Helicity fractions of W bosons from top quark decays

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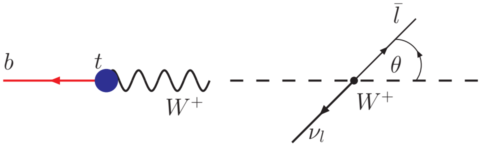
Introduction

- couplings are very susceptible to “new physics”
- dominant decay mode: $t \longrightarrow b + W^+$
- polarisation allows for detailed study of decay process
- LHC will produce many top quarks
- total width is hard to measure directly at hadron colliders
- helicity fractions can be measured

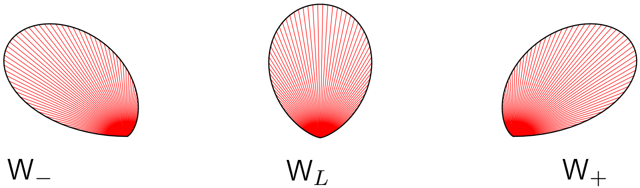
$t \longrightarrow b + W^+$

W has spin 1 $\rightsquigarrow$ 3 polarisation states



$\rightsquigarrow$ different contributions to angular distribution

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta}:$$



Effect of radiative corrections

bottom quark has to be left-handed

additional gluon or $m_b \neq 0$ leads to $\Gamma_+ \neq 0$

Experimental results

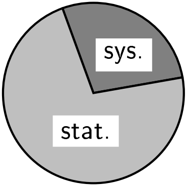
leading order Standard Model predictions ($m_b = 0$)

$$\mathcal{F}_L = 0.697\,,\ \mathcal{F}_- = 0.303\,,\ \mathcal{F}_+ = 0$$

Tevatron [CDF '07] [D0 '05, '07]

$$\mathcal{F}_L = 0.85 \pm 0.28\,,\ \mathcal{F}_+ = 0.05 \pm 0.12$$

$$\mathcal{F}_L = 0.56 \pm 0.31\,,\ \mathcal{F}_+ = 0.056 \pm 0.10$$



LHC with 10 fb^{-1} [Aguilar-Saavedra *et al.* '07]

$$\Delta\mathcal{F}_L = 1.9\%\,,\ \Delta\mathcal{F}_- = 1.8\%\,,\ \Delta\mathcal{F}_+ = 0.21\%$$

State of the art

unpolarised decay

$\mathcal{O}(\alpha_s)$ correction with full m_b and m_W dependence

$\mathcal{O}(\alpha_s^2)$ correction as expansion in m_W/m_t

Ježabek
Kühn
Czarnecki
Blokland
Ślusarczyk
Tkachov
Pak

polarised decay

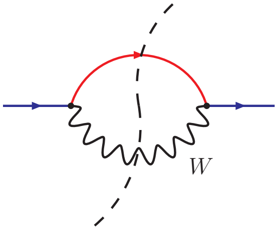
$\mathcal{O}(\alpha_s)$ correction with full m_b and m_W dependence

subsequent decay of W boson

electroweak and finite width

Fischer
Groote
Körner
Mauser
Lampe
Do

How can we reach $\mathcal{O}(\alpha_s^2)$ for polarised decay?



apply optical theorem and expansion by regions

do not sum over polarisations

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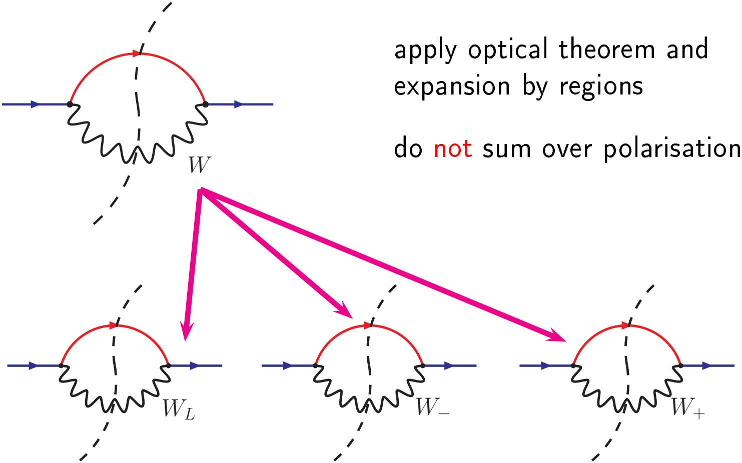

apply optical theorem and expansion by regions

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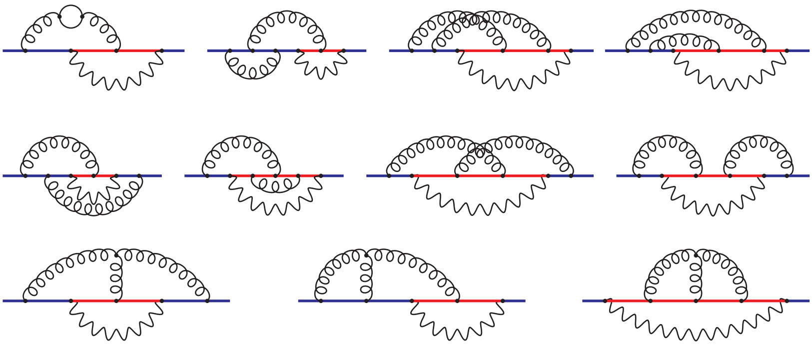
3-loop diagrams



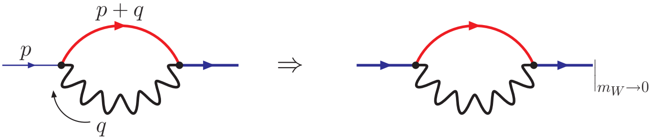
$m_b = 0 \rightsquigarrow$ two scales: m_t, m_W

$\rightsquigarrow$ use expansion by regions to reduce to single scale integrals

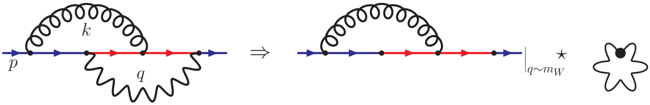
Expansion by regions

on-shell integrals with two masses and large mass on-shell
$\rightsquigarrow$ two regions:

- hard: $q_i \sim m_t$ $\rightsquigarrow$ expand in m_W



- soft: $q_i \sim m_W$ $\rightsquigarrow$ expand in q



How does polarisation enter the game?

polarisation sum

$$\epsilon^*_{L\mu}\,\epsilon_{L\nu} + \epsilon^*_{+\mu}\,\epsilon_{+\nu} + \epsilon^*_{-\mu}\,\epsilon_{-\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}$$

$\rightsquigarrow$ numerator of W boson propagator

pick out individual terms:

$$\epsilon^*_{L\mu}\,\epsilon_{L\nu} = \frac{(m_W^2\, p^\mu - p\cdot q\, q^\mu)(m_W^2\, p^\nu - p\cdot q\, q^\nu)}{m_W^2 m_t^2\, |\vec{q}\,|^2}$$

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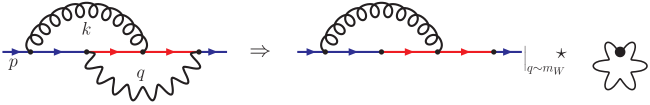
complication: appearance of $|\vec{q}| = \sqrt{q_0^2 - m_W^2}$ in denominator

Soft region

⇝ $|\vec{q}|^2 = q_0^2 - m_W^2$ cannot be expanded, but W loop factorises:



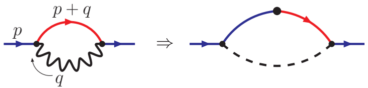
⇝ calculation just as in unpolarised case, but with different vacuum integral:

$$\int \frac{\mathrm{d}^d q}{(q^2 - m_W^2)\,(q_0^2 - m_W^2)} = i\pi^{d/2}\,\frac{2\,\Gamma(\epsilon)}{1-2\epsilon}\,\frac{1}{(m_W^2)^{\epsilon}}$$

Hard region

rewrite $|\vec{q}|^2$ in terms of p^μ and q^μ

$$|\vec{q}|^2 = q_0^2 - m_W^2 = \frac{(2p\cdot q)^2}{4m_t^2} - m_W^2$$



we only need the imaginary part $\rightsquigarrow$ $q^2 = m_W^2$:

$$|\vec{q}|^2 = \frac{1}{4m_t^2}\left(q^2 + 2p\cdot q - m_W^2\right)^2 - m_W^2$$

expansion in terms of top propagators

$$\frac{1}{|\vec{q}|^2} = \frac{4m_t^2}{(q^2+2p\cdot q)^2}\sum_{i=0}^{\infty}\left(\frac{\dots}{(q^2+2p\cdot q)^2}\right)^i$$

Results

- checked expansion of $\mathcal{O}(\alpha_s)$ up to $(m_W/m_t)^{16}$
- obtained analytical results for all three-loop master integrals
- calculated $\mathcal{O}(\alpha_s^2)$ corrections up to $(m_W/m_t)^{10}$

Numerical evaluation

Helicity fractions

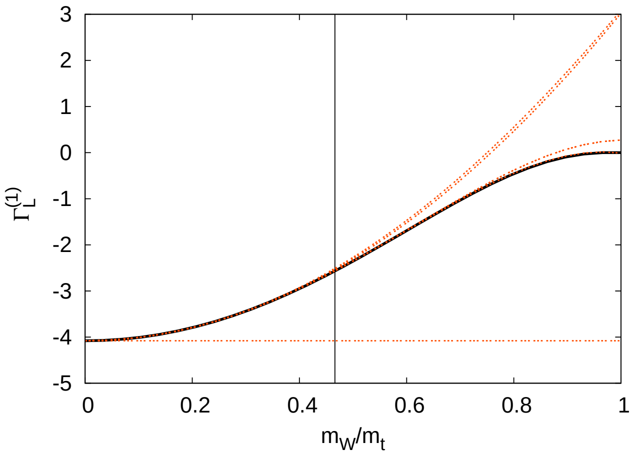
$$
\begin{aligned}
\mathcal{F}_L &= 0.6971\,(1 - 0.0108 - 0.0034) = 0.6872(6) \\
\mathcal{F}_- &= 0.3029\,(1 + 0.0214 - 0.0070) = 0.3115(5) \\
\mathcal{F}_+ &= 0 + 0.00103 + 0.00023 = 0.00126(6)
\end{aligned}
$$

estimated error from expansion: $\sim 10^{-6}$

expected uncertainties at LHC:

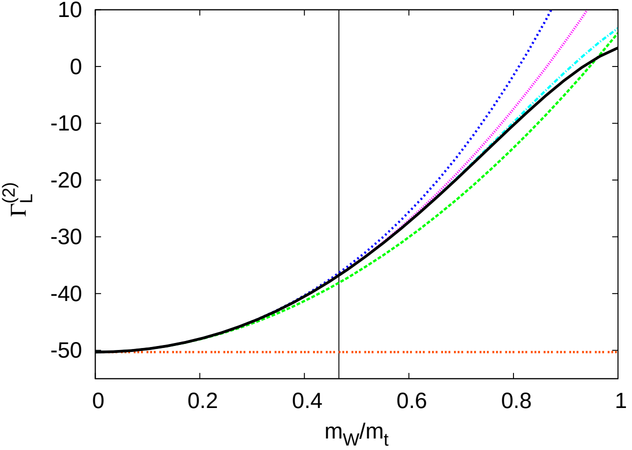
$$\Delta\mathcal{F}_L = 1.9\%\,,\ \Delta\mathcal{F}_- = 1.8\%\,,\ \Delta\mathcal{F}_+ = 0.21\%$$

$\mathcal{O}(\alpha_s)$ correction as function of m_W/m_t



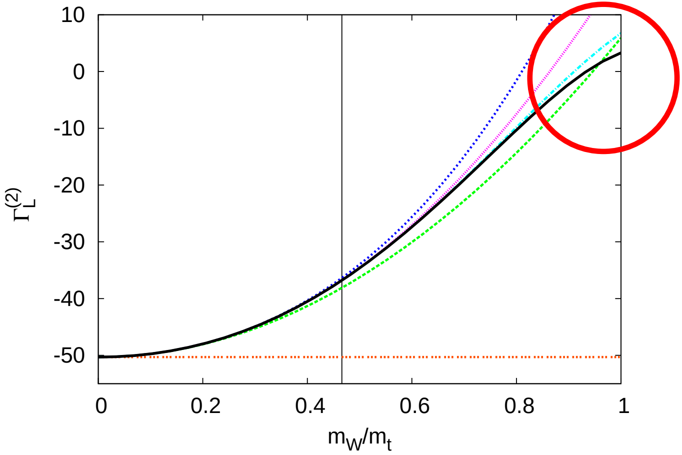
$$\frac{\Gamma_L^{(1)}(0.466)}{\alpha_s C_F G_0} = -4.078 + 1.543 + 0.027 - 0.055 - 0.002 + \cdots = -2.567$$

$\mathcal{O}(\alpha_s^2)$ correction as function of m_W/m_t



$$\frac{\Gamma_L^{(2)}(0.466)}{\alpha_s^2 C_F G_0} = -50.316 + 12.214 + 1.736 - 0.381 - 0.044 - 0.003 + \ldots$$

$\mathcal{O}(\alpha_s^2)$ correction as function of m_W/m_t



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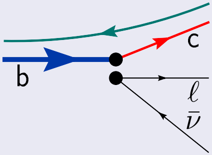
Mass-dependent NNLO corrections [arXiv:0803.0960]

Semi-leptonic B-meson decay



Clean decay

HQET: $\Gamma(b \to c\ell\bar{\nu})$

$\mathcal{O}(\alpha_s^2) \Rightarrow |\mathbf{V_{cb}}|, \ldots$

$m_c/M_b \sim 0.3$

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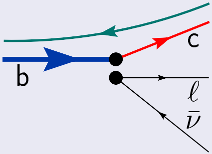

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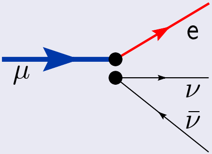
Muon decay



- $\Gamma(\mu \to e\nu\bar{\nu}) \sim \mathbf{G_F^2}$
- Need $\mathcal{O}(\alpha^2)$
- MuLan@PSI: <ppm
- $m_e/M_\mu \sim 0.005$

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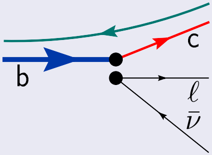

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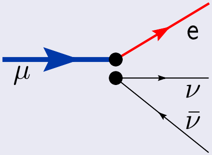

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NNLO calculations:

[van Ritbergen, Stuart '00]: $m = 0$

[Pak, Czarnecki '08]: expand in $\frac{m}{M}$

- Agrees with K.Melnikov's results
- Sub-percent accuracy for b-decay:

  $\mathcal{O}\left[\left(\frac{m_c}{M_b}\right)^2 \ln^2 \frac{m_c}{M_b}\right]$, $\overline{\mathrm{MS}}$ scheme
- 0.43 ppm correction to μ decay:

  $\mathcal{O}\left(\frac{m_e}{M_\mu}\right)$, pole mass scheme

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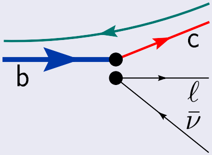

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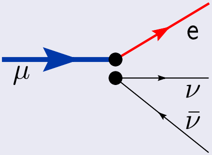

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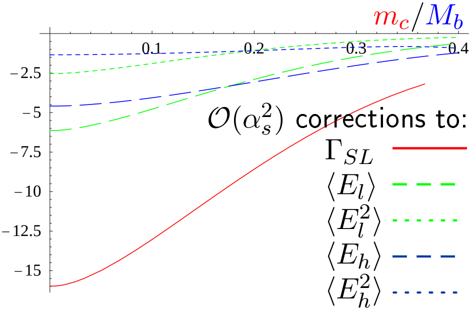
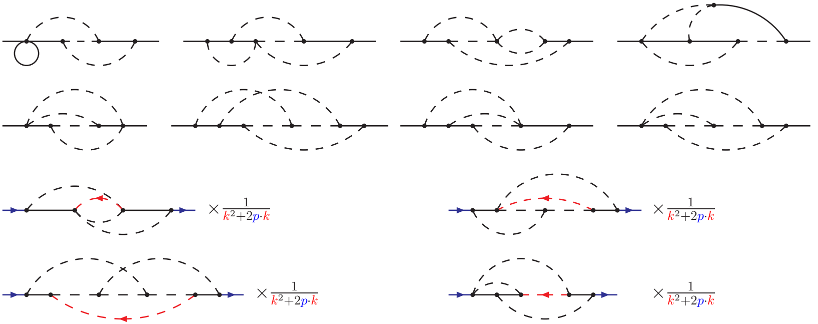


BLM

estimate of higher order effects: [Brodsky, Lepage, Mackenzie '82]

$$\begin{aligned}
\Gamma_L^{(3)} &= -0.0186_{\mathrm{BLM}} + 0.0043 = -0.0143 \\
\Gamma_-^{(3)} &= -0.0052_{\mathrm{BLM}} + 0.0013 = -0.0039 \\
\Gamma_+^{(3)} &= 0.00029_{\mathrm{BLM}} - 0.00013 = 0.00016
\end{aligned}$$

Master integrals

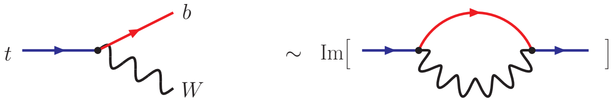


analytical results via:

- cutting rules [Cutkosky '60]
- differential equations [Kotikov '91] [Gehrmann, Remiddi '00]

Optical theorem

Optical theorem relates decay width to imaginary part of top quark self-energy:



Advantages:

- real radiation is automatically taken into account
- "technology" for multi-loop calculations can be used

Expansion by regions

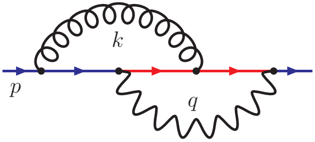
general rules:

- use dimensional regularisation
- identify all regions which contribute to the given integral
- expand integrand in small quantities of each region
- set scaleless integrals to zero
- integrate loop momenta from $-\infty$ to $+\infty$
- sum contributions from all regions

[Smirnov, *Applied Asymptotic Expansions in Momenta and Masses*]

Higher orders

consider both regions for all loop momenta



$\rightsquigarrow$ integrals become scaleless if gluon momentum is soft:

$$\int \frac{\mathrm{d}^d k\, \mathrm{d}^d q}{k^2\,(k^2+2p\cdot k)\,(p+k+q)^2\,(p+q)^2\,(q^2-m_W^2)}$$

$$\sim \int \frac{\mathrm{d}^d k}{k^2\,(2p\cdot k)} \times \begin{cases} \int \frac{\mathrm{d}^d q}{((p+q)^2)^2\, q^2} & q \text{ hard} \\ \frac{1}{(p^2)^2} \int \frac{\mathrm{d}^d q}{(q^2-m_W^2)} & q \text{ soft} \end{cases} + \ldots$$

Treatment of γ_5

- replace γ_5 with ε tensor:

$$\gamma_\mu \gamma_5 \to \frac{i}{3!}\, \varepsilon_{\mu\alpha\beta\delta}\, \gamma^\alpha \gamma^\beta \gamma^\delta$$

- contract ε tensor with projector $\rightsquigarrow$ metric tensors
- renormalise axial-vector current
- restore anti-commutativity with additional finite renormalisation

[Larin, Vermaseren '91]

Unpolarised case

average over spin states of top quark

$$\overline{u}(p)[\dots]u(p) \to \frac{1}{2}\,\mathrm{Tr}\left\{(\not{p}+m_t)\,[\dots]\right\}$$

sum over polarisations of W boson

$$\sum_{\text{polarisations}} \epsilon^*_\mu(q)\,\epsilon_\nu(q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)$$

$\rightarrow$ numerator of W propagator in unitary gauge

Longitudinally polarised W boson

kinematics

top quark is at rest: $p^\mu = (m_t, 0, 0, 0)$

W momentum in z direction: $q^\mu = (q_0, 0, 0, |\vec{q}\,|)$

polarisation vector: $\epsilon_L^\mu(q) = (|\vec{q}\,|, 0, 0, q_0)\,/m_W$

$$\Rightarrow \; \epsilon_L^\mu = -\frac{m_W^2\, p^\mu - p\cdot q\, q^\mu}{m_W m_t\, |\vec{q}\,|}$$

$$\rightsquigarrow \; \epsilon_L^{\mu *}(q)\, \epsilon_L^\nu(q) = \frac{(m_W^2\, p^\mu - p\cdot q\, q^\mu)(m_W^2\, p^\nu - p\cdot q\, q^\nu)}{m_W^2 m_t^2\, |\vec{q}\,|^2}$$

Polarisation

use projection operators:

$$\mathbb{P}_0^{\mu\nu}(q) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2}$$

$$\mathbb{P}_L^{\mu\nu}(q) = \frac{(m_W^2\, p^\mu - p\cdot q\, q^\mu)(m_W^2\, p^\nu - p\cdot q\, q^\nu)}{m_W^2 m_t^2\, |\vec{q}|^2}$$

$$\mathbb{P}_F^{\mu\nu}(q) = -\frac{i}{m_t\, |\vec{q}|}\, \varepsilon^{\mu\nu\sigma\rho}\, p_\sigma q_\rho$$

$$\mathbb{P}_\pm^{\mu\nu} = \mathbb{P}_0^{\mu\nu} - \mathbb{P}_L^{\mu\nu} \pm \mathbb{P}_F^{\mu\nu}$$

complications:

- appearance of $|\vec{q}| = \sqrt{q_0^2 - m_W^2}$ in denominator
- treatment of γ_5 in dimensional regularisation