Precise Predictions for LHC using a GOLEM

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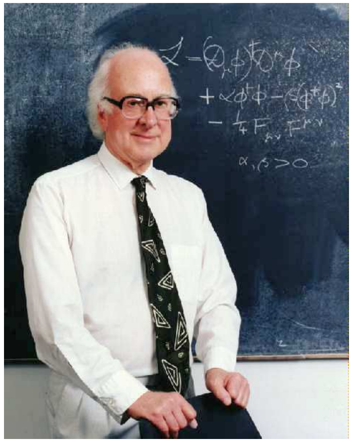
Loops and Legs in Quantum Field Theory
Sondershausen, Germany

Content:

- Motivation: LHC @ NLO
- Framework for one-loop amplitudes: the GOLEM project
- Applications for LHC
- Summary

The advent of the LHC era

LHC: Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, start 2008



What do we expect?

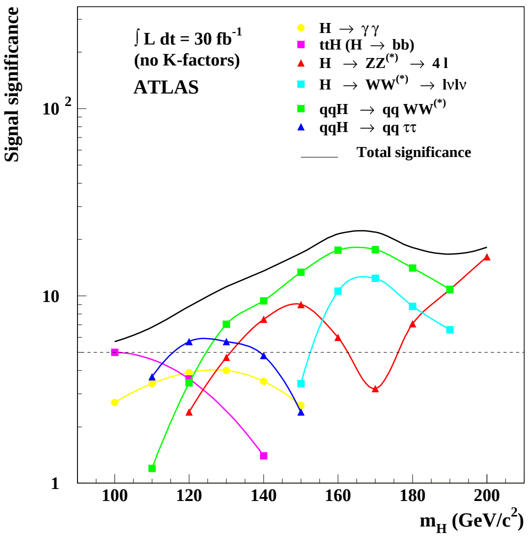
- test Higgs mechanism
  - $\longrightarrow$ SM Higgs boson: 114.4 GeV $< m_H <$ 200 GeV (!)
  - $\longrightarrow$ $V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3 \, H^3 + \lambda_4 \, H^4$
    SM: $\lambda_4 = \lambda_3 / v = 3 \, M_H^2 / v^2$
- explore physics beyond the Standard Model
  - $\longrightarrow$ SM $\subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
    SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
  - $\longrightarrow$ BSM something around 1 TeV (?)
- nothing ?!
  - $\longrightarrow$ hint of a hidden sector (?)
  - $\longrightarrow$ hint of strong interactions in the e.w. sector (?)

Discovery potential of the Higgs boson at the LHC

- most studies based on LO Monte Carlo tools

  $\longrightarrow$ large uncertainties

  $\longrightarrow$ some loop induced LO processes not included [e.g. $gg \to ZZ$]



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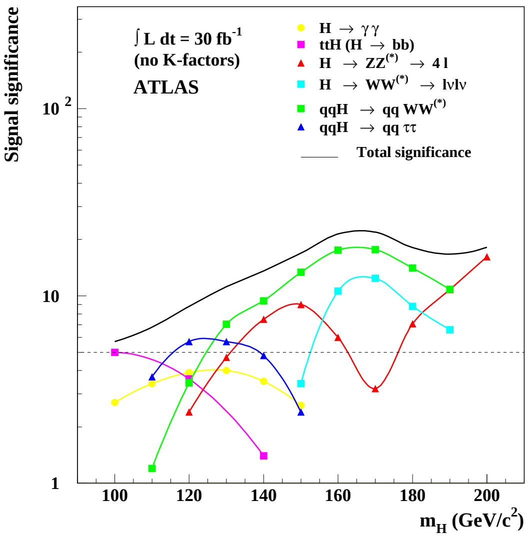
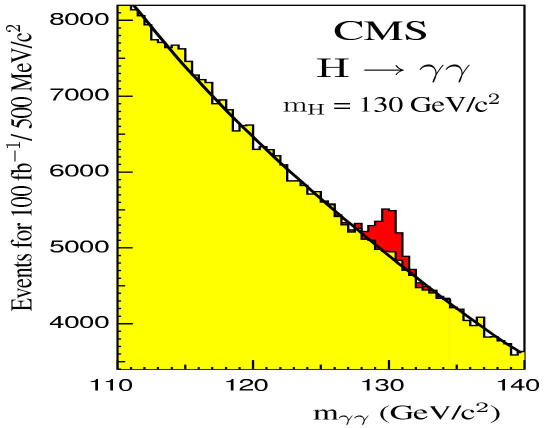
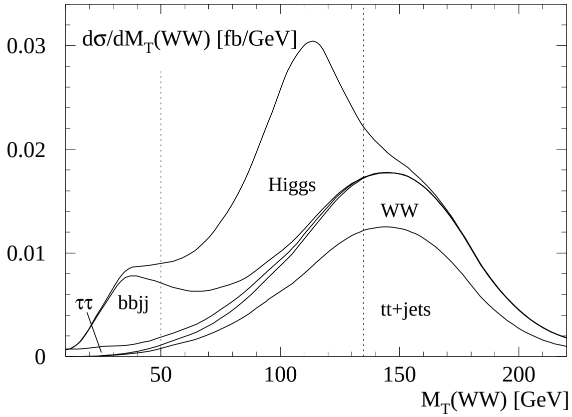


- Not all backgrounds can be measured
- Nothing @ LHC = Bkgnd(experiment) - Bkgnd(theory) !
- Quantitative analysis of SM/BSM physics needs background control

$PP \to H + X \to \gamma\gamma + X$

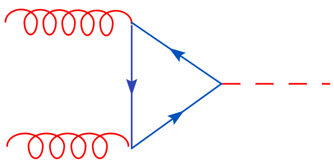
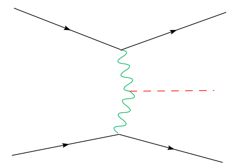
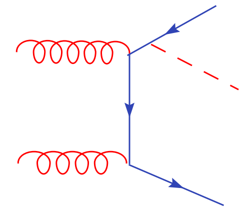
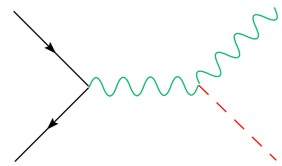


WBF: $H \to WW \to l^+l^- + \not{p}_T$



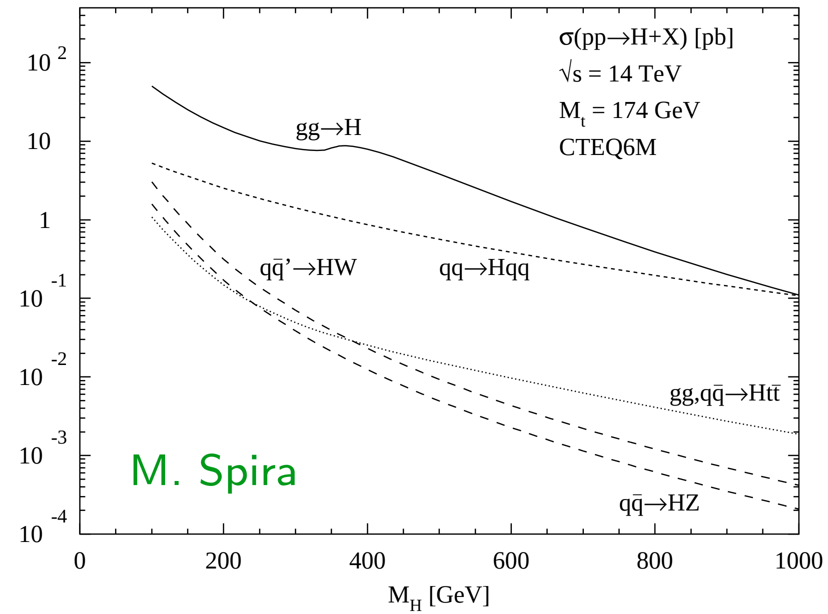
Kauer, Plehn, Rainwater, Zeppenfeld (2001)

S+B for the Higgs boson

Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$



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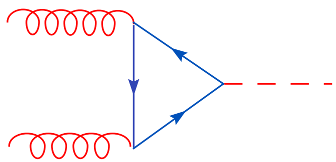
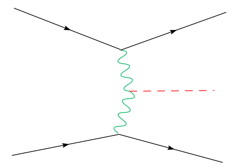
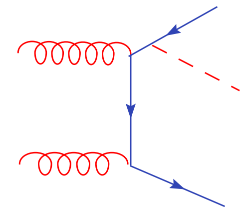
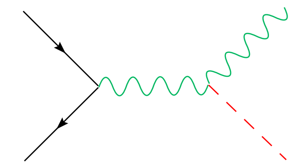

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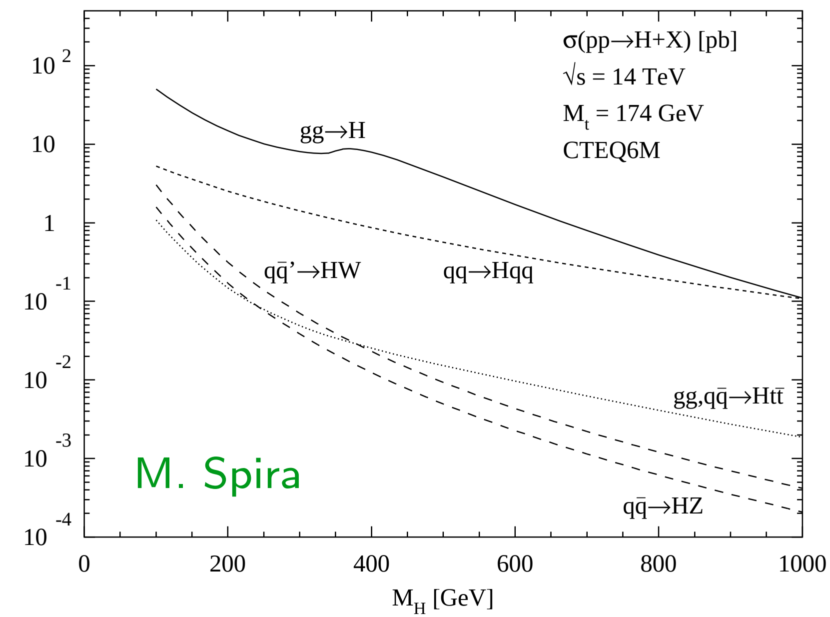
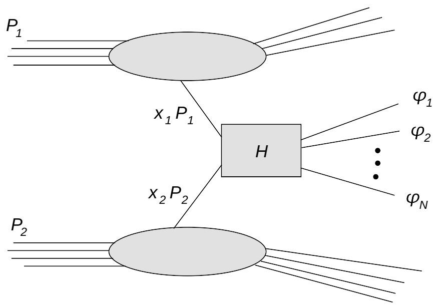


Backgrounds:

- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to V +$ up to 3 jets $\quad (V = \gamma, W, Z)$
- $PP \to VVV + 0, 1$ jet

Parton model and scale uncertainties



$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\mathrm{parton}_j(x_1 P_1) + \mathrm{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

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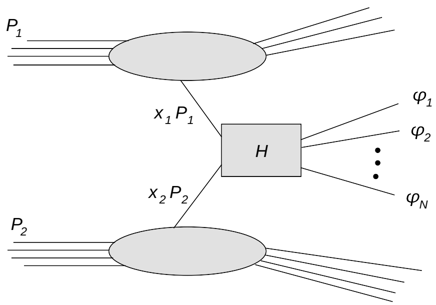

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Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

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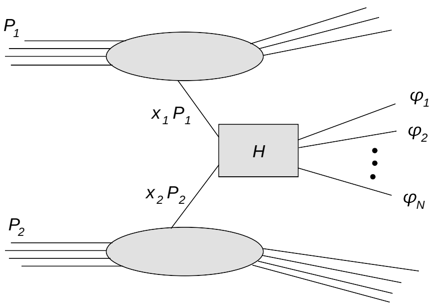

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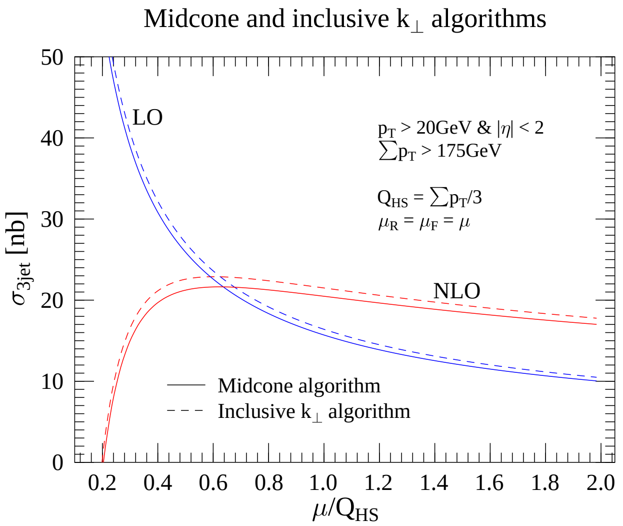
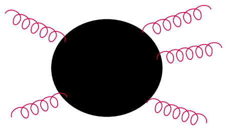
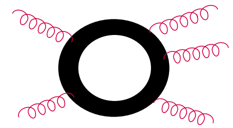
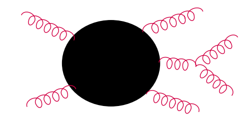
Example: 3 jet cross section at NLO

[Z. Nagy, Phys.Rev. D68 (2003)]



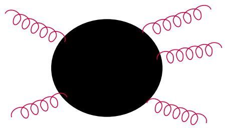
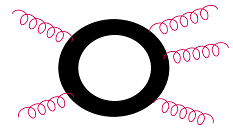
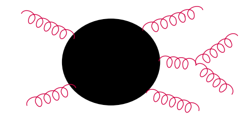
Higher order QCD calculations are mandatory to soften scale dependence !!!

Framework for NLO calculations

$\mathcal{M}_{\mathrm{LO}}$:  $\mathcal{M}_{\mathrm{NLO,virtual}}$:  $\mathcal{M}_{\mathrm{NLO,real}}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \int dPS_N \, \frac{1}{2s} \, \mathcal{O}_N(\{p_j\}) \; |\mathcal{M}_{\mathrm{LO}}|^2 \\
\sigma_{NLO} &= \int dPS_N \, \frac{1}{2s} \, \alpha_s \, \Big(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\mathrm{LO}} \mathcal{M}^*_{\mathrm{NLO,V}} + \mathcal{M}^*_{\mathrm{LO}} \mathcal{M}_{\mathrm{NLO,V}} \right] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \; |\mathcal{M}_{\mathrm{NLO,R}}|^2 \Big)
\end{aligned}
$$

Framework for NLO calculations

$\mathcal{M}_{\rm LO}$:  $\mathcal{M}_{\rm NLO,virtual}$:  $\mathcal{M}_{\rm NLO,real}$: 

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- For IR-safe observables, $\mathcal{O}_{N+1} \underset{IR}{\rightarrow} \mathcal{O}_N$, IR divergences cancel
- treelevel LO, NLO contributions technically unproblematic
- IR subtraction: e.g. dipole method à la Catani, Seymour (massless); Dittmaier, Trocsanyi, Weinzierl, Phaf (massive).
- automated dipole subtraction Gleisberg, Krauss (2007); Seymour, Tevlin (2008).
- Bottleneck: virtual corrections

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after 2005:

- $pp \to HHH$ (2005)
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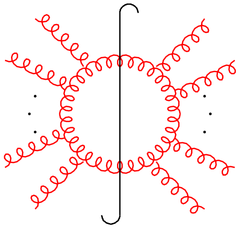
$\mathcal{A}_{\text{1-loop}} \sim \sum_C \int d\text{PS}_C$  $+\mathcal{R}$

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$2 \to 4:$ No LHC cross section done yet!

- 6 photon amplitude (2007) cut-construction, Feynman diagrams, OPP
- 6 gluon amplitude (1994-2006) cut-construction, . . .
- $N > 6$ gluon amplitudes evaluated (2008) (see talk by Daniel Maitre)

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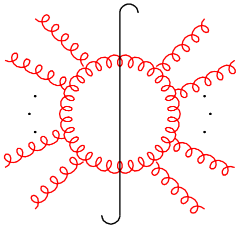
$\mathcal{A}_{\text{1-loop}} \sim \sum_C \int d\text{PS}_C$  $+\mathcal{R}$

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Full $2 \to 4$ one-loop calculations for e^+e^-, $\gamma\gamma$ colliders:

- $\mathcal{O}(\alpha)$ $e^+e^- \to f\bar{f}f'\bar{f}'$ Denner, Dittmaier, Roth, Wieders (2005)
- $\mathcal{O}(\alpha)$ $e^+e^- \to HH\nu\nu$ GRACE collaboration (2005)
- $\mathcal{O}(\alpha_s)$ $\gamma\gamma \to b\bar{b}t\bar{t}$ Lei, Wen-Gan, Liang, Ren-You, Yi (2007)

The GOLEM project

General 0ne Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

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"GOLEM"...

- ... refers in the Bible to embryonic / incomplete substance.
- ... is created from clay
- ... maybe a creation of overambition $\rightarrow$ Mary Shelley's Frankenstein.
- ... need to be instructed wisely $\rightarrow$ Goethes Zauberlehrling (The Sorcerer's Apprentice).
- ... "Wie er in die Welt kam", film by Paul Wegener 1920.

Feynman diagrammatic approach:

$$\Gamma^{c,\lambda}(p_j, m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \dots D_N} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\dots,\mu_R} \, I_N^{\mu_1 \cdots \mu_R}(p_j, m_j)$$

$$I_N^{\mu_1 \cdots \mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1} \dots k^{\mu_R}}{D_1 \dots D_N} \, , \; D_j = (k - r_j)^2 - m_j^2, \; r_j = p_1 \dots + p_j$$

- Passarino-Veltman: $\to 1/\det(G)^R$, $G_{ij} = 2\, r_i \cdot r_j$ induce numerical problems
- projection on helicity amplitudes reduces $2\, k \cdot r_j = D_N - D_j + r_j \cdot r_j$
- Lorentz Tensor Integrals $\longrightarrow$ form factor representation à la Davydychev
- Reduction in Feynman parameter space

$$I_N^{\mu_1 \cdots \mu_R} = \sum \tau^{\mu_1 \cdots \mu_R}(r_{j_1}, \dots, r_{j_r}, g^m) \, I_N^{n+2m}(j_1, \dots, j_r)$$

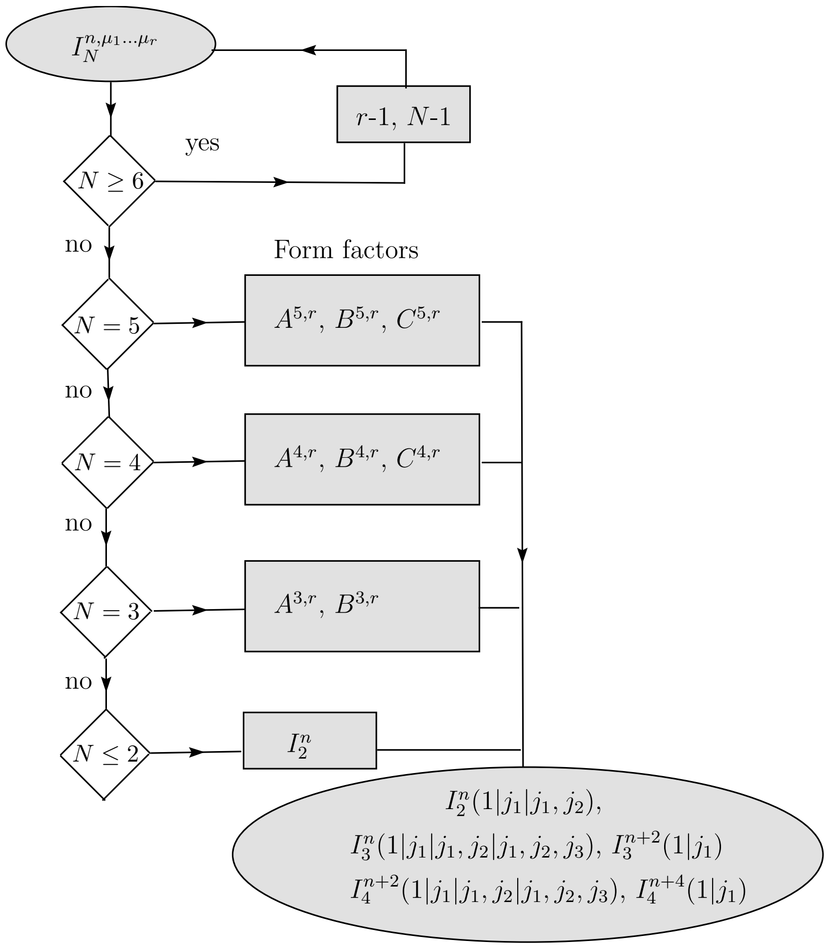
$$I_N^D(j_1, \dots, j_r) = (-1)^N \Gamma(N - \frac{D}{2}) \int_0^\infty d^N z \, \delta(1 - \sum_{l=1}^N z_l) \frac{z_{j_1} \dots z_{j_r}}{(-\frac{1}{2} z \cdot \mathcal{S} \cdot z)^{N - D/2}}$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

Schematic overview of N-point tensor integral reduction

T.B., J.P. Guillet, G. Heinrich (2000); T.B., Guillet, Heinrich, Pilon, Schubert (2005).

- works for general N
- no inverse Gram determinants
- isolation of IR divergences simple
- tractable expressions
- form factors for $N \leq 6$ implemented in Fortran90 code "golem90"
- optional reduction to scalar integrals
- evaluation of rational terms



$$I_{N=3,4}^{n,n+2}(j_1,\ldots,j_r) \quad \sim \quad \int_0^1 \prod_{i=1}^{4} dz_i\,\delta(1-\sum_{l=1}^{4} z_l)\,\frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2}\,z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$

Implementation of the algorithm

Preparation:

- Diagram generation: `QGRAF` P. Nogueira, `FeynArts 3.2` T. Hahn
- Perform colour algebra
- Determine integral basis
- Projection on helicity amplitudes

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From here two independent set-ups:

a) Symbolic reduction to scalar integrals based on FORM and MAPLE
   - $\mathcal{M}^{\{\lambda\}} \to C_{box} I_4^{d=6} + C_{tri} I_3^{d=4-2\epsilon} + C_{bub} I_2^{d=4-2\epsilon} + C_{tad} I_1^{d=4-2\epsilon} + \mathcal{R}$
   - automated method to evaluate $\mathcal{R}$ T.B., Guillet, Heinrich (2006)
   - introduces $1/\det G$ but allows to apply symbolic simplifications

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b) Convert to form factor representation, link to `Fortran90` library "golem90"

- $\mathcal{M}^{\{\lambda\}} \to C_{box}^{ijk} I_4^{n+2,n+4}(x_i x_j x_k) + C_{tri}^{ijk} I_3^{n,n+2}(x_i x_j x_k) + \dots$
- In numerically critical phase space regions:
  – compile/run code in quadruple precision
  – use one-dimensional integral representations for $I_{N=3,4}^{n+2,n+4}(x_i x_j x_k)$

Remark on one-loop scalar integrals

- 't Hooft, Veltman (1979), all finite triangles and boxes $\sim \log$, dilog box integrals only for $p_i^2 > 0$.
- Denner, Nierste, Scharf (1991), IR finite boxes, arbitrary real kinematics
- Bern, Dixon, Kosower (1994), IR divergent box integrals, $m_j^2 = 0$
- Ellis, Zanderighi (2007), all IR divergent triangle/box integrals

Remark on one-loop scalar integrals

- 't Hooft, Veltman (1979), all finite triangles and boxes $\sim \log$, dilog box integrals only for $p_i^2 > 0$.
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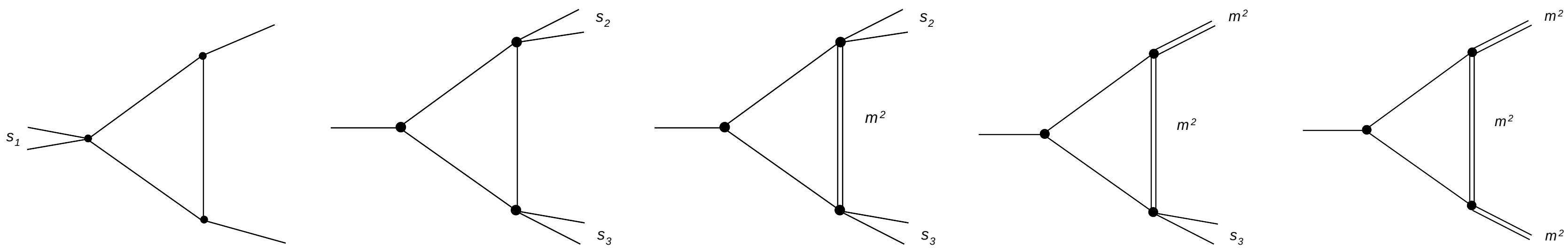
All you need: Five divergent triangle integrals . . .



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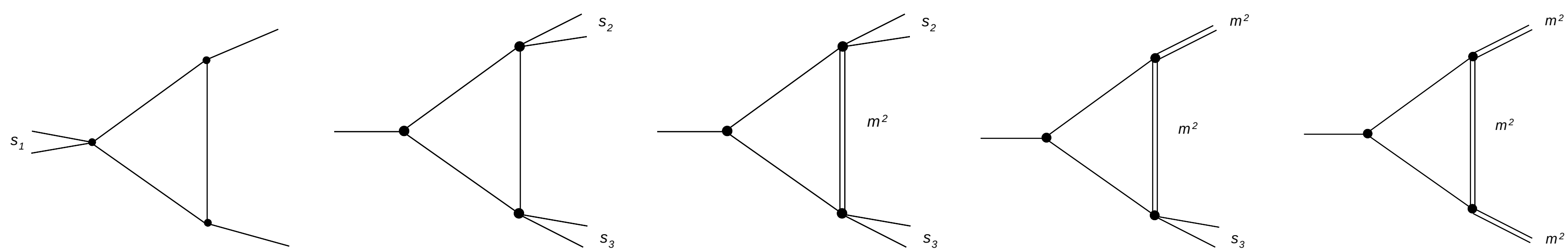
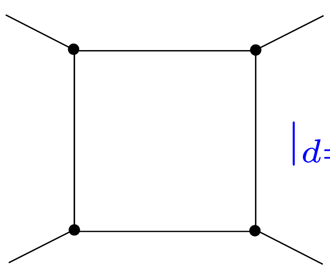
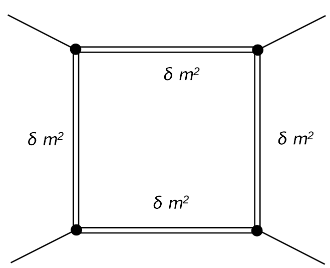


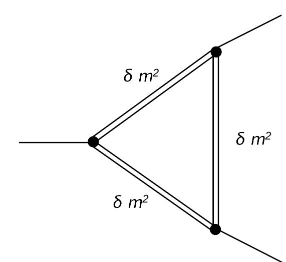
... and `LoopTools` T. Hahn [Using: Oldenburgh FF, Beenakker, Denner et al.]



$\big|_{d=6} \sim$



$\big|_{d=4} - \sum_j b_j$



$+\mathcal{O}\left(\frac{\delta m^2}{Q^2}\right)$

$\longrightarrow$ test for scalar integrals in GOLEM

Computations with GOLEM:

Algorithm coded in FORM and FORTRAN 90

some recent applications ...

- $gg \to W^*W^* \to l\nu l'\nu'$
  T.B., M. Ciccolini, M. Kramer, N. Kauer (2006)
- $gg \to HH, HHH$
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- $pp \to Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
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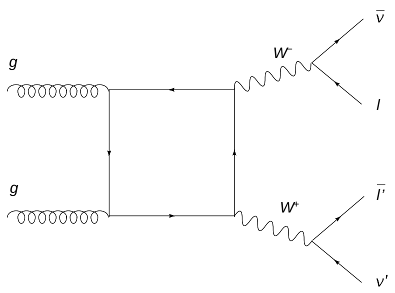

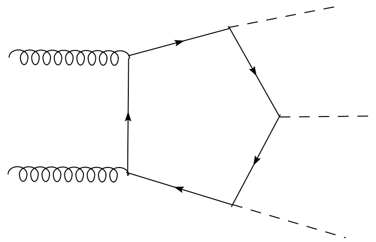

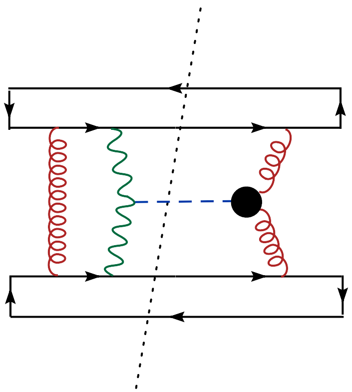

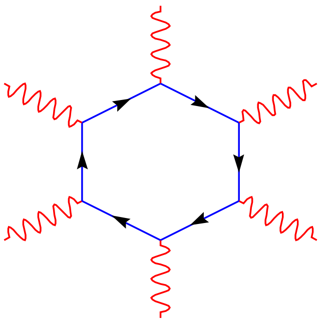

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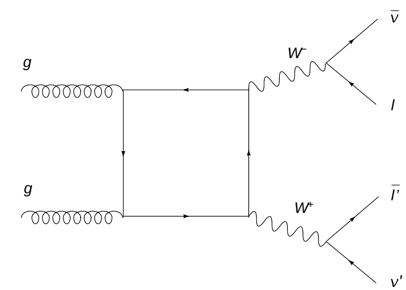

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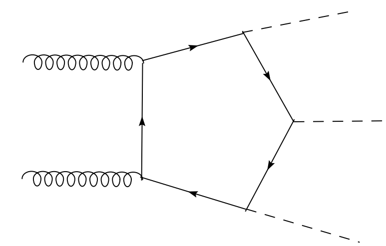

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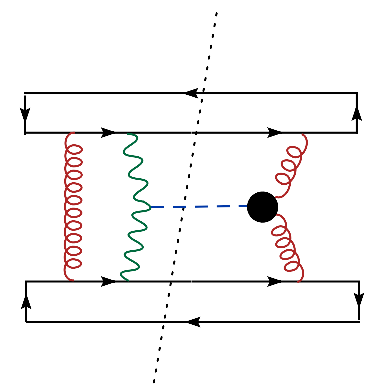
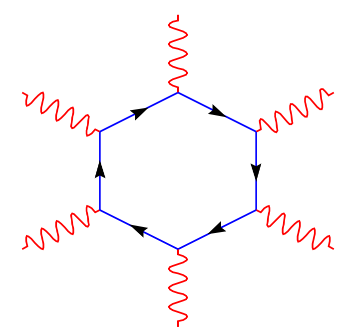


... and ongoing work:



- $gg \to Z^*Z^*, \gamma^*Z^*, \gamma^*\gamma^* \to l\bar{l}l'\bar{l}'$
- $pp \to WWj, ZZj$
- $u\bar{u} \to d\bar{d}b\bar{b}$, goal: $pp \to jjbb, bbbb$

The $gg \to Z^*Z^* \to l\bar{l}l'\bar{l}'$ process

- missing background for $gg \to H \to Z^*Z^*$
- result known since a long time, but no code available
- Z's on-shell: Dicus, Kao, Repko (1987), Glover, v.d.Bij (1989).
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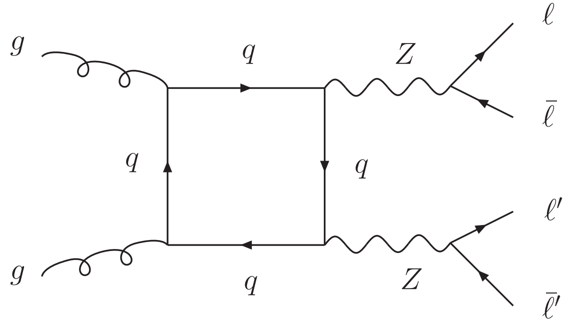

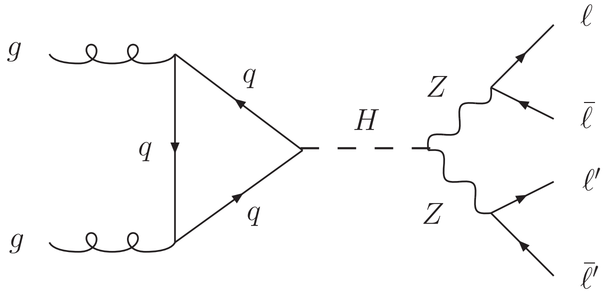

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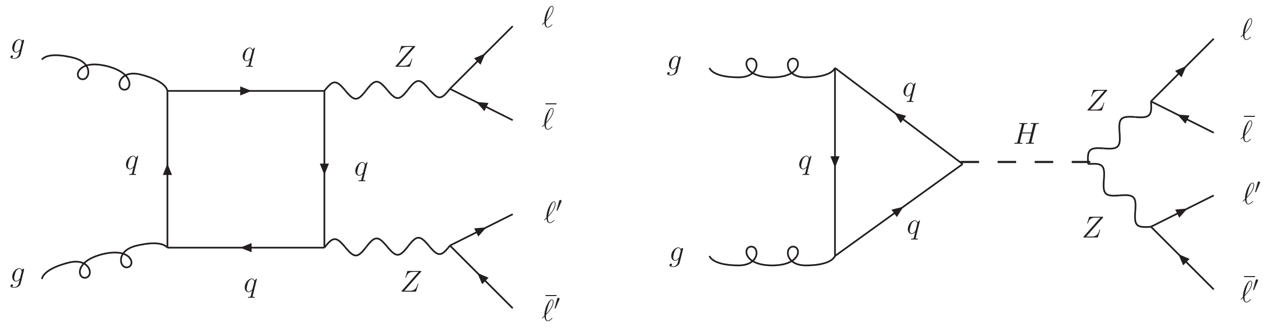
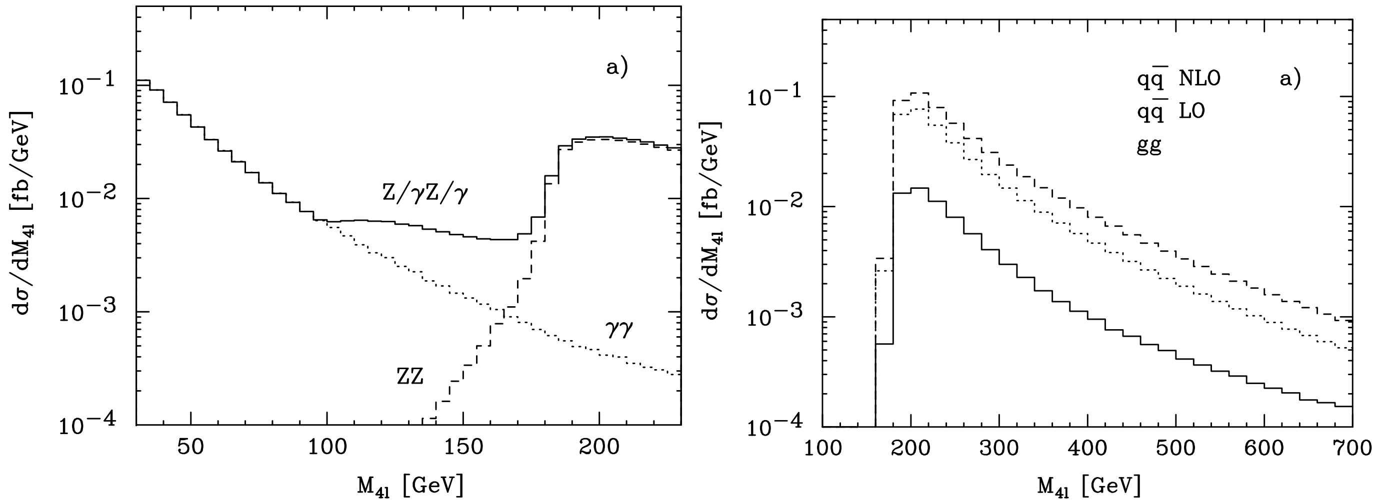


- interference between Higgs signal and background
- relevant below ZZ threshold

Background to $pp \to H \to \ell^+\ell^-\ell^+\ell^-$

std. cuts applied: $p_{T\ell} > 20$ GeV, $|\eta_\ell| < 2.5$, 75 GeV $< M_{\ell^+\ell^-} < 105$ GeV

	$\sigma(pp \to Z^*(\gamma^*)Z^*(\gamma^*) \to \ell\bar{\ell}\ell'\bar{\ell}')$ [fb]				
	gg	$q\bar{q}$		$\frac{\sigma_{\rm NLO}}{\sigma_{\rm LO}}$	$\frac{\sigma_{\rm NLO+gg}}{\sigma_{\rm NLO}}$
		LO	NLO		
$\sigma_{\rm std}$	1.492(2)	7.343(1)	10.953(2)	1.49	1.14



a) minimal cuts: $M_{\ell^+\ell^-} > 5$ GeV
b) std. cuts applied

TB, N. Kauer, P. Mertsch (2008)

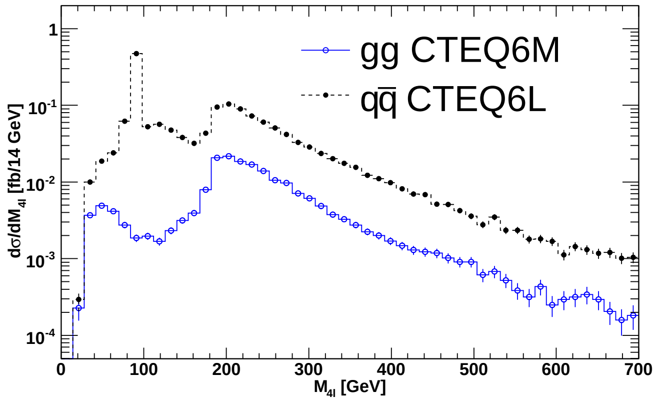
GG2ZZ parton-level MC integrator & event generator

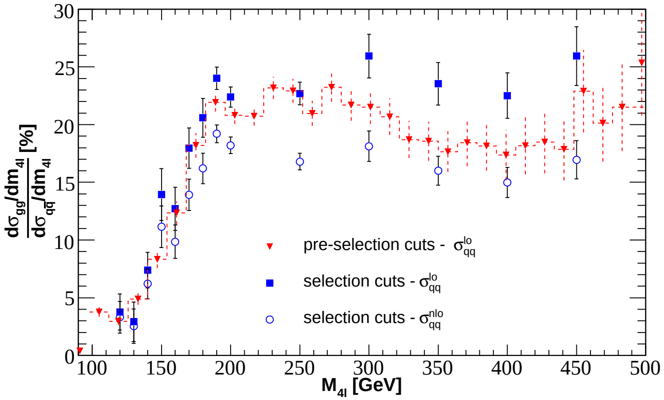
http://hepsource.sf.net/programs/ implemented by N. Kauer

- include full spin correlations, off-shell & interference effects
- generate unweighted events in Les Houches standard format
- user-friendly specification of selection cuts and histograms
- adaptive MC integration with parallel mode (OmniComp-Dvegas)

used by ATLAS and CMS for $H \to ZZ$ studies

$gg \to Z^*(\gamma^*)Z^*(\gamma^*)$ background simulation for Higgs boson search





Giordano [CMS] (2008)

Status of processes with vector bosons plus jet(s)

Leptons, missing energy and jets are generic backgrounds for New Physics

- Still wanted for LHC: $pp \to VVV$, $VV + j$, $VV + jj$, $V + jjj$ at NLO
- $pp \to VVV$ see talk of Costas Papadopoulos
  Lazopoulos, Melnikov, Petriello (2007), Hankele, Zeppenfeld (2007)
- $pp \to VVj$ see talk of Stefan Kallweit
  Dittmaier, Kallweit, Uwer (2007); Campbell, Ellis, Zanderighi (2007)
  T.B., S. Karg, N. Kauer, J.Ph.-Guillet, G. Sanguinetti (in progress)
- $pp \to VVjj$ vector boson fusion contribution, including leptonic decays
  B. Jäger, Oleari, Zeppenfeld (2006), Bozzi, B. Jäger, Oleari, Zeppenfeld (2007)

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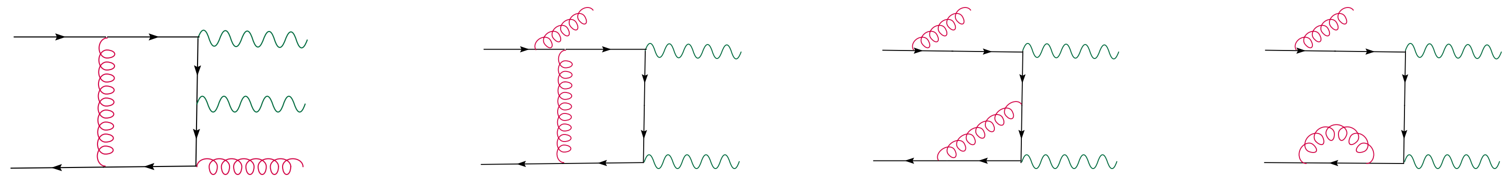
Process $pp \to WW/ZZ + j$ needs loop corrections for:

$$q(p_1, \lambda_1, c_1) + \bar{q}(p_2, \lambda_2, c_2) + V(p_3, \lambda_3) + \bar{V}(p_4, \lambda_4) + g(p_5, \lambda_5, a_5) \to 0$$



Algebraic evaluation of $q\bar{q}VVg \to 0$

- t'Hooft-Veltman scheme, γ_5 rules:
  $k_j = \hat{k}_j$, $k = \hat{k} + \tilde{k}$, $\gamma = \hat{\gamma} + \tilde{\gamma}$, $\{\gamma_5, \hat{\gamma}\} = 0$, $[\gamma_5, \tilde{\gamma}] = 0$
- 36 helicity amplitudes, 3 colour structures

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$$k_3 = \frac{1}{2\beta}[(1+\beta)\,p_3 - (1-\beta)\,p_4], \quad k_4 = \frac{1}{2\beta}[(1+\beta)\,p_4 - (1-\beta)\,p_3],$$

$$k_3^2 = k_4^2 = 0, \ \beta = \sqrt{1 - 4\,M_V^2/s_{34}}$$

$$\varepsilon^{+}_{3\,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 4^-|\mu|3^-\rangle}{\langle 43\rangle}, \quad \varepsilon^{-}_{3\,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 3^-|\mu|4^-\rangle}{[34]}, \quad \varepsilon^{0}_{3\,\mu} = \frac{(1+\beta)k_{3\,\mu} - (1-\beta)k_{4\,\mu}}{2\,M_V}$$

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Use to define projectors on helicity amplitudes, schematically:

$$\begin{aligned}\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} &= \mathcal{P}^{\lambda_3\lambda_4\lambda_5}_{\mu_3\mu_4\mu_5}\,\langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle \\ &= (\text{global spinorial factor}) \times (\text{contracted tensor integrals})\end{aligned}$$

- Lorentz indices saturated, at most rank 1 5-point functions
- spinor products can be treated as global factors
- analytical expressions allow for further simplifications (Maple/Mathematica)
- evaluation time of full amplitude about 0.6 seconds per PS point

Results for virtual corrections to $\mathcal{M}(pp \to ZZj)$

- UV renormalisation in 'tHooft-Veltman scheme
- Axial vector coupling needs finite renormalisation $\sim 1 - \frac{\alpha_s}{\pi} C_F$
- IR dipole subtraction a la Catani/Seymour, $1/\epsilon$ poles contained in $\mathbf{I}(\epsilon)$

For $N_F = 0$ with $p_{Tjet} > 100$ GeV, $p_{TZ} > 5$ GeV, $|\eta| < 5$, UV/IR subtraction applied:

$$\sigma_{LO}(ZZj) = 990.3 \pm 2.2 \text{ fb}$$

$$\sigma_{LO+NLO,Virtual}(ZZj) = 897.7 \pm 4.7 \text{ fb}$$

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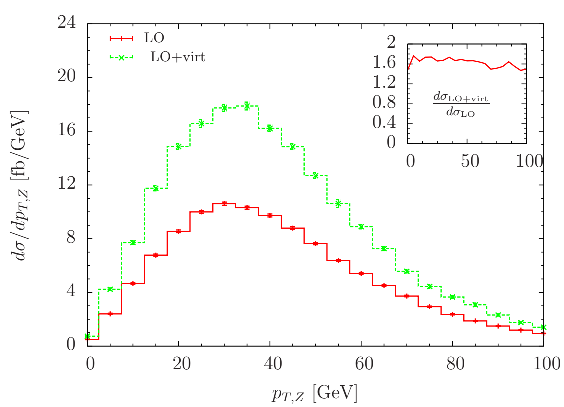
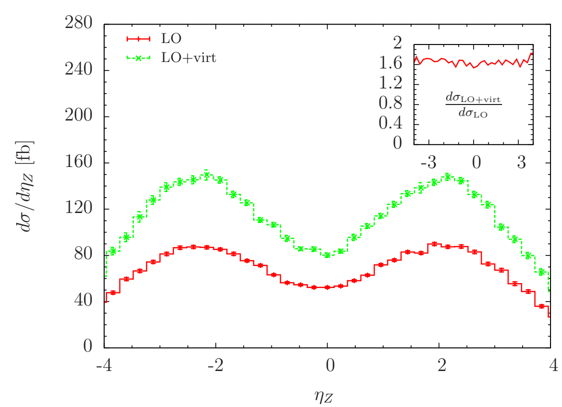
E.g. $\sigma^{--+++}_{LO+NLO,V}(q\bar{q} \to ZZg)$ [μ, μ_F independent parts]:





- Virtual part ready and tested
- Real emission part under construction

WW + jet: Tuned comparison of three groups

Results for a single phase-space point of bosonic virtual corrections:

$$2\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{\mathrm{LO}}\} = e^4 g_s^2 \Gamma(1+\epsilon)(4\pi\mu_{\mathrm{ren}}^2/M_W^2)^\epsilon \left(c_{-2}/\epsilon^2 + c_{-1}/\epsilon + c_0\right)$$

Dittmaier, Kallweit, Uwer [1];
Campbell, K. Ellis, Zanderighi [2];
Binoth, Guillet, Karg, Kauer, Sanguinetti [3]

	$c_{-2}[\mathrm{GeV}^{-2}]$	$c_{-1}^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$	$c_0^{\mathrm{bos}}[\mathrm{GeV}^{-2}]$
$u\bar{u} \to W^+W^-g$			
[1]	$-1.08069930550876 \cdot 10^{-4}$	$7.84286190526307 \cdot 10^{-4}$	$-3.38291091542537 \cdot 10^{-3}$
[2]	$-1.08069930550587 \cdot 10^{-4}$	$7.84286190527672 \cdot 10^{-4}$	$-3.38291091546403 \cdot 10^{-3}$
[3]	$-1.08069930550881 \cdot 10^{-4}$	$7.84286190526329 \cdot 10^{-4}$	$-3.38291091561624 \cdot 10^{-3}$
$ug \to W^+W^-u$			
[1]	$-1.67502983350323 \cdot 10^{-5}$	$1.23626843013156 \cdot 10^{-4}$	$-5.41712094792788 \cdot 10^{-4}$
[2]	$-1.67502983350126 \cdot 10^{-5}$	$1.23626843012411 \cdot 10^{-4}$	$-5.41712094800408 \cdot 10^{-4}$
[3]	$-1.67502983350329 \cdot 10^{-5}$	$1.23626843013193 \cdot 10^{-4}$	$-5.41712094818452 \cdot 10^{-4}$

Les Houches 2007 proceedings arXiv:0803.0494 [hep-ph]

The $u\bar{u} \to d\bar{d}b\bar{b}$ amplitude

T.B., J.Ph.-Guillet, A. Guffanti, G. Heinrich, T. Reiter, J. Reuter

- Goal: $pp \to jjbb, bbbb$ at NLO
- $\sim$ 250 diagrams, 25 pentagon and 8 hexagon diagrams, 8 independent scales
- two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- t'Hooft-Veltman scheme
- six different colour structures: $\mathcal{A} = \sum_{j=1,6} |c_j\rangle \mathcal{A}_j$, IR subtraction operator:

$$\langle c_j|\mathbf{I}(\epsilon)|c_k\rangle = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\frac{\mathcal{V}_q}{C_F}\sum_{I,J}\langle c_j|\mathbf{T}_I\cdot\mathbf{T}_J|c_k\rangle\left(\frac{4\pi\mu^2}{2p_I\cdot p_J}\right)^{\epsilon}$$

$$\mathcal{V}_q = C_F\left(\frac{1}{\epsilon^2}+\frac{3}{2}\frac{1}{\epsilon}+5-\frac{\pi^2}{2}\right)$$

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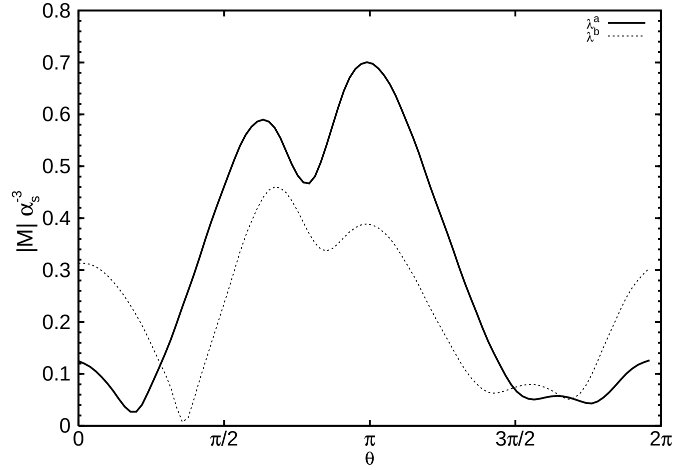
$$\mathcal{V}_q = C_F\left(\frac{1}{\epsilon^2}+\frac{3}{2}\frac{1}{\epsilon}+5-\frac{\pi^2}{2}\right)$$

- $\Rightarrow$ cancellation of IR/UV poles
- Two completely independent evaluations of the amplitude:
  a) algebraic reduction $\to$ Master integrals
  b) semi-numerical reduction with Fortran 90 code golem90
- Amplitude evaluation $\mathcal{O}(1)$ s, rank 3 6-point form factor $\mathcal{O}(10)$ ms

Virtual NLO corrections of $u\bar{u} \rightarrow d\bar{d}b\bar{b}$ (preliminary)

Behaviour of amplitude along trajectory in phase space:



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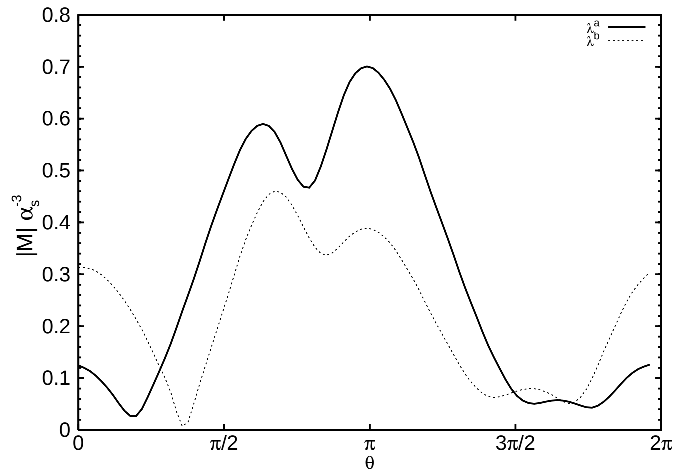
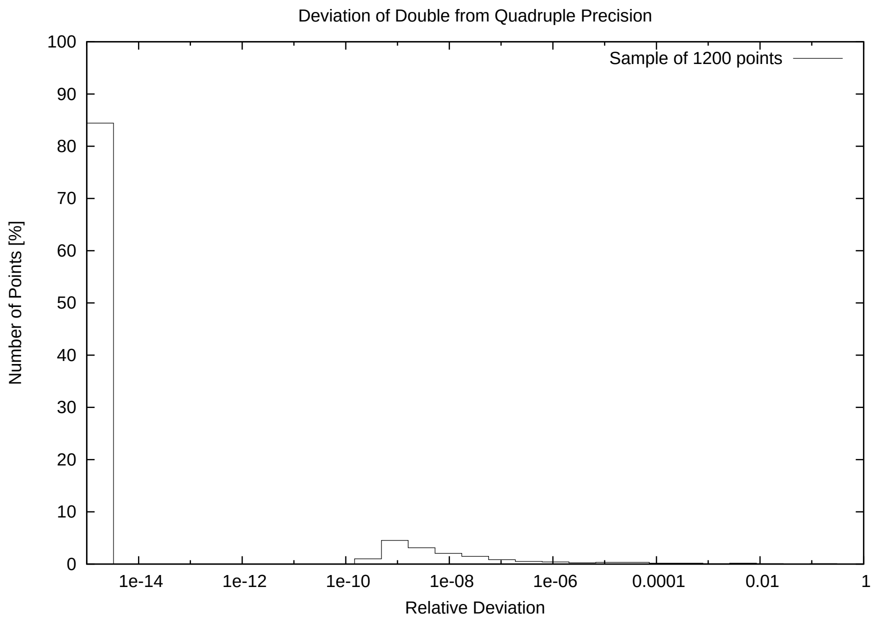


Accuracy of 1200 random points double/quadruple precision:



Virtual NLO corrections of $u\bar{u} \to d\bar{d}b\bar{b}$ (preliminary)

Integrated result using Whizard by W. Kilian, T. Ohl, J. Reuter
applying cuts $p_{T\,j} > 50$ GeV, $|\eta| < 3$, $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} > 0.3$:

$$\sigma_{LO} = 112.7 \text{ fbarn}$$
$$\sigma_{LO+NLO,virt} = 142.9 \text{ fbarn}$$

pdfs: CTEQ61L, $\mu = \mu_F = 100$ GeV, $\alpha_s(M_Z) = 0.1187$.

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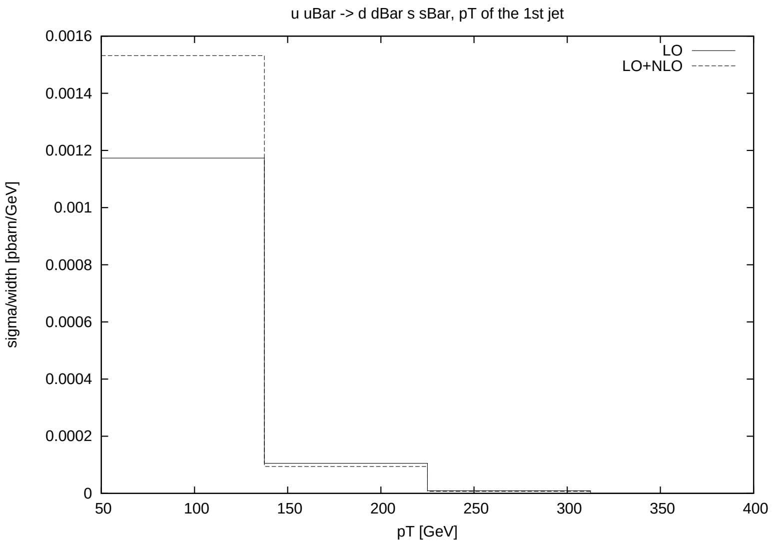
$$\sigma_{LO+NLO,virt} = 142.9 \text{ fbarn}$$

pdfs: CTEQ61L, $\mu = \mu_F = 100$ GeV, $\alpha_s(M_Z) = 0.1187$.
p_T distribution of the leading jet:



$\longrightarrow$ see Thomas Reiters talk at the Loopfest in May !

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LHC = Long and Hard Calculations ...

- ...but results are coming in now !