3609 \pm 25 \text{ MeV}$
- $m_b(m_t) = 2703 \pm 18 \pm 19 \text{ MeV}$
- $m_t/m_b = 59.8 \pm 1.3$

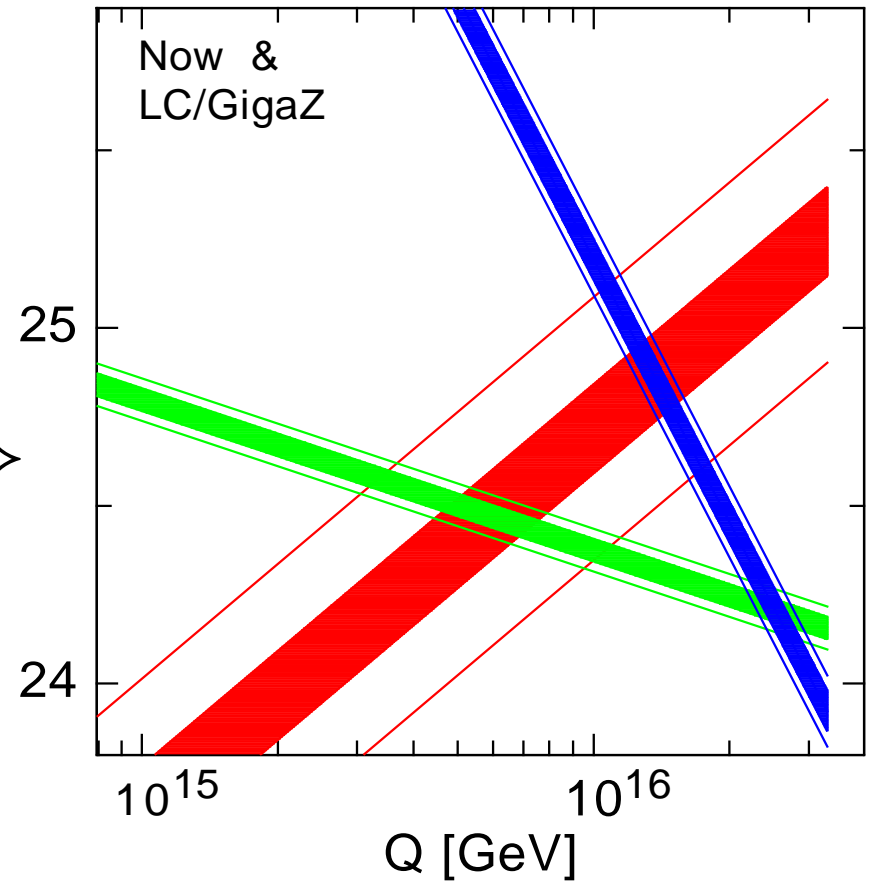
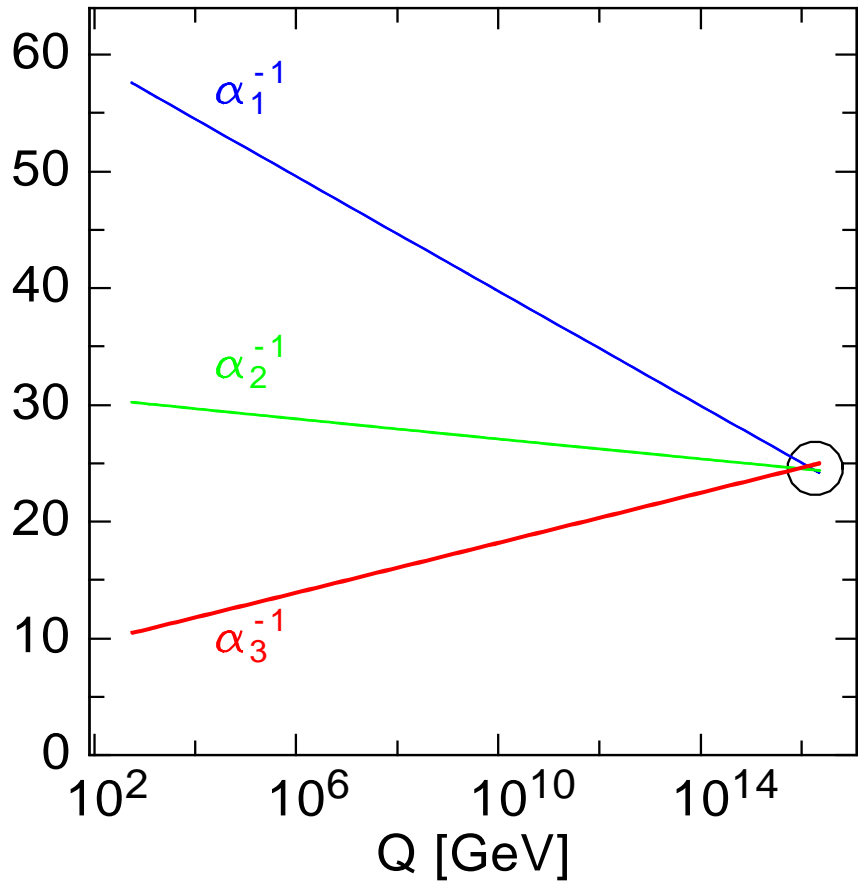
$$m_c(3 \text{ GeV}) = 0.986(13) \text{ GeV}$$
$$m_c(m_c) = 1.286(13) \text{ GeV}$$

$$m_b(10 \text{ GeV}) = 3.609(25) \text{ GeV}$$
$$m_b(m_b) = 4.164(25) \text{ GeV}$$

II.

The Strong Coupling Constant

in N^3LO



$$\alpha_s = 0.1183 \pm 0.0027 \quad \text{vs.} \quad \pm 0.0009$$

Status 2007

α_s from LEP

$$R_\ell = \Gamma_h / \Gamma_\ell = 20.767 \pm 0.025 \quad (1.2 \text{ ‰} !)$$

($\sim 10^6$ leptonic events)

$$\alpha_s = 0.1226 \pm 0.0038 \begin{matrix} +0.0028 \\ -0.0 \end{matrix} (M_H = \begin{matrix} 900 \\ 100 \end{matrix} \text{ GeV})$$

$$\sigma_\ell \sim \frac{\Gamma_\ell^2}{\Gamma_{tot}^2} = 2.003 \pm 0.0027 \text{ pb}$$

(luminosity)

$$\alpha_s = 0.1183 \pm 0.0030 \begin{matrix} +0.0022 \\ -0.0 \end{matrix} (M_H = \begin{matrix} 900 \\ 100 \end{matrix} \text{ GeV})$$

SM-fit:

$$\alpha_s = 0.1185 \pm 0.0026$$

based on $\sim 10^7$ Z-events/experiment

GIGA-Z: 10^9 events

$$\Rightarrow \delta\alpha_s = 0.0009 \quad \text{M. Winter}$$

α_s based on

$$\Gamma_{\text{had}} = \Gamma_0 (1 + a_s + 1.409 a_s^2 - 12.767 a_s^3)$$

+ small corrections

$$\left(a_s \equiv \frac{\alpha_s}{\pi} \right)$$

α_s from τ -decays

one of the most precise results for α_s

$$\frac{\Gamma(\tau \rightarrow h_{s=0} \nu)}{\Gamma(\tau \rightarrow l \bar{\nu} \nu)} = |V_{ud}|^2 S_{EW} R_\tau = 3.471 \pm 0.011$$

$$R_\tau = 3 \left(1 + \delta_P + \underbrace{\delta_{EW}}_{\text{small}} + \underbrace{\delta_{NP}}_{0.003 \pm 0.003} \right)$$

$$\delta_P = 0.1998 \pm 0.043 \text{ (exp)}$$

- previous fixed order perturbation theory:

$$\delta_P = a_s + 5.202 a_s^2 + 26.37 a_s^3 + ?$$

- previous contour improved perturbation theory:

$$\delta_P = 1.364 a_s + 2.54 a_s^2 + 9.71 a_s^3 + ?$$

previously:

estimates for α_s^4 (and α_s^5) terms only (FAC, PMS)

questions:

- are FAC/PMS supported by higher order calculations
- does the difference between fixed order (FOPT) and CIPT decrease upon inclusion of α_s^4 ?

aim: evaluate α_s^4

⇒ absorptive part of 5-loop correlators

**The long march towards α_s^4
has ended**

consider $D_1(Q^2) \equiv -12\pi^2 Q^2 \frac{d}{dQ^2} \Pi_1 = \int_0^\infty ds \frac{Q^2}{(Q^2+s)^2} R(s)$

(Adler function, μ independent)

$$D_1(q^2) = 1 + a_s + a_s^2 (-0.1153 n_f + 1.968)$$

$$+ a_s^3 (0.08621 n_f^2 - 4.216 n_f + 18.24)$$

$$+ a_s^4 (-0.010 n_f^3 + 1.88 n_f^2 - 34.4 n_f + 135.8)$$

relation to FAC/PMS

n_f	$d_4^{\text{FAC/PMS}}$	d_4^{exact}	$r_4^{\text{FAC/PMS}}$	r_4^{exact}
3	27 ± 16	49.08	-129 ± 16	-106.88
4	8 ± 28	27.39	112 ± 30	-92.90
5	-8 ± 44	9.21	97 ± 44	-79.98

impact on α_s from Z -decays

$$R(s) = D(s) - \pi^2 \beta_0^2 \left\{ d_1/3a_s^3 + \left(d_2 + \frac{5\beta_1}{6\beta_0} d_1 \right) a_s^4 \right\}$$

$$\Rightarrow \delta\alpha_s(M_Z) = 0.0005$$

$$\alpha_s(M_Z)^{\text{NNNLO}} = 0.1190 \pm 0.0026$$

impact on α_s from τ -decays

$$\frac{\Gamma(\tau \rightarrow h_{s=0} \nu)}{\Gamma(\tau \rightarrow l \bar{\nu} \nu)} = |V_{ud}|^2 S_{EW} 3 \left(1 + \delta_P + \underbrace{\delta_{EW}}_{\text{small}} + \underbrace{\delta_{NP}}_{0.003 \pm 0.003} \right)$$

$$R_\tau = 3.471 \pm 0.011$$

(Davier, Höcker, Zhang; ALEPH, OPAL, CLEO, ...)

$$\delta_P = 0.1998 \pm 0.043 \text{ (exp) scale } \mu^2/M_\tau^2 = 0.4 - 2$$

	$\alpha_s^{FO}(M_\tau)$	$\alpha_s^{CI}(M_\tau)$
no α_s^4	$0.337 \pm 0.004 \pm 0.03$	$0.354 \pm 0.006 \pm 0.002$
$d_4 = 25$	$0.325 \pm 0.004 \pm 0.02$	$0.347 \pm 0.006 \pm 0.009$
$d_4 = 49.08$	$0.322 \pm 0.004 \pm 0.02$	$0.342 \pm 0.005 \pm 0.01$

use mean value between FOPT and CIPT

$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{\text{exp}} \pm 0.015_{\text{theo}}$$

four-loop running ($\beta_0, \beta_1, \beta_2, \beta_3$)

four-loop matching at quark thresholds

$$(m_c(m_c) = 1.286(13) \text{ GeV}, m_b(m_b) = 4.164(25) \text{ GeV})$$

$$\alpha_s(M_Z) = 0.1202 \pm 0.006_{\text{exp}} \pm 0.018_{\text{theo}} \pm 0.0003_{\text{evol}}$$

$$= 0.1202 \pm 0.0019$$

consistent with α_s from Z

$\delta\alpha_s$ from τ dominated by theory.

$\delta\alpha_s$ from Z dominated by statistics.

Summary

- Adler function, $R(s)$, R_τ available to $\mathcal{O}(\alpha_s^4)$

- First and only N³LO results

$$\alpha_s(M_Z) = \begin{cases} 0.1190 \pm 0.0026 & \text{from } Z \\ 0.1202 \pm 0.0019 & \text{from } \tau \end{cases}$$

- α_s^4 terms move Z and τ closer together

combined

$$\alpha_s(M_Z) = 0.1198 \pm 0.0015$$